Transport Barriers and Self-Organisation
A nonlinear, nonlocal, stochastic perspective

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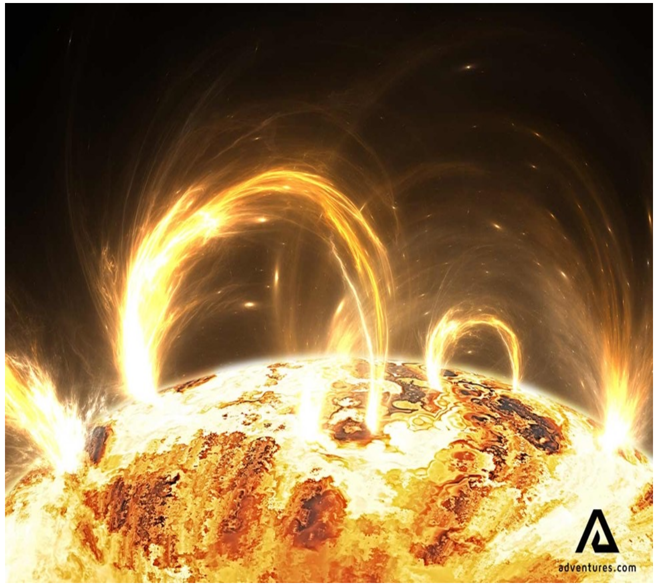
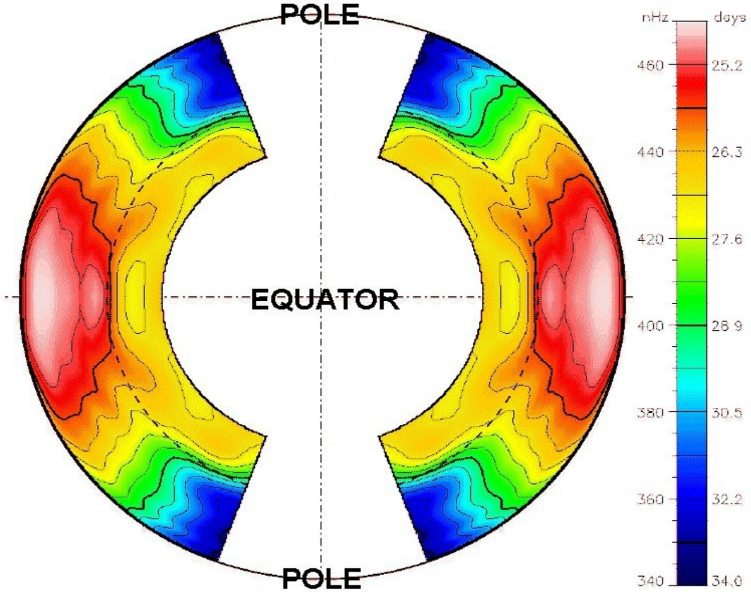
Turbulence, bursts, self-organisation

Sun

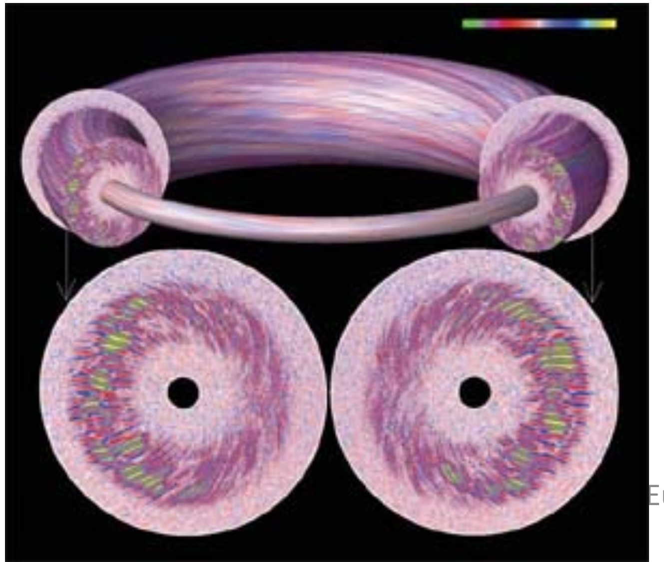


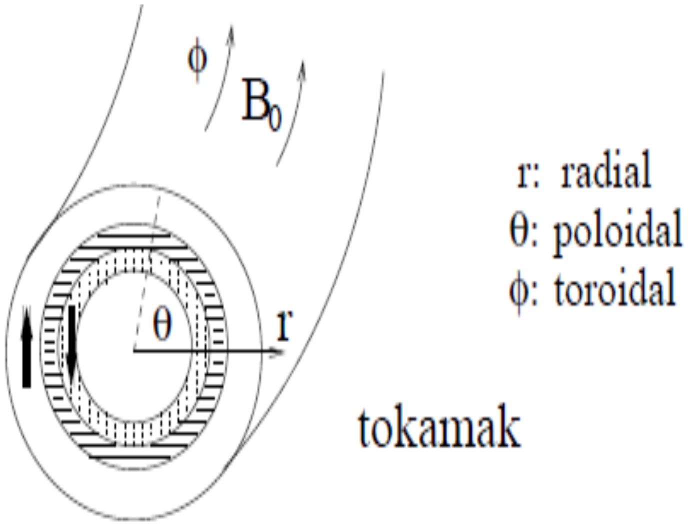


tokamak

Bursts: flares, edge-localized modes (ELMs)

shear flows, self-organisation (differential rotation, zonal flows)





Roadmap of the lecture

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1 **Transport barriers**

Basic nature and diversity of transport barriers

2 **Nonlinear coupling and self-organisation**

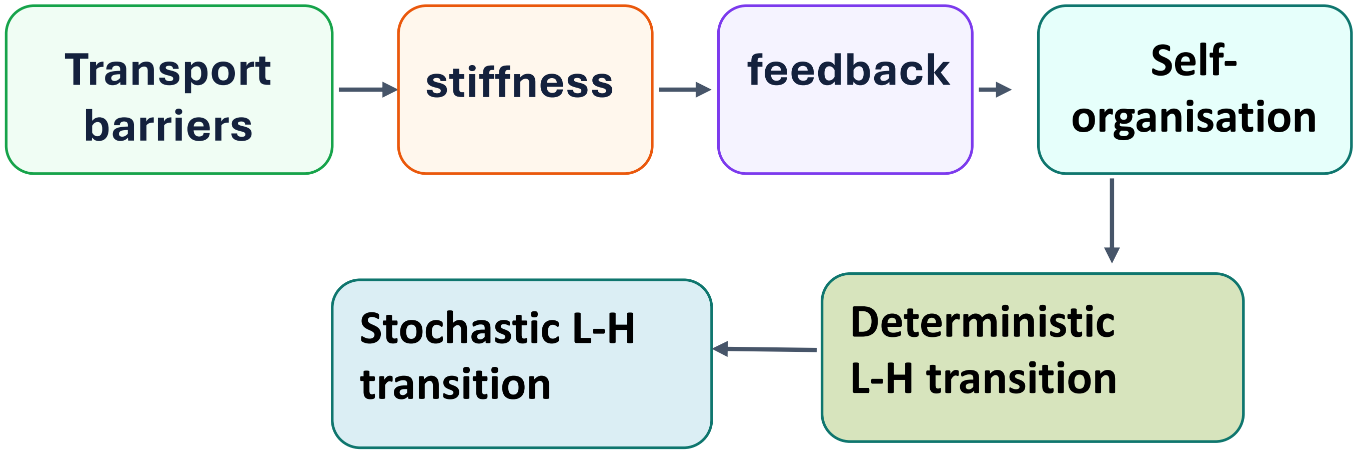
Understand profile resilience/stiffness, bifurcation, self-organisation, self-similar statistical property.

3 **Self-organisation= feedback-created structure**

Turbulence, flows, and gradients are coupled in non-equilibrium.

4 **L-H transition and stochastic dynamics**

Theoretical and experimental results



Part I –Transport barriers

Basic nature and diversity of transport barriers

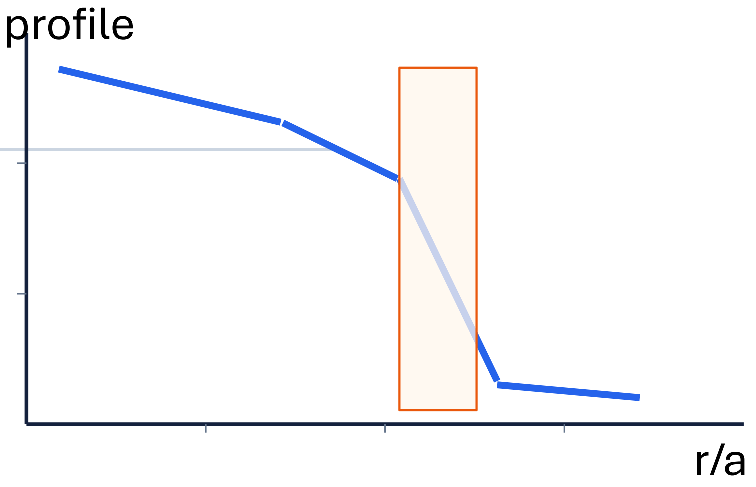
What is transport barrier?

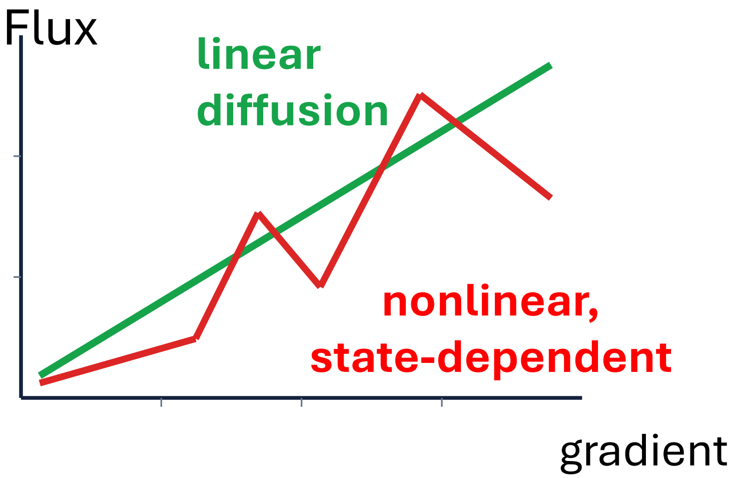
A transport barrier is not just a steep profile (top figure)

- Sources build gradients; fluxes relax them
- Linear closures give the baseline:

$$\Gamma n = -D \partial n/\partial r,\ q = -n\chi\ \partial T/\partial r$$

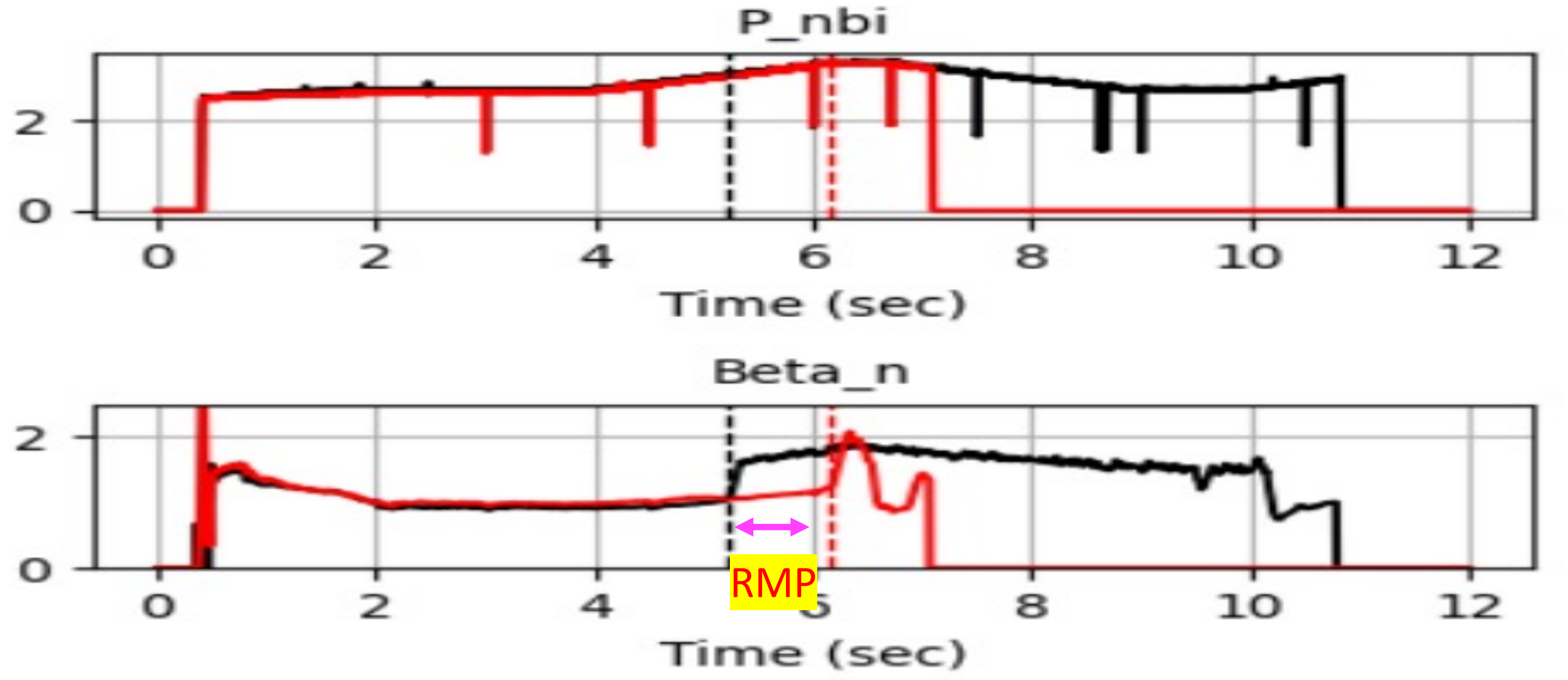
- Flux rises proportionally with gradient (green line in bottom figure).
- But flux–gradient relations can be nonlinear and state-dependent (red curve in bottom figure)
- Transport can depend on turbulence state, flows, history, and nonlocal events (examples in the next slides)
- A barrier forms when the gradient steepens without a proportional rise in flux
- Equivalently: reduced flux per unit gradient
- ⇒"reduced effective diffusivity / conductivity"





Example of history dependent barriers:
RMPs applied in L-mode can change the subsequent H-mode pedestal

- Conventional discharge (#35641): with no RMPs in L-mode, plasmas transition to H-mode at t=5.23 secs (black curve in bottom figure).
- Pre-emptive RMP discharge (#37404): RMP are applied in the L-mode for 1 secs at t=[5,6] secs: after switching off RMPs, plasma transition to H-mode at 6.2 sec (red in the bottom figure).
- Same NBI heating until t=7 secs (top), and same plasma parameters until t=5 secs (bottom);no RMP from t=6 secs



Different H-modes (bottom)

➔ L-H transition depends on plasma history

RMPs modify E_r shear, edge conditions, thereby changing the L-mode conditions for H mode access and the subsequent evolution.

Fig. Time-traces of plasma parameters for conventional #35641 (black) and pre-emptive ERMP #37404 (red) discharges. L-H transition time is marked by red(black) vertical line. LH transition at t=5.23 secs for #35641 and t=6.2 secs for #37404 [KSTAR, E Kim et al 2025].

Barriers are not one object

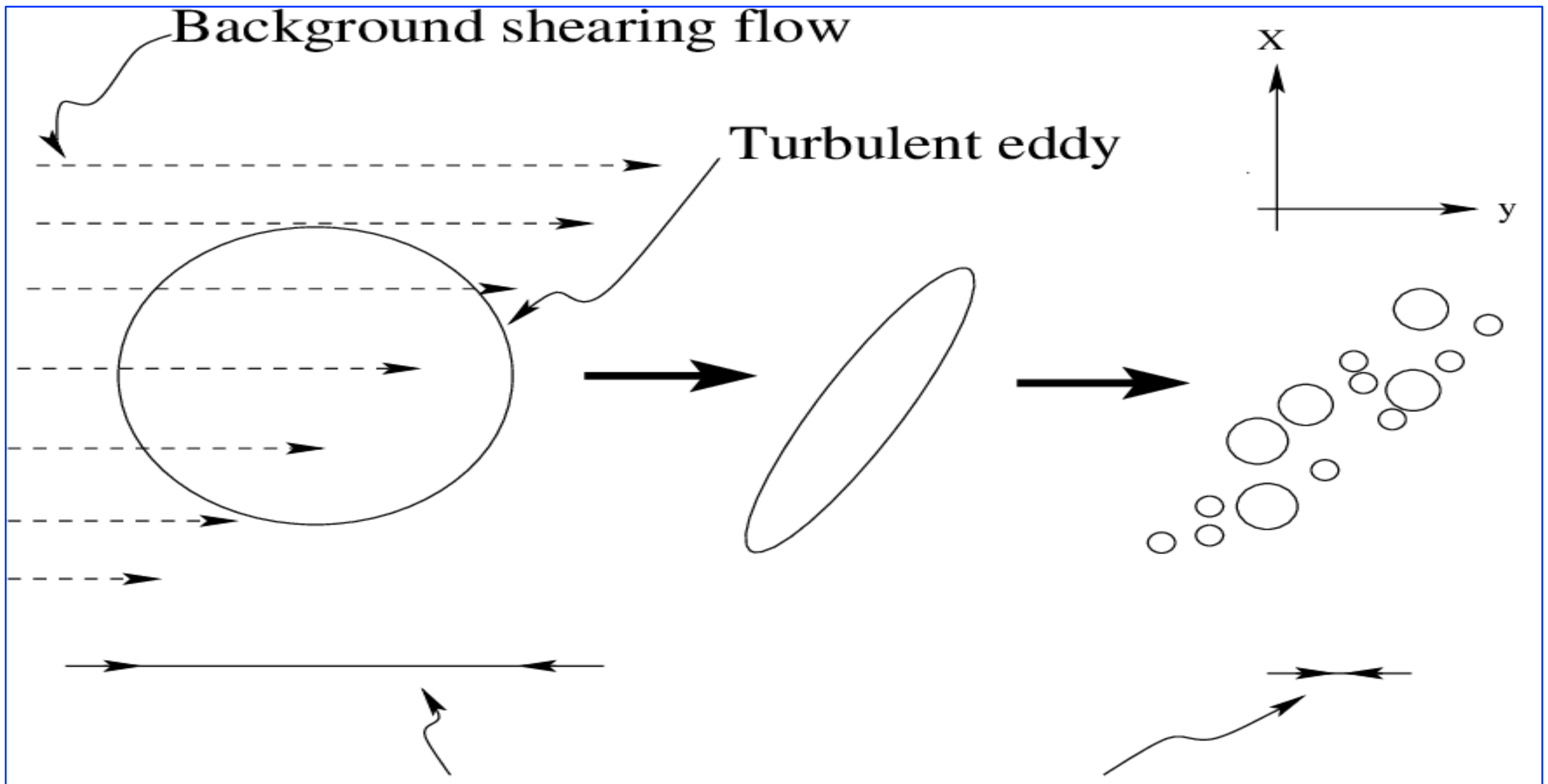
- **H-mode / ETB pedestal:** Edge barrier through steep pressure gradients limited by pedestal stability and ELMs.
- **ELM-free or small-ELM H-mode variants: QH-mode** and **Enhanced D-alpha (EDA) H-mode** maintain good confinement while providing continuous edge transport through coherent edge activity (e.g, DIII-D, Alcator C-Mod).
- **I-mode:** Improved energy confinement without a strong particle barrier (e.g., DIII-D, JET, ASDEX Upgrade, JT-60U, KSTAR).
- **Internal transport barriers, ITBs**: Core or mid-radius barriers often linked to magnetic shear, q-profile, rotation, current drive, and rational surfaces.
- **Hybrid / advanced scenario:** Improved confinement with tailored current profile and weak shear by combining pedestal, core transport, stability, and current-profile control.
- **Turbulence staircases:** Multiple mesoscale transport layers separated by jets or shear zones.
- **Different channels can form different barriers:** Particle, impurity, momentum, electron-heat, and ion-heat barriers can occur at different radial locations.
- **No single “barrier type” exists in terms of spatial locations and channels.**

Refs: Wolf, PPCF 45, R1 (2003); Rice et al., Nucl. Fusion 47, 1618 (2007); Angioni & Peeters, Phys. Plasmas 15, 052307 (2008).

E x B shear flows

Turbulence regulation (reduction) by shear flow

K Burrell 1997, TS Hahm 1994, Z Lin 1998, PH Diamond 1994;2011, E Synakowski 1997, H Biglari 1990, X Garbet 2001, E Kim 2000;2002;2004;2006;2007, Leprovost 2006;2007;2008; C Hidalgo 2000, Y Idomura 2005



Typical distance an eddy can transport a passive scalar field

From N Leprovost & E Kim, PRE 78, 016301, 2008

Contributing factors to barrier access and sustainment

- $E \times B$ shear can suppress turbulence: reduces radial correlation length and cross-phase, but may also drive secondary instabilities
- q-profile sets magnetic shear and rational surfaces: controls radial mode coupling, resonance structure, and wave–particle interactions
- Weak / reversed magnetic shear can favor ITBs by reducing radial coupling and allowing gradients to steepen
- Divertor geometry and drift direction shape ETB access: ion ∇B drift toward the X-point often gives more favorable H-mode access
- **Current diffusion and history:** Current diffusion is slow so barriers have memory, meaning that the barrier can depend on how the q-profile was prepared
- **Collisionality, fast ions, impurities:** modify turbulence, stability, and transport channels
- **Core–edge–exhaust coupling:** barriers must fit pedestal, SOL, divertor, and PFC constraints
- Core → pedestal / ETB → separatrix → SOL → divertor → PFC

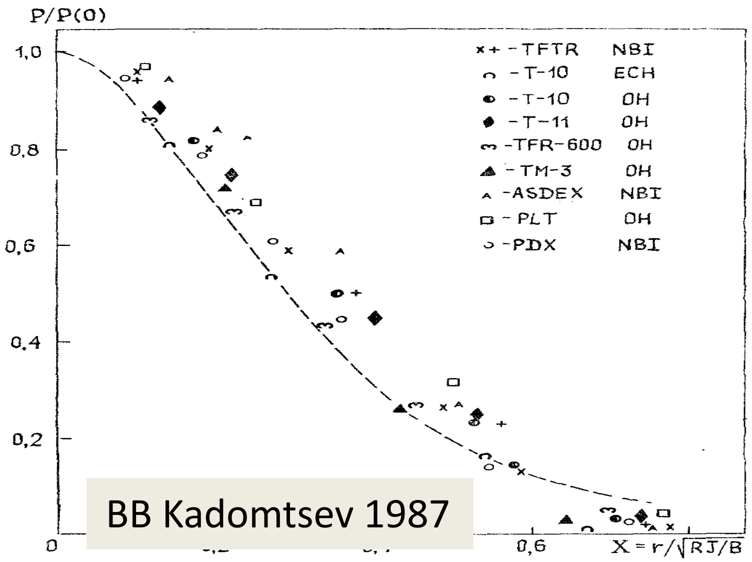
Part II – Nonlinear coupling and self-organisation across

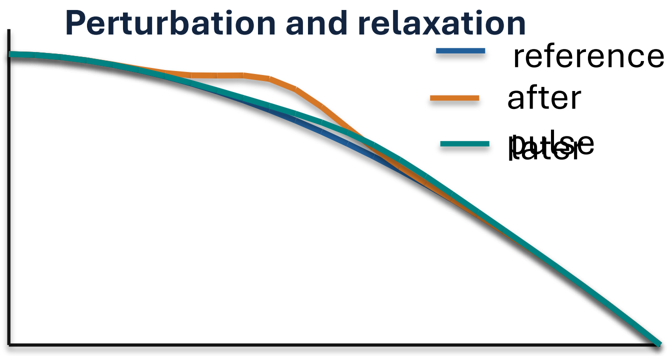
How does the plasma organize itself to produce, maintain, or lose barriers?

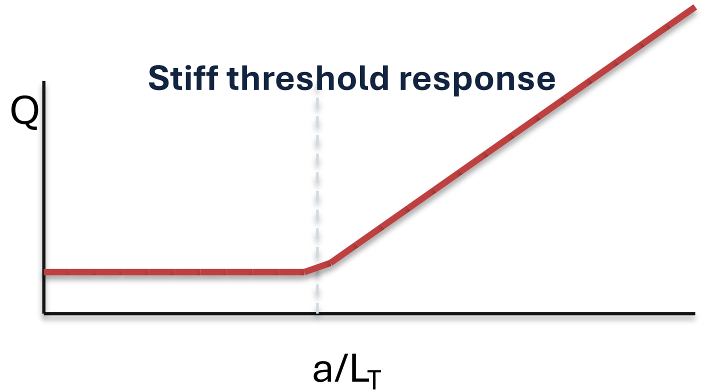
1. Profile resilience and stiffness
2. Bifurcation
3. Self-organisation
4. Statistical similarity

1. Profile consistence/resilience

- **Profiles often show preferred shapes** despite changes in heating or fuelling. This does not mean profiles never change; it means transport, sources, sinks, and boundary conditions interact so the plasma often returns toward similar profile shapes.
- **Resilience:** after a perturbation, the profile tends to relax back toward a preferred shape.
- **Stiffness:** once a gradient exceeds a threshold, a small further increase can produce a large transport response.
- **Key distinction:** a resilient profile may be stiff, but resilience is broader than local diffusivity. It involves transport, sources, sinks, boundary conditions, and feedback.







2. Nonlinear coupling allows the plasma to access distinct confinement states

- **Transport is not a fixed material response**
  In a turbulent plasma, turbulent transport coefficients depend on the turbulence state, flows, gradients, geometry, and history.
- **Nonlinear coupling can create multiple stable states**
  Small changes in heating, fuelling, torque, density, or shear can push the plasma from one confinement branch to another.
- **Similar control parameters can support different regimes**
  The plasma may access L-mode, H-mode, I-mode, or ITB barriers under similar operating conditions.
- **Transitions can show thresholds and hysteresis**
  The threshold to enter a state can differ from the threshold to leave it. The path matters.
- Nonlinear transport can select different confinement regimes.

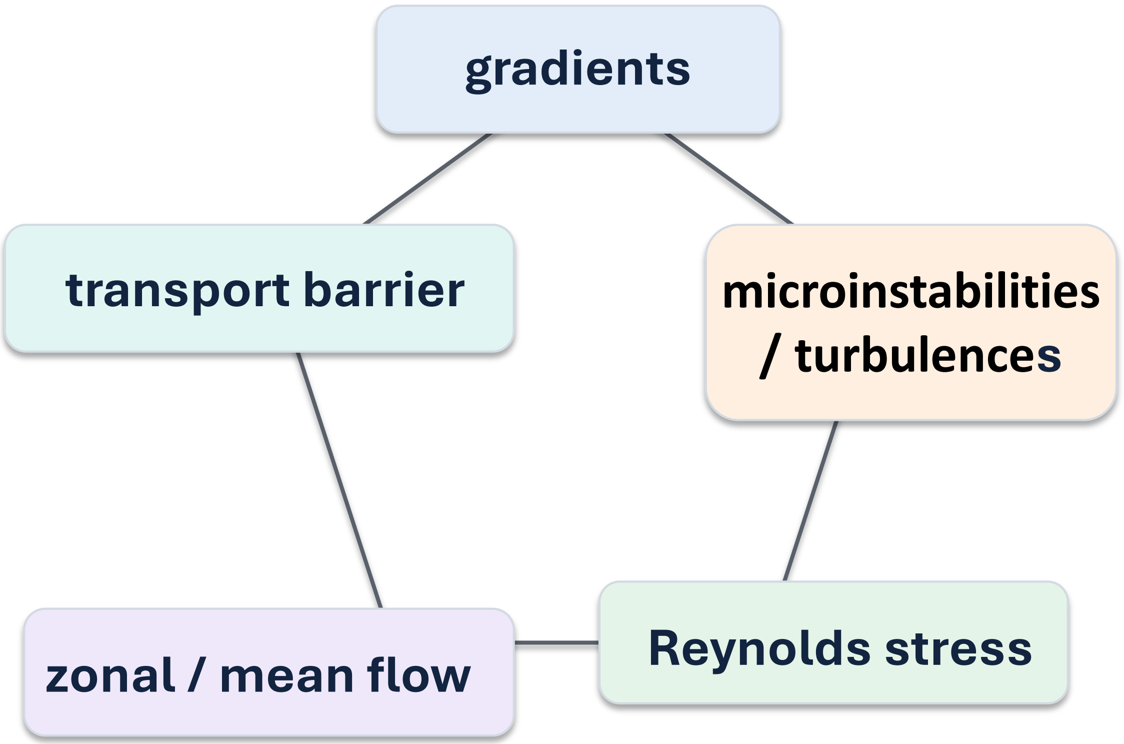
Nonlocality: transport barriers are not purely local objects

- We need to relax another assumption of local transport theory: that the flux at a point is determined only by the gradient at that same point.
- **A barrier may be triggered, weakened, or destroyed by nonlocal events**

- **Sawtooth-triggered L–H transition / ETB formation:** Core sawtooth crashes can launch heat pulses toward the edge; near threshold, the transient edge power / shear change can trigger H-mode access (Meyer et al., *NF* 2011; Shao et al., *Phys. Lett. A* 2020).
- **ELM crashes relax the H-mode pedestal / ETB:** Type-I ELMs periodically relax pedestal gradients and can temporarily weaken or collapse the edge transport barrier (Snyder et al., IAEA 2006; Kurzan et al., IAEA 2010.)
- **Magnetic islands can trigger or reshape ITBs:** NTM island can trigger an L-mode → ITB bifurcation; the ITB forms just inside the island boundary and depends on island width. (Mao et al., 2025)
- **Turbulence spreading can affect barrier boundaries:** Turbulence generated in an unstable region can spread into stable or barrier-adjacent regions, changing the effective barrier foot and transport scaling (Hahm et al., *PPCF* 2004; Singh & Diamond, *PoP* 2020).

3. Self-organisation = feedback-created structure

- Gradients drive turbulence
- Turbulence drives fluxes and Reynolds stress-driven-flows regulate turbulence
- Transport reshapes the gradients
- Barriers emerge when feedback reduces radial transport
- Self-organisation is maintained by continual energy transfer, not by thermodynamic equilibrium
- The "structure" may be a zonal flow, staircase, barrier, streamer, blob population



Self-organisation means dynamically maintained structure produced by feedback among gradients, turbulence, flows, and transport.

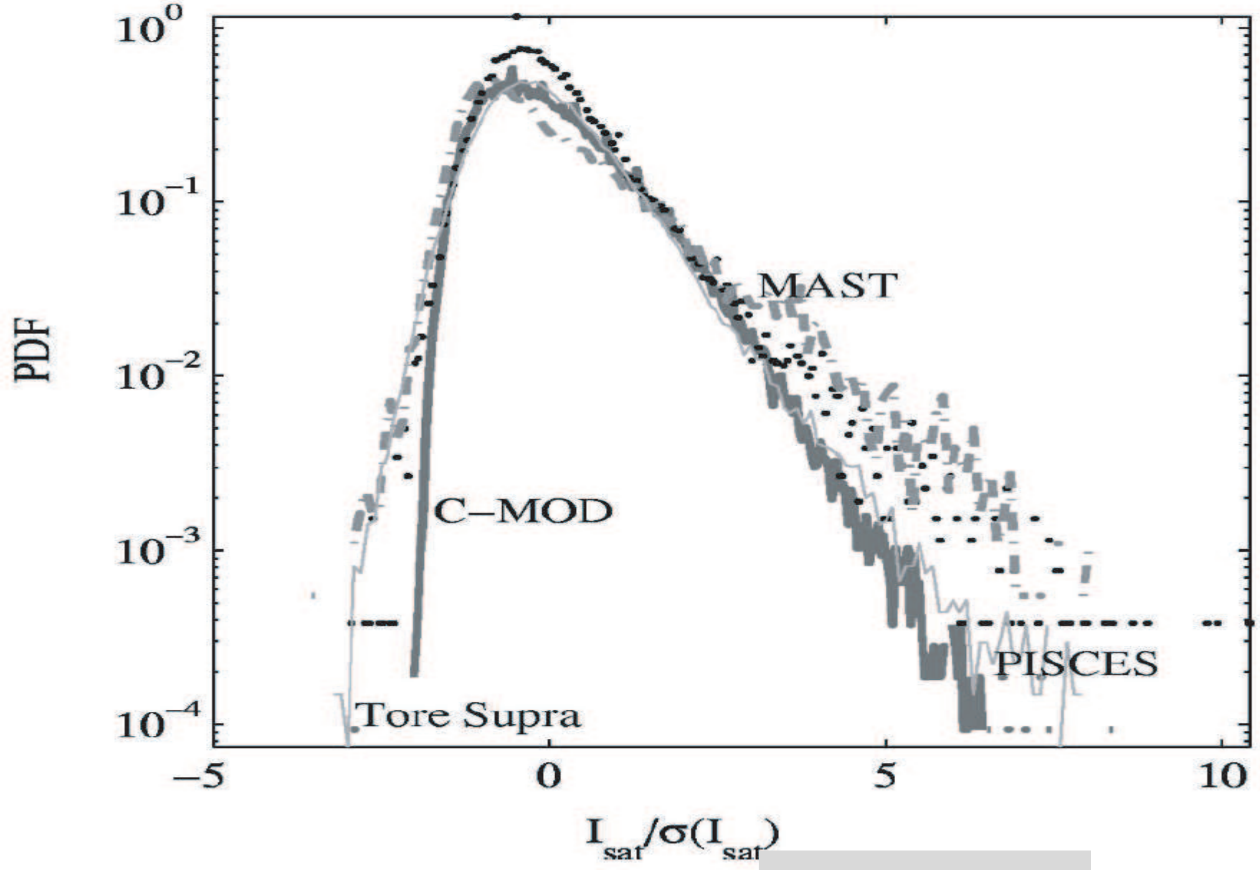
4. Statistical property can also be similar

- Similar PDF of the ion saturation current normalized by the standard deviation in the Tore Supra (solid line), Alcator C-Mod (thick solid line), MAST (dashed-dotted line), and PISCES (dots).

- Normalization by uncertainty

- Similarity in rescaled turbulent flux PDFs (B PH Van Milligen 2005, C. Hidalgo 2002, OF Castellanos 2006)

- Universal PDFs for bursty transport (I Sandberg et al 2009)



G Antar 2003

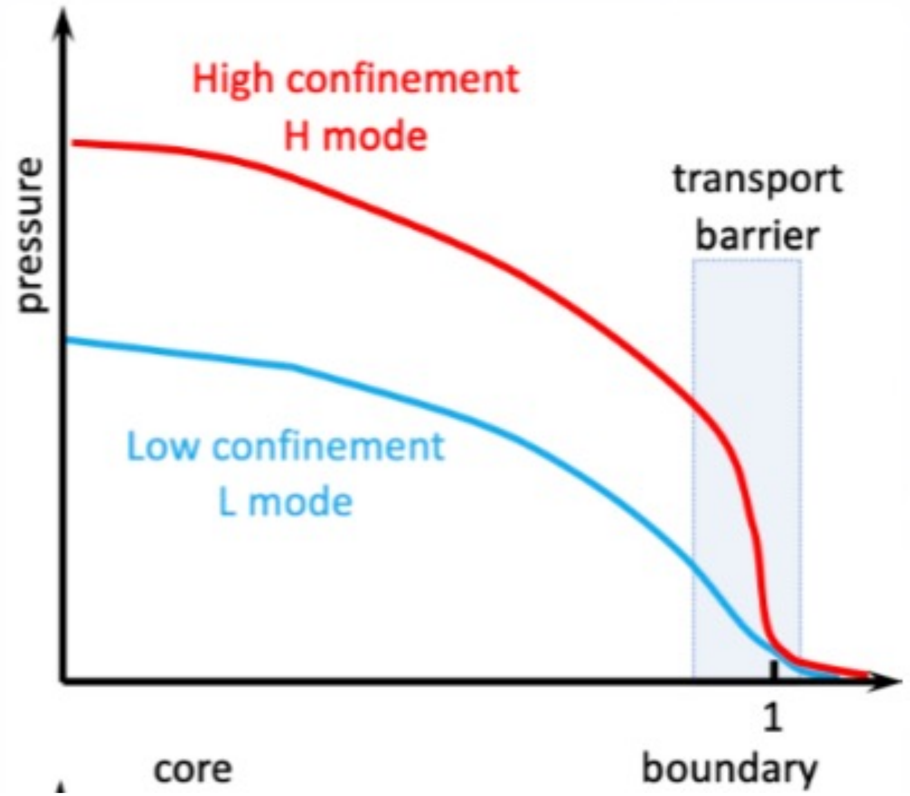
Part III – L-H transition and stochastic dynamics

1. L-H transition
2. Deterministic 3-variable prey-predator model
3. Stochastic L-H transition
- 1-variable model
- 3-variable prey-predator model

1. L-H transition is not fully understood

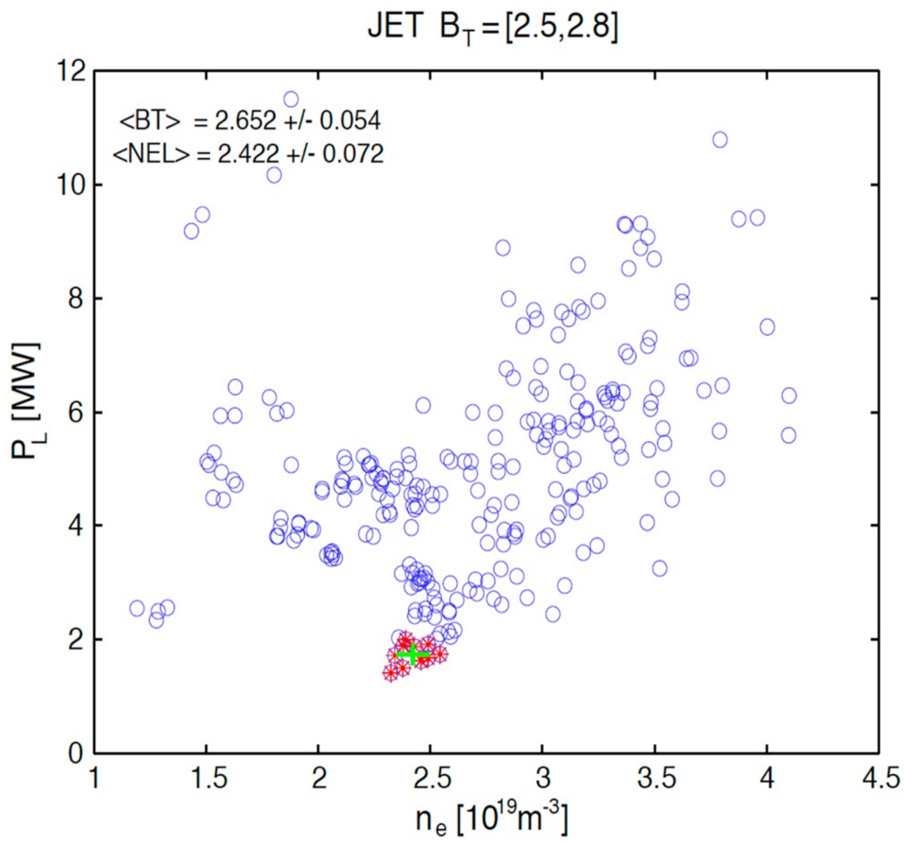
- Bifurcation at power threshold Q_c
- Edge transport barrier formation
- Order parameter: ExB flow shear
- Power threshold scaling by mean values
- Uncertainty (Martin 2008; Plank 2023; Andrew 2022)
- Hidden variables (divertor, isotopes, impurity, safety factor, stochasticity, gas puffing, displacing plasma column)
- Different transitions/turbulence characteristics
- Trigger mechanism: Causal relation?

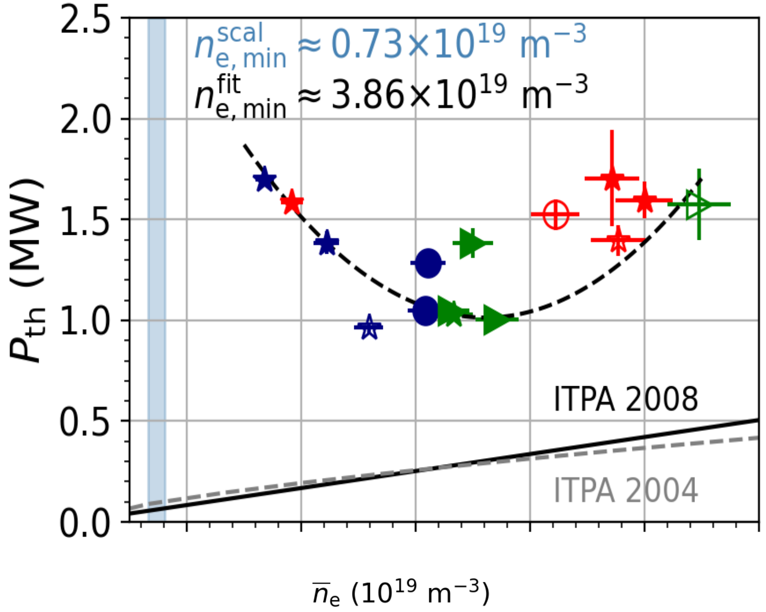
- Probability theory of L-H transition: stochasticity is a part of physics (external perturbation, fluctuating energy flux, stochastic magnetic fields, mini-avalanches on time scales t ~ O(0.1) ms)

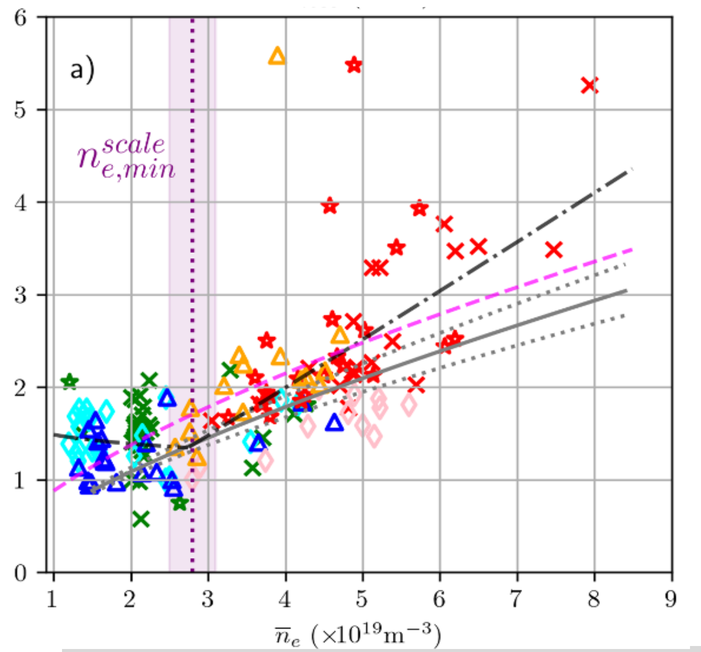


https://www.lpp.polytechnique.fr/The-localization-of-the-transition-from-low-to-high-confinement-mode-in-the?lang=fr

L-H transition power threshold P_{LH} against density has a U-shape: Minimum power threshold at a rollover density







MAST-U [C Jones…**E Kim** et al, PPCF 2025]

DIII-D [T Ashton-Key…**E Kim** et al, PPCF 67, 025027, 2024]

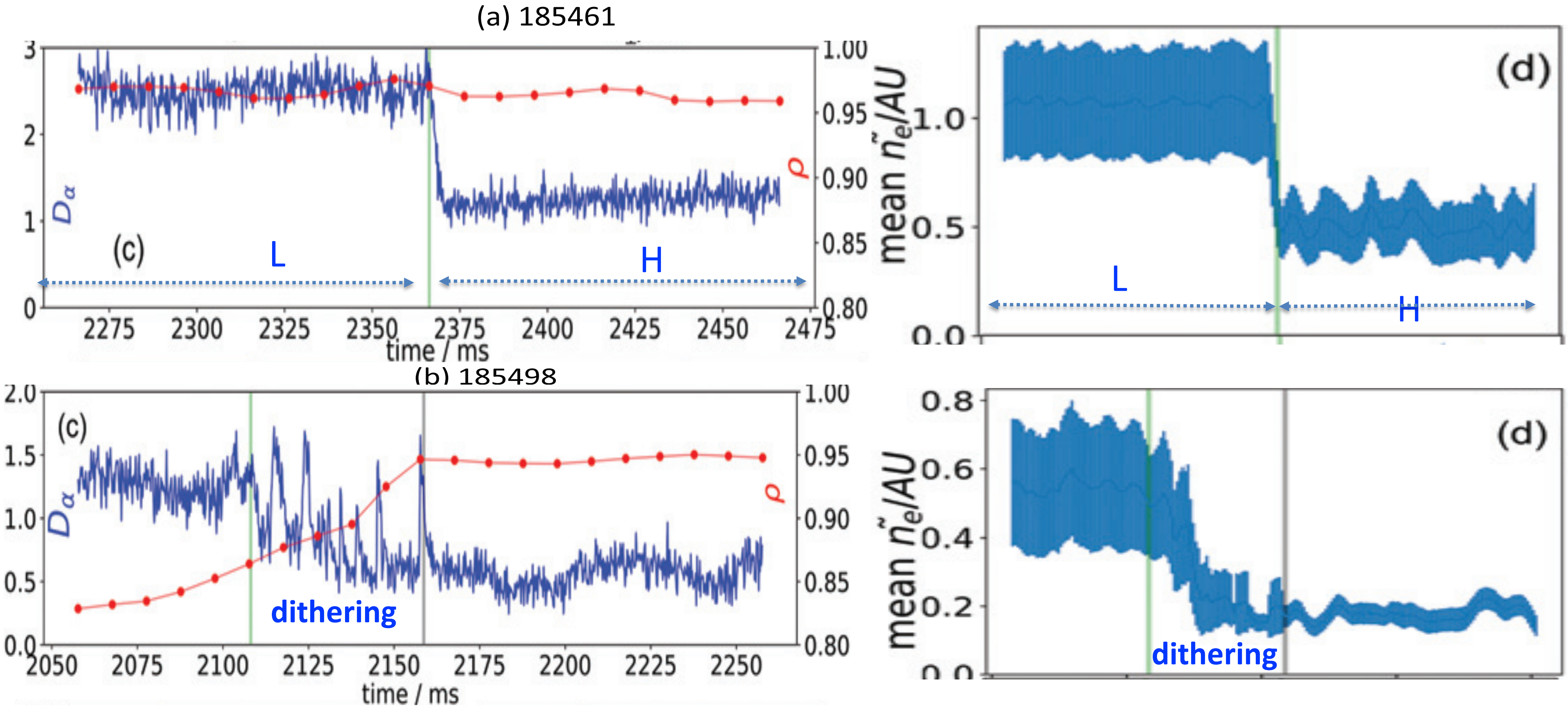
JET: The L–H transition threshold power vs plasma density ne shows significant variation in the JET tokamak [Martin et al., J. Phys.: Conf. Ser. 123, 012033 (2008)].

- High-density branch: P_{LH} scaling by mean values n_e, B_T, S

$$P^{scal}_{th} = 0.0488 \times 10^{\pm0.057} \bar{n}_e^{0.717\pm0.035} B_T{}^{0.803\pm0.032} S^{0.941\pm0.019}$$

$$P_{LH} = P_{ohm} + P_{NBI} - \frac{dW_{mhd}}{dt} - P_{rad}$$

L (H) modes present different turbulence characteristics



HJ Farre-Kaga, Y Andrew… E Kim et al (EPL 142, 64001, 2023) [see also Howlett et al., NF 63, 052001 (2023)].

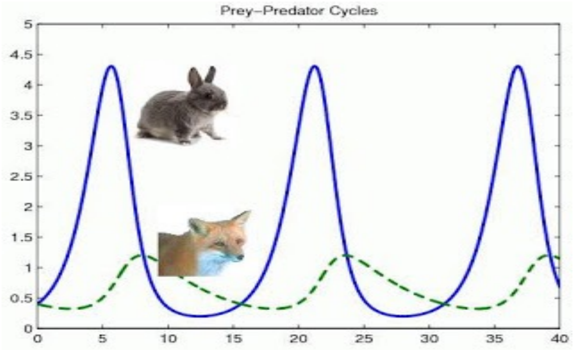
State-of-the art of numerical simulations of the L-H transition:
routine predictive P_{LH} modelling across devices remains an open challenge

- **Gyrokinetics / XGC:** ETB formation in diverted geometry with $E\times B$ shear and neoclassical orbit loss (Chang et al., *PRL* 2017; Ku et al., *PoP* 2018).
- **Flux-driven gyrokinetic simulations / GYSELA** show spontaneous barrier onset, but not yet a full predictive H-mode model (Dif-Pradalier et al., *Communications Physics* 2022).
- **EM drift-fluid / GRILLIX:** Electromagnetic, drift-fluid model; evolves electron and ion temperature dynamics, $E\times B$ flows, edge-SOL geometry, and includes extensions beyond standard collisional Braginskii closures (Zholobenko et al., *PRL* 2026).
- **Two-fluid simulations / GBS:** edge–SOL simulations show suppressed-transport regimes, hysteresis (Giacomin & Ricci, *JPP* 2020; De Lucca et al., arXiv 2026).
- **Common message: the LH transition is a nonlinear transition involving feedback**
- **Predator–prey and stochastic models:** Isolate the feedback structure, explain the organising principles behind the simulations, capture transition dynamics, hysteresis, probabilistic thresholds, and noise-triggered transition variability (Kim et al PRP 2020; Fuller et al PoP 2024; Kim & Thiruthummal, PRE 2024; PPCF 2025).

2. Deterministic 3-variable prey-predator L-H transition model

- Isolate the feedback structure
- Explain the organizing principles behind the simulations: thresholds, limit-cycle oscillations, hysteresis, and bifurcation between confinement states.
- Three variables: turbulence amplitude E, zonal flows V_{ZF}, mean pressure gradient N (mean flow $V=N^2$)

- Input power (heating) drives turbulence (prey) via instabilities
- (Meso-scale) Zonal flow $V_{ZF}=v$ generated by turbulence regulate turbulence
- (Macro-scale) Mean flow shears V (driven by density/pressure gradient) regulate turbulence and zonal flow (super-predator, lions)



$$\partial_t \mathcal{E} = \mathcal{E}\mathcal{N} - a_1\mathcal{E}^2 - a_2 V^2 \mathcal{E} - a_3 V_{\rm ZF}^2 \mathcal{E},$$

$$\partial_t V_{\rm ZF} = b_1 \frac{\mathcal{E} V_{\rm ZF}}{1 + b_2 V^2} - b_3 V_{\rm ZF},$$

$$\partial_t \mathcal{N} = -c_1 \mathcal{E}\mathcal{N} - c_2 \mathcal{N} + Q.$$

$\epsilon = x^2$: Turbulence amplitude (prey: rabbits)
v: zonal flows (predator: foxes)
N: Density (pressure) gradient
Q: Input power (constant + fluctuation)
$V = dN^2$: mean flow shear (super predator: lions)

E Kim & PH Diamond, PRL90, 185006, 2003

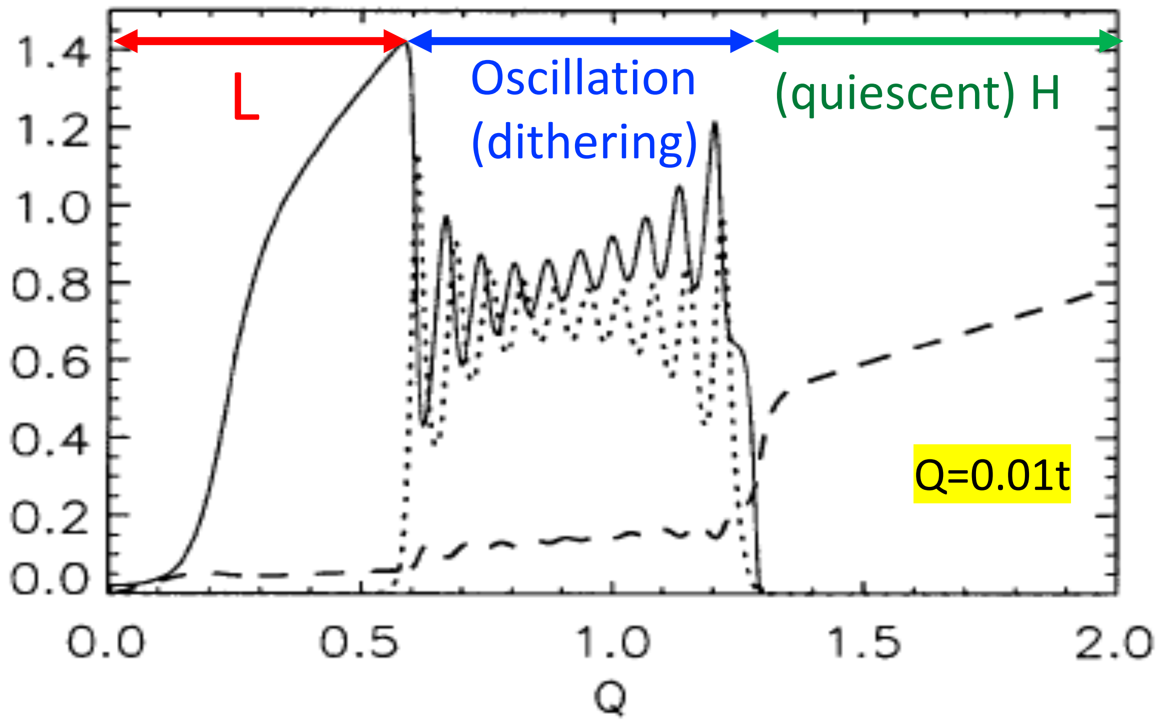


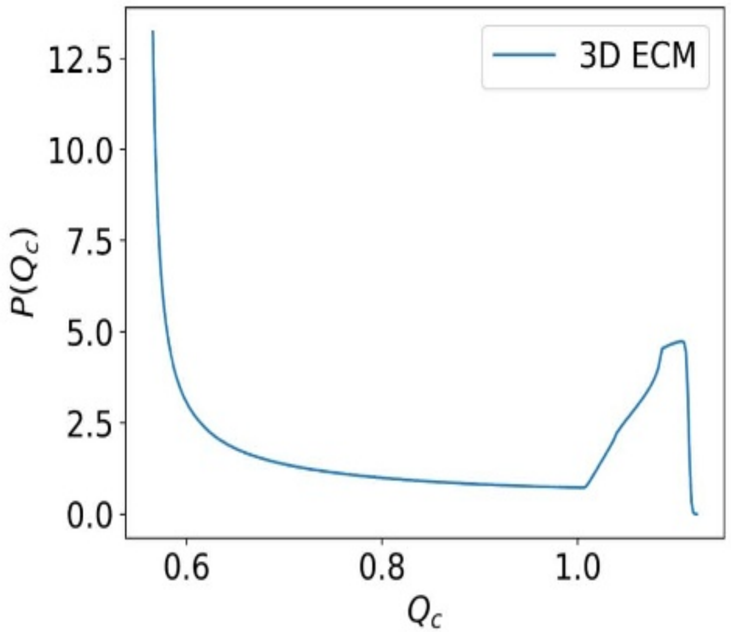
FIG. 1. Evolution of $\mathcal{E}$ (solid line), V_{ZF} (dotted line), and $\mathcal{N}/5$ (dashed line) as a function of input power $Q = 0.01t$. Parameter values are $a_1 = 0.2$, $a_2 = a_3 = 0.7$, $b_1 = 1.5$, $b_2 = b_3 = 1$, $c_1 = 1$, $c_2 = 0.5$, and $d = 1$.

L mode: high turbulence, small zonal flow

Dithering: finite turbulence, finite zonal flow

H mode: zero turbulence, zero zonal flow

PDF of power threshold Qc



BUT, the L–H transition power thresholds is determined not only by the instantaneous external power but also by plasma state before the power ramp (initial conditions). [E Kim et al PRE2024; PPCF2025]

3. Stochastic L-H transition: bridge to statistical theory

- Noise represents turbulent forcing, finite-size fluctuations, or unresolved fast variables
- The same feedback can yield very different transition statistics when noise is included
- PDFs can become non-Gaussian and multimodal during L-H-like transitions
- This topic is introduced here, then reappears in the statistical lecture in turbulence and statistical theory (information-geometry)

- Langevin approach: stochastic differential equations (SDE) for trajectories
- Fokker-Planck approach: partial differential equation for PDF

1-variable stochastic model of the L-H transition

[S-I Itoh, K Itoh & S Toda, PRL 89, 215001, 2002]

- Langevin equation for $X \propto E_r$, noise w

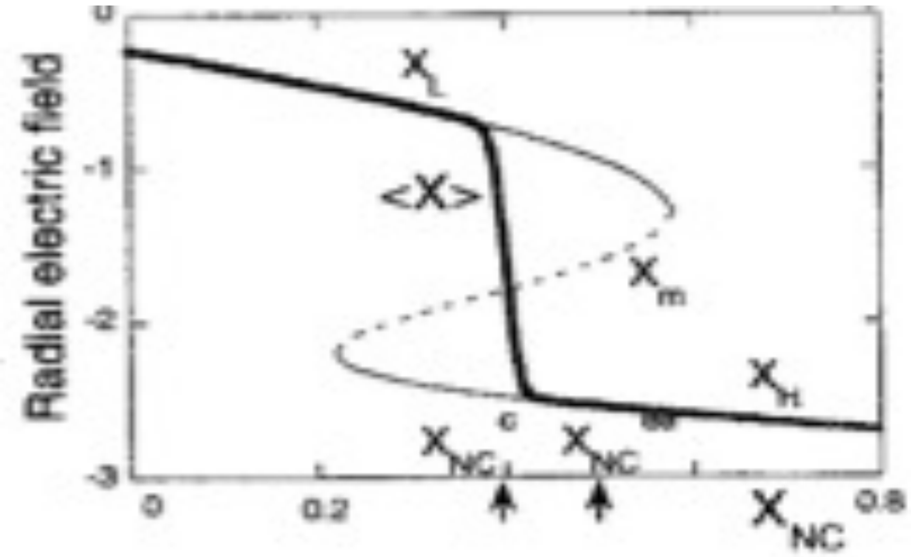
$$\frac{dX}{dt} + \Lambda(X) = g(X)w$$

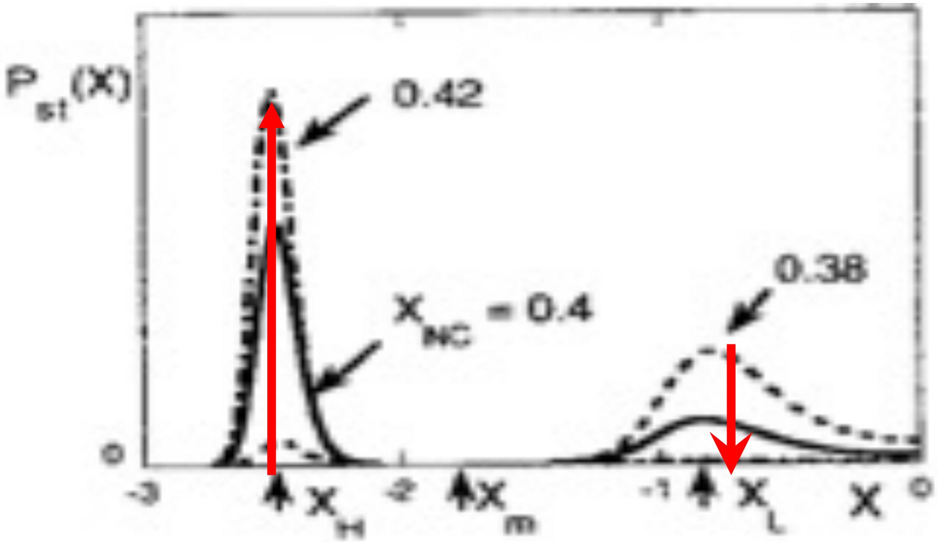
Here, $\Lambda(X), g(X)$ are deterministic force (bulk viscosity of ions, ion orbit loss, eddy damping); w is random kick (Stratonovich calculus).

- The Fokker-Planck Equation for $p(X,t)$

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}[\Lambda p] + \frac{\partial}{\partial X}\left[g \frac{\partial}{\partial X}(gp)\right]$$

- $p_s(X) \propto g^{-1}e^{-S(X)}$ for $\frac{\partial p}{\partial t} = 0$
- $S(X)$ has minima at $X = X_L, X_H$
- Bimodal PDF with two peaks at L & H mode solutions X_L, X_H





3-variable prey-predator stochastic L-H transition model

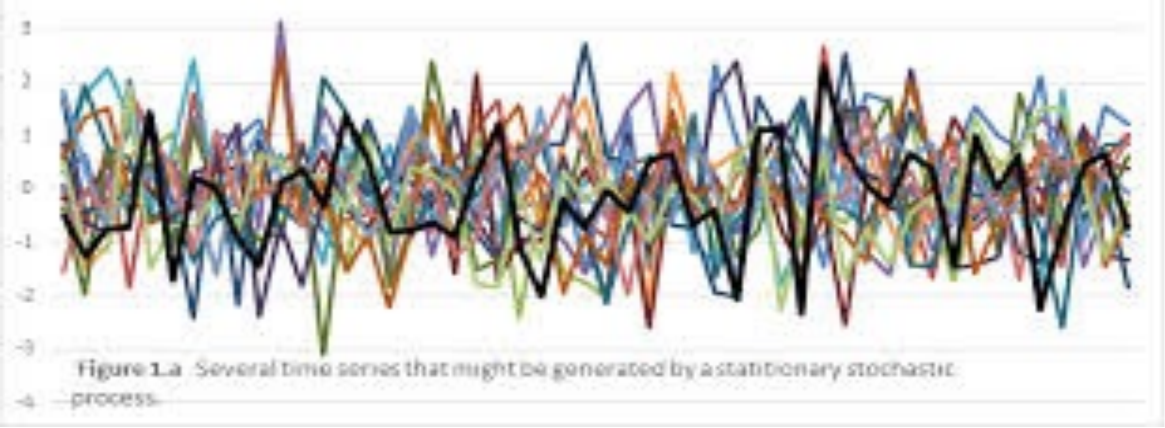
[E Kim et al, PPCF 2025; PRE 2024; Entropy 2024, P Fuller et al 2024, E Kim et al PRP 2020, R Hollerbach & Kim POP 2021]

$$\frac{d\epsilon}{dt} = N\epsilon - a_1\epsilon^2 - a_2V^2\epsilon - a_3v^2\epsilon + \xi_1\epsilon,$$

$$\frac{dv}{dt} = \frac{b_1\epsilon v}{1+b_2V^2} - b_3v + \xi_2,$$

$$\frac{dN}{dt} = -c_1\epsilon N - c_2N + Q + \xi_3.$$

ξ_i



Time

Here, stochastic noises ξ_i (i=1,2,3) are assumed to be independent Gaussian noises with a short correlation time and amplitude D_i.

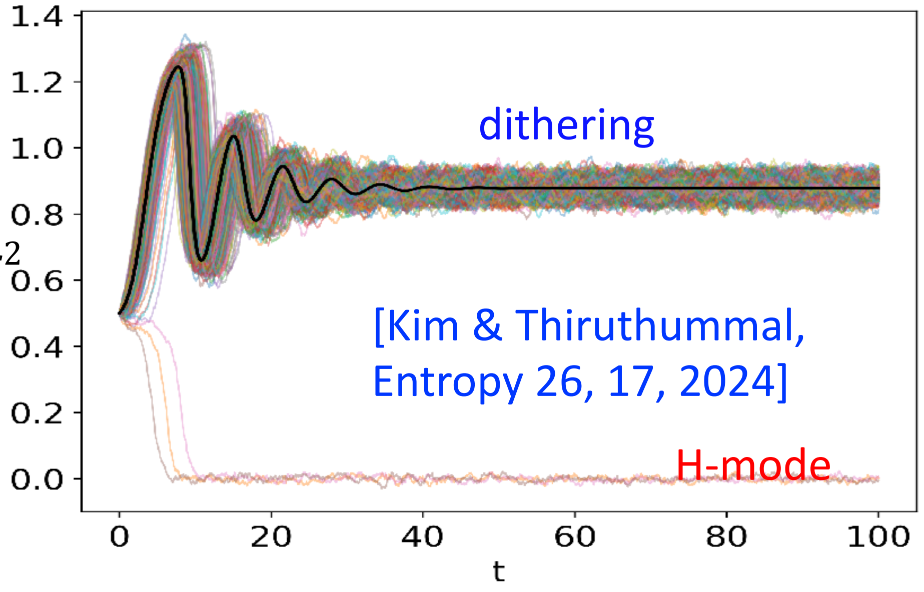
$$\langle \xi_i(t)\xi_j(t')\rangle = 2D_i\delta_{ij}\delta(t-t'), \quad \langle \xi_i \rangle = 0.$$

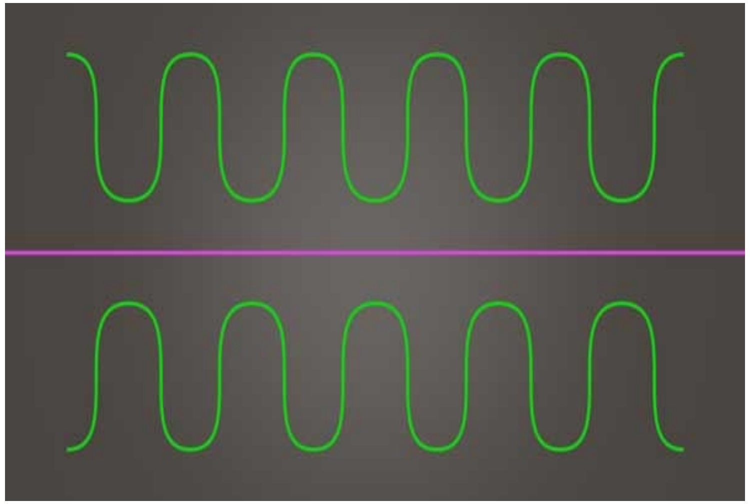
[ξ_i models stochasticity caused by external stochastic perturbation, fluctuating energy flux, stochastic magnetic fields, mini-avalanches on time scales t ~ O(0.1) ms.]

3. Stochastic L-H transition: bridge to statistical theory

- Stochasticity caused by external stochastic perturbation, transports, fluctuating energy flux, pellet pacing, stochastic magnetic fields, mini-avalanches on time scales t ~ O(0.1) ms (e.g. ECEI on KSTAR, gyrokinetic simulations).
- The same initial condition leads to different trajectories, equilibria.

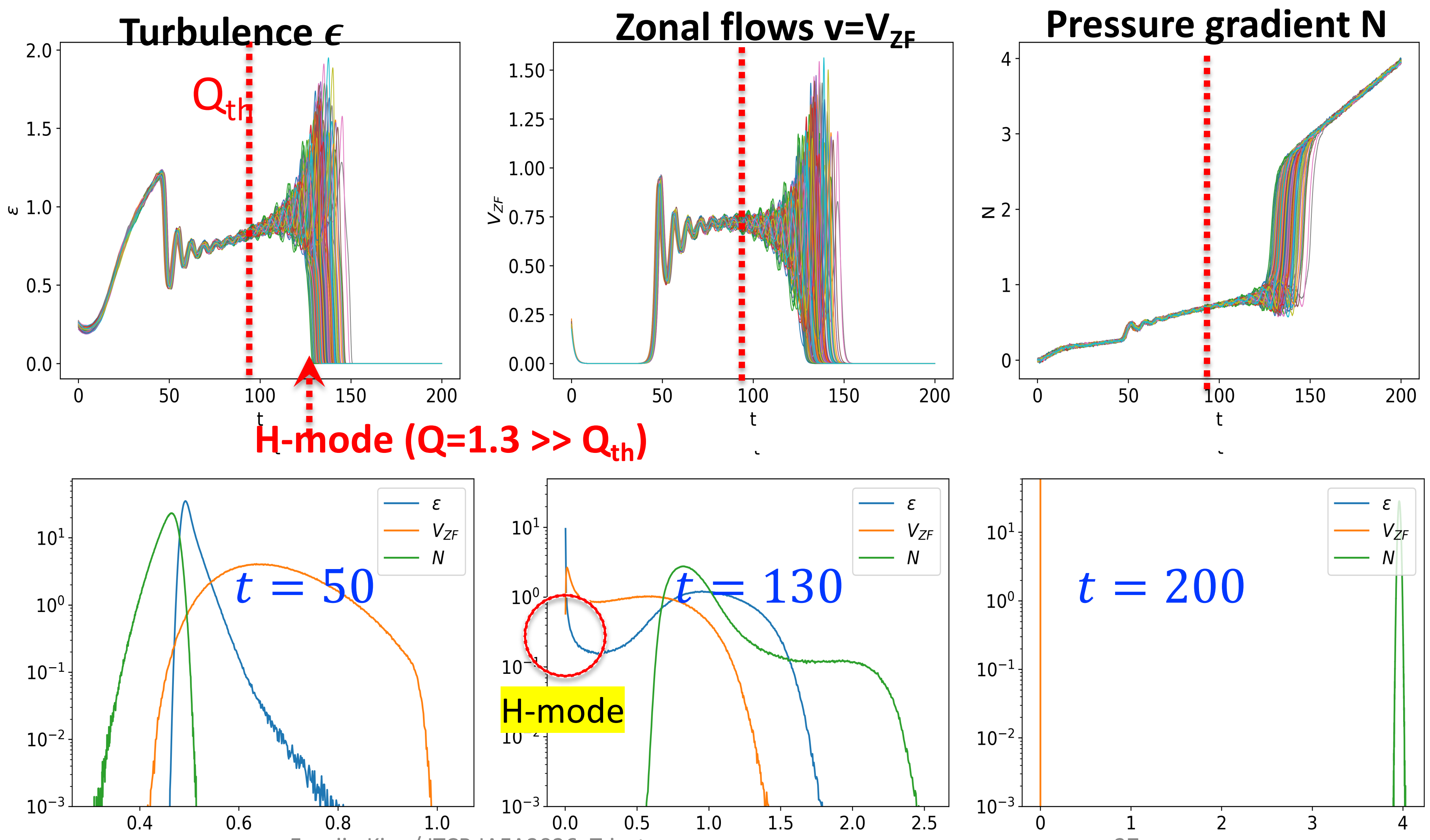
$\sqrt{\epsilon} = x^2$

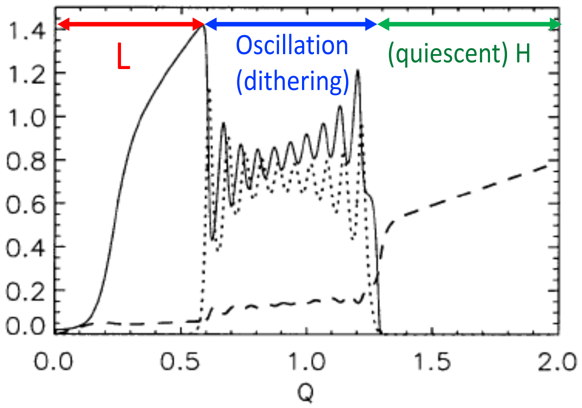




- Phase mixing: the sum of two waves of 180 phase difference leads to cancellation.

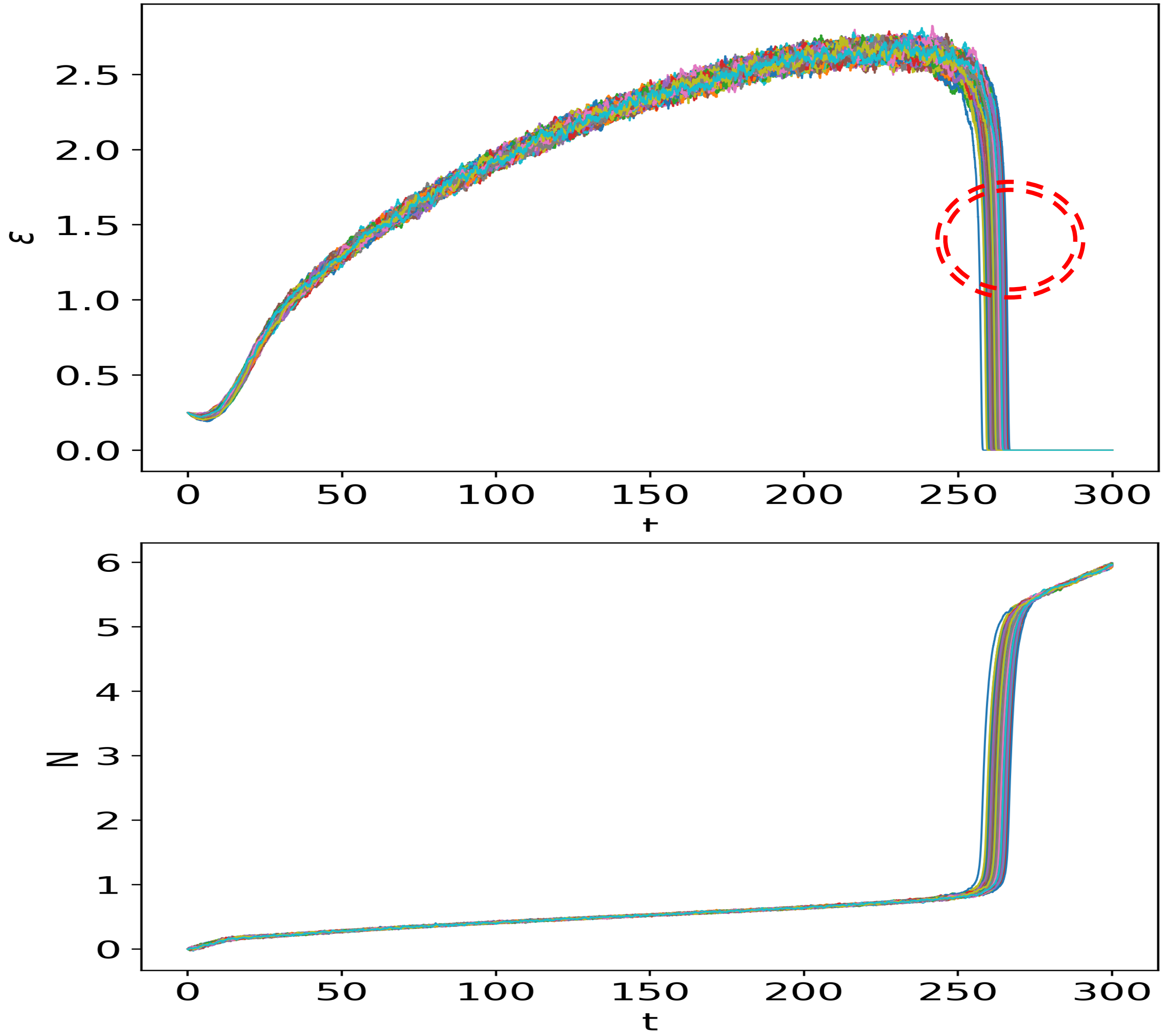
Q=0.01t: Stochastic L-H transition model can have a mixture of dithering and H-mode at a fixed time (bimodal PDFs)





Cf. Deterministic model

Note: without zonal flows, information rate forms a sharp peak at the transition and the transition occurs at higher Qc ~ 2.6-2.7

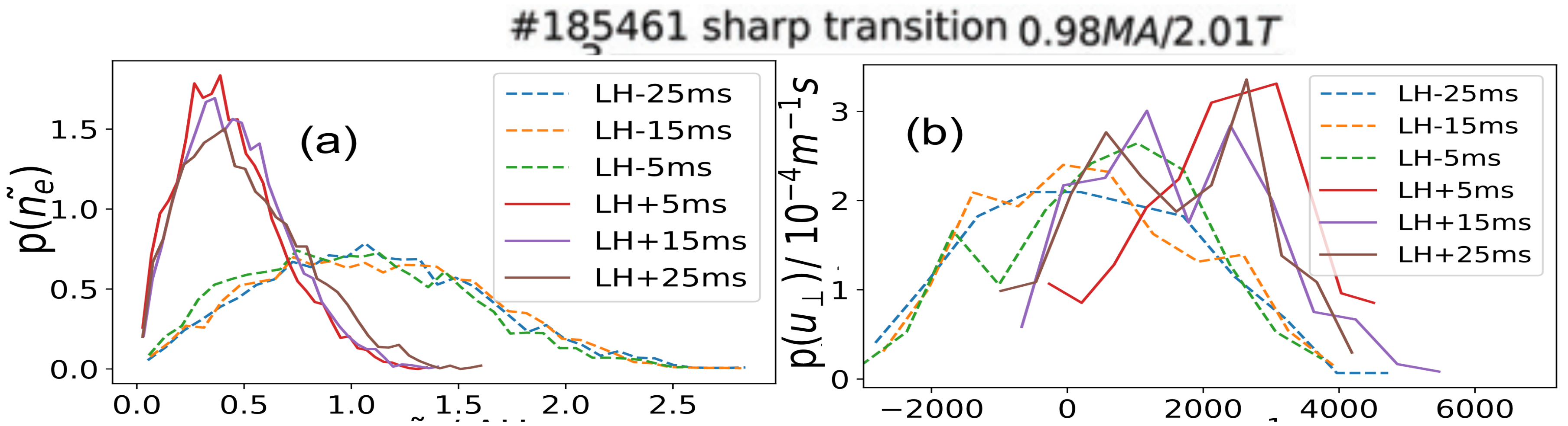


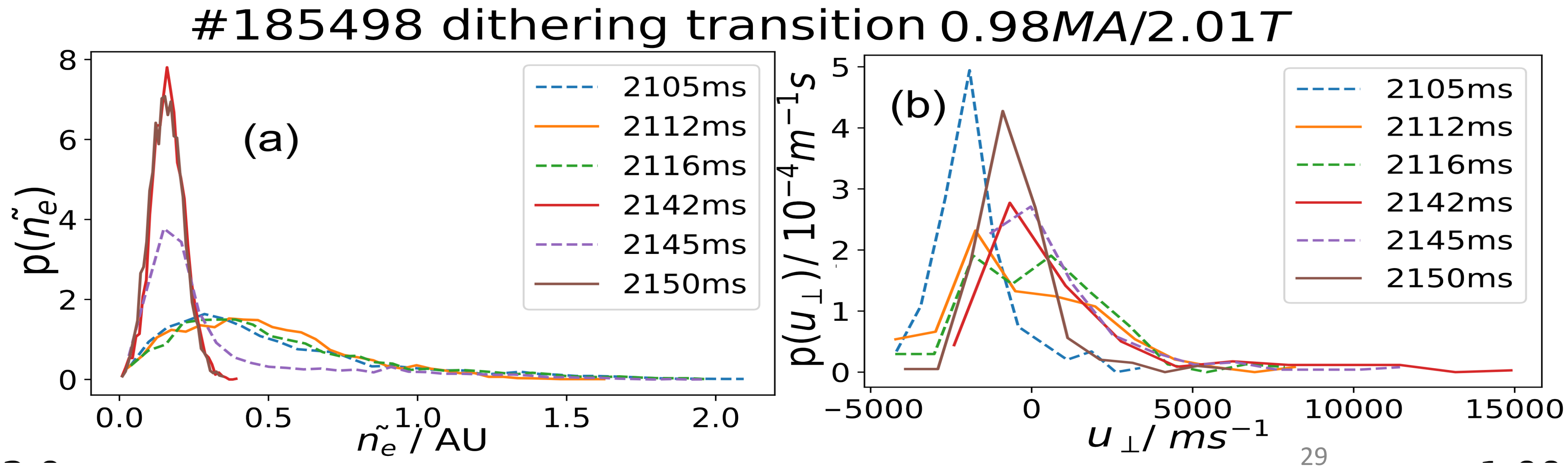
Coexistence of L and H mode solutions, similarly to 1-variable stochastic Itoh 2002 model
[E Kim & A Thiruthumal PRE 2024]

Time-dependent PDF analysis of DIII-D data

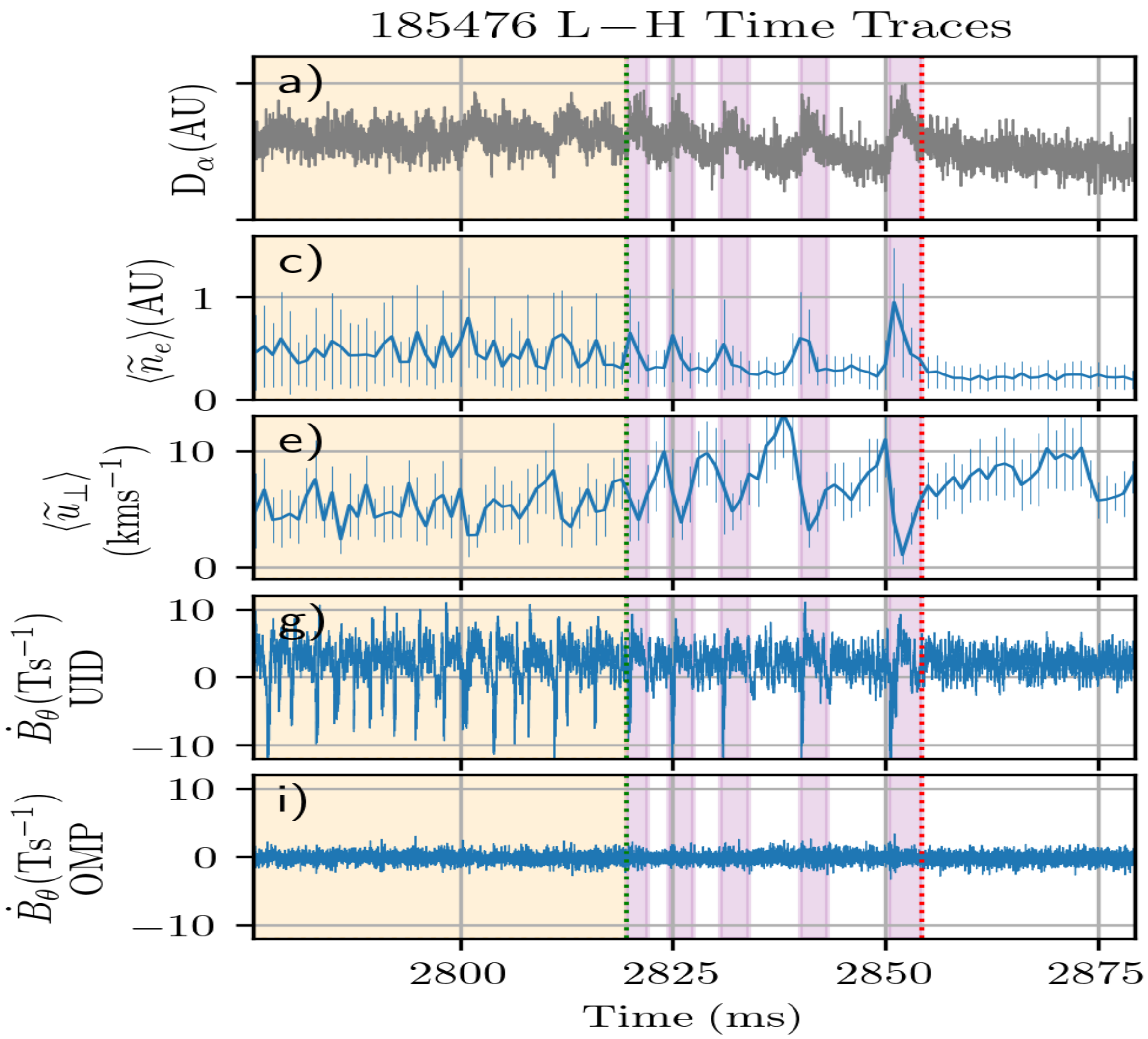
[HJ Farre-Kaga, Y Andrew… E Kim, TL Rhodes, L Schmitz, Z Yan, EPL 142, 64001, 2023]

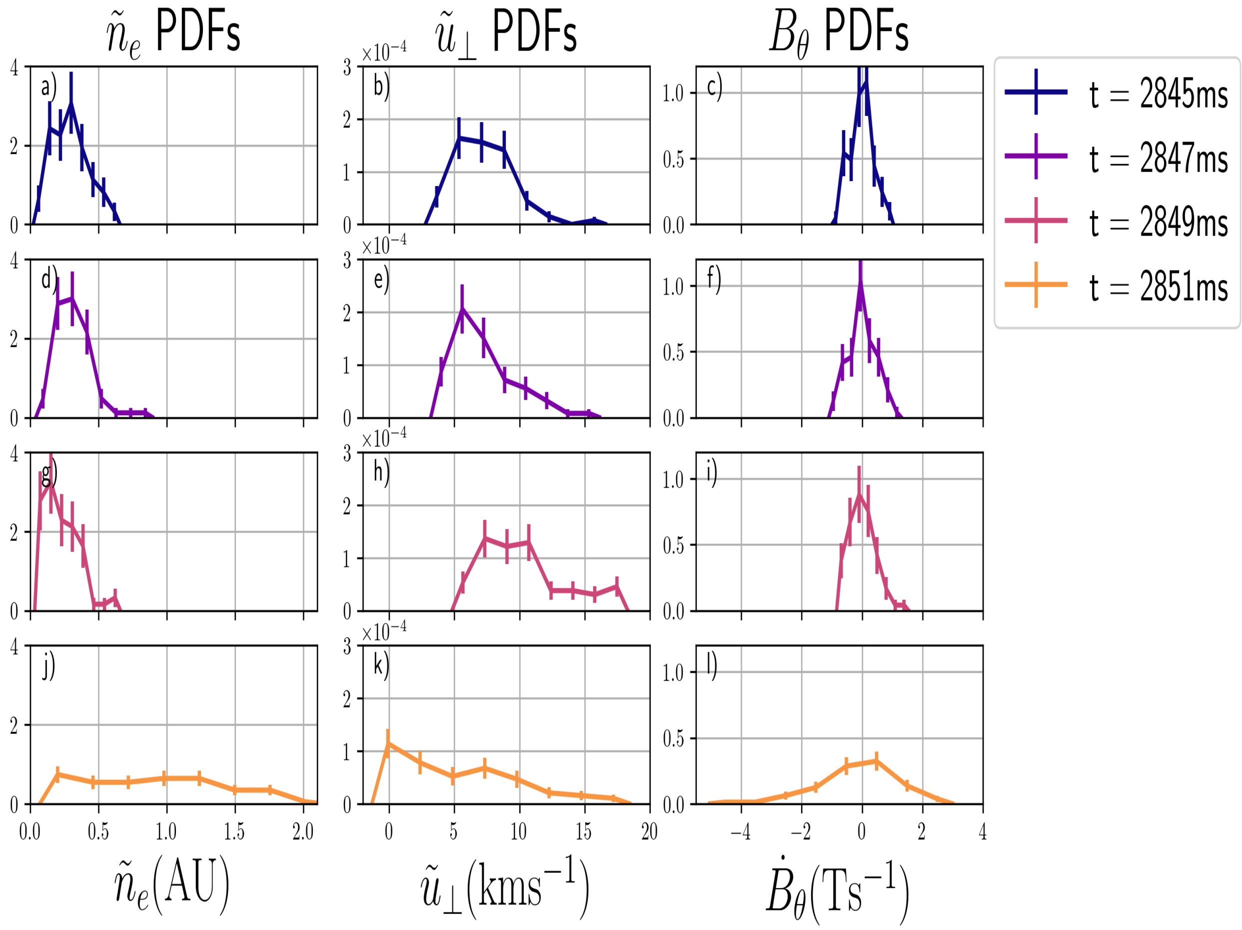
Time-sliding windows are used to construct time-dependent PDFs from time-series DBS data at a fixed spatial location





T. Ashton-Key, Y. Andrew… E. Kim, et al (PPCF 2025)



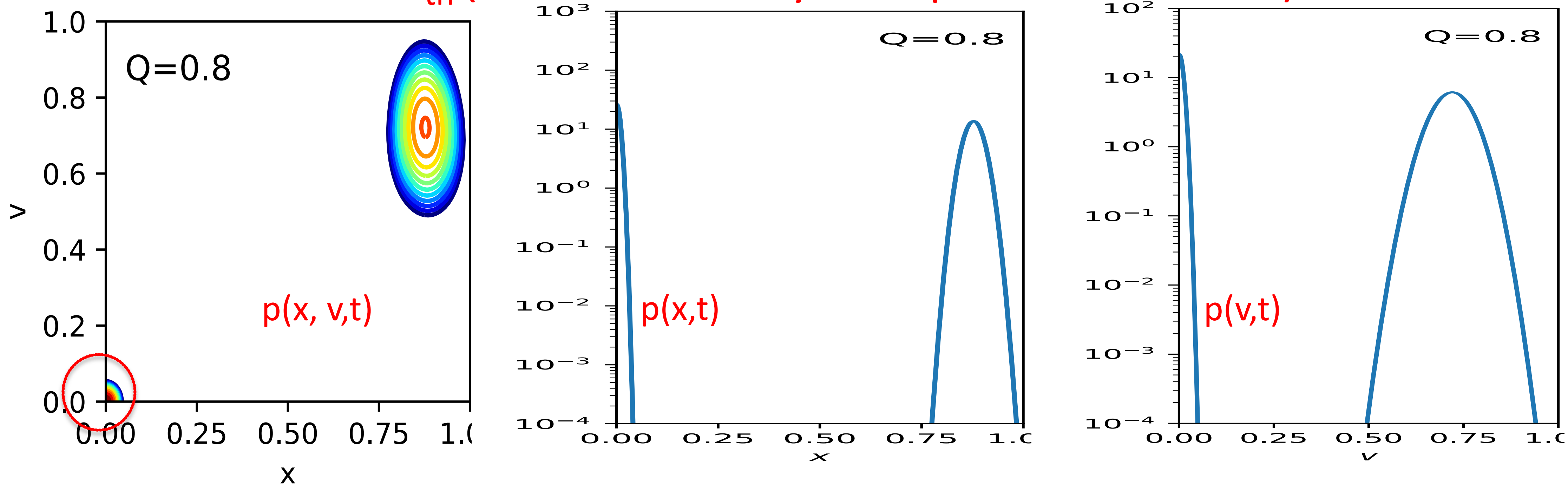


2780 2800 2820 2840 2860
Time (ms)

Constant Q: Fokker-Plank approach: time-dependent PDFs (x=turbulence, v=zonal flows)

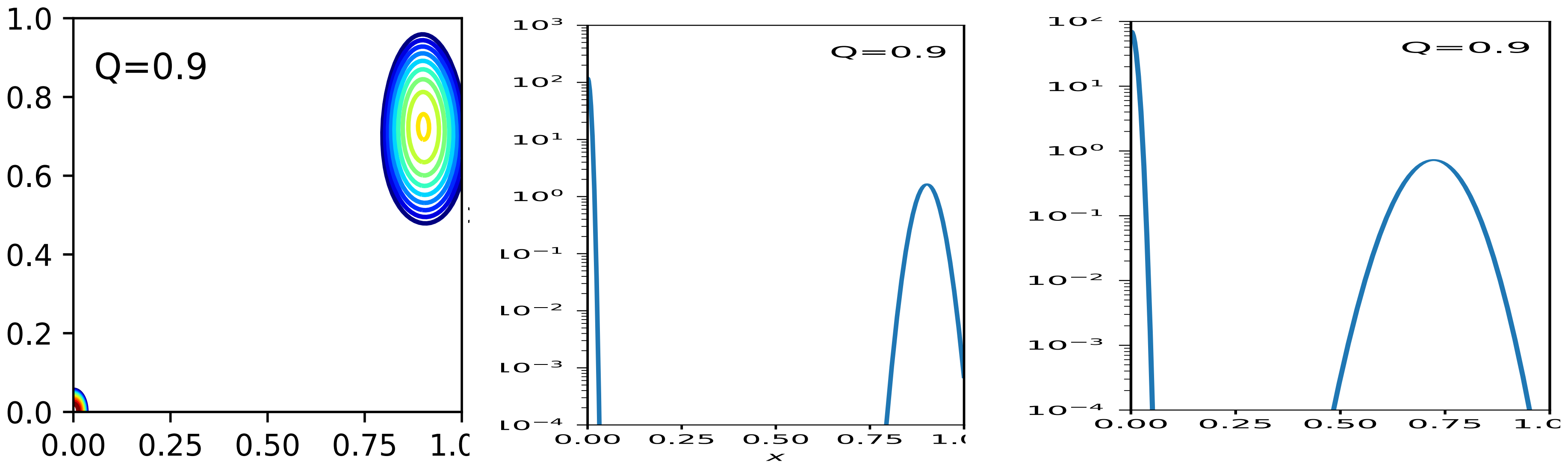
Stationary PDFs for constant Q (Dx=Dv=10^{-4}) in 2D model [For Dx=Dv=0, power threshold Q_{th} =0.832]

$Q=0.8 < Q_{th}$ (deterministic system power threshold)

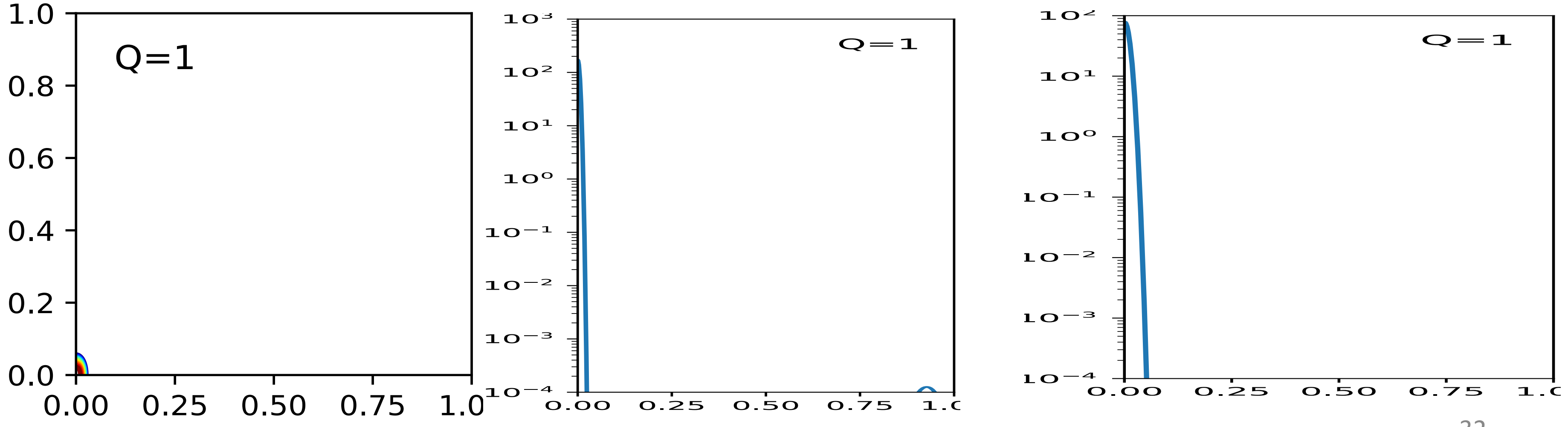


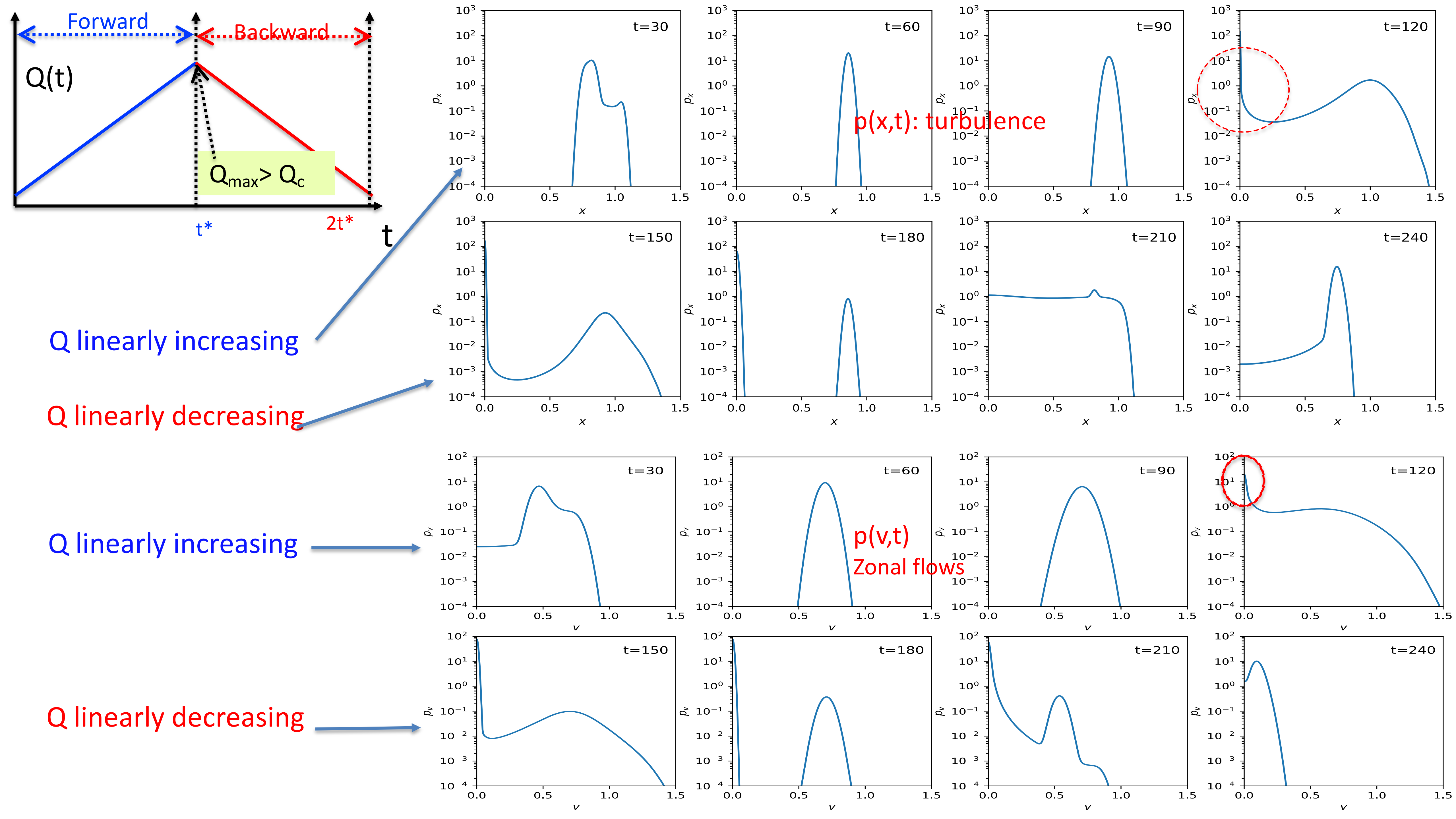
- PDF: two peaks for H-mode (x=v=0) and dithering (finite values of x, v) : "mixed H-mode & dithering"
- H-mode characteristics appears at a lower power threshold

Q=0.9 > Q_{th} : "mixed H-mode & dithering" persists



Q =1 : clear H-mode further above from power threshold





Forward
Backward
Q(t)
$Q_{max} > Q_c$
t*
2t*
t
Q linearly increasing
Q linearly decreasing
p(x,t): turbulence
Q linearly increasing
Q linearly decreasing
p(v,t)
Zonal flows
t=30
t=60
t=90
t=120
t=150
t=180
t=210
t=240

Initial conditions (plasma state before the power ramp) and stochastic noise cause uncertainty in Qc: how to calculate P(Qc)? (Kim et al PPCF 2025)

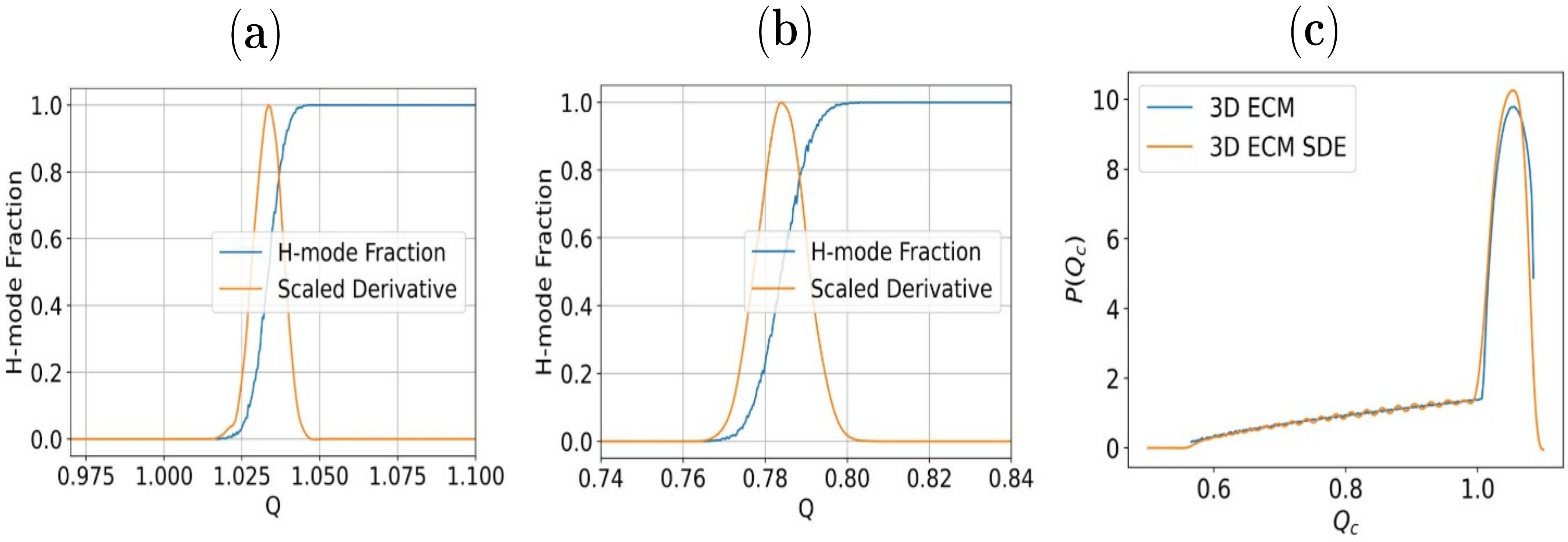


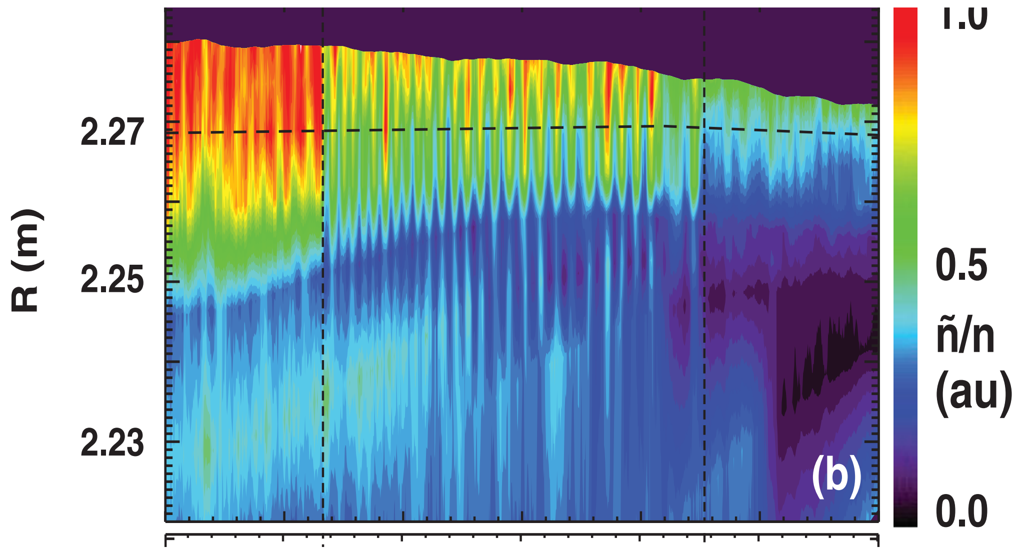
Figure 3. H-mode fraction (blue) and PDF of power threshold $p(Q_c)$ (orange) for a delta-initial condition with: (a) $\epsilon(0) = 0.25, v(0) = 0.2$ as used in figure 2; (b) $\epsilon(0) = 0.25^2, v(0) = 0.2$; (c) $p(Q_c)$ against Q_c sampled over $(x_0, v_0) = [0.01, 0.7]$ with $x_0 \geqslant v_0$ for deterministic (blue) and stochastic (orange) systems. $D_1 = D_2 = D_3 = 10^{-4}$.

Summary of stochastic L-H transition

- Stochastic noise leads to some trajectories undergoing the transition at a lower input power while delaying the complete transition to H-mode at a higher input power
    - Stochastic noise leads to "mixed H-mode & dithering" or "mixed H-mode & L-mode" with a bimodal PDF at input power ($Q< Q_c$)
    - As Q increases further from power threshold, H-mode characteristic becomes prominent at higher $Q>Q_c$
- Time dependent input power Q leads to delay in response, causing further "uncertainty" in power threshold
- Initial conditions cause uncertainty in power threshold.
- Power threshold is found as a distribution.

Open questions in L-H transition physics

- What are the mechanisms behind different types of L-H transitions?
- How to track the spatio-temporal evolution of turbulence, flows, and gradients to identify causal pathways in core-edge coupling?
- What are the role of electromagnetic fluctuations, neutrals, and impurities?
- Why do isotope, density, drift-direction, and geometry dependences vary across devices?
- How do stochastic fluctuations trigger, delay, or smear the transition?
- Can simulations predict LH access and H-mode pedestal evolution in reactor-relevant conditions?
- What is the role of self-organisation in other confinement (hybrid, FIRE) modes?
- How do barriers co-evolve with the propagation of ExB shear flows and turbulence spreading (G Dif-Pradalier 2022, MJ Choi 2022, S Yi 2024)?



DIII-D (L Schmitz et al, PRL 108, 155002, 2012)

Further reading

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