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Title: Extremal Sasaki Twins.

Abstract: Let (g, ω) be a Kähler metric (and form) on a compact complex manifold (M, J). We know from the famous work of Yamabe, Trudinger, Aubin, and Schoen that there always exists some positive smooth function $f : M \to \mathbb{R}^+$ such that the metric $h = f^{-2}g$ has constant scalar curvature.
LeBrun and Apostolov-Calderbank-Gauduchon observed that if (M, J, g, ω) is a compact Kähler surface and f is a killing potential, then h is part of a solution to the Riemannian version of the so-called Einstein-Maxwell equations. Such Einstein-Maxwell solutions are called Strongly Hermitian Einstein-Maxwell solutions while h is called a conformally Kähler Einstein-Maxwell metric. LeBrun constructed many examples of such solutions on Hirzebruch surfaces and on the first Hirzebruch surface he discovered an interesting bifurcation: a pair of extra solutions "pop up" when we reach a certain subcone of the Kähler cone. Whenever the cohomology class, $\left[\frac{\omega}{2\pi}\right]$, is an integer class, these extra solutions turn out to provide non-trivial examples of the so-called extremal Sasaki Twins (introduced by Boyer-Legendre-Huang-TF).
In this talk, which is mainly based on joint work with Boyer, Legendre, and Huang, I will introduce the definitions of twins in the Kähler/Sasaki setting and then discuss some examples (starting with LeBrun's) both for dimensions four/five and higher dimensions.