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Title: Harmonic maps into high-dimensional spheres.

Abstract: I will survey recent developments related to harmonic maps from surfaces to high-dimensional Euclidean spheres, which establish new connections between geometry and probability.
Given a closed Riemann surface and a unitary representation of its fundamental group, classical variational theory produces a corresponding equivariant harmonic map from the Poincar´e disk to a Euclidean sphere. In general not much can be said about the geometry of such maps. However, I will explain that by leveraging the theory of random matrices, if the representation is sufficiently generic, then the corresponding harmonic map becomes very close to an immersed hyperbolic plane inside a sphere. Moreover, the asymptotic limit is essentially unique. People have shown that all of this implies a strong positive answer to a question from Yau's 1982 list about negatively curved minimal surfaces in spheres.
Partly joint with Riccardo Caniato and Xingzhe Li.