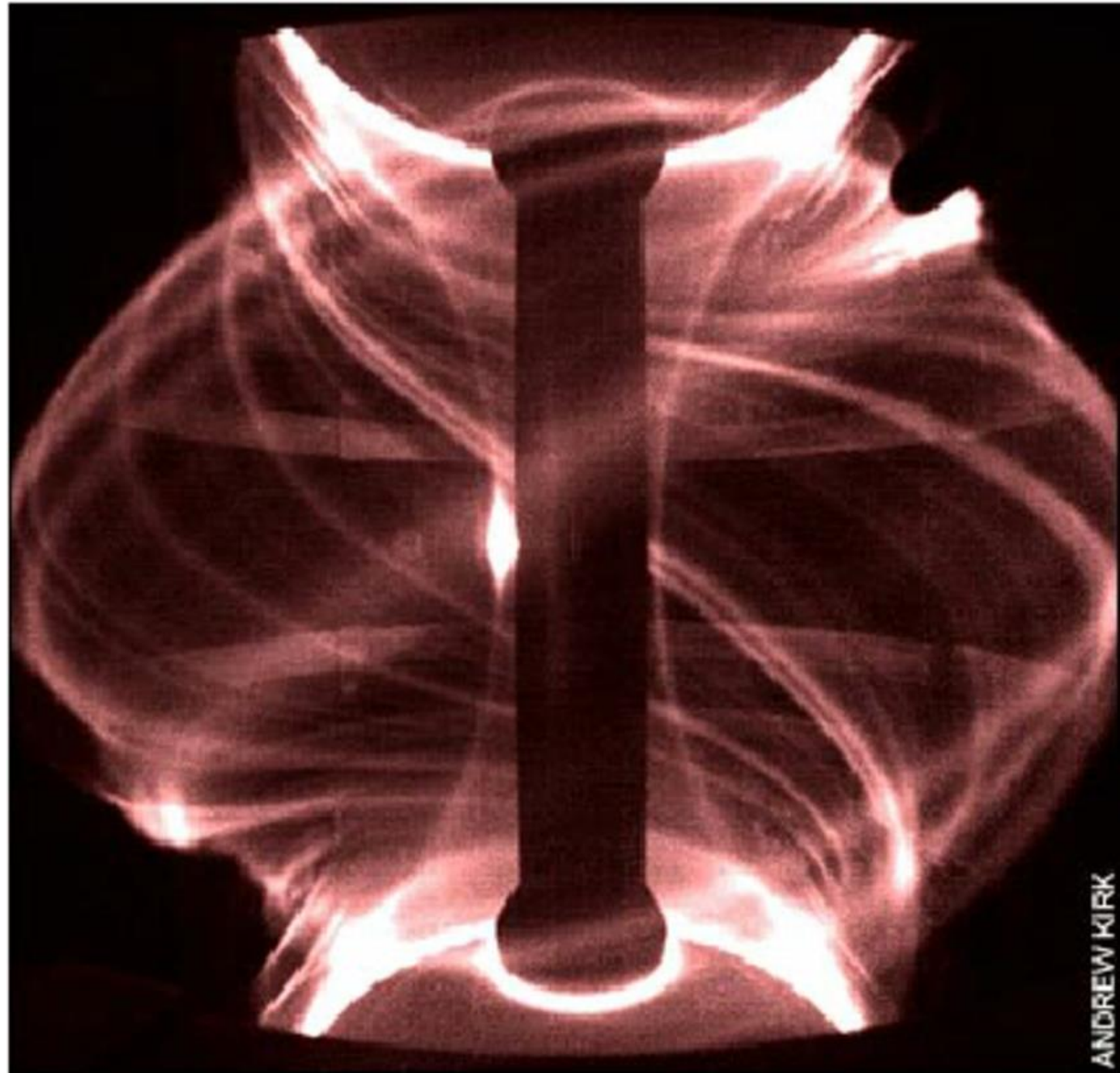


IMPERIAL

# Introduction to Plasma Physics

Yasmin Andrew



ANDREW KIRK

# ICTP and plasma physics

- The International Centre for Theoretical Physics (ICTP) first ran an international conference on plasma physics in 1964 with the International Atomic Energy Agency (IAEA)
- This event was also the inauguration of the ICTP
- A notable contribution from ICTP to global plasma physics; closely tied to the development of fusion plasma physics over the next decades
- The ICTP has continued to support research and training in plasma physics and fusion through the ICTP-IAEA school on Plasma Physics

## Reading list

- Introduction to Plasma Physics and Controlled Fusion, Francis Chen 3<sup>rd</sup> edition, Cham, Switzerland, Springer [2018]
- Plasma Physics: an introductory course, Richard Denedy, Cambridge, Cambridge University Press [1999]
- The Physics of Plasmas, T.J.M. Boyd, J.J. Sanderson, Cambridge, Cambridge University Press [2003]

# Introduction to Plasma Physics and Controlled Fusion

*Third Edition*

# Learning Objectives:

- Develop an understanding of what plasmas are
- Explore different applications of plasmas

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- Develop an understanding of what plasmas are
- Explore different applications of plasmas

# Temperature

- Temperatures conventionally measured in K
- Typically, plasma temperature measured in eV
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

## Maxwellian velocity distribution

- Maxwell-Boltzmann distribution of particle velocities in an idealised gas
- The width of the distribution is characterised by T
- Average kinetic energy of the particles,

$$E_{av} = \frac{3}{2} KT,$$

where,  $K = 1.38 \times 10^{-23} \text{ J K}^{-1}$  and T is in K, for three degrees of freedom

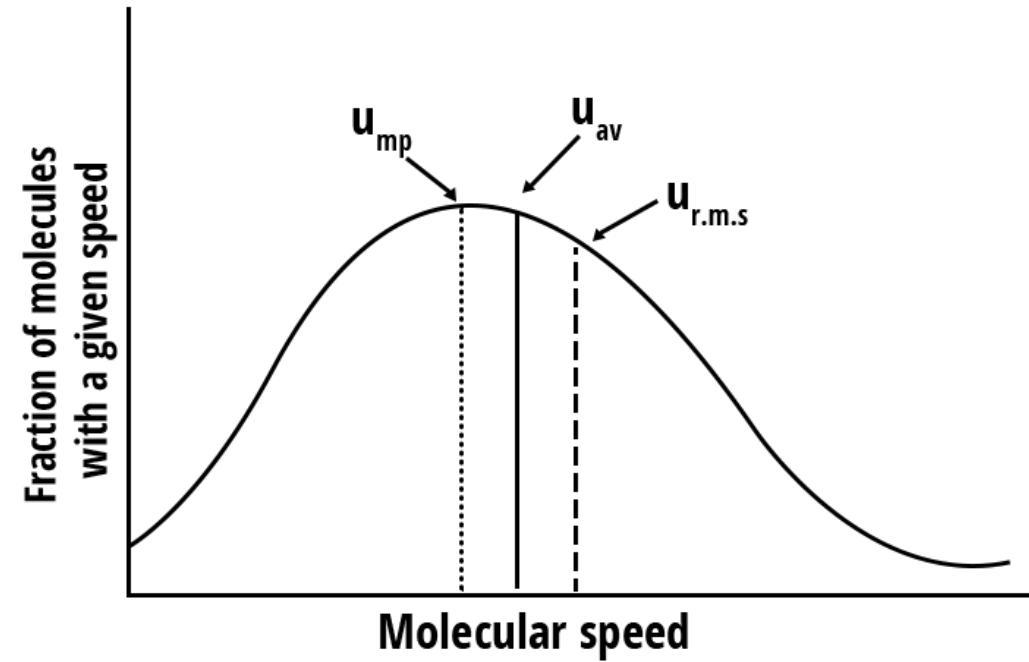
$$E = KT$$

$$T = E/K$$

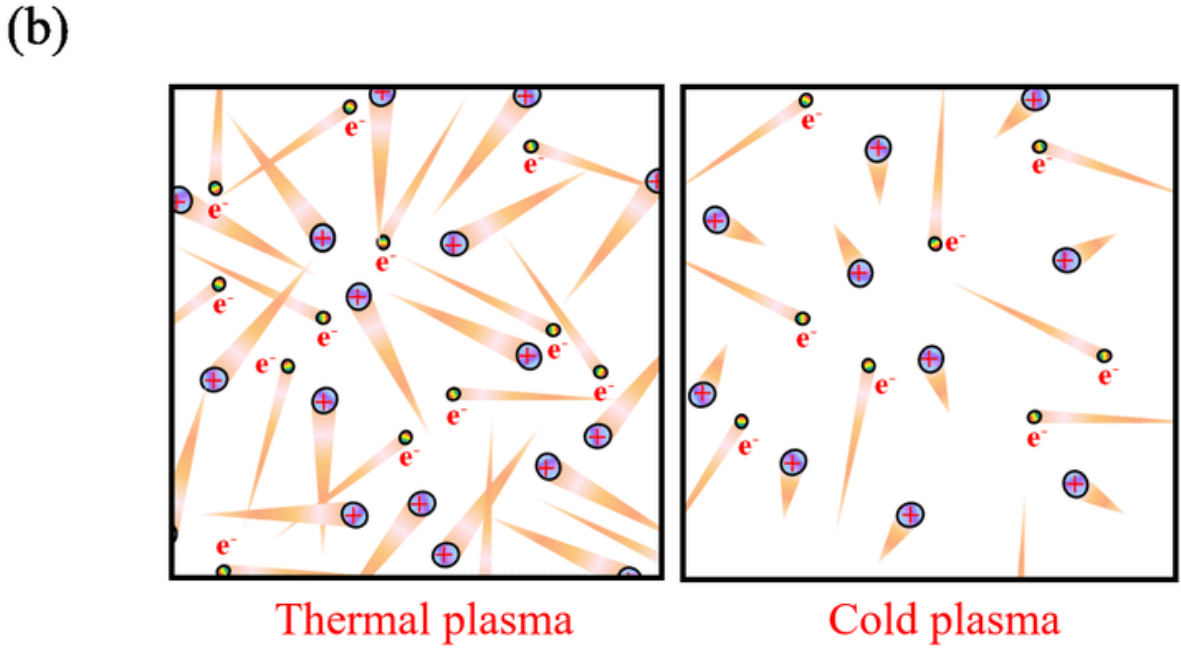
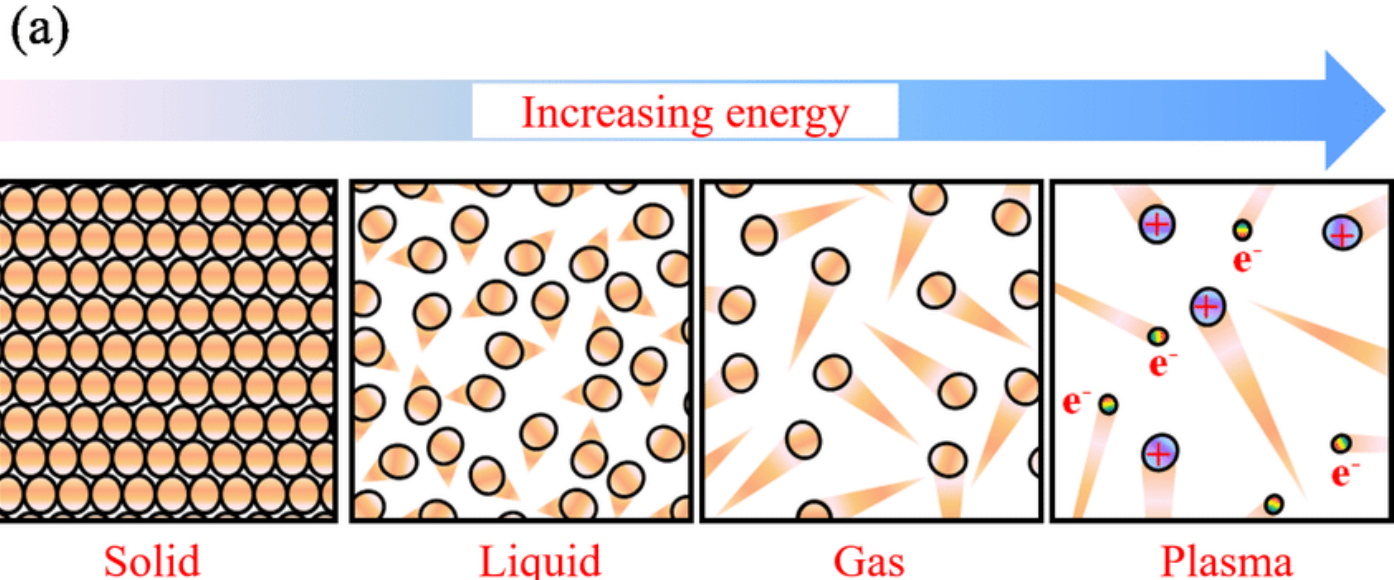
$$T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}}$$



$$1 \text{ eV} = 11\,600 \text{ K}$$

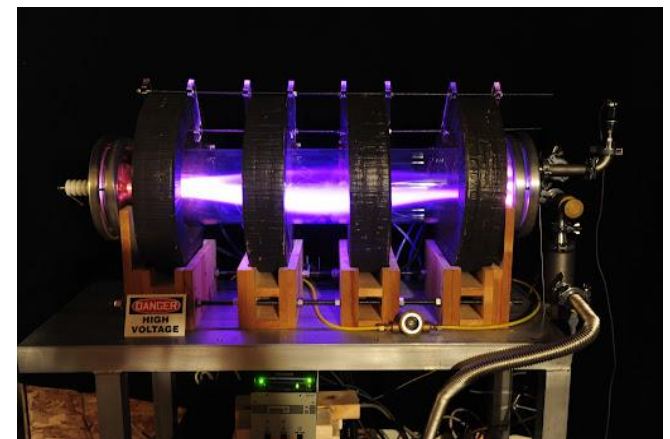
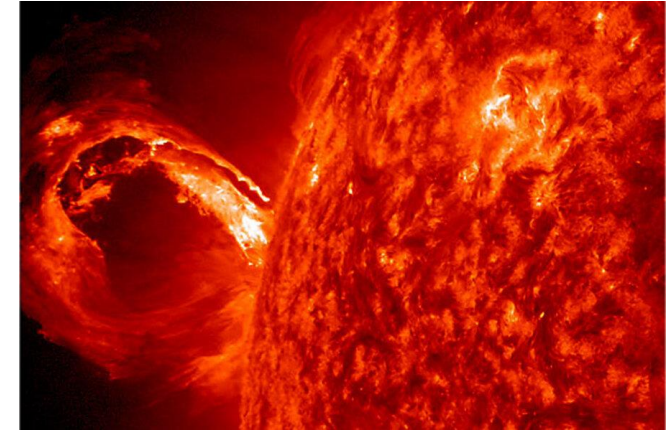
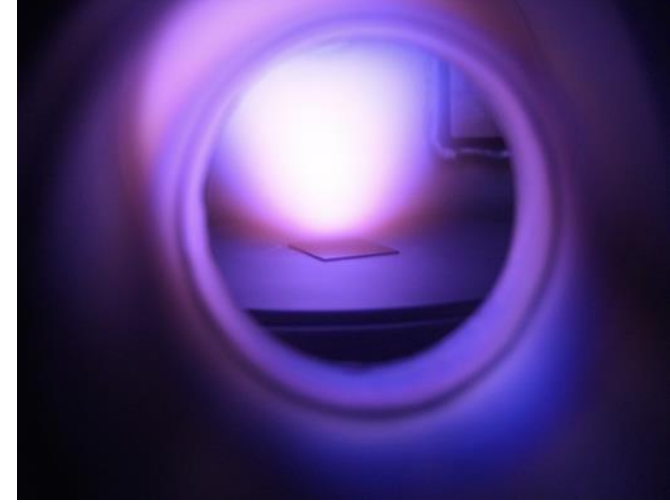


# Plasma



# Plasma

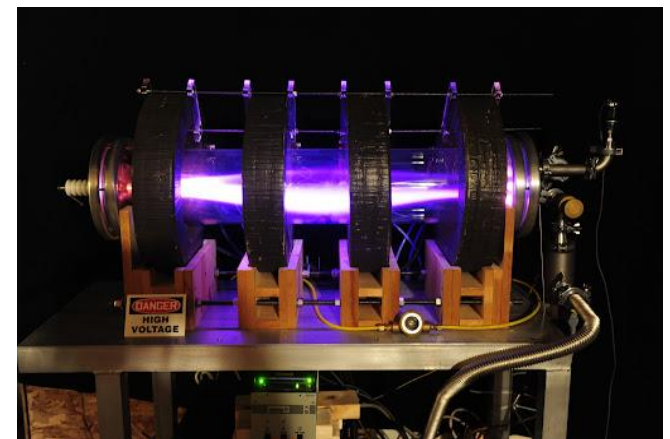
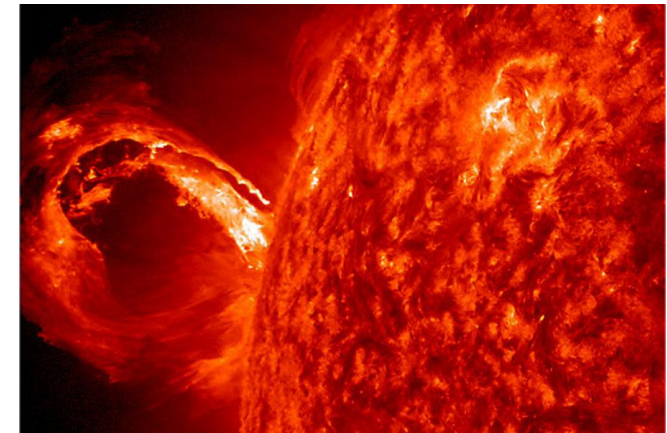
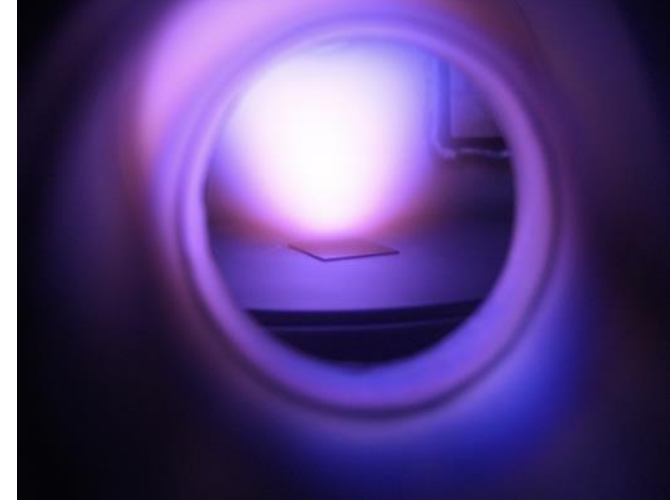
- To create a plasma, gas needs to be ionised
- For example,  $E = 13.65 \text{ eV}$  is needed to ionise a hydrogen atom
- However, an ionised gas is not necessarily a plasma



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*A plasma is a quasineutral gas of charged and neutral particles that shows collective behaviour.*

## Quasineutrality

- There needs to be approximately equal numbers of positive and negative charges on a time scale much longer compared with collective interaction time scale for a plasma to be **quasineutral**.

$$Q = -en_e + Zen_i$$
$$Q = 0 \text{ C}$$

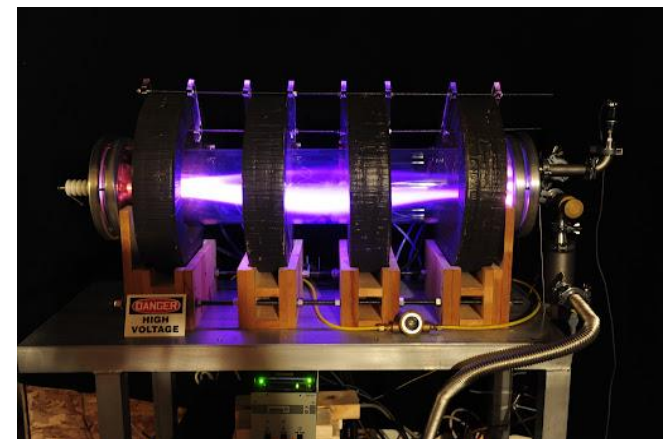
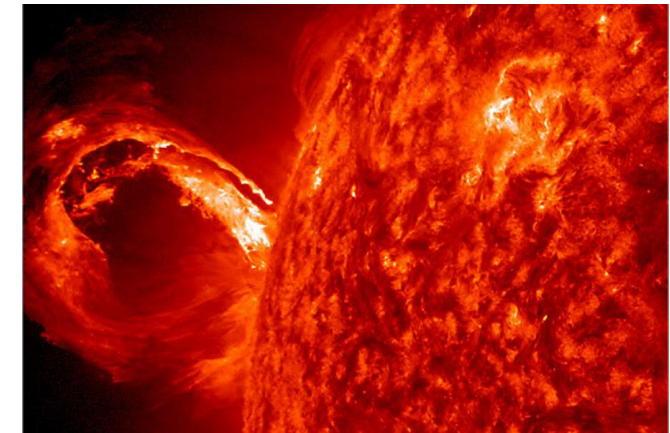
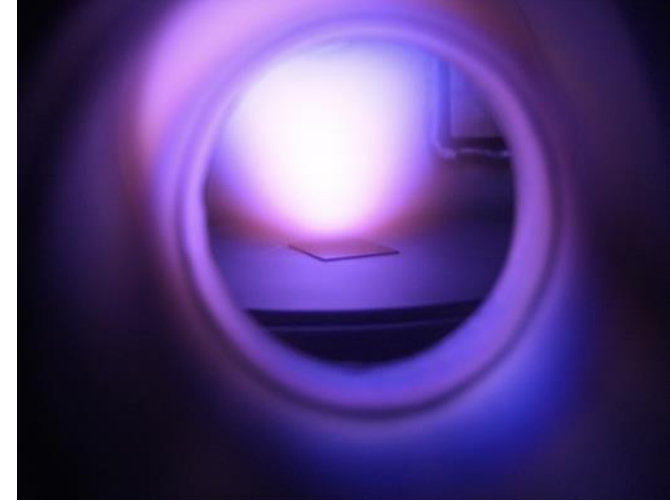
Q is the total charge (C ),

$e = 1.6 \times 10^{-19} \text{ C}$

Z: ion charge (C )

$n_e$ : plasma electron (number) density ( $\text{m}^{-3}$ )

$n_i$ : plasma ion number density ( $\text{m}^{-3}$ )



# Plasma

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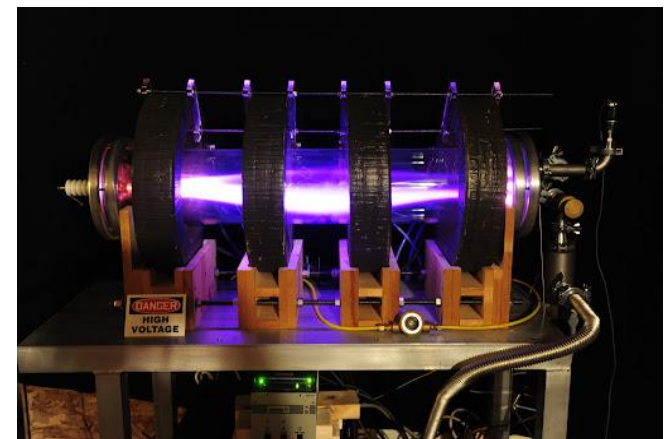
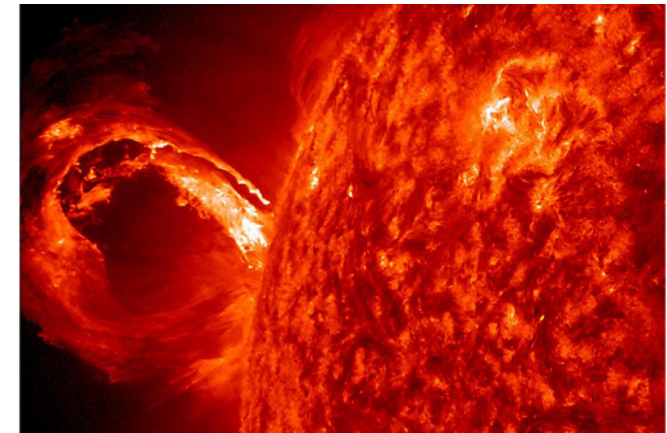
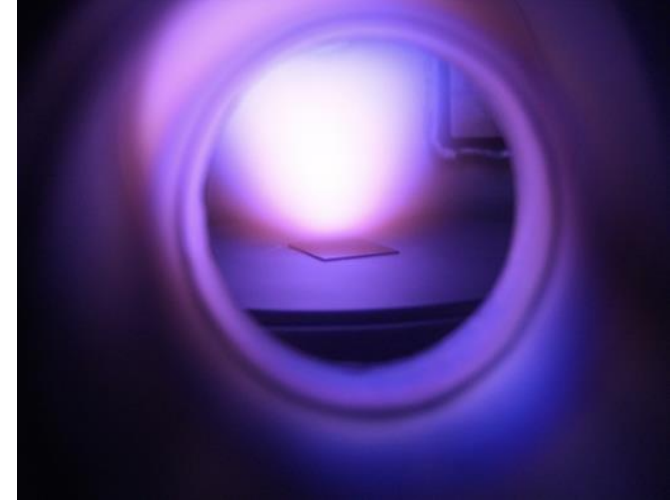
Q is the total charge (C),  $e = 1.6 \times 10^{-19} \text{ C}$ , Z: ion charge (C)

$n_e$ : plasma electron (number) density ( $\text{m}^{-3}$ ),  $n_i$ : plasma ion number density ( $\text{m}^{-3}$ )

- Due to differences in ion and electron motion, transport and plasma fluctuations local differences arise in  $n_e$  and  $n_i$
- This can lead to localized electric fields in the plasma, so that:

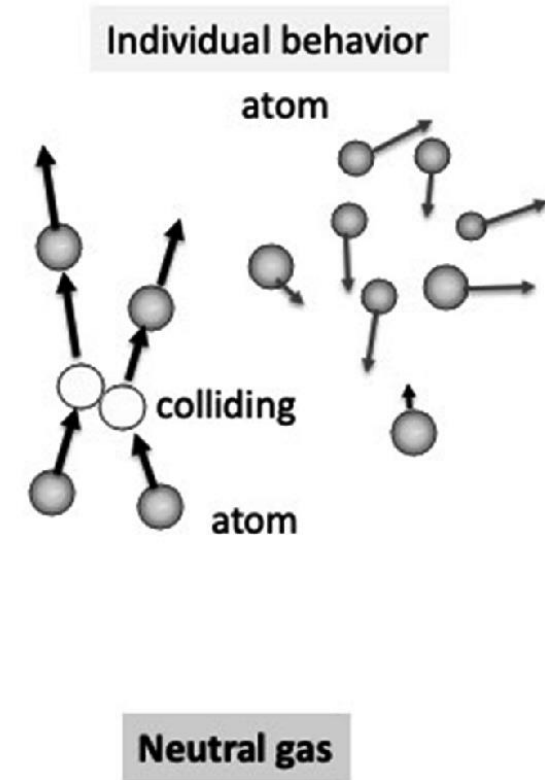
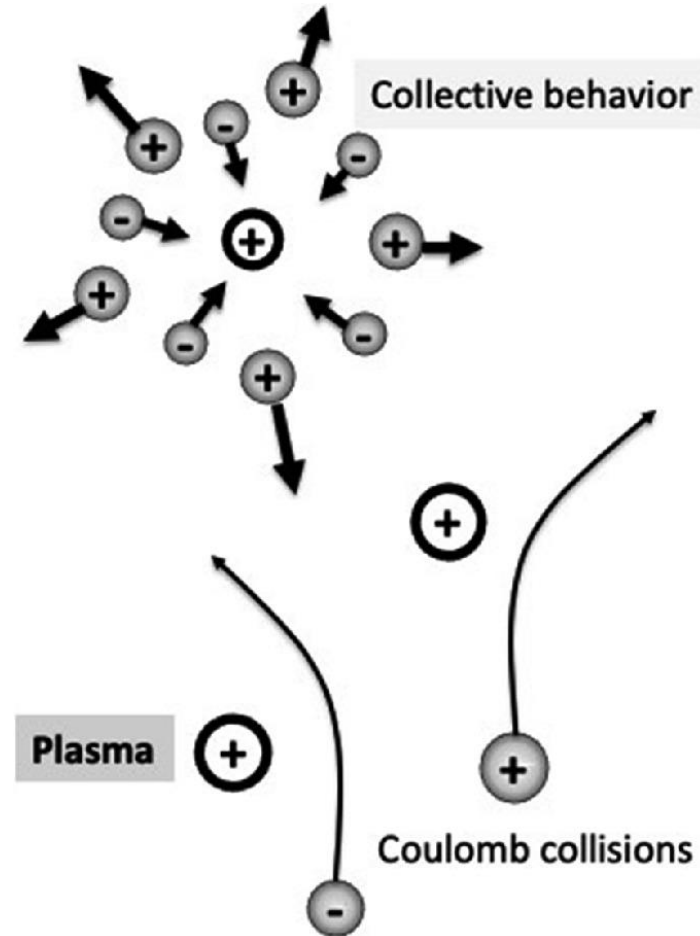
$$Q \neq 0 \text{ C}$$

- Therefore, the plasma is referred to as **quasineutral** on a macroscopic level.



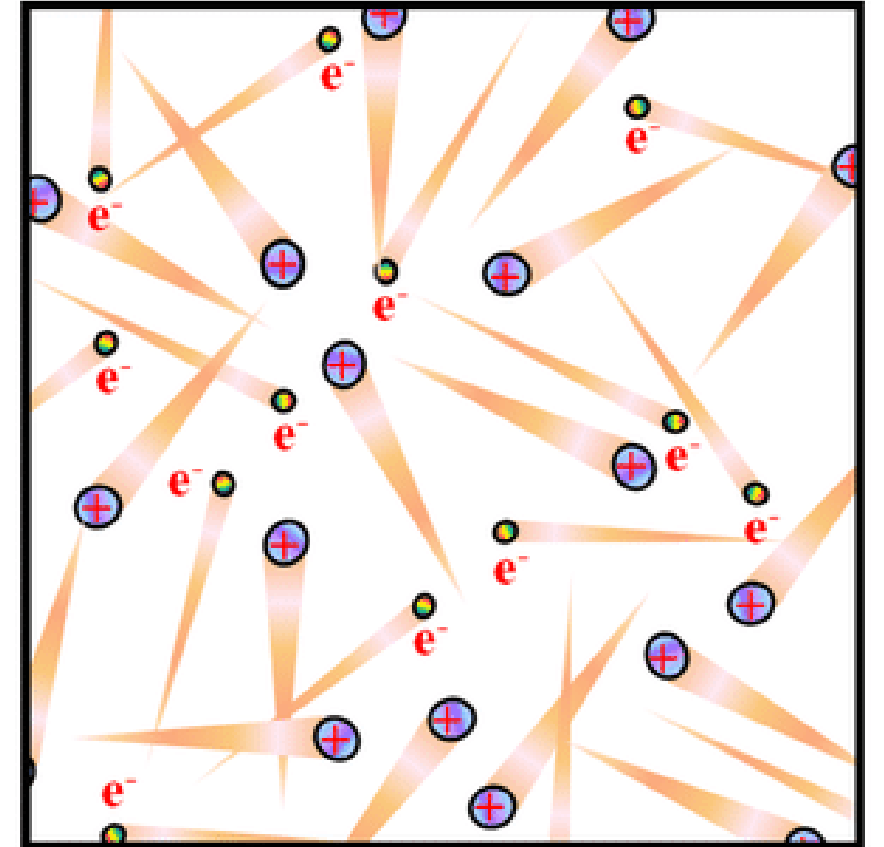
# Collective behaviour

- The charged particles in a plasma can generate localised concentrations of charges which give rise to electric fields
- Motion of charge particles also generates currents and hence magnetic fields
- These electric and magnetic fields affect charged particles at a distance
- Long range Coulomb forces between charge particles can lead to a range of particle motions encountered in plasma physics
- Collective behaviour refers to both plasma particle motion on a short, local scales and in remote regions.



# Collective behaviour

- Approximately equal numbers of positive and negative charges exist on a time scale much longer than the collective interaction time scale, for a plasma to be quasineutral.
- A charged particle would move through a plasma medium and interact simultaneously with many other charged particles.
- Each particle interaction is more than 2-body
- Each particle has a collective interaction
- There is a maximum limit to the scale length of the collective interaction in a plasma defined by the **Debye shielding**

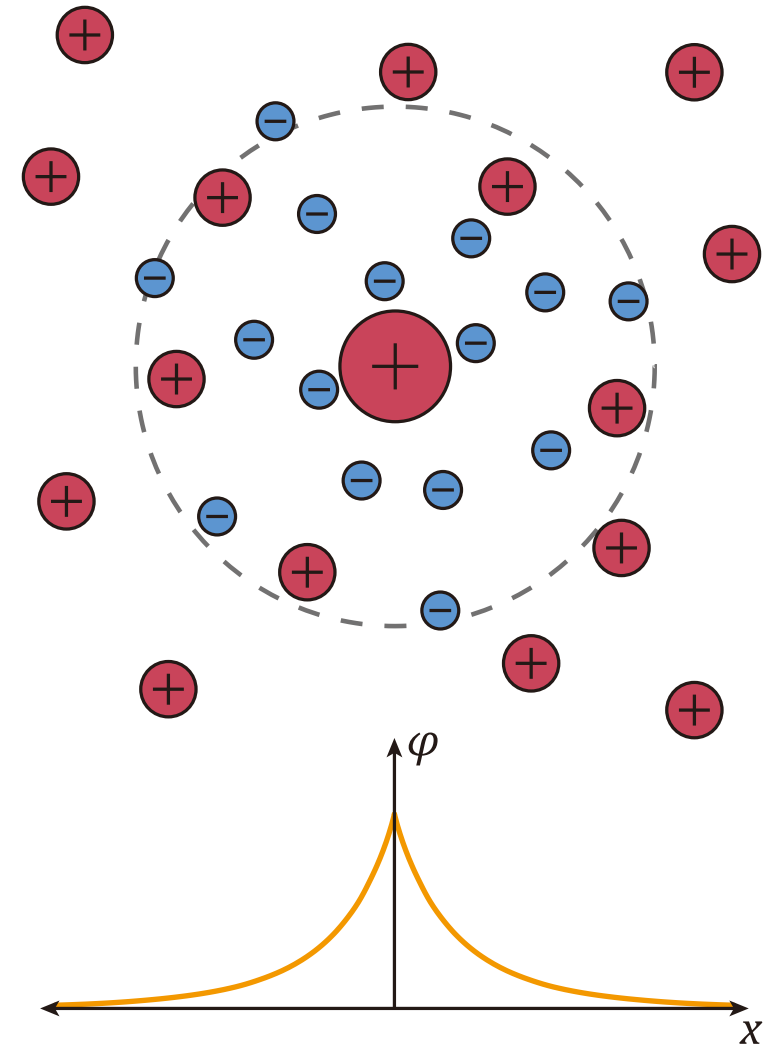


# Debye Shielding

- Consider a neutral plasma,  $Q = 0 \text{ C}$ , with  $T = 0 \text{ eV}$
- If a positively charged object is introduced
- Since,  $m_e \ll m_i$ , the electrons are more mobile and will move towards the positively charged object
- The electrons act to screen the rest of the plasma by forming a cloud around the positive object, so within the dashed circle:

$$n_e = n_o \gg n_i$$

- In a plasma with  $T = 0 \text{ eV}$ , the shielding effect is perfect, and the negatively charged plasma is infinitely thin.
- However, in plasmas with  $T \neq 0 \text{ eV}$ , this is never the case



# Debye length

- In a thermal plasma with particles described by a Maxwell-Boltzmann velocity distribution,  $n_e$  is given by:

$$n_e = n_0 e^{-E/KT} \dots\dots\dots(1)$$

$$n_e = n_0 e^{e\phi/KT_e} \dots\dots\dots(2)$$

where,  $E$  is energy in J,  $\phi$  is the potential in V and  $T_e$  is the electron temperature in K

- Consider Poisson's equation:

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \dots\dots\dots(3)$$

$$\nabla^2 \phi = -\frac{(n_i Z q - n_e q)}{\epsilon_0} \dots\dots\dots(4)$$

where,  $\rho$  is the volume charge density ( $\text{C m}^{-3}$ ) and vacuum permittivity,  $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

- Assume  $n_i Z = n_0$  and substitute (2) into (4):

$$\nabla^2 \phi = -\frac{en_0}{\epsilon_0} \left( 1 - e^{\frac{e\phi}{KT_e}} \right) \dots\dots(5)$$

# Debye length

- If we assume spherical symmetry, equation (5) can be re-written as:

$$\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) = -\frac{en_0}{\epsilon_0} \left( 1 - e^{\frac{e\phi}{KT_e}} \right) \dots\dots(5)$$

- At a point far away from the positive charge  $\phi \rightarrow 0$ , so  $e\phi \ll KT_e$ :

$$e^{\frac{e\phi}{KT_e}} = 1 + \frac{e\phi}{KT_e}$$

- Equation 5 can be re-written as:

$$\begin{aligned} \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) &= -\frac{en_0}{\epsilon_0} \left( 1 - 1 - \frac{e\phi}{KT_e} \right) \\ \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) &= -\frac{en_0}{\epsilon_0} \left( -\frac{e\phi}{KT_e} \right) \dots\dots\dots(6) \end{aligned}$$

$$\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) = \frac{e^2 n_0}{KT_e \epsilon_0} \phi \dots\dots\dots(7)$$

# Boundary conditions

- Far away from the positively charged object:

$$\phi \rightarrow 0 \dots\dots(7)$$

- Close to the positively charged object:

$$\phi \rightarrow \frac{q}{\epsilon_0 4\pi r} \dots\dots(8)$$

- Simplify equation (6) :

$$\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 d\phi}{dr} \right) = \alpha^2 \phi, \dots\dots(9)$$

$$\text{where, } \alpha^2 = \frac{n_0 e^2}{\epsilon_0 K T_e}$$

- Rearrange equation (9) :

$$\frac{d^2}{dr^2} (r\phi) = \alpha^2 r\phi, \dots\dots(10)$$

- Let  $\varphi = r\phi$  so that the solution is of the form  $Ae^{-\alpha r}$  and using the boundary condition (7) and (8):

$$\varphi = \frac{q}{4\pi\epsilon_0 r} e^{-\frac{r}{\lambda}}, \dots\dots(11)$$

$$\text{where, } \alpha = 1/\lambda$$

Coulomb potential      Screening effect

- Then:

$$\frac{d^2\varphi}{dr^2} = \frac{1}{\lambda^2} \frac{q}{4\pi\epsilon_0 r} e^{-\frac{r}{\lambda}} \dots\dots(12)$$

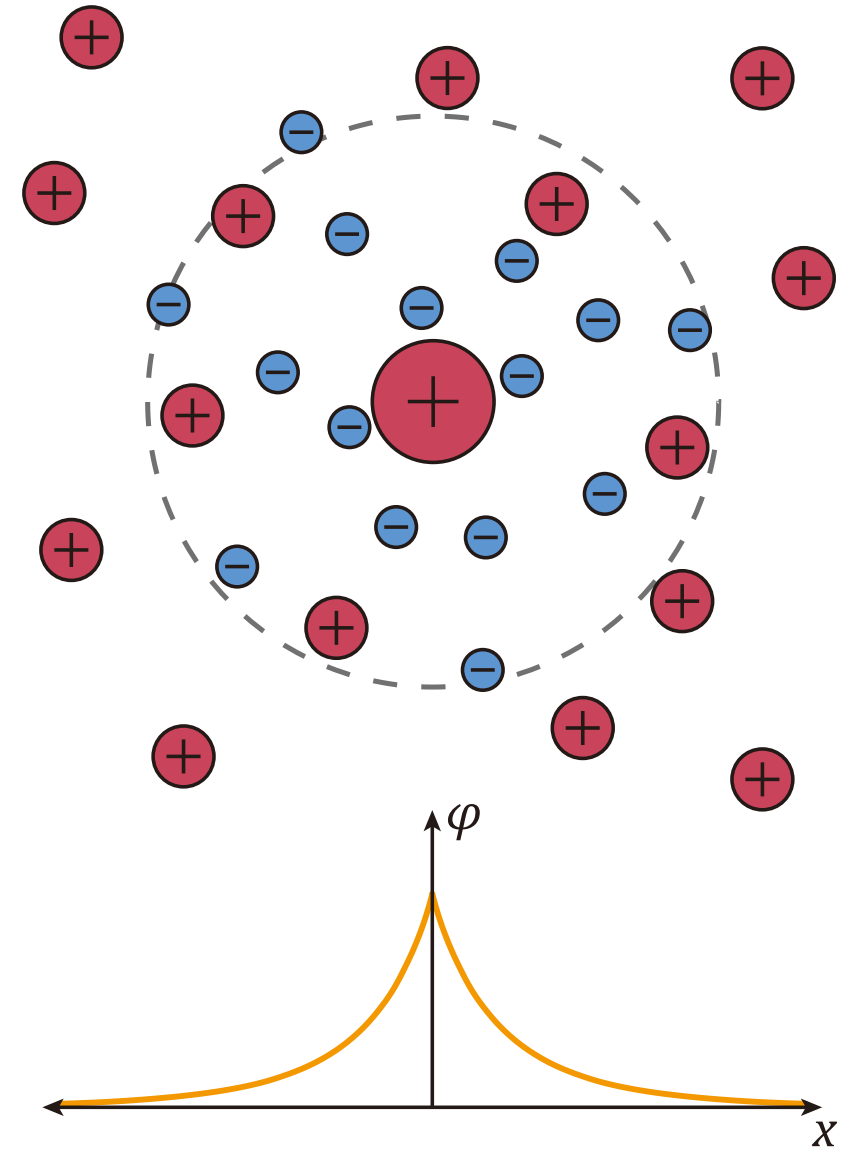
$$\frac{d^2\varphi}{dr^2} = \frac{n_0 e^2}{\epsilon_0 K T} \frac{q}{4\pi\epsilon_0 r} e^{-\frac{r}{\lambda}} \dots\dots(13)$$

or:

$$\frac{1}{\lambda^2} = \frac{n_0 e^2}{\epsilon_0 K T_e} \dots\dots(14)$$

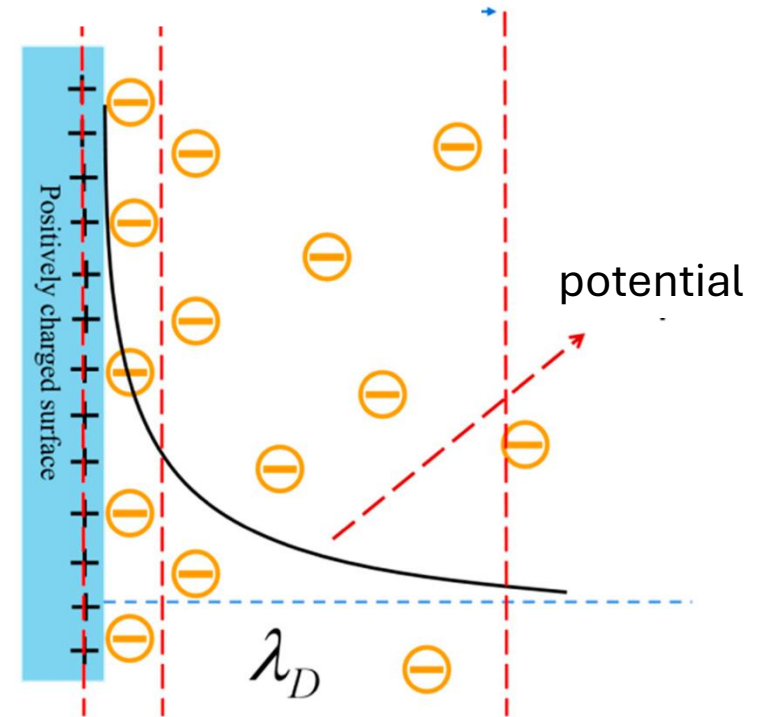
- Equation (14) yields the **Debye length** :

$$\lambda_D = \left( \frac{\epsilon_0 K T_e}{n_0 e^2} \right)^{1/2}$$



# Debye Shielding

- A point charge,  $q$ , is shielded by a cloud of charged particles with radius,  $\lambda_D$ , and a total charge of  $-q$
- At distances shorter than  $\lambda_D$  individual particles are visible to the point charge
- At distances greater than  $\lambda_D$  the plasma exhibits **collective behaviour**, individual particles are invisible to the point charge
- As a result, the plasma is described as quasineutral for  $L \gg \lambda_D$  for the system
- There needs to be enough particles inside the Debye Sphere,  $N_D$ , to shield the bulk plasma from the positive charge



# Number of particles in a Debye Sphere, $N_D$

- Assume a sphere of volume:

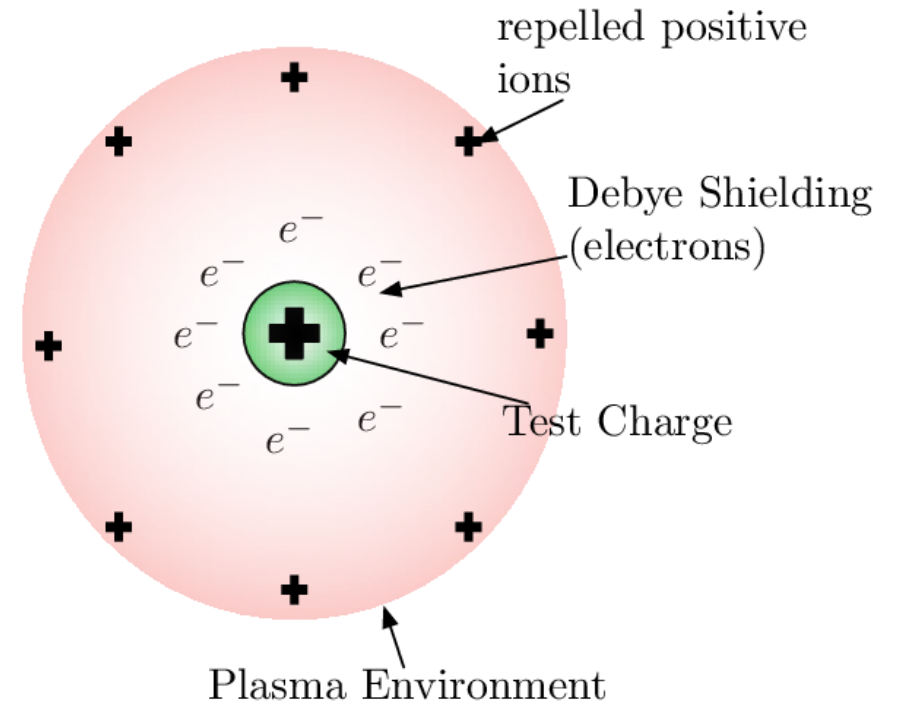
$$V = \frac{4}{3}\pi r^3$$

- If we use the plasma density,  $n_e$  then:

$$N_D = n_e V$$

$$N_D = \frac{4}{3}\pi \lambda_D^3 n_e$$

$$N_D = \frac{4}{3}\pi \left( \frac{\epsilon_0 K T_e}{n_0 e^2} \right)^{3/2} n_e$$



# Plasma frequency

- What happens if electrons in a plasma are displaced by a small amount?
- If an electron moves away from its original position, an electric field is created due to localised charge imbalance and the electron will be pulled back towards its original location
- Inertia can cause the electron to go slightly past its starting location and a restoring force means the electron starts to oscillate with a frequency
- This is the plasma frequency:

$$\omega_p = \left( \frac{n_e e^2}{m \epsilon_0} \right)^{\frac{1}{2}}$$

The plasma frequency provides a characteristic timescale over which the plasma responds and is linked to the plasma collective behaviour

# Debye Length and plasma frequency

- The values of  $\lambda_D$  and  $\omega_p$  provide information on spatial and temporal scale lengths over which plasma quasineutrality is valid:

Charge imbalance  
can exist in a plasma

$$L < \lambda_D < L$$

$$t < 1/\omega_p < t$$

Quasineutrality is a  
valid description

- The values of  $\lambda_D$  and  $\omega_p$  are related through the plasma electron thermal velocity:

$$v_{th} = \lambda_D \omega_p$$

$$v_{th} = \left( \frac{\epsilon_0 K T_e}{n_0 e^2} \right)^{1/2} \left( \frac{n_0 e^2}{m \epsilon_0} \right)^{1/2}$$

$$v_{th} = \left( \frac{K T_e}{m} \right)^{1/2}$$

The value of  $\lambda_D$  is the distance an average plasma electron moves over a plasma time period.

# Learning Objectives:

- Develop an understanding of plasmas
- Explore different applications of plasmas

# Applications of plasmas

- Plasmas include a very wide range of  $n$  and  $KT$ :

$$10^6 < n_e < 10^{34}$$

$$10^{-1} < T_e < 10^6$$

- This large range in parameters mean that plasmas have a very wide range of applications



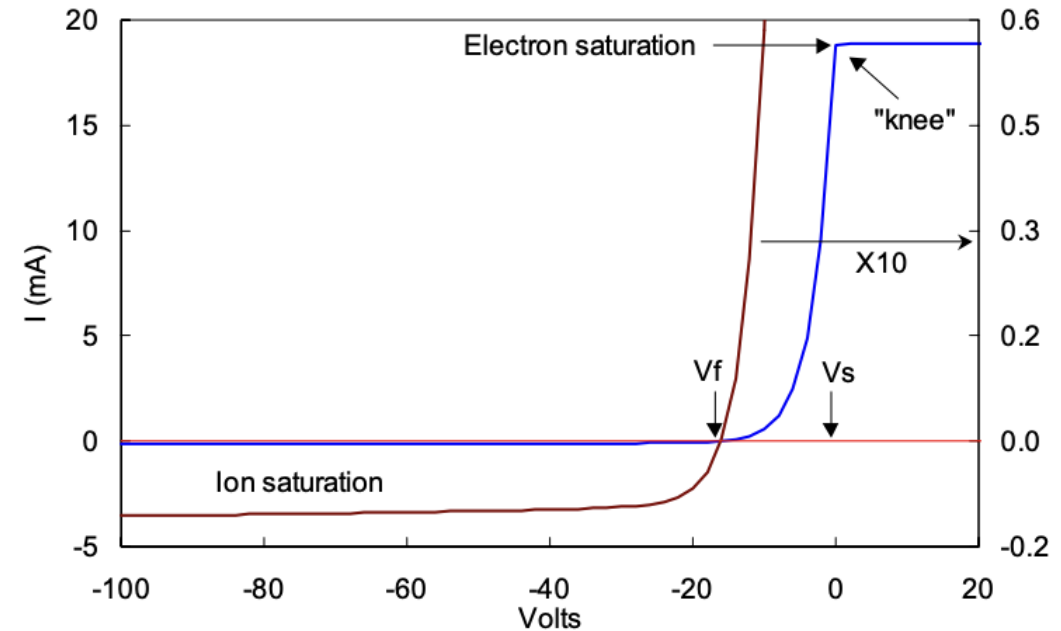
# Low temperature plasmas

- Low temperature have typical  $n$  and  $KT$  values of:

$$10^{14} < n_e < 10^{18} \text{ m}^{-3}$$

$$T_e \approx 2 \text{ eV}$$

- These plasmas were some of the earliest, Langmuir, Tonks and collaborators worked on these weakly ionized glow discharges in the 1920s
- The Debye shielding was directly observed as the dark plasma sheath in front of the DC electrode
- Direct applications of this work are lighting e.g. mercury lamps, fluorescent and neon lighting
- A wider commercial application is the semiconductor industry, etching and metal deposition for circuits in chips
- Is an important and wide academic area of research due to its commercial value

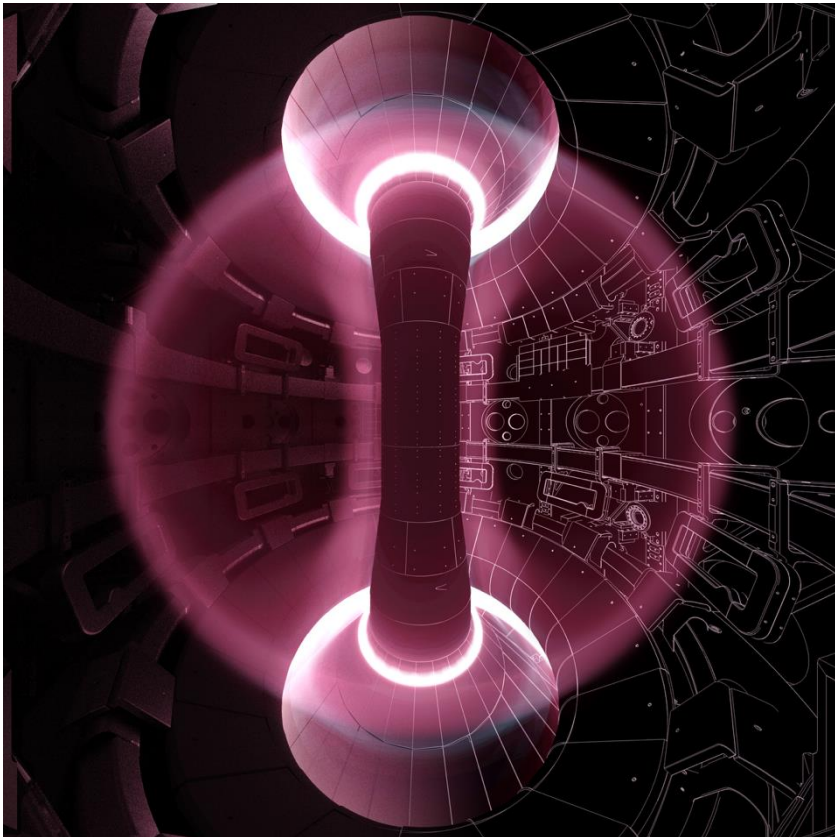


# Controlled Nuclear Fusion

- Fusion plasmas have typical  $n$  and  $KT$  values of:

$$10^{18} < n_e < 10^{31} \text{ m}^{-3}$$

$$T_e \approx 30 \text{ keV}$$



# Space Plasma Physics

- An example of space plasma physics is the study of Earth's environment in space
- A continuous stream of plasma flows towards Earth from the Sun
- The magnetosphere protects Earth by trapping the charge particles

$$n_e \approx 10^7 - 10^{11} \text{ m}^{-3}$$

$$T_e \approx 10 \text{ eV}$$

$$B \approx 10 \text{ nT}$$

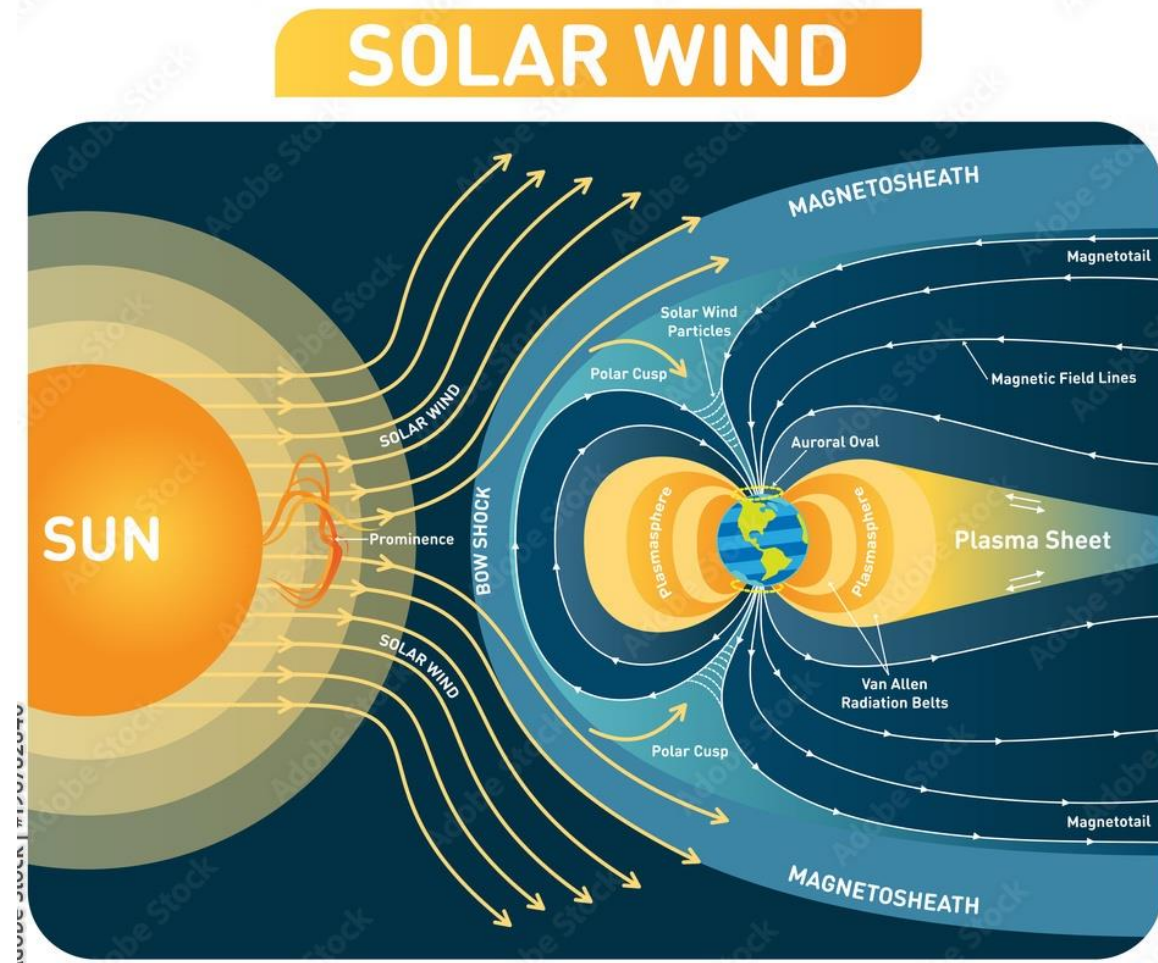
$$v_{drift} \approx 450 \text{ km s}^{-1}$$

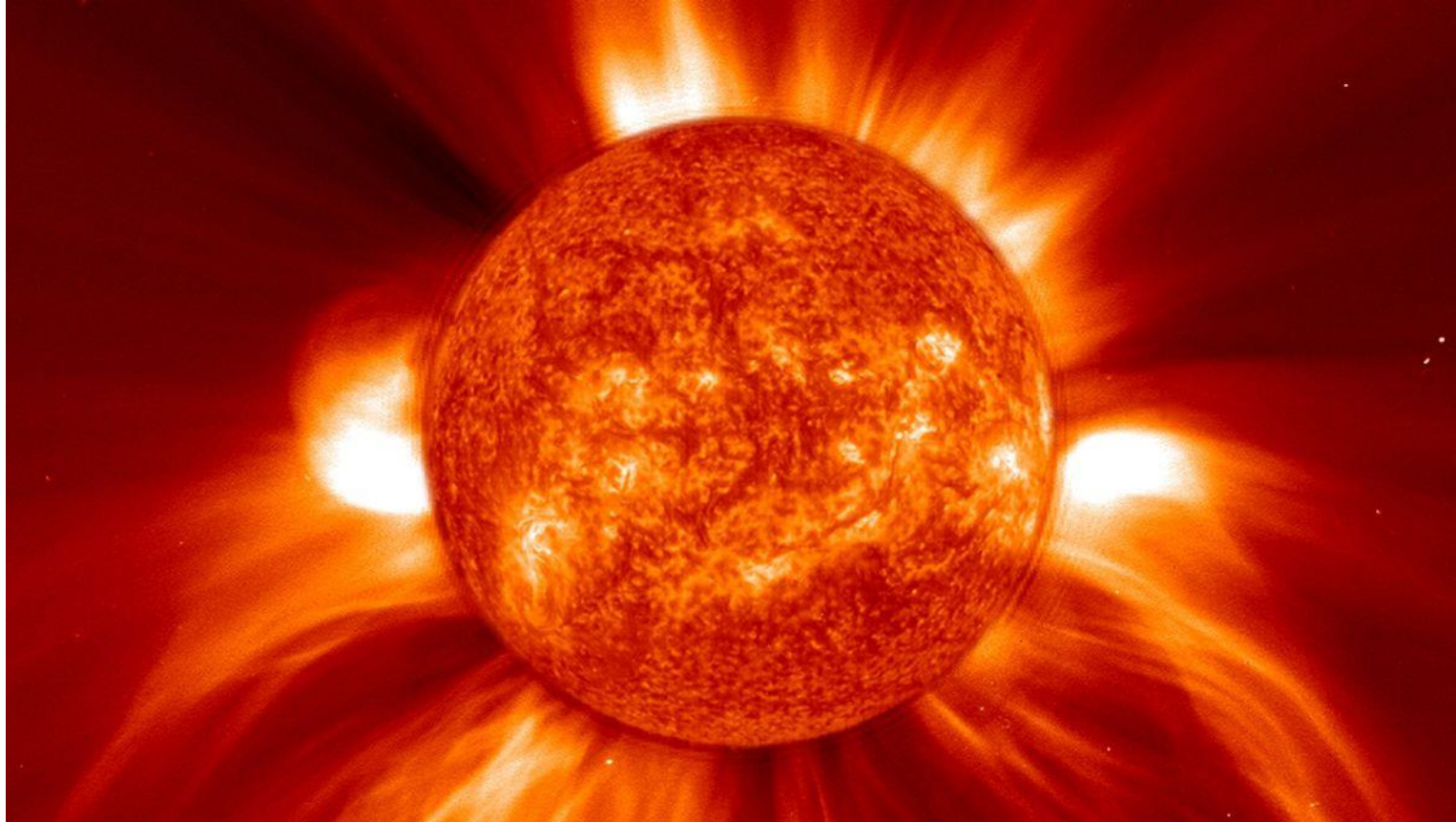
- The ionosphere exists 50 km from the Earth's surface
- Plasma values vary with altitude, typical values:

$$n_e \approx 10^{12} \text{ m}^{-3}$$

$$T_e \approx 0.1 \text{ eV}$$

- Other planets such as, Jupiter and Saturn, also have plasmas in their environments





# Astrophysics

- The interiors and environment of stars are hot enough to be plasmas
- For example, in the core of the Sun temperature:

$$T_e \approx 2 \text{ keV}$$

- Nuclear fusion reactions release energy radiated from stars
- Astrophysics requires an understanding of plasma physics

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