

Abstract template for Youth in High-Dimensions

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Large language models demonstrate remarkable ability in factual recall, but the fundamental limits of memorizing and retrieving many input–output associations remain unclear. We study these limits in a minimal setting: a linear associative memory that must map p input embeddings in d to their corresponding p -dimensional targets via a single linear map, while keeping each mapped input well separated from all other targets. Unlike in supervised classification, this strict separation condition induces p constraints per association and produces correlations between constraints through the shared outputs. Here, we characterise the storage *capacity* p^* of a linear associative memory, i.e. the maximum number of input-output patterns it can store reliably, in the following ways. We introduce a simpler “decoupled” capacity problem in which, for each input, the full set of competing output patterns is independently re-sampled. We find numerically that the original and the decoupled problem exhibit a striking similarity. We then characterise the capacity p^* of this decoupled model using tools from the statistical physics of disordered systems. Our results clarify the fundamental scaling law governing linear associative memories, and provide a starting point for the analysis of more complex models.