

On the Haline Contribution to Conservative Temperature and Its Implications

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Motivation: OGCM temperature equation

Numerical Ocean General Circulation Models (OGCMs) have traditionally used potential temperature θ as their prognostic temperature variable and assumed it to obey

$$\frac{D\theta}{Dt} = \nabla \cdot (\mathbf{K}_\theta \nabla \theta), \quad F_\theta = \frac{Q_{net}}{c_{p0}}(z = 0)$$

while in reality [Bacon and Fofonoff, 1996]

$$\frac{D\theta}{Dt} = \nabla \cdot (\mathbf{K}_\theta \nabla \theta) + \dot{\theta}_{irr}^{visc} + \dot{\theta}_{irr}^{diff}, \quad F_\theta = \frac{Q_{net}}{c_p}$$

[McDougall, 2003] and TEOS10 argue that the above equation and boundary condition are more accurately satisfied by Conservative Temperature $\Theta = h(\eta, S, p_0)/c_{p0}$

Aim: show that Θ has an overlooked haline conservative term comparable in magnitude as θ non-conservation

What is 'heat'? What is 'heat' non-conservation?

Total Energy = Mechanical Energy + 'Heat'

Energy conservation principle has two equivalent forms:

Conservative form

$$\frac{\partial}{\partial t} \left[\rho \left(E_k + \Sigma - \frac{p}{\rho} \right) \right] + \nabla \cdot [\rho(E_k + \Sigma)\mathbf{v} + \mathbf{J}_k + \mathbf{J}_h] = 0$$

Bernoulli form (Bernoulli = Kinetic energy E_k + static energy Σ)

$$\frac{\partial}{\partial t} [\rho(E_k + \Sigma)] + \nabla \cdot [\rho(E_k + \Sigma)\mathbf{v} + \mathbf{J}_k + \mathbf{J}_h^{(T)} + \mathbf{J}_h^{(S)}] = \frac{\partial p}{\partial t}$$

For inviscid, steady, adiabatic and isohaline motions

$$\frac{D}{Dt}(E_k + \Sigma) = 0$$

Bernoulli theorem

'Heat' as a subcomponent of static energy

$$\Sigma(\eta, S, p, \Phi) = \Sigma_a(\eta, S, p, \Phi) + \Sigma_r(\eta, S)$$

Mechanical Energy Budget

$$\frac{\partial}{\partial t} [\rho(E_k + \Sigma_a)] + \nabla \cdot [\rho(E_k + \Sigma_a)\mathbf{v} + \mathbf{J}_k + \mathbf{J}_a] = \frac{\partial p}{\partial t} - \mathcal{D}_K - \mathcal{D}_a$$

Heat budget

$$\frac{\partial}{\partial t} (\rho \Sigma_r) + \nabla \cdot [\rho \Sigma_r \mathbf{v} + \mathbf{J}_r] = \mathcal{D}_K + \mathcal{D}_a$$

Non-conservation of heat $\mathcal{D} = \mathcal{D}_K + \mathcal{D}_a =$ conversion with mechanical energy (dissipation of mechanical energy into heat)

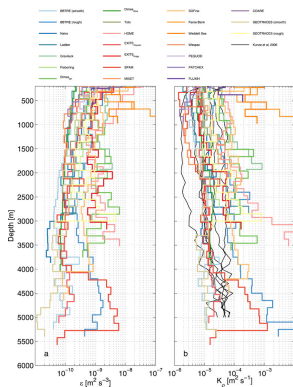
Scaling estimates for 'heat' nonconservation

$$\varepsilon_k \sim 10^{-12} - 10^{-8} \text{ m}^2 \text{s}^{-3}$$

$$\varepsilon_k / c_p \sim$$

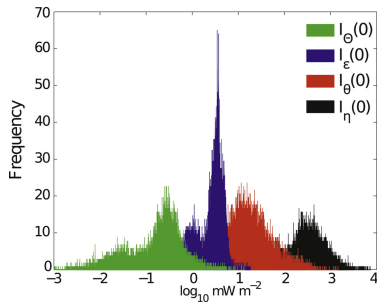
$$2.5 \times 10^{-16} - 10^{-12} \text{ K s}^{-1} \sim$$

$$7.5 \times 10^{-6} - 10^{-2} \text{ K (kyr)}^{-1}$$



[Waterhouse and al, 2014]

Kinetic energy of a fluid parcel traveling at 2 m/s is 2 J/kg. If instantaneously dissipated into heat $\Rightarrow \Delta T = \frac{2}{4000} = \mathbf{0.5 \text{ mK}}$
 $\Rightarrow$ non-conservation of θ
 $\sim O(1 \text{ mK})$ at most.



[Graham and McDougall, 2013]

Error associated with OGCM treatment of heat variables

Writing

$$\mathbf{J}_r = (1 - \Upsilon_T) \mathbf{J}_h^{(T)} + (1 - \Upsilon_S) \mathbf{J}_h^{(S)}$$

Exact equation for heat has the general form

$$\frac{\partial}{\partial t}(\rho \Sigma_r) + \nabla \cdot [\rho \Sigma_r \mathbf{v} + \mathbf{J}_h^{(T)}] = \mathcal{R}$$

$$\mathcal{R} = \nabla \cdot [\Upsilon_T \mathbf{J}_h^{(T)} + (\Upsilon_S - 1) \mathbf{J}_h^{(S)}] + \mathcal{D}_k + \mathcal{D}_a$$

Term neglected by numerical OGCMs includes both conservative and non-conservative terms.

Published assessments concluding that $\mathcal{R}$ is smaller for potential enthalpy than for $c_{p0}\theta$ overlook the presence of the haline term. Does it matter?

Surface 'heat' flux

Local budget equation for $\Sigma_r(\eta, S)$ is

$$\frac{D\Sigma_r}{Dt} = \underbrace{\frac{\partial \Sigma_r}{\partial \eta}}_{T_r} \frac{D\eta}{Dt} + \underbrace{\frac{\partial \Sigma_r}{\partial S}}_{\mu_r} \frac{DS}{Dt} \quad (1)$$

This implies for the surface flux of Σ_r :

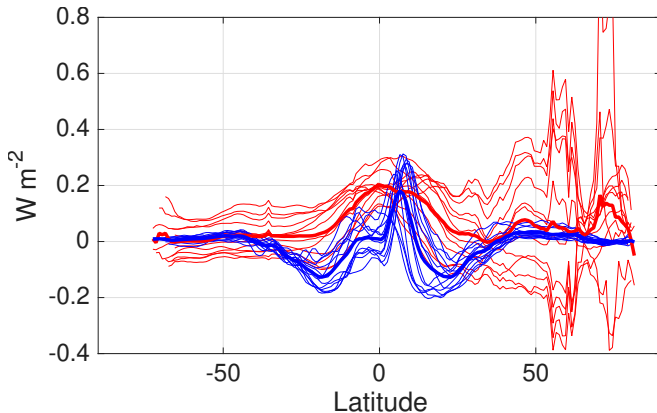
$$\begin{aligned} \mathcal{J}_r &= \frac{T_r}{T} Q_{net} + \left(\mu_r - T_r \frac{\partial \mu}{\partial T} \right) \rho_f S (E - P) \\ &= Q_{net} + \underbrace{\left(\frac{T_r - T}{T} \right) Q_{net} + \left(\mu_r - T_r \frac{\partial \mu}{\partial T} \right) \rho_f S (E - P)}_{\text{Surface flux error}} \end{aligned}$$

Surface flux errors for $c_{p0}\theta$ and potential enthalpy:

$$E_\theta = \left(\frac{c_{p0} - c_p}{c_p} \right) Q_{net}, \quad E_\Theta = \left(\mu - T \frac{\partial \mu}{\partial T} \right) \rho_f S (E - P)$$

Comparison of surface flux errors for θ and Θ

- ▶ NOCS1.1 seasonal heat and freshwater fluxes
- ▶ Error for $c_{p0}\Theta$ versus Error for $c_{p0}\theta$

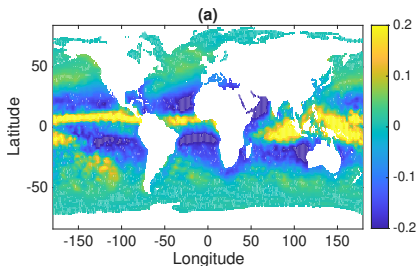


Errors comparable in magnitude!

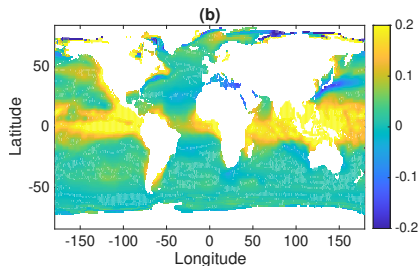
Comparison of surface flux errors (mW m^{-2})

NOCS1.1 heat and freshwater fluxes, time mean

$$E_{\theta} = \left(\frac{c_{p0} - c_p}{c_p} \right) Q_{net} \quad \text{versus} \quad E_{\Theta} = \left(\mu - T \frac{\partial \mu}{\partial T} \right) \rho_f S (E - P)$$



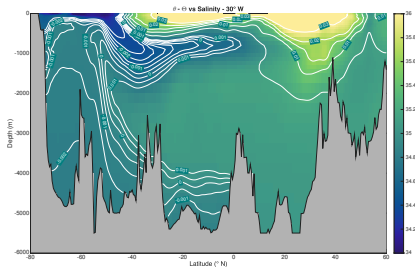
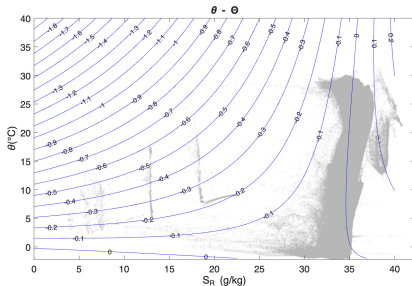
Conservative Temperature



Potential Temperature

Implications for the interpretation of $\theta - \Theta$

Widely promoted as "illustrating the non-conservative production of potential temperature at the surface"
[Graham and McDougall, 2013] and TEOS10 literature. However, non-conservation of heat should rarely exceed 0.1mK .



$\theta - \Theta$ is in fact primarily dominated by the conservative haline term, as confirmed by the Atlantic 30°W section.

Consequences of haline contamination of Θ

- ▶ Differential diffusion
- ▶ Uncertainties associated with initial conditions

Differential Diffusion

Turbulent mixing coefficients K_θ and K_S for heat and salt do not need to be equal $\Rightarrow$ depends on turbulent regime

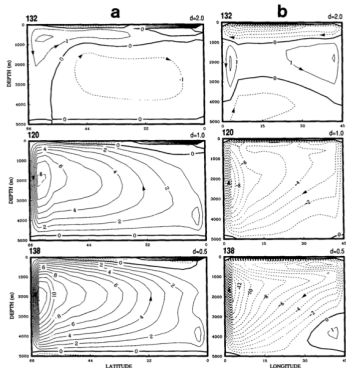


FIG. 3. (a) Meridional and (b) zonal overturning streamfunctions for experiments with $\bar{\tau} = 1$ and varying d ($\bar{\tau}$). Upper panels: run 132, $d = 2$; middle panels: "standard" run 120, $d = 1$; lower panels: run 138, $d = 0.5$. Contour interval (c.l.) is 1 Sv ; solid contours denote counterclockwise flow.

Figure 1: AMOC sensitivity to $d = K_S/K_T$
[Gargett and Holloway, 1992]

Estimated possible ranges for $d = K_S/K_T$ as a function of Reynolds buoyancy number

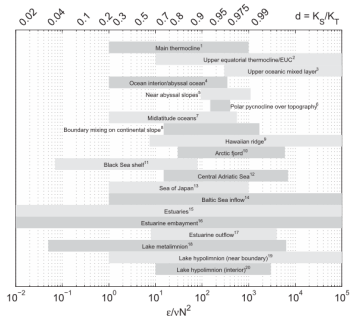


Figure 2:
[Jackson and Rehmann, 2009]

Differential diffusion

If θ and S diffuse differently

$$\frac{D\theta}{Dt} = \nabla \cdot (\mathbf{K}_\theta \nabla \theta) + \dot{\theta}_{irr}, \quad \frac{DS}{Dt} = \nabla \cdot (\mathbf{K}_S \nabla S)$$

the implication for Conservative Temperature $\Theta = \Theta(\theta, S)$ is

$$\begin{aligned} \frac{D\Theta}{Dt} &= \frac{\partial \Theta}{\partial \theta} \left(\nabla \cdot (\mathbf{K}_\theta \nabla \theta) + \dot{\theta}_{irr} \right) + \frac{\partial \Theta}{\partial S} \nabla \cdot (\mathbf{K}_S \nabla S) \\ &= \nabla \cdot \left(\frac{\partial \Theta}{\partial \theta} \mathbf{K}_\theta \nabla \theta + \frac{\partial \Theta}{\partial S} \mathbf{K}_S \nabla S \right) \neq \nabla \cdot (\mathbf{K}_\Theta \nabla \Theta) \end{aligned}$$

Θ can not simply be substituted for θ if $\mathbf{K}_\theta$ distinct from $\mathbf{K}_S$: Turbulent flux of Θ no longer function of Θ alone $\Rightarrow$ Special treatment!

Digression: Accounting for the non-conservation of θ

Non-conservation of θ making Θ exactly conservative

$$\begin{aligned}\dot{\theta}_{irr} &= (\mathbf{K}_\theta \nabla \theta) \cdot \nabla \frac{\partial \Theta}{\partial \theta} + (\mathbf{K}_S \nabla S) \cdot \nabla \frac{\partial \Theta}{\partial S} = \frac{\partial^2 \Theta}{\partial \theta^2} (\mathbf{K}_\theta \nabla \theta) \cdot \nabla \theta \\ &+ \frac{\partial^2 \Theta}{\partial S \partial \theta} [(\mathbf{K}_\theta \nabla \theta) \cdot \nabla S + (\mathbf{K}_S \nabla S) \cdot \nabla \theta] + \frac{\partial^2 \Theta}{\partial S^2} (\mathbf{K}_S \nabla S) \cdot \nabla S\end{aligned}$$

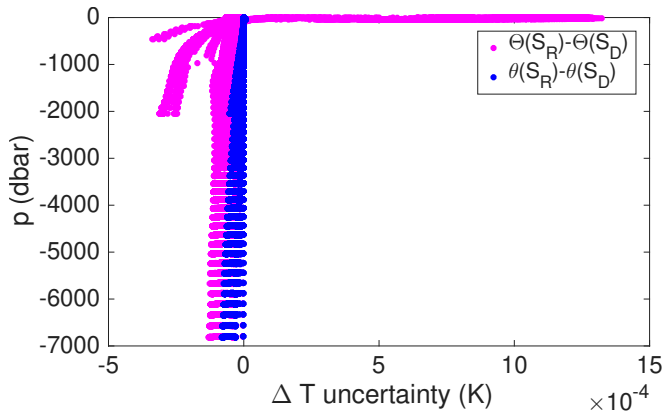
Depend on dissipations of temperature and salinity variances, as well as co-dissipation of θ and $S \Rightarrow$ Useful diagnostics of mixing

Accounting for non-conservation of θ in OGCMs would be straightforward if one wanted to empirically ascertain its importance, while also providing a useful discriminating metric of mixing
[Graham and McDougall, 2013, Tailleux, 2015]

Uncertainties in initial conditions

$$\delta\theta = \theta(T, S_R, p) - \theta(T, S_A, p), \quad \delta\Theta = \Theta(T, S_R, p) - \Theta(T, S_A, p)$$

S_R = reference composition salinity, $S_A = S_D$ = Absolute Salinity (or Density Salinity), accounts for spatial variations in composition



Summary and Conclusions

- ▶ Θ is more nearly conservative than θ , but it has a stronger salinity dependence
- ▶ A consistent Θ heat budget requires the additional haline term and its corresponding boundary flux.
- ▶ Without these terms, replacing θ with Θ does not, by itself, produce a more accurate heat-budget equation
- ▶ Even after correction, Θ couples thermal and haline uncertainties more strongly, with possible consequences for modelling and data assimilation.
- ▶ Thermal and haline errors are significantly more correlated with Θ than with θ : Implications for Data Assimilation?

The estimated effects are generally small; their practical importance therefore requires quantitative, application-specific assessment.

Implications

Choice of variables

- ▶ Θ is not universally preferable to θ ; the appropriate variable depends on the scientific objective.
- ▶ Use θ when separating thermal and haline effects is the main priority. Use Θ when its improved conservation properties are more important, while retaining all required source and boundary terms.

Modelling and data assimilation

- ▶ Assess whether the stronger coupling of thermal and haline errors affects parameter estimation, model initialisation, or data assimilation.

Community actions

- ▶ Consider adding θ -based routines in the GSW library.
- ▶ Review the relevant TEOS-10 documentation to make the additional haline term and associated boundary flux explicit.

References I



Bacon, S. and Fofonoff, N. (1996).
Oceanic heat flux calculation.
J. Atm. Ocean. Tech., 13:1327–1329.



Gargett, A. E. and Holloway, G. (1992).
Sensitivity of the gfdl ocean model to different diffusivities for heat and salt.
J. Phys. Oceanogr., 22:1158–1177.



Graham, F. and McDougall, T. J. (2013).
Quantifying the nonconservative production of conservative temperature, potential temperature and entropy.
J. Phys. Oceanogr., 43:838–862.



Jackson, P. R. and Rehmann, C. R. (2009).
Theory for differential transport of scalars in sheared stratified turbulence.
J. Fluid Mech., 621:1–21.



McDougall, T. J. (2003).
Potential enthalpy: a conservative oceanic variable for evaluating heat content and heat fluxes.
J. Phys. Oceanogr., 33:945–963.



Tailleux, R. (2015).
Observational and energetic constraints on the non-conservation of potential/conservative temperature and implications for ocean modelling.
Ocean Modell., 88:26–37.



Waterhouse, A. F. and al (2014).
Global patterns of diapycnal mixing from measurements of the turbulent dissipation rate.
J. Phys. Oceanogr., 44:1854–1872.