Robert Bryant

Title: Curvature-homogeneous metrics in dimension 4.

Abstract: A Riemannian manifold ($M^n$, g) is said to be curvature-homogeneous if, for any two points p and q in M , there is a g-isometry $\iota : T_pM \to T_qM$ Such that $\iota^*(\mathrm{Riem}(g)_q) = \mathrm{Riem}(g)_p$. Of course, when n = 2, curvature-homogeneity is equivalent to having constant Gauss curvature, while, when n = 3, curvature-homogeneity is equivalent to the eigenvalues of the Ricci-tensor being constant. In higher dimensions, curvature-homogeneity is more subtle. For example, it is not yet known which curvature tensors are possible for curvature-homogeneous manifolds, or even which curvature tensors can occur for curvature-homogeneous manifolds that are not locally homogeneous. In this talk, I will survey the known results on the associated classification problem, which has been of interest since at least the early work on I. Singer on the subject, and I will discuss recent joint work with Renato Bettiol yielding some surprising new results and examples in dimension 4. The techniques used involve classic results of ´E. Cartan on the solvability of overdetermined systems of partial differential equations.