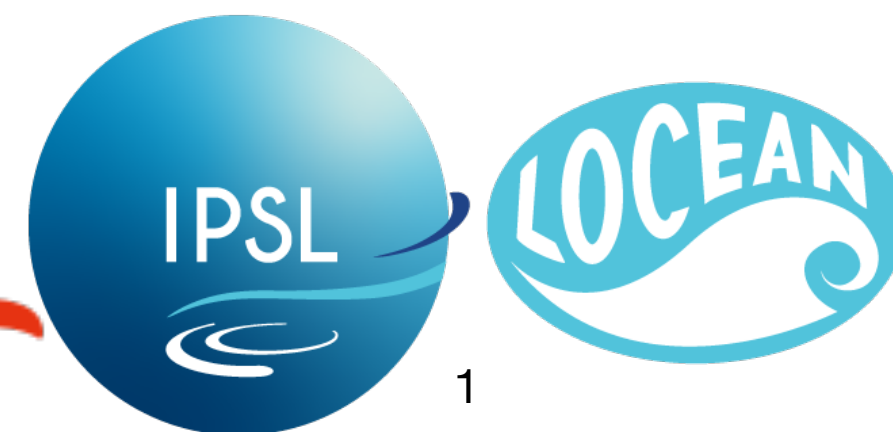
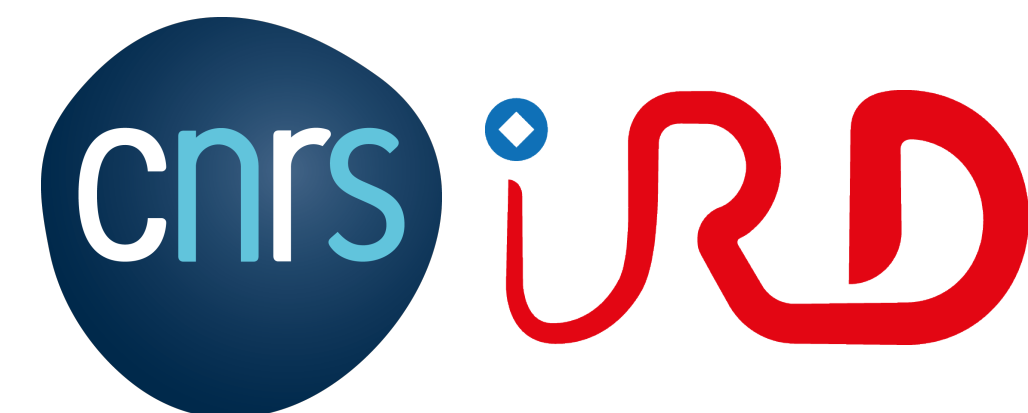


Porous Cavities and Smooth Boundaries: Brinkman Volume Penalization for Ice-Shelf–Ocean Interactions in Ocean General Circulation Models

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COMMODORE Workshop

Tuesday 22, September 2026
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- 1) Introduction
 - 2) Overview of the Brinkman Volume Penalization (BVP) method
 - 3) Benefit of the BVP for the ice-shelf melt rates produced in Z models
- General Conclusions and Perspectives

Topography is a major constraint for the world ocean circulation

- Refers to the land/ocean frontier
 - coastlines
 - slopes of the ocean floor
 - straits, passages, sills
 - mid-ocean ridges
 - seamounts, ...
- Marine currents tend to follow f/h contours

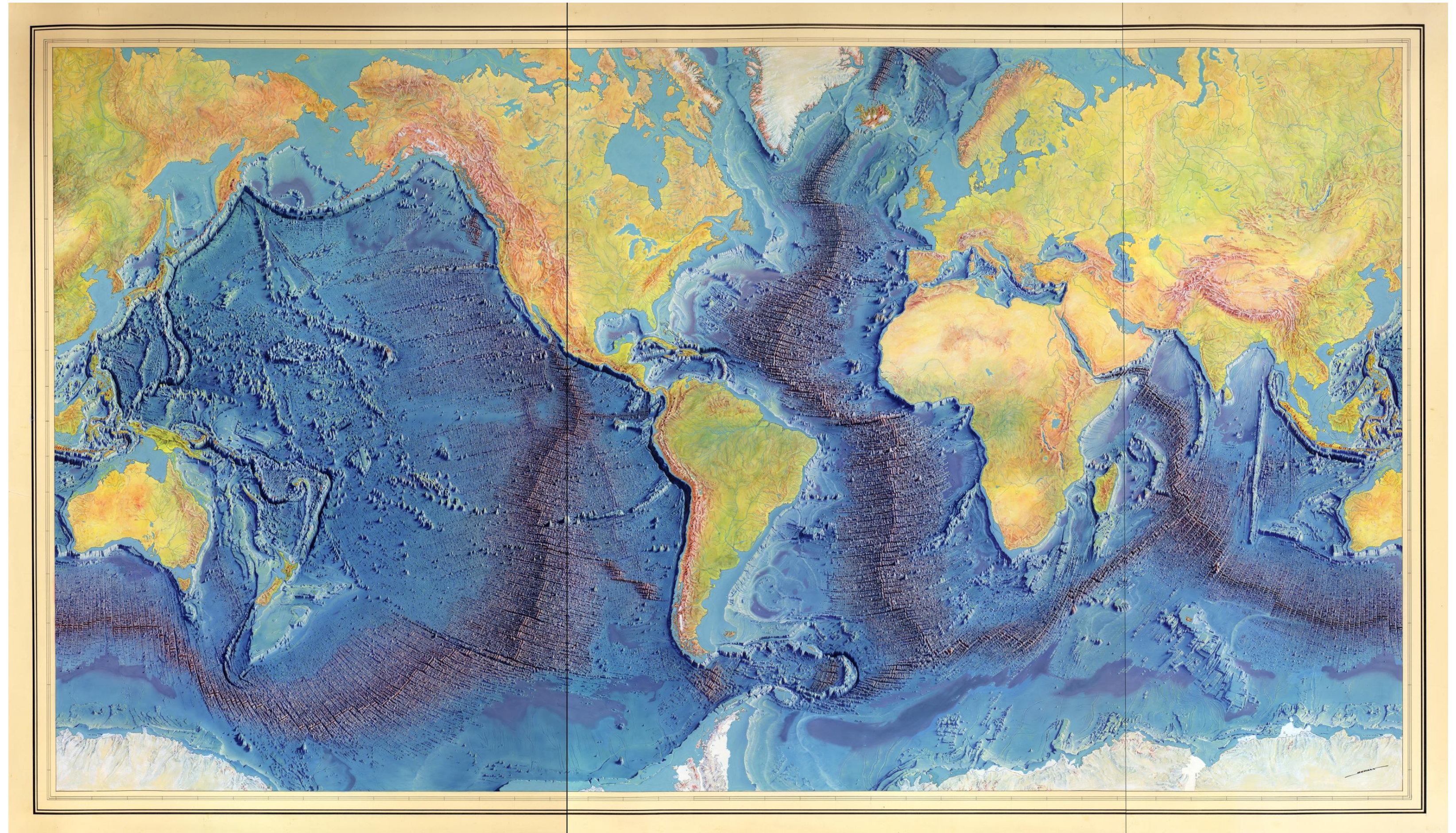


Figure : "World Ocean Floor" map.
From (Berann et al., 1977)

Overflows are critical steps for the general ocean circulation

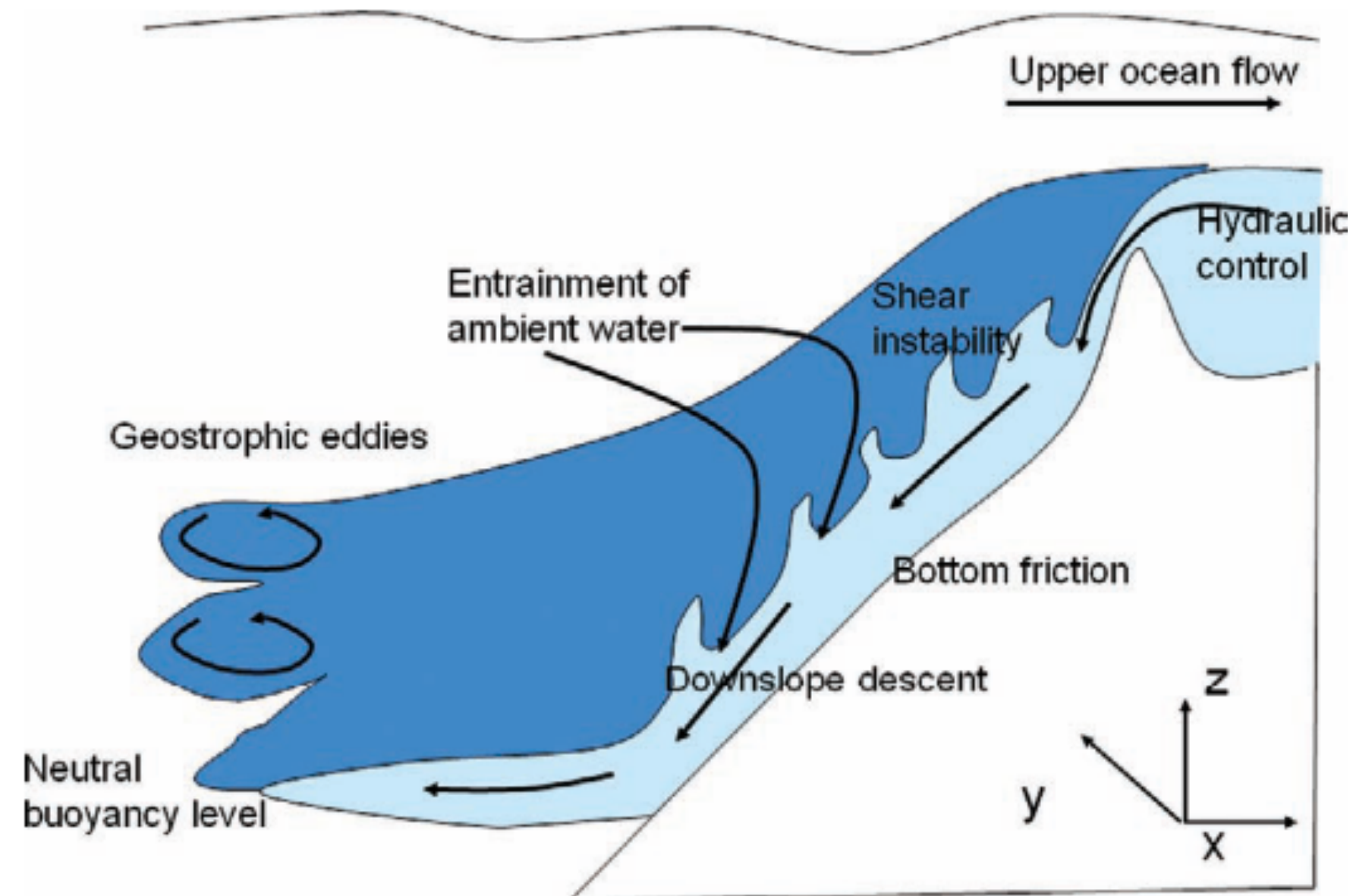


Figure : Physical processes taking place in overflows.

From Legg et al. (2009).

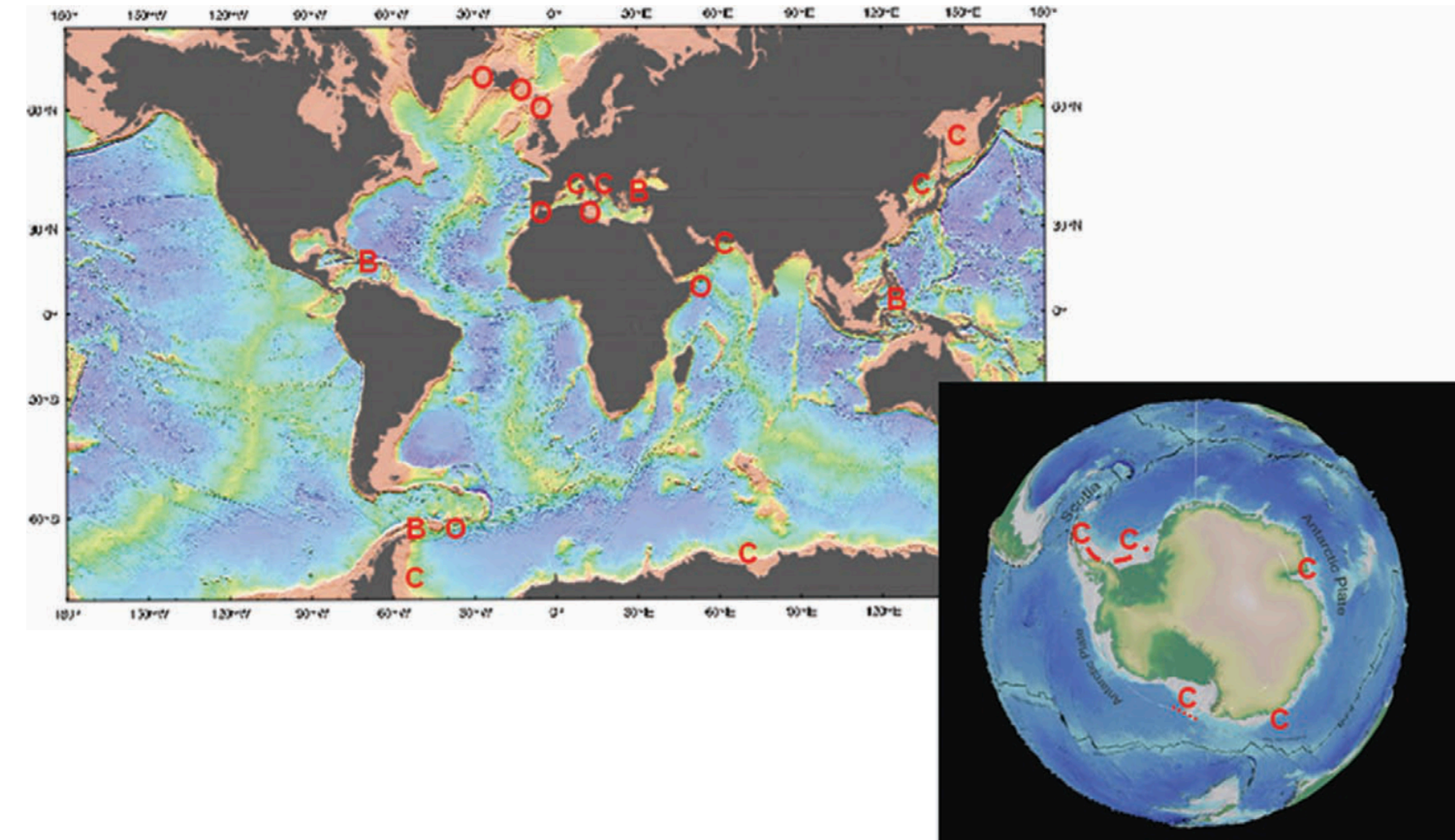


Figure: Location of major overflows across the ocean.

From Legg et al. (2009).

Modelling the melting of the Antarctic Ice Sheet to predict future sea level rise

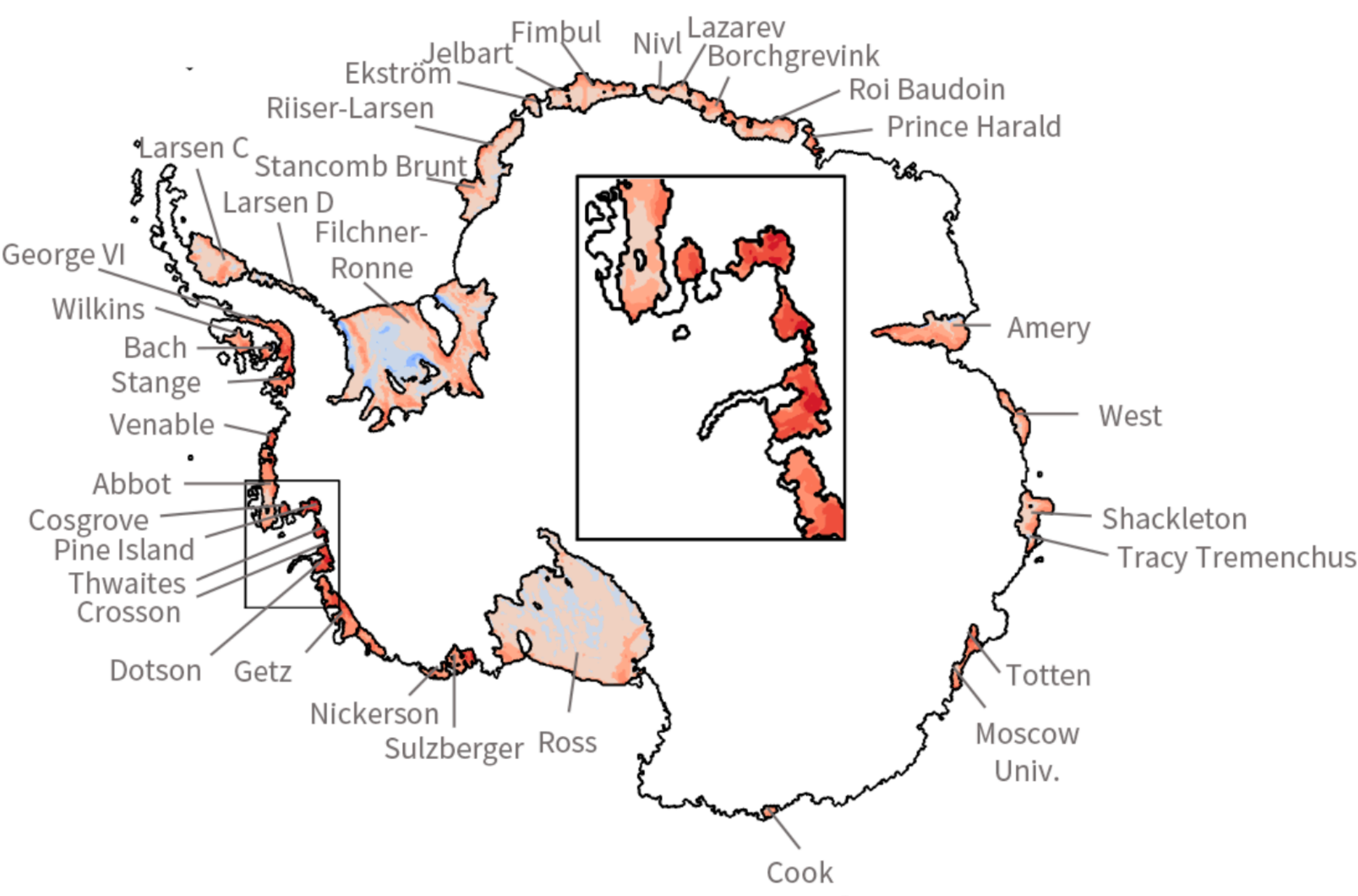


Figure: Spatial distribution of sub-shelf melt rates
From (Burgard et al. 2022)



Figure: Picture of the Filchner-Rone ice shelf.
From DPA

Ice shelf cavity geometry significantly influences basal melting patterns

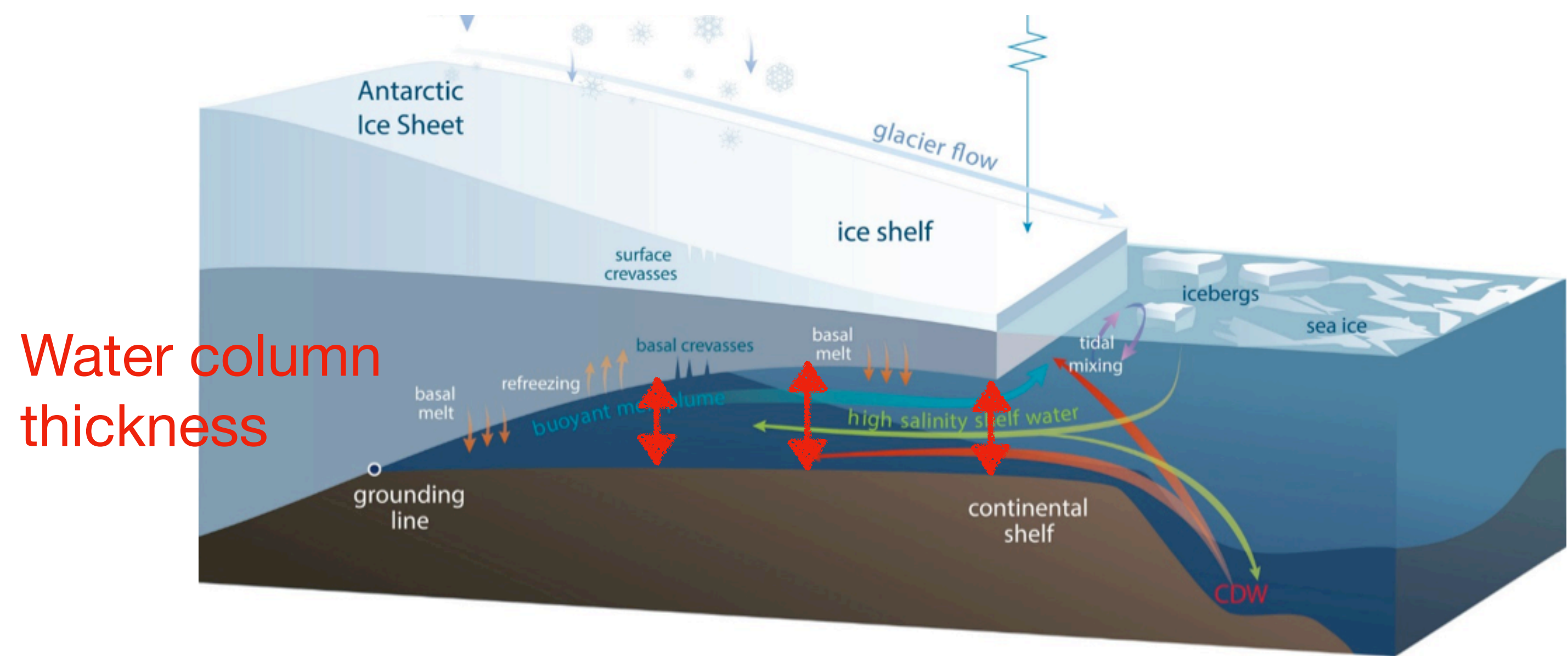


Figure: Schematic diagram of an Antarctic ice shelf.
Credits: Helen Amanda Fricker

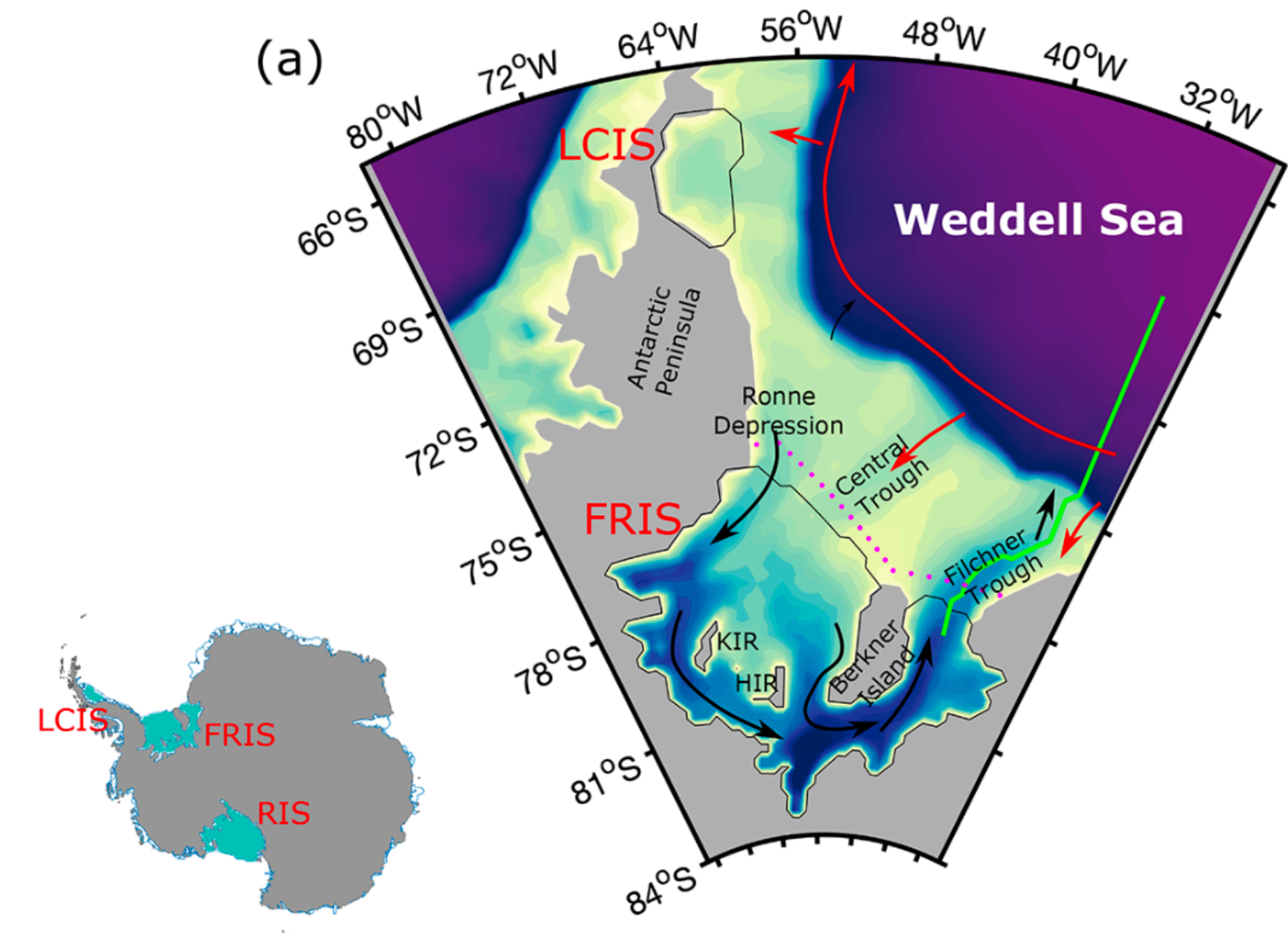


Figure: Water column thickness under ice cavities
in the Weddell sea and Ross sea.
Adapted from (Hutchinson et al. 2023)

S-coordinate: a natural framework for representing topographic flows

s-coordinate

« Terrain-following approach »

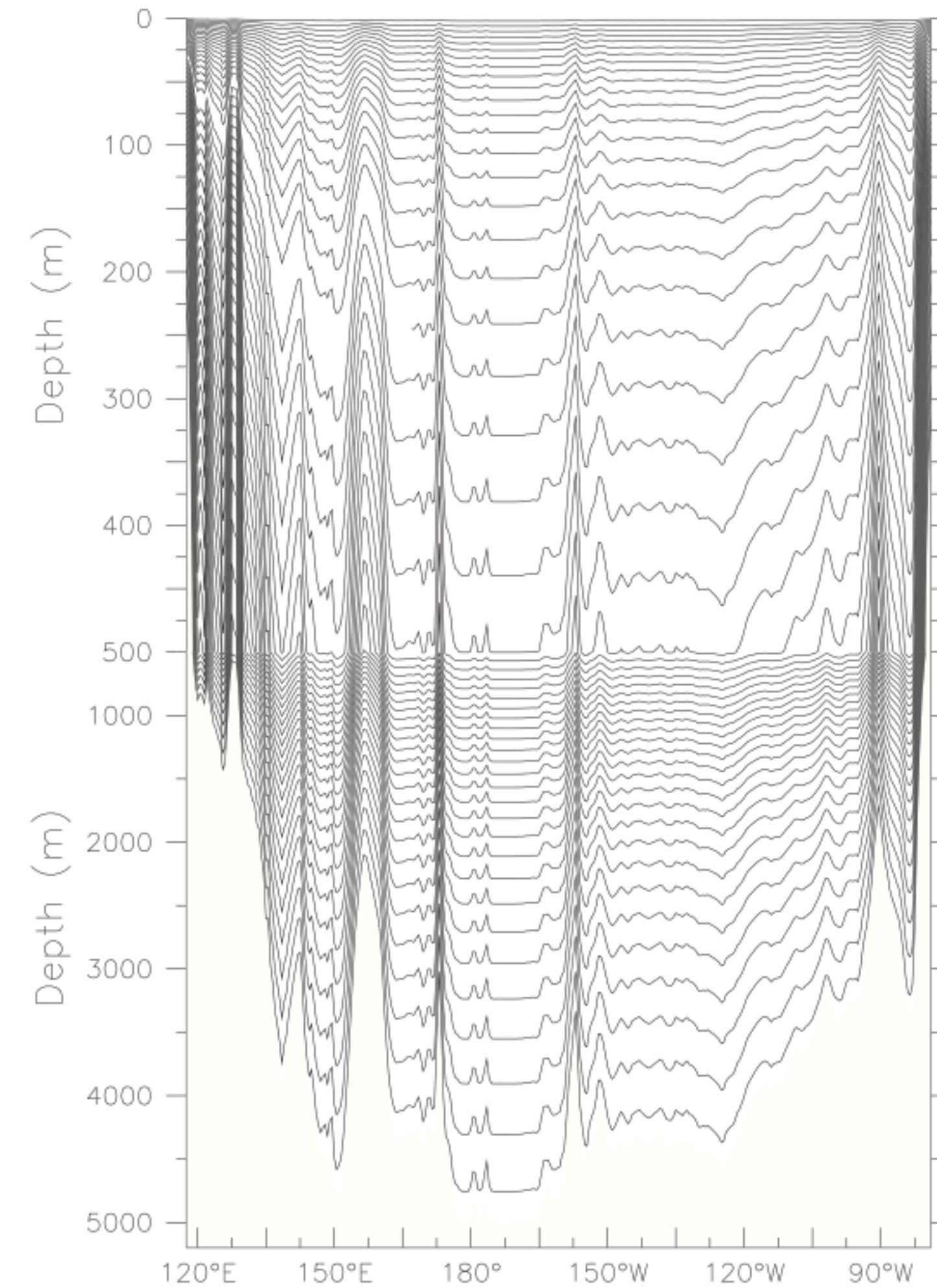
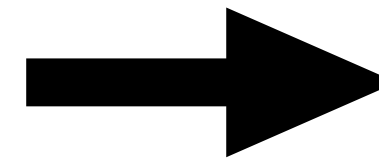
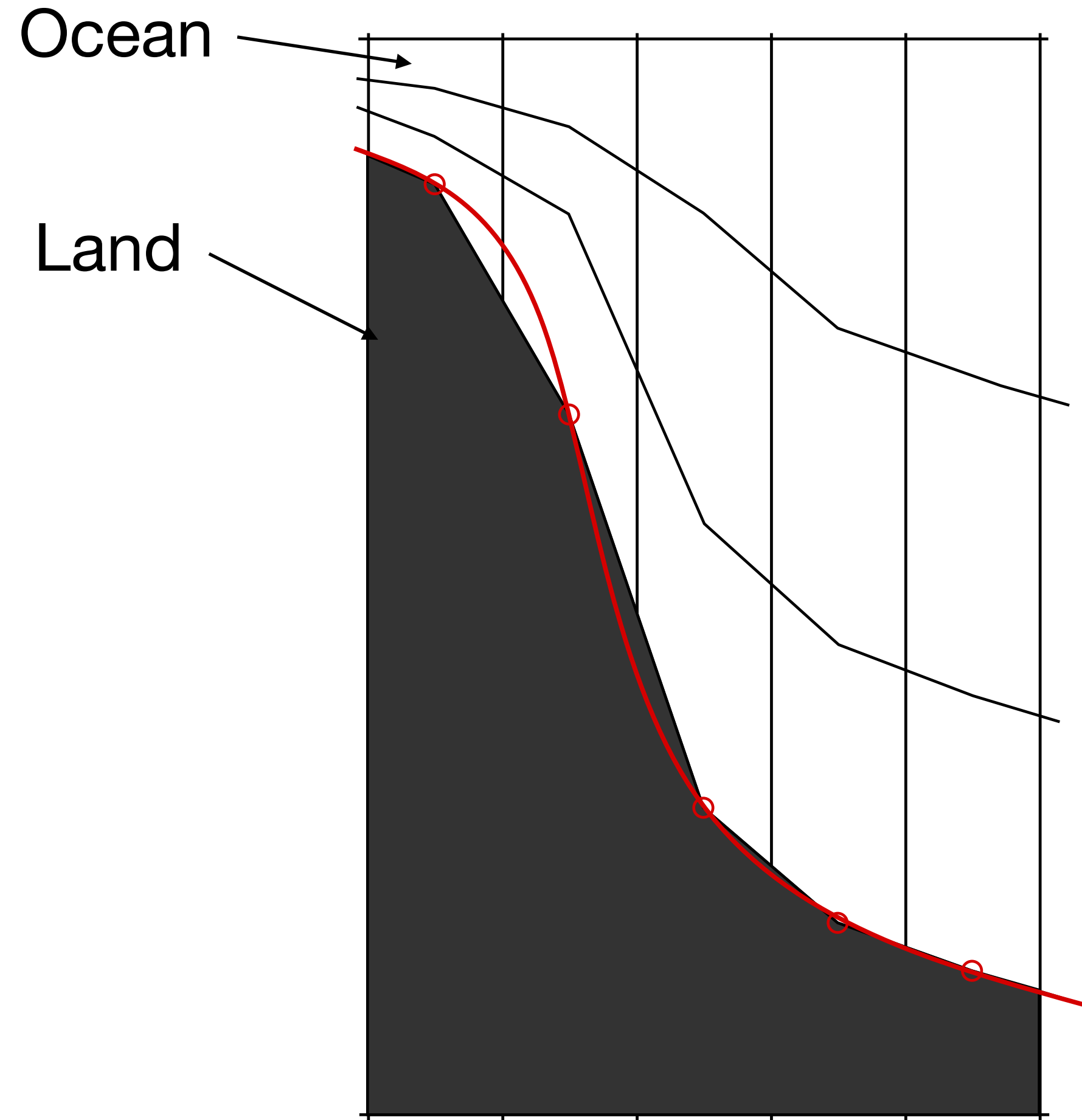


Figure: Model σ levels

From Lemarié et al. (2012).

Terrain-following topographies suffer from HPG errors at steep slopes

s-coordinate

« Terrain-following approach »

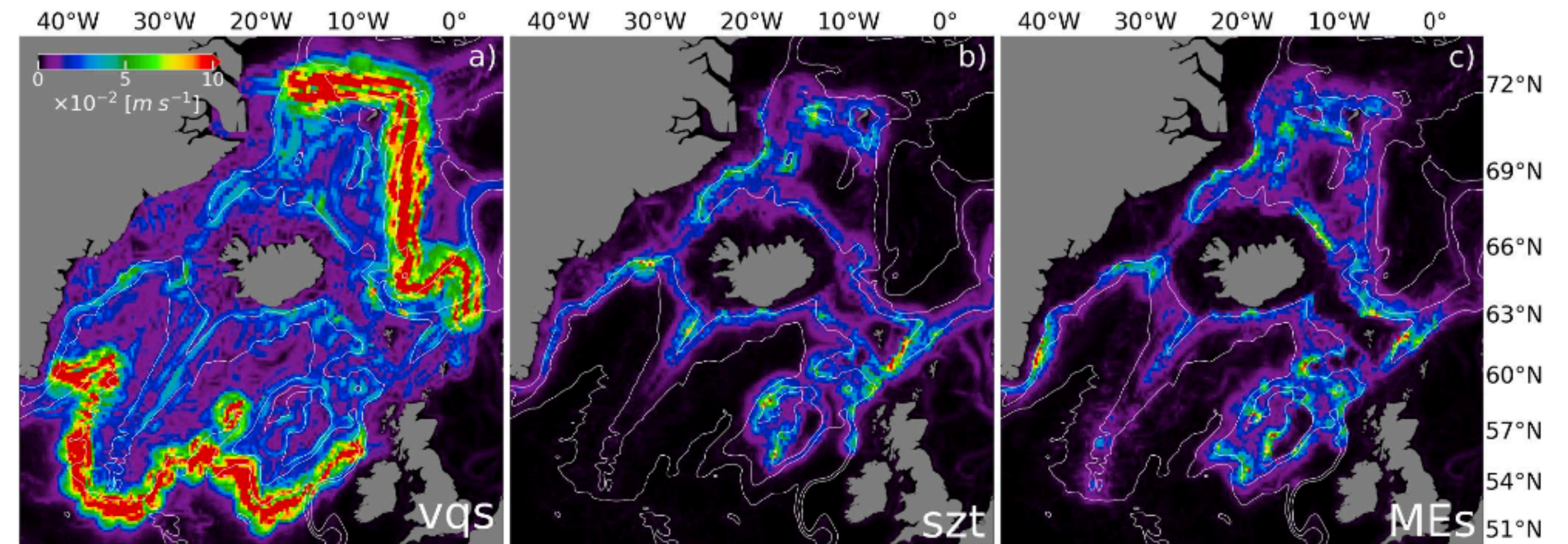
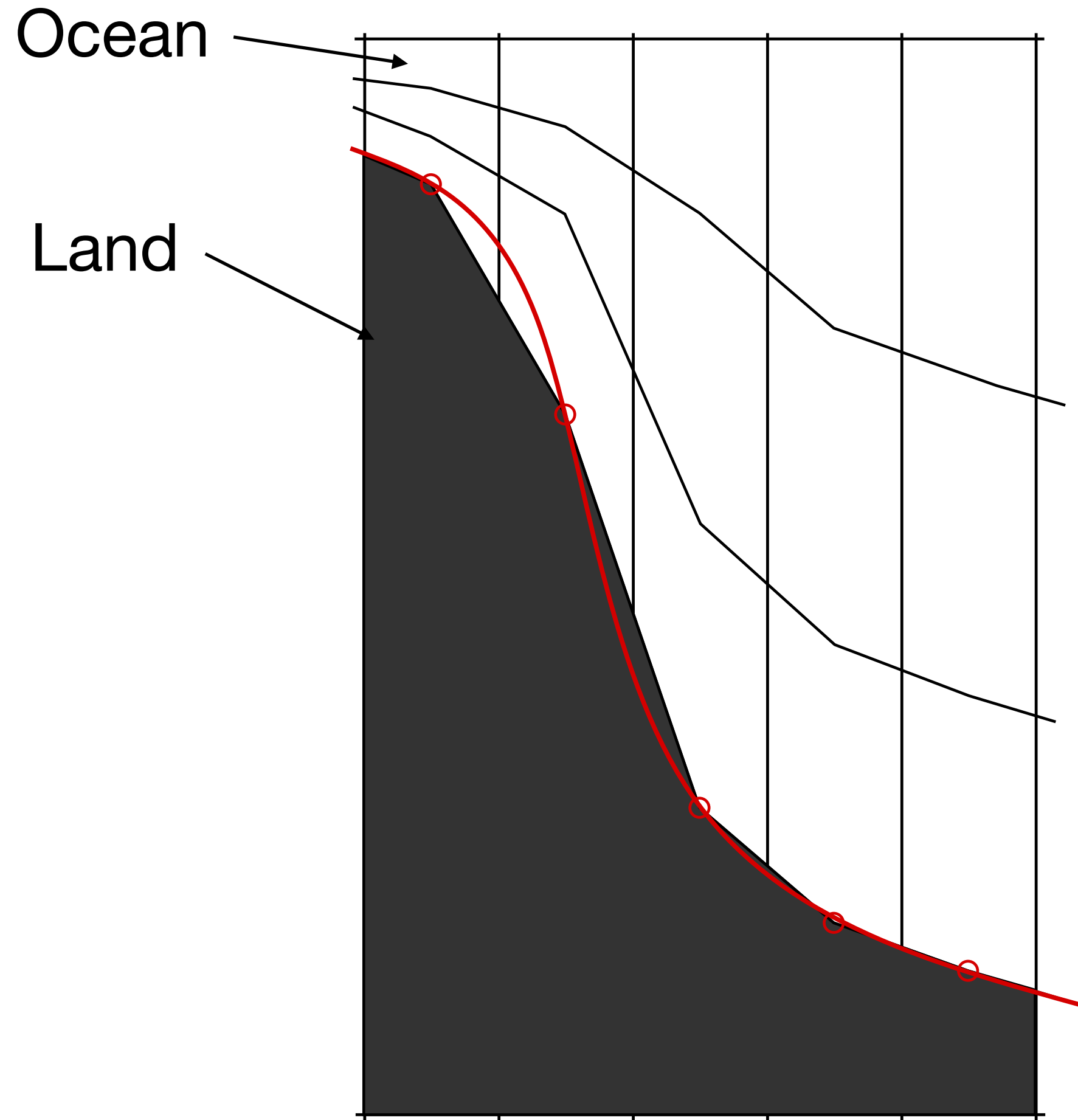


Figure: Map of spurious currents after 90 days.
From Bruciaferri et al. (2024).

Z-models produce « lego-like » or « step-like » topographies

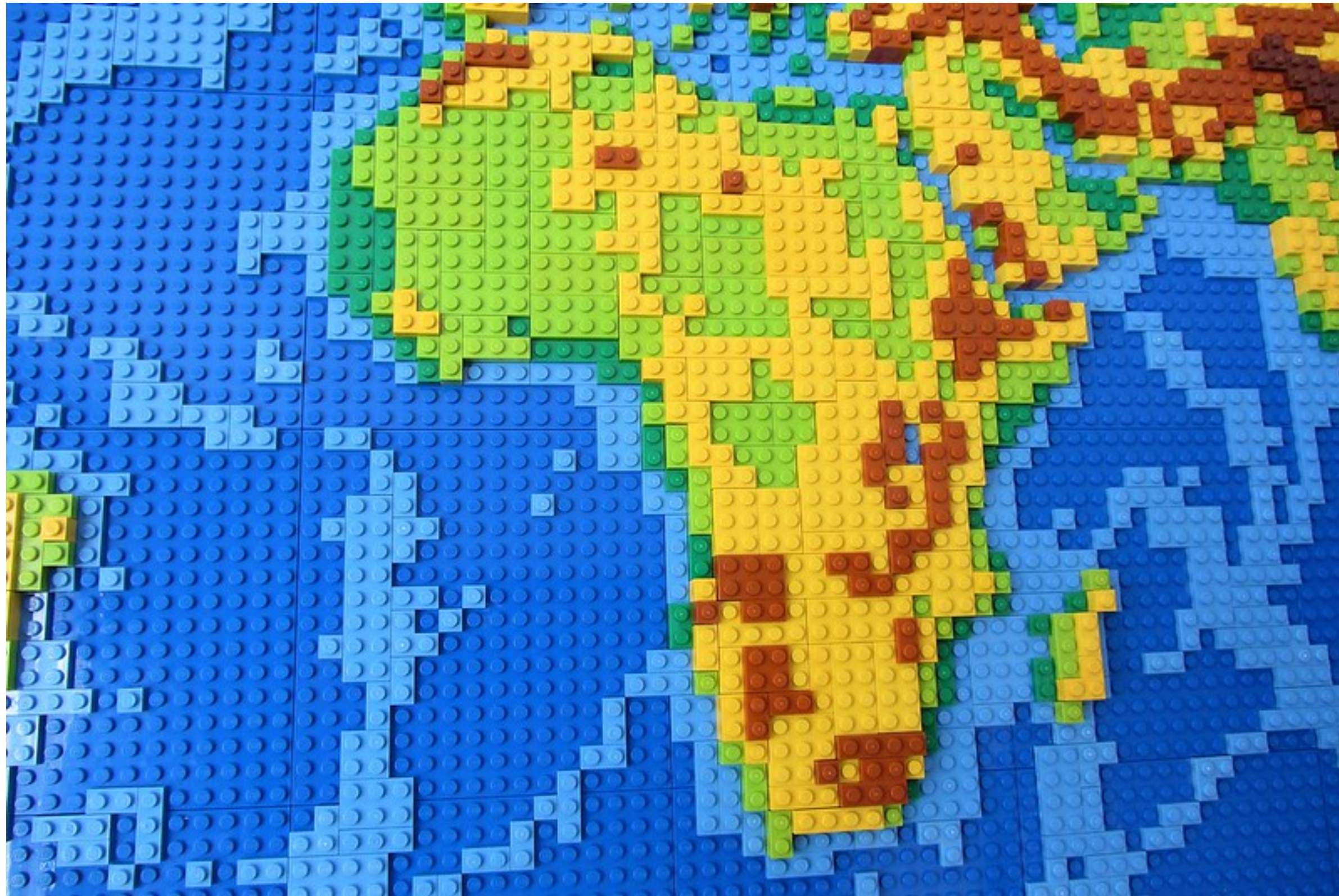


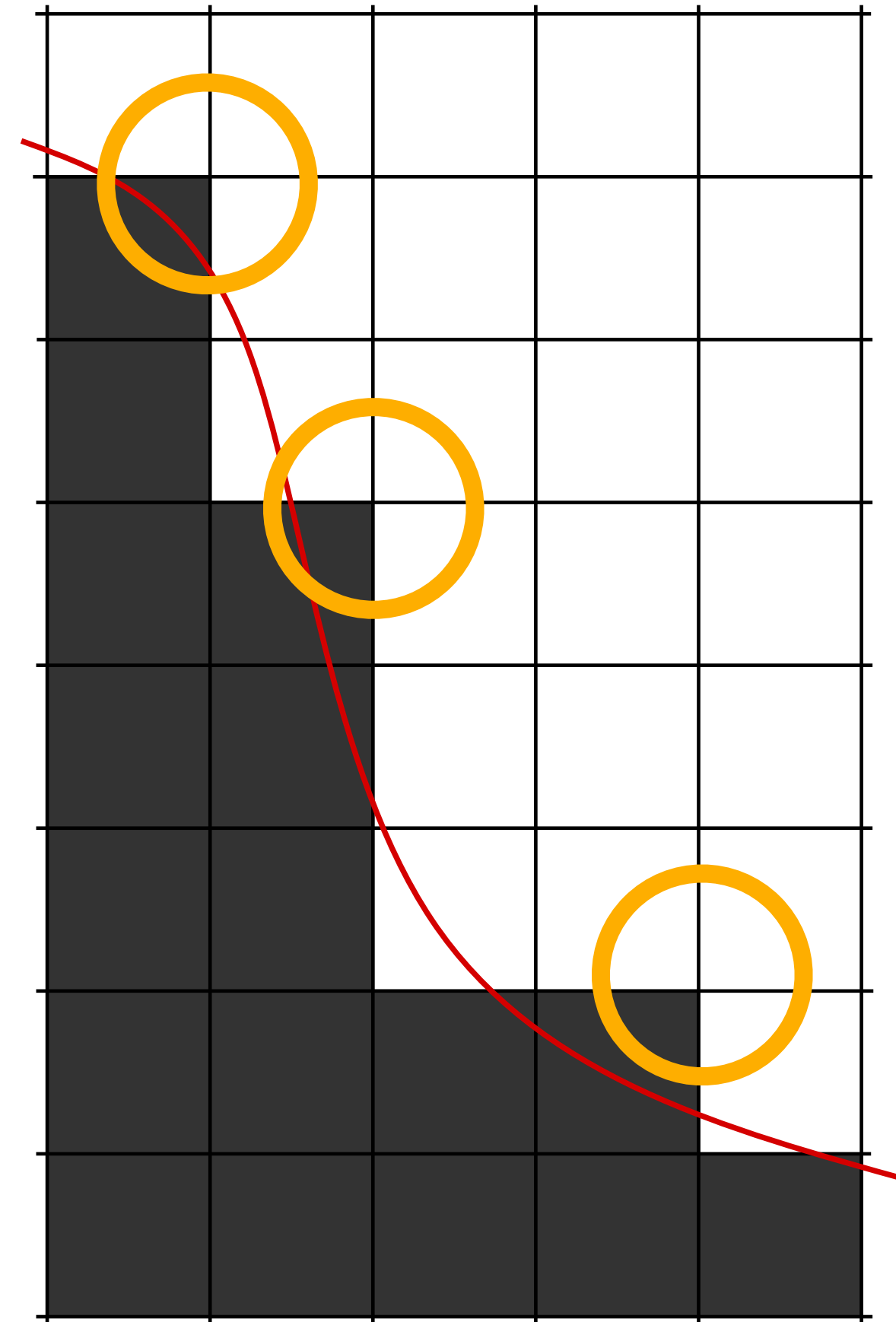
Figure : Lego map of Africa
By dirkb86



Figure : River Valley - Minecraft

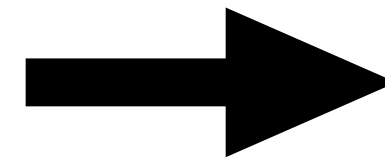
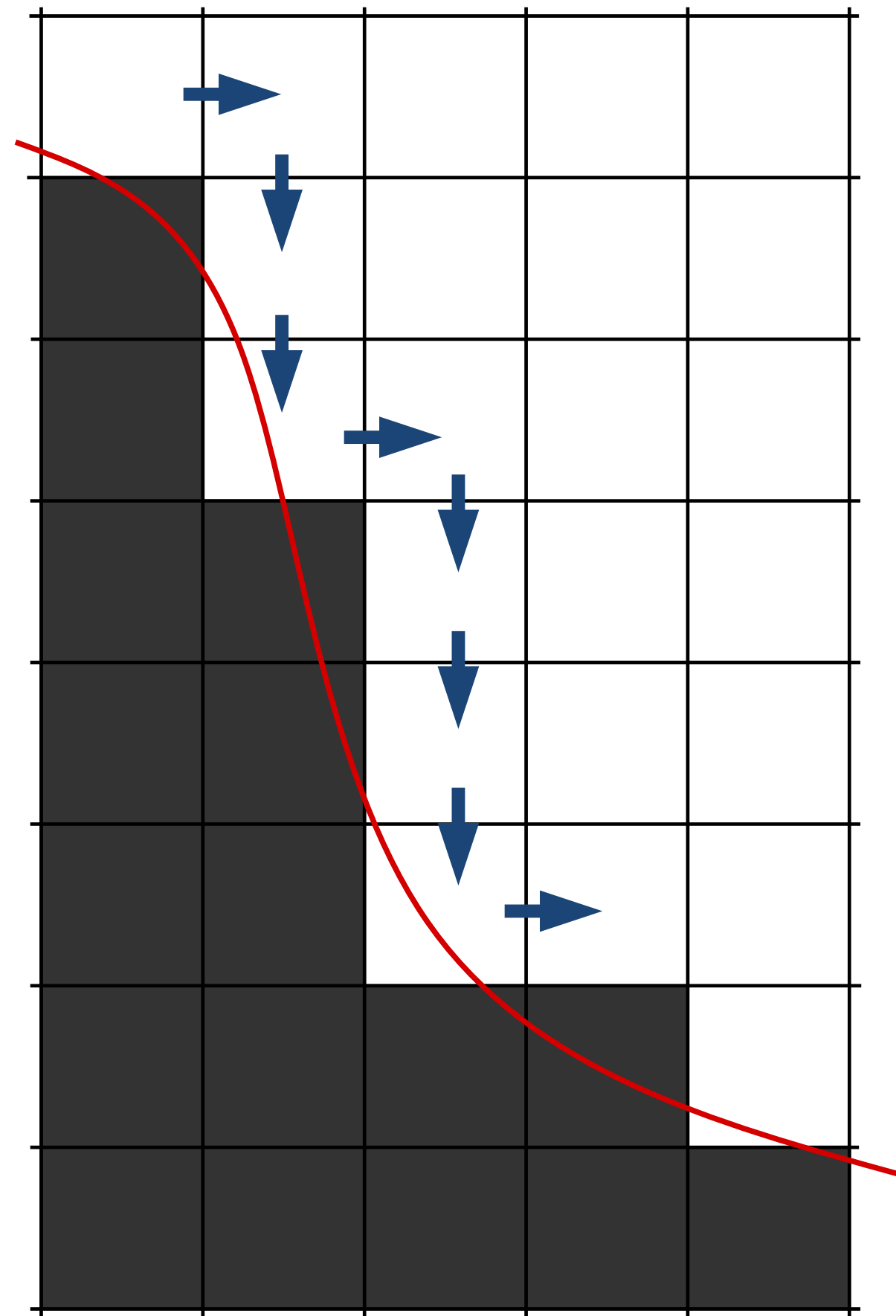
Z-coordinate models face a number of recurring challenges

Finding solutions to mitigate the adverse effects step-like topographies on flows is a **recurrent challenge**.

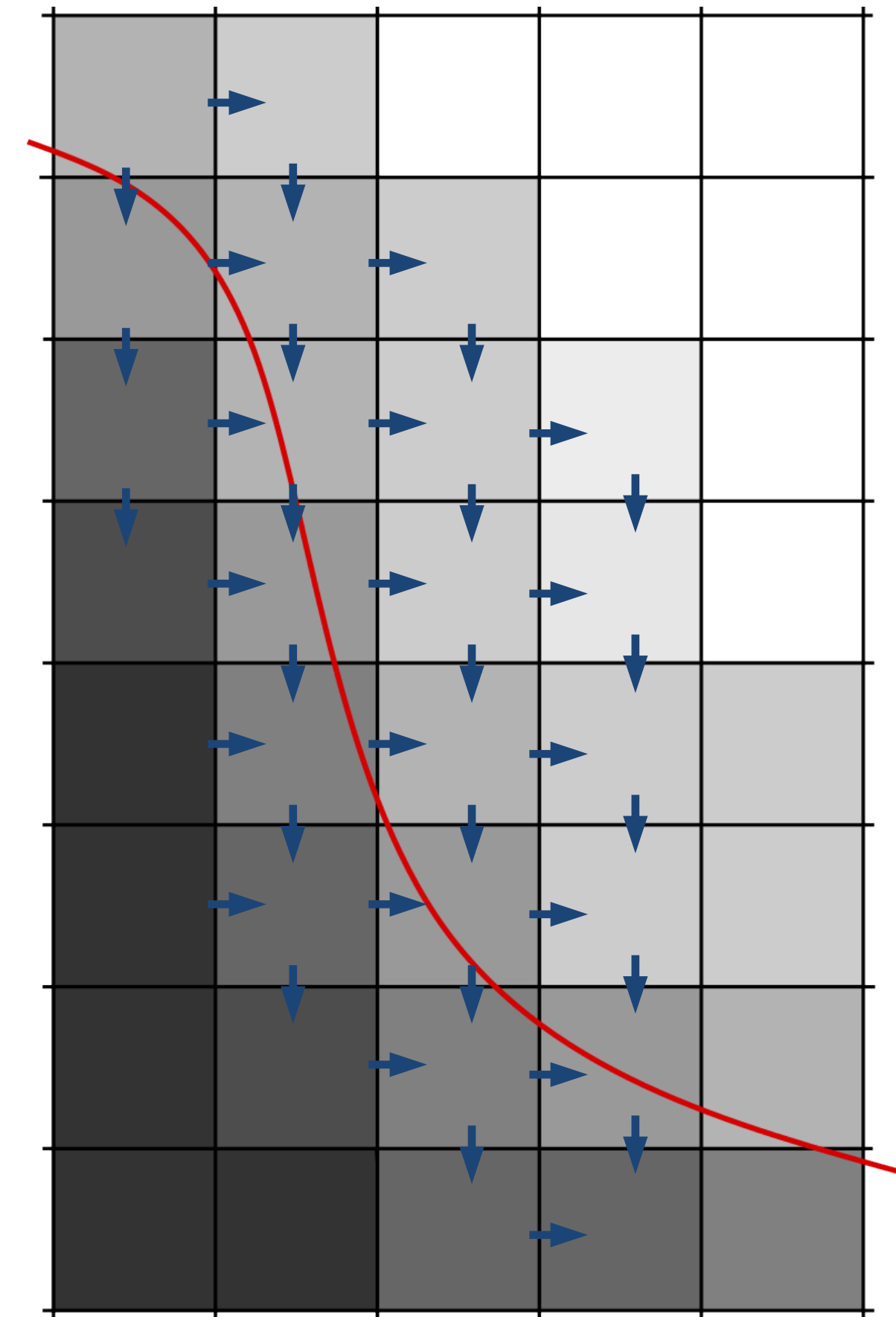


Exploring the potential of the Brinkman Volume Penalisation method

Traditional 'lego-like'
topography



Penalised
topography



Kevlahan et al. (2015),
Debreu et al. (2020, 2022), Nasser (2023)

Overview of the Brinkman Volume Penalisation (BVP) method

- Half-land / half-ocean medium

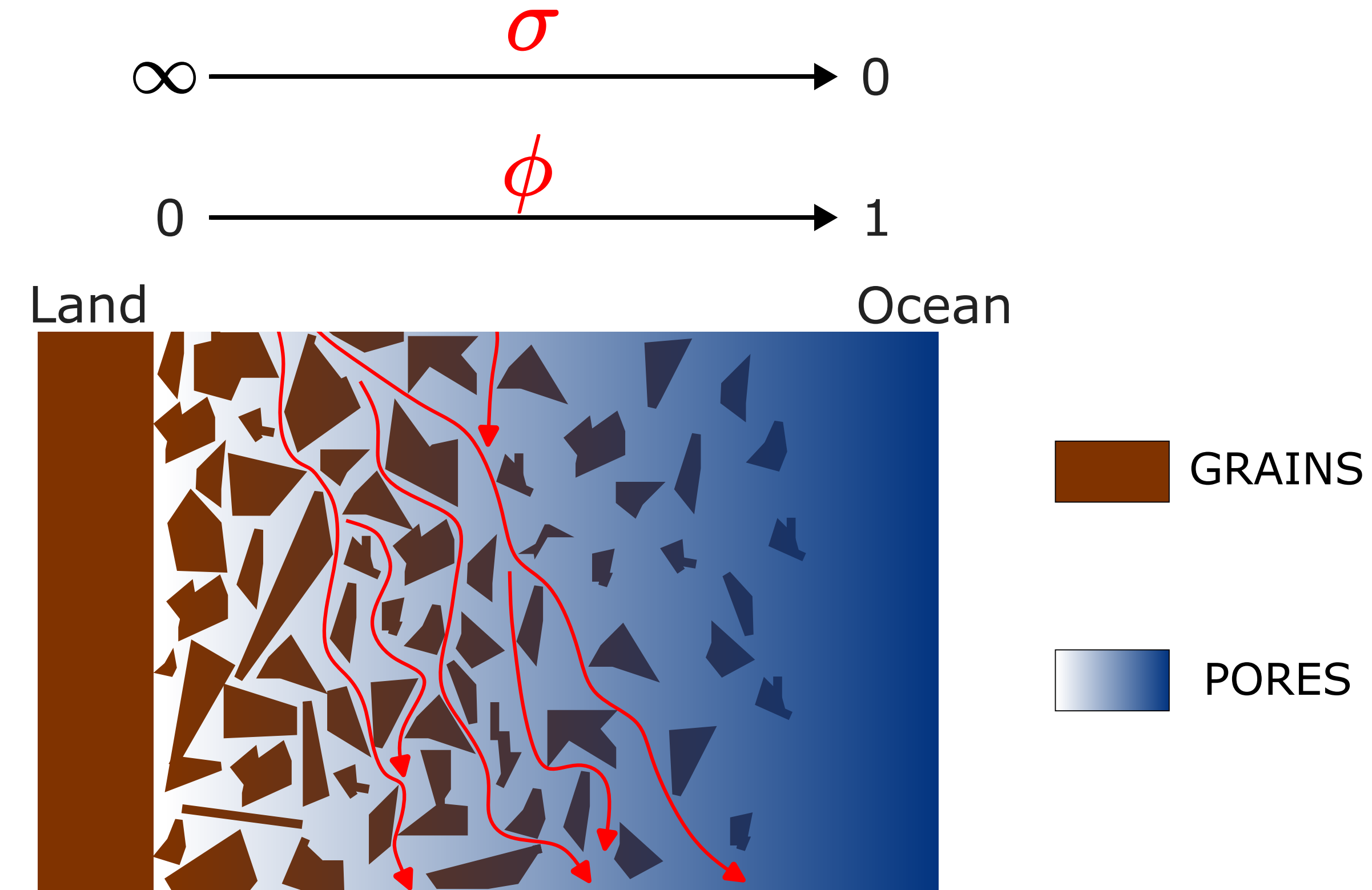
ϕ • Porosity (liquid portion)

➔ Modifies the effective volume

σ • Friction (drag in porous region)

➔ Decelerates the flow

- Light developments in existing code !



$$\partial_t \phi \mathbf{U}_h + \nabla \cdot (\phi \mathbf{U} \otimes \mathbf{u}_h) = \frac{\phi h}{\rho_0} \nabla P - \sigma \phi \mathbf{U}_h$$

$$\frac{1}{h} \partial_t (\phi h \rho) + \nabla \cdot (\phi \mathbf{U} \rho) = 0$$

$$\nabla \cdot \phi \mathbf{U} = 0$$

$$\partial_z P = -\rho g$$

Improved Gulf Stream separation through Brinkman penalization

Traditional
s-coordinate

s-coordinate
with BVP on top

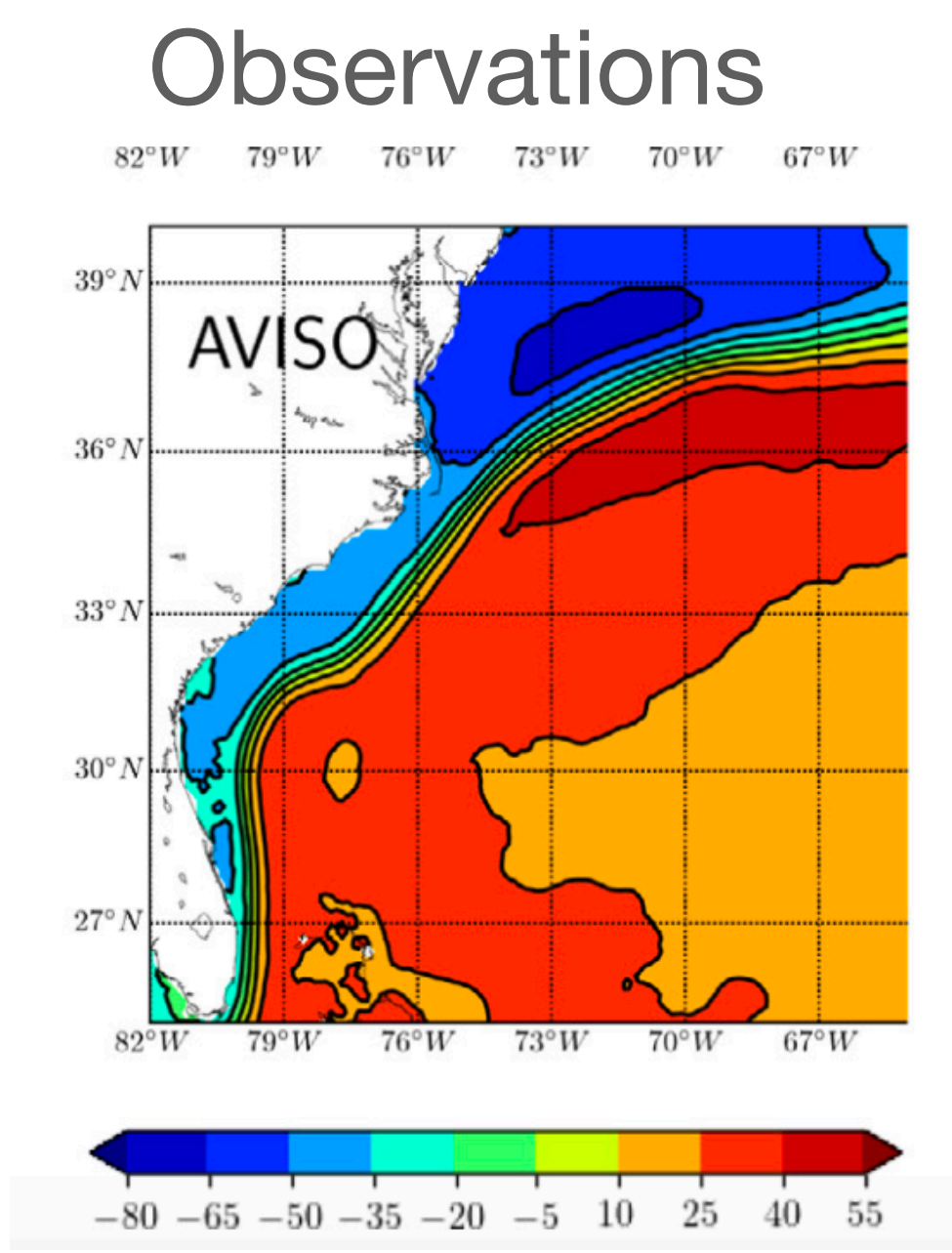
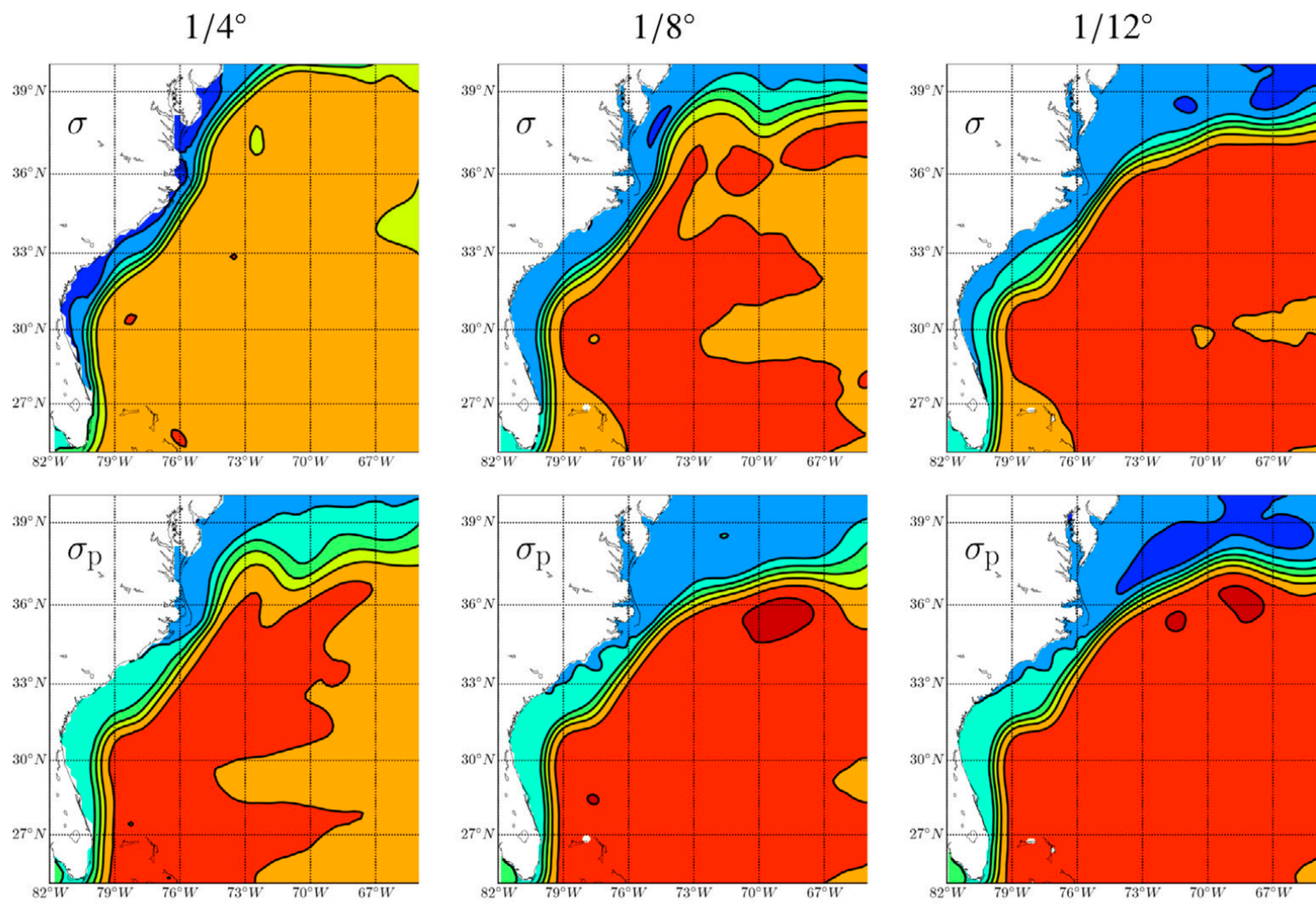


Figure: SSH in CROCO simulations of the Gulf Stream
From Debreu et al. (2022).

Overflows suffer from excessive mixing over step-like topographies

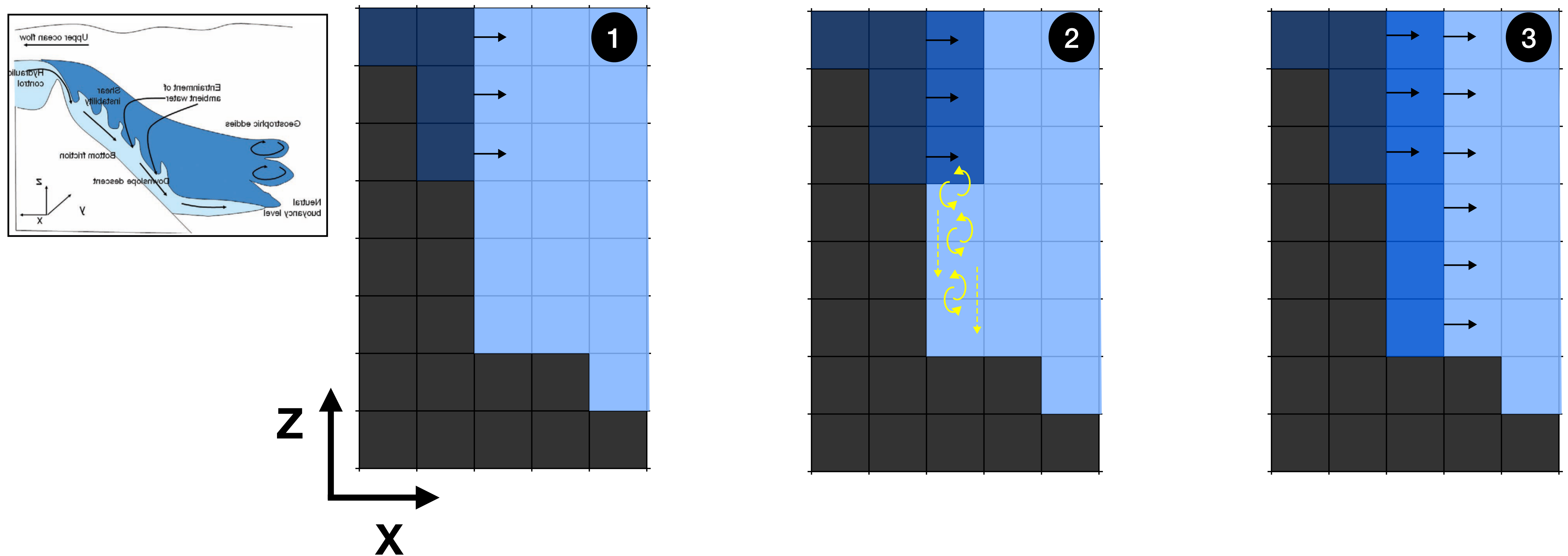


Figure: Illustration of the vertical mixing occurring during a downflow of dense waters along a step-like topography.

A s-coordinate provides a natural representation of bottom currents

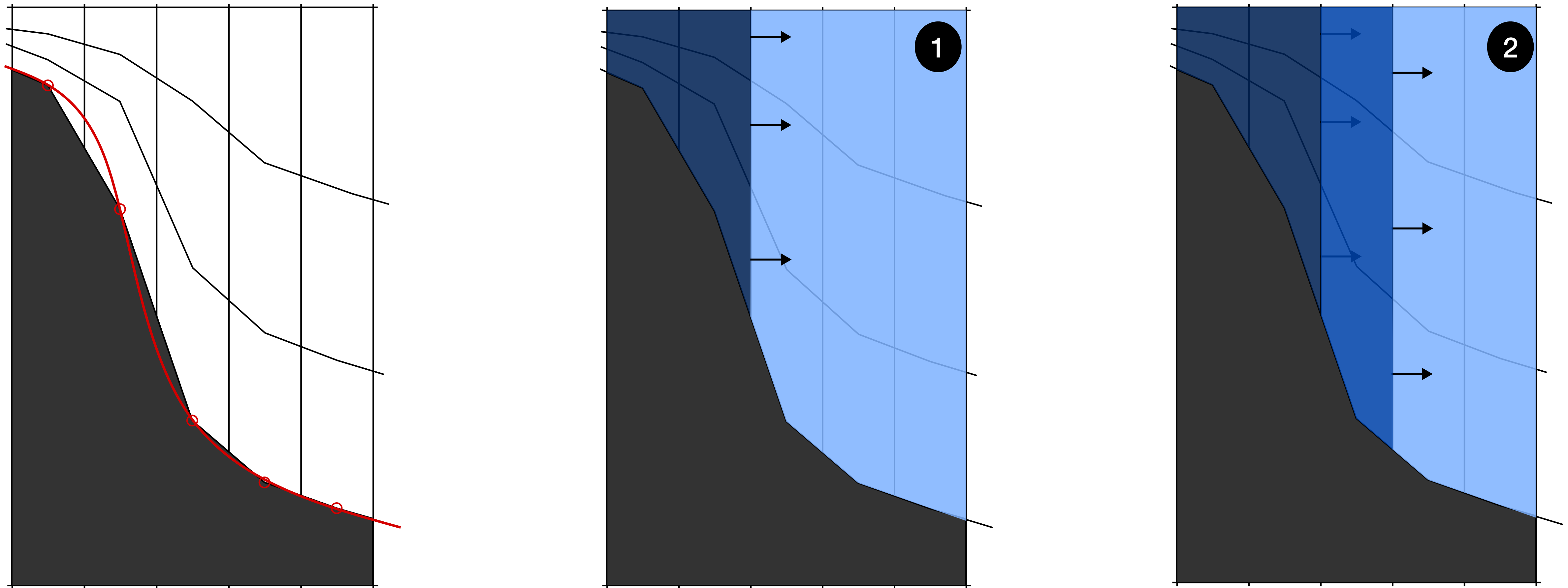
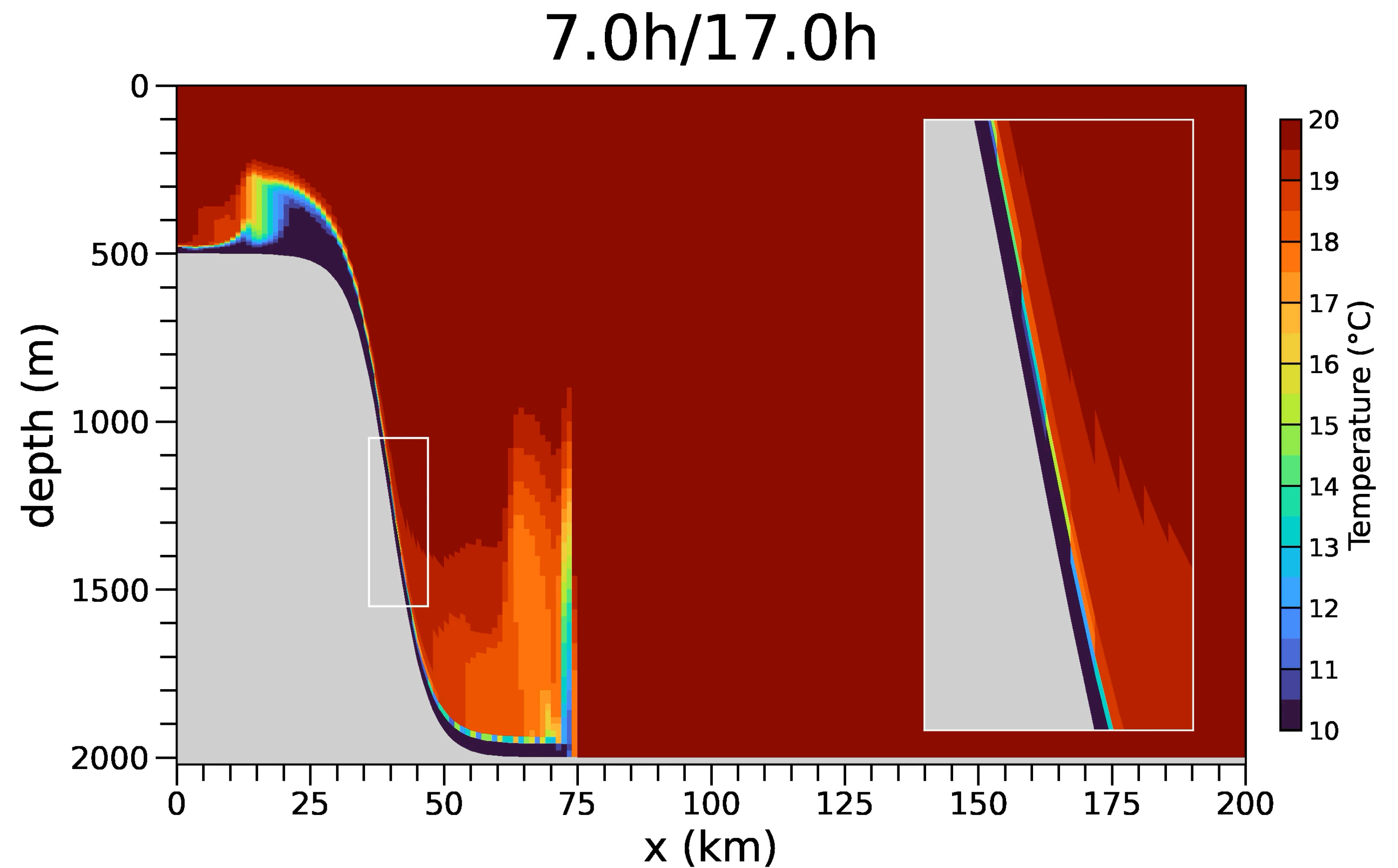
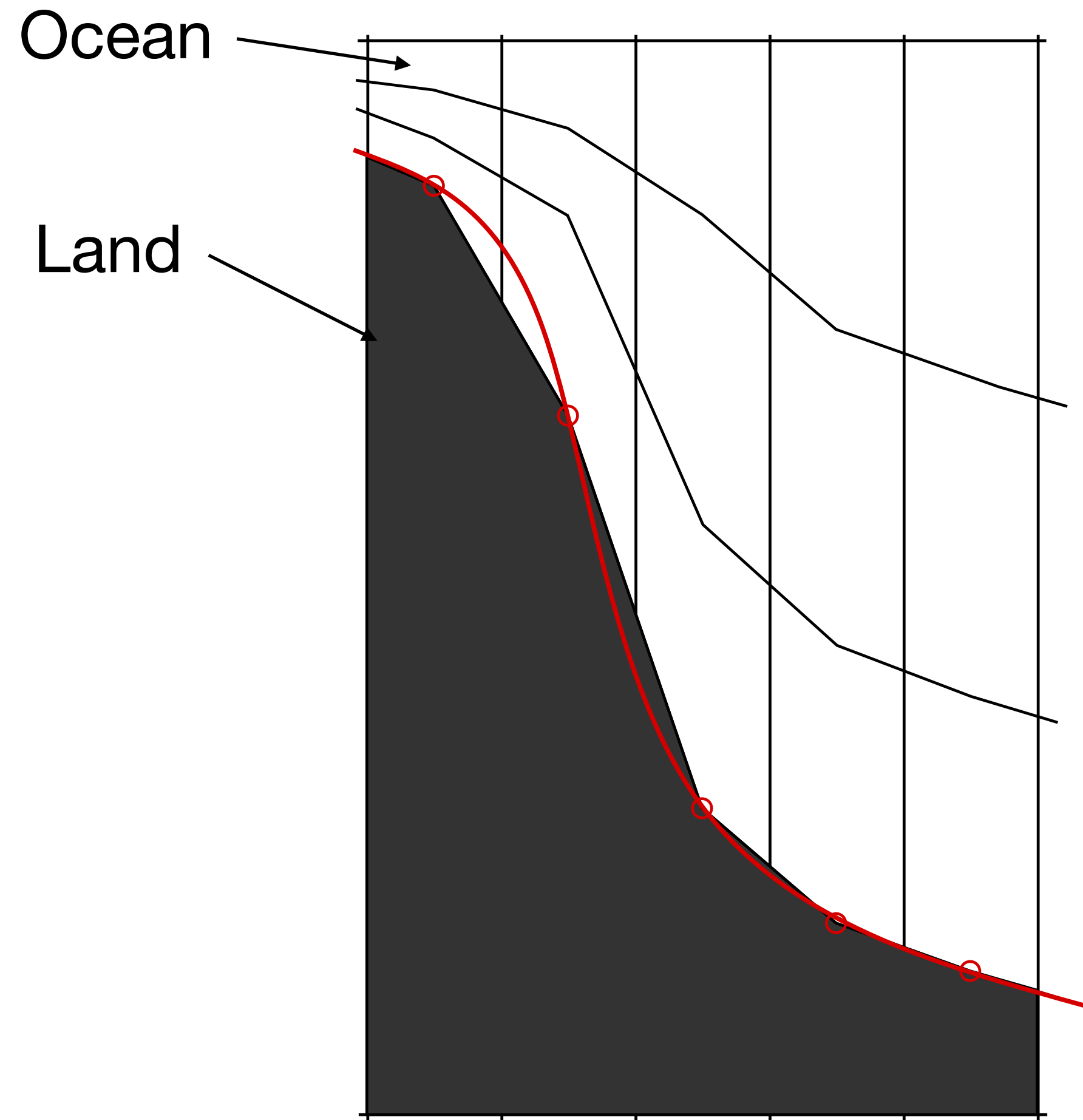


Figure: Illustration of the downflow of dense waters in s-coordinate.

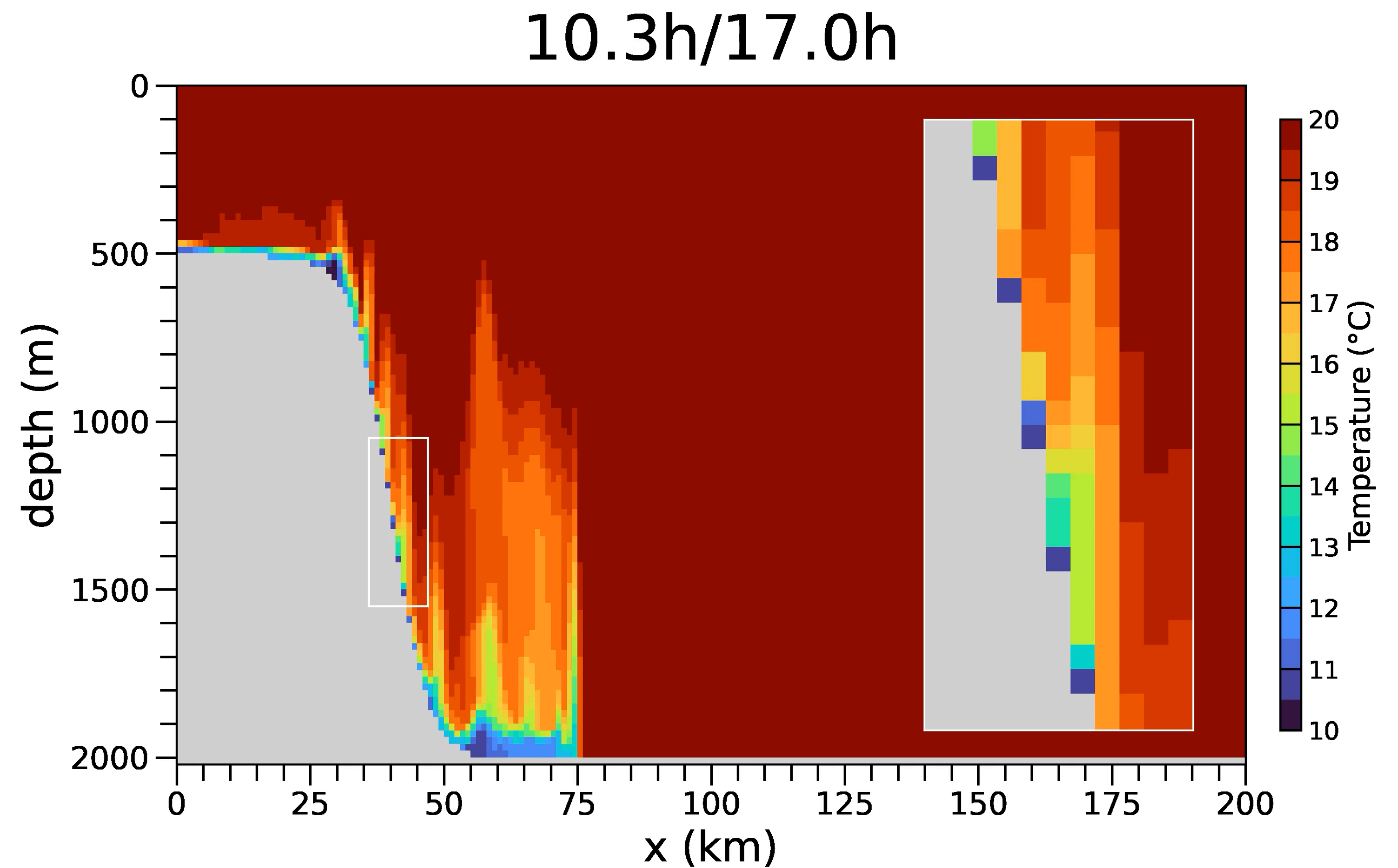
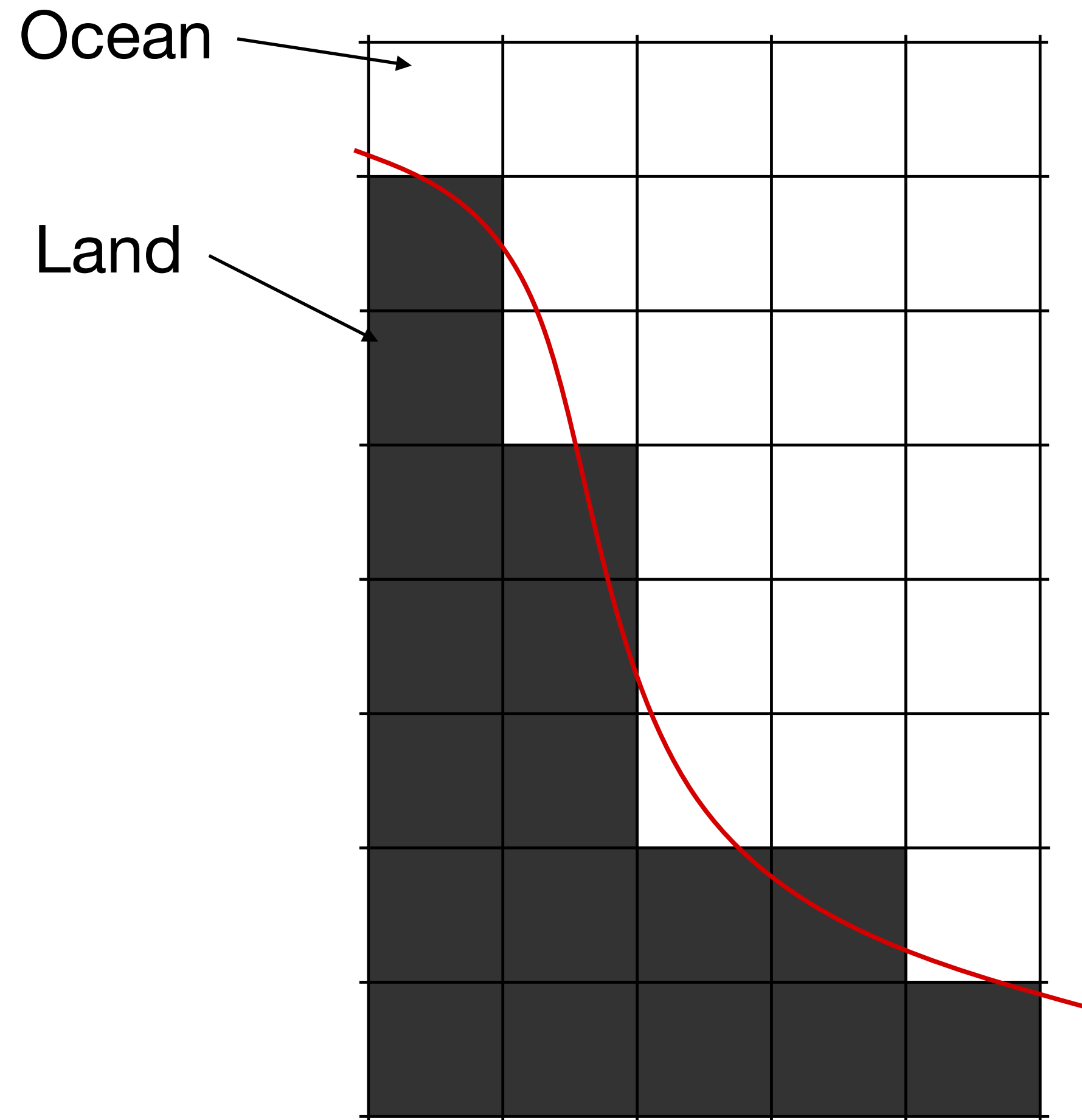
s-coordinate provides a natural representation of topographic flows

s-coordinate



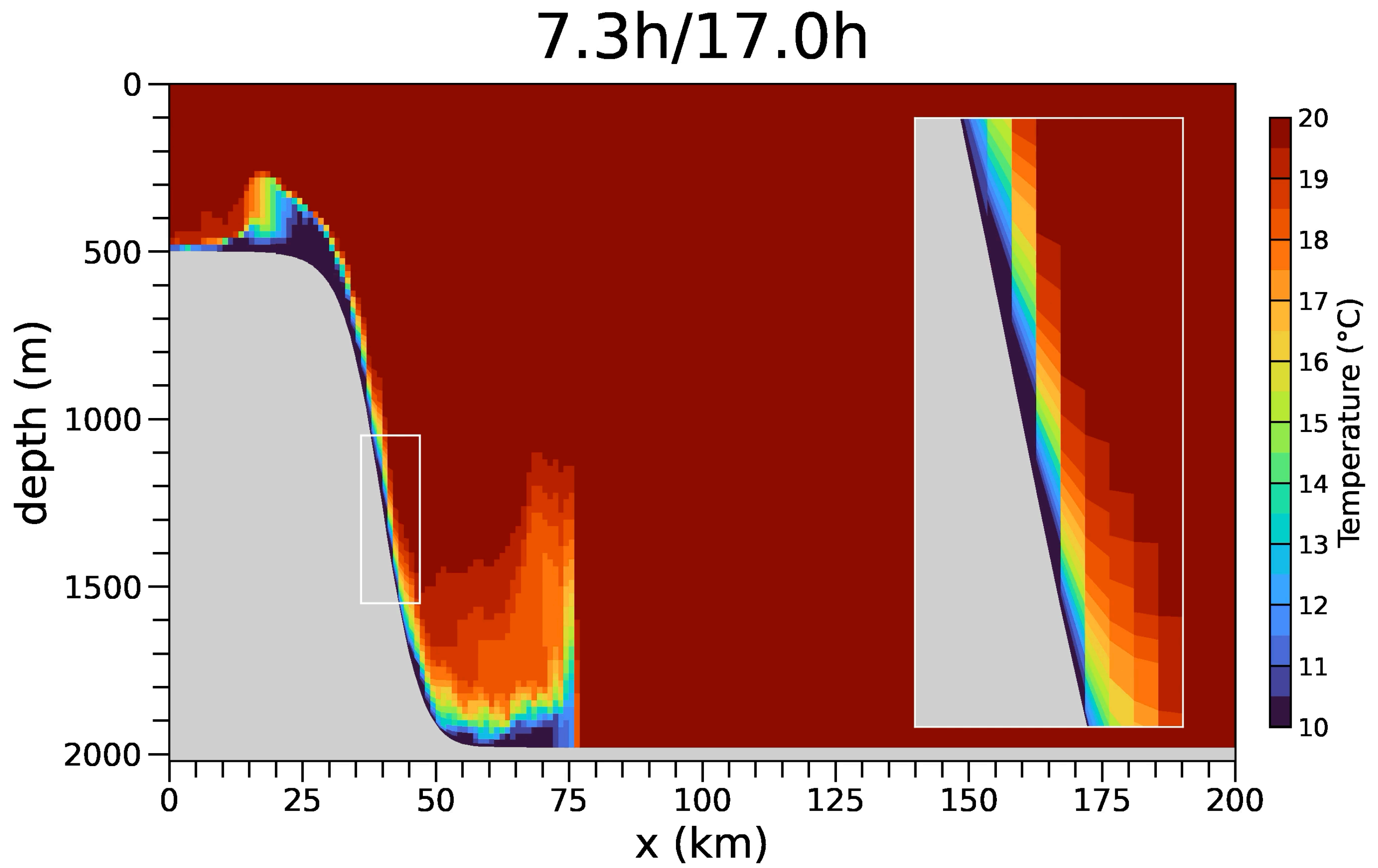
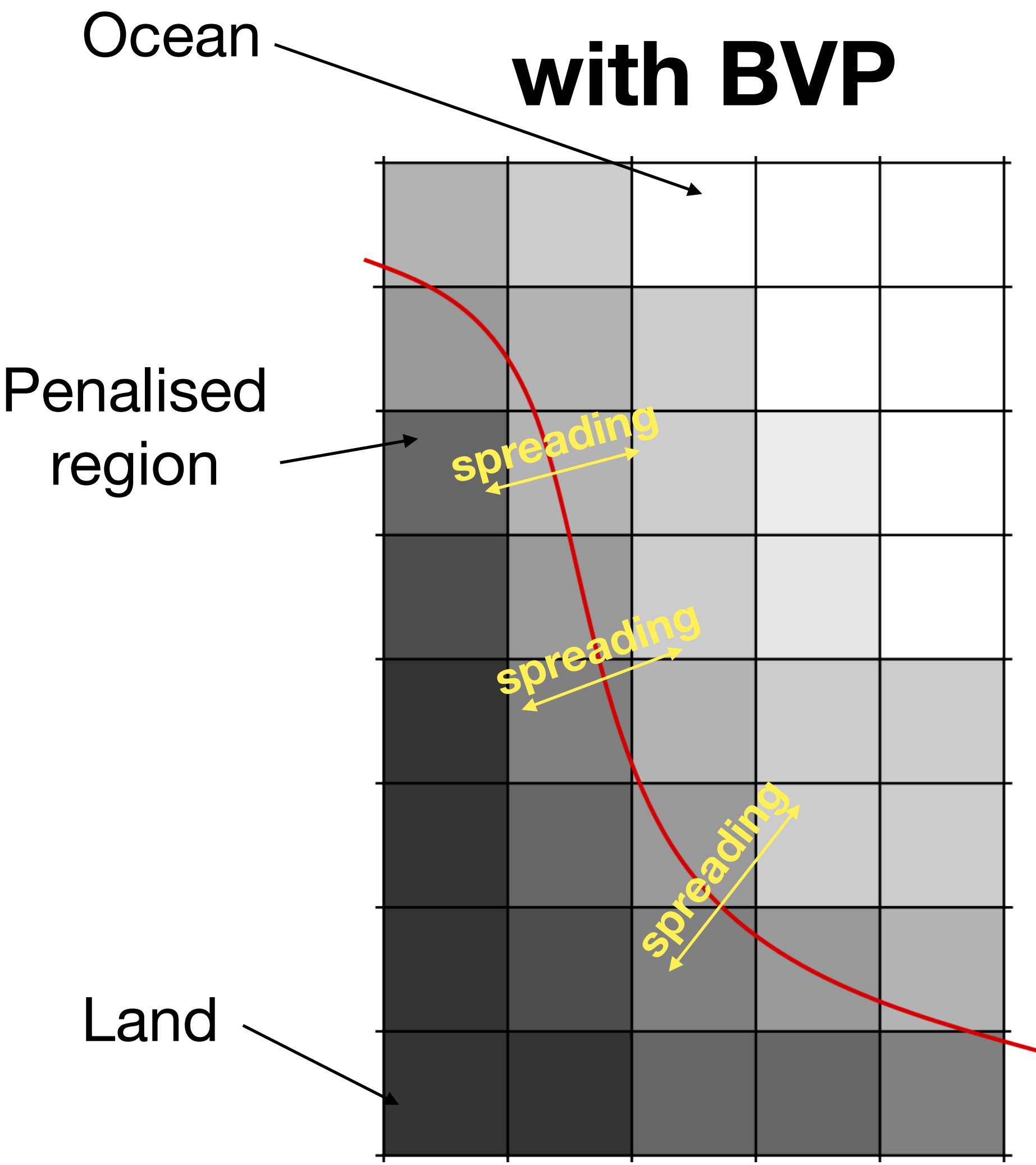
Traditional simulation in z-coordinate

z-coordinate



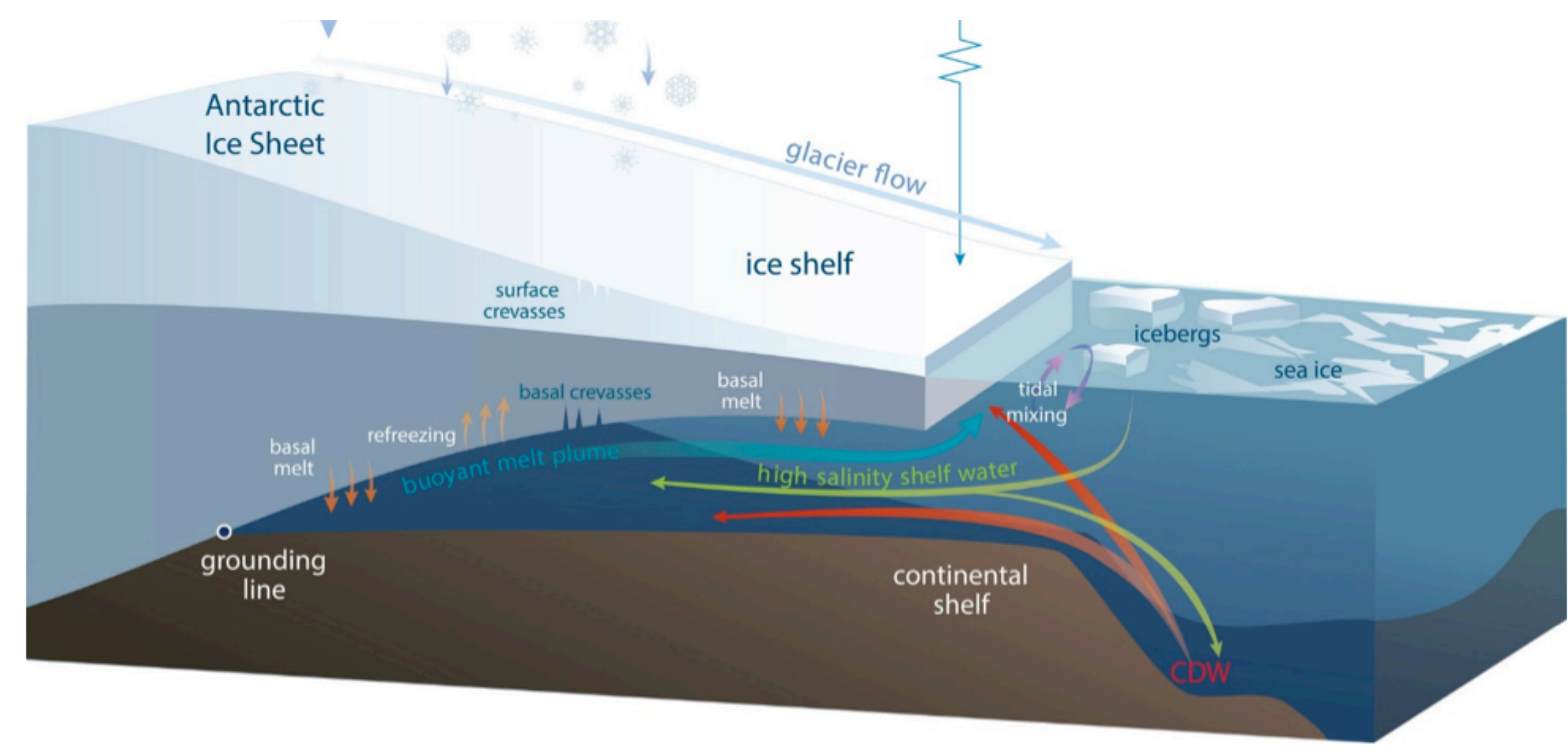
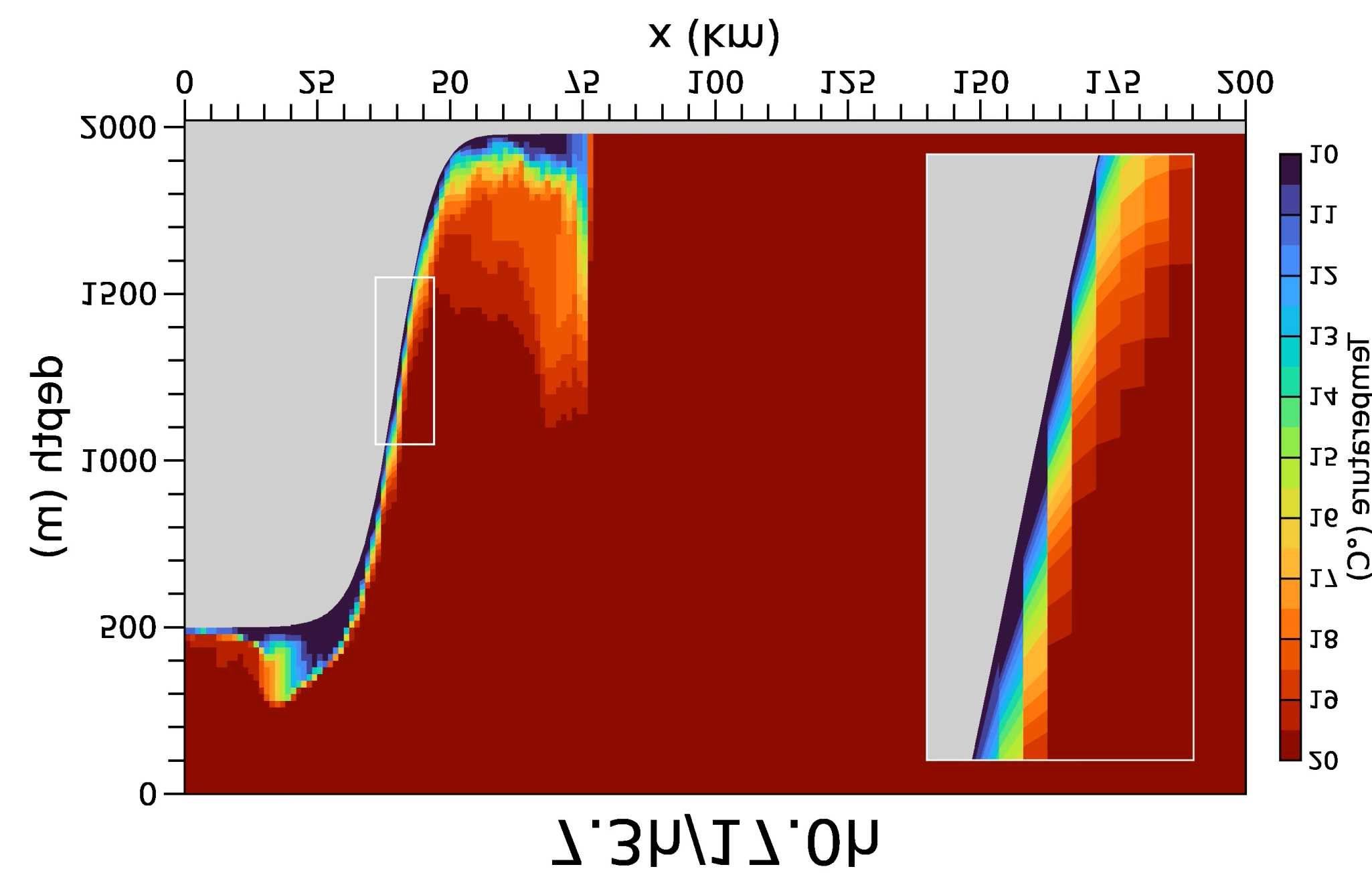
Overflow resorting to the BVP method

Z-coordinate with BVP



Circulation under iceshelves face similar challenges

Ice draft acts as an inverted topography on marine currents



Numerical representation of basal melt induce excessive mixing

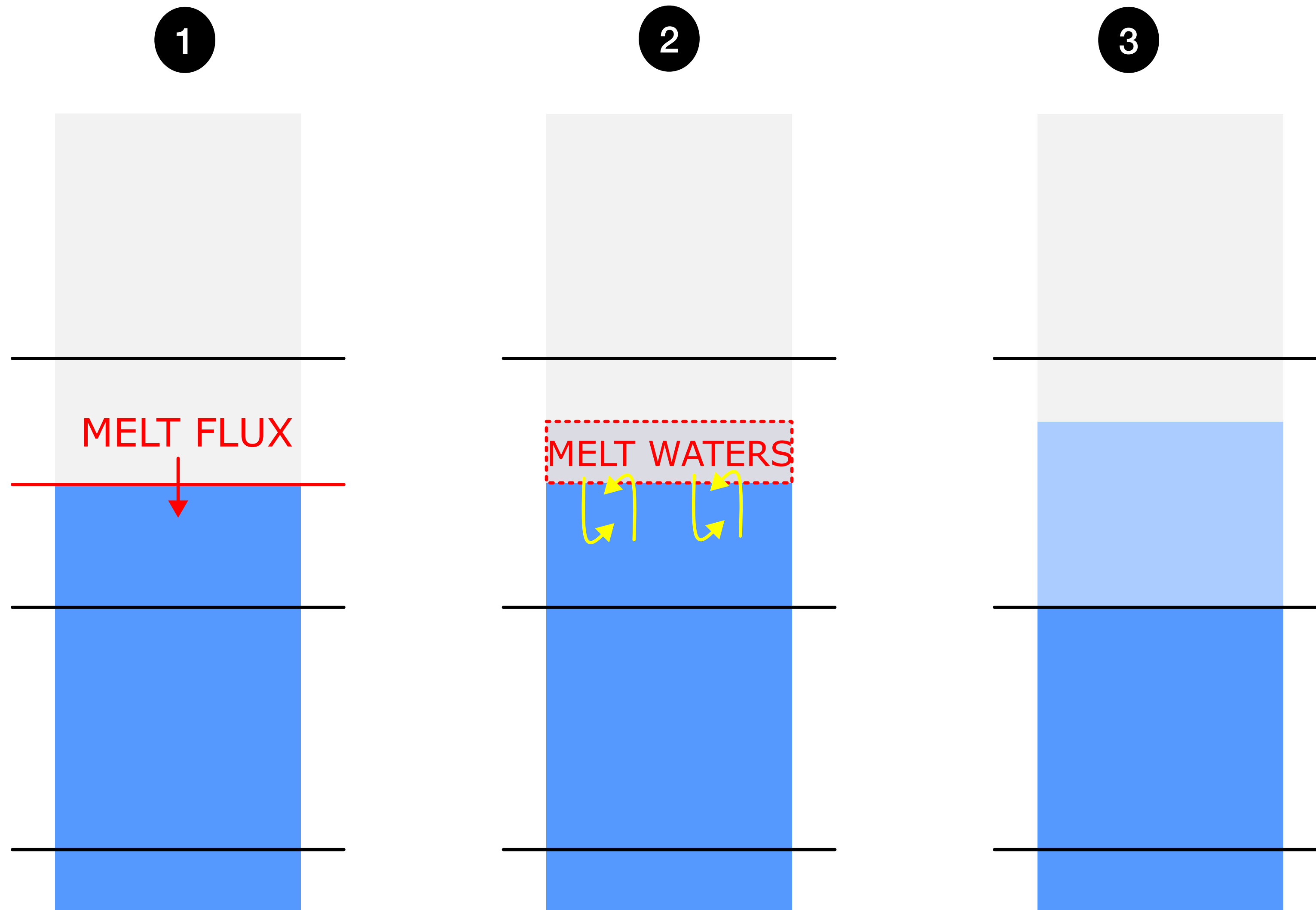
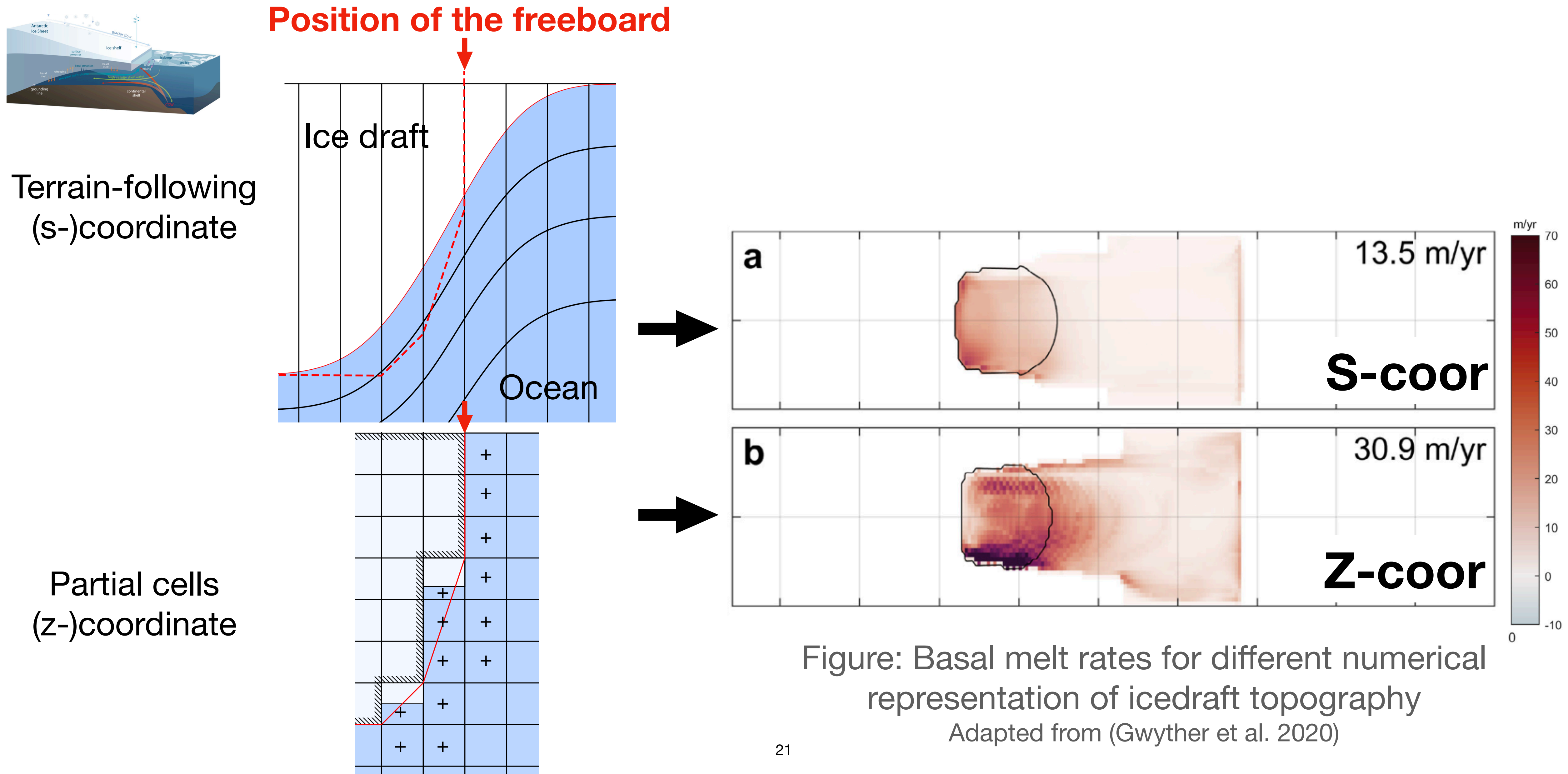
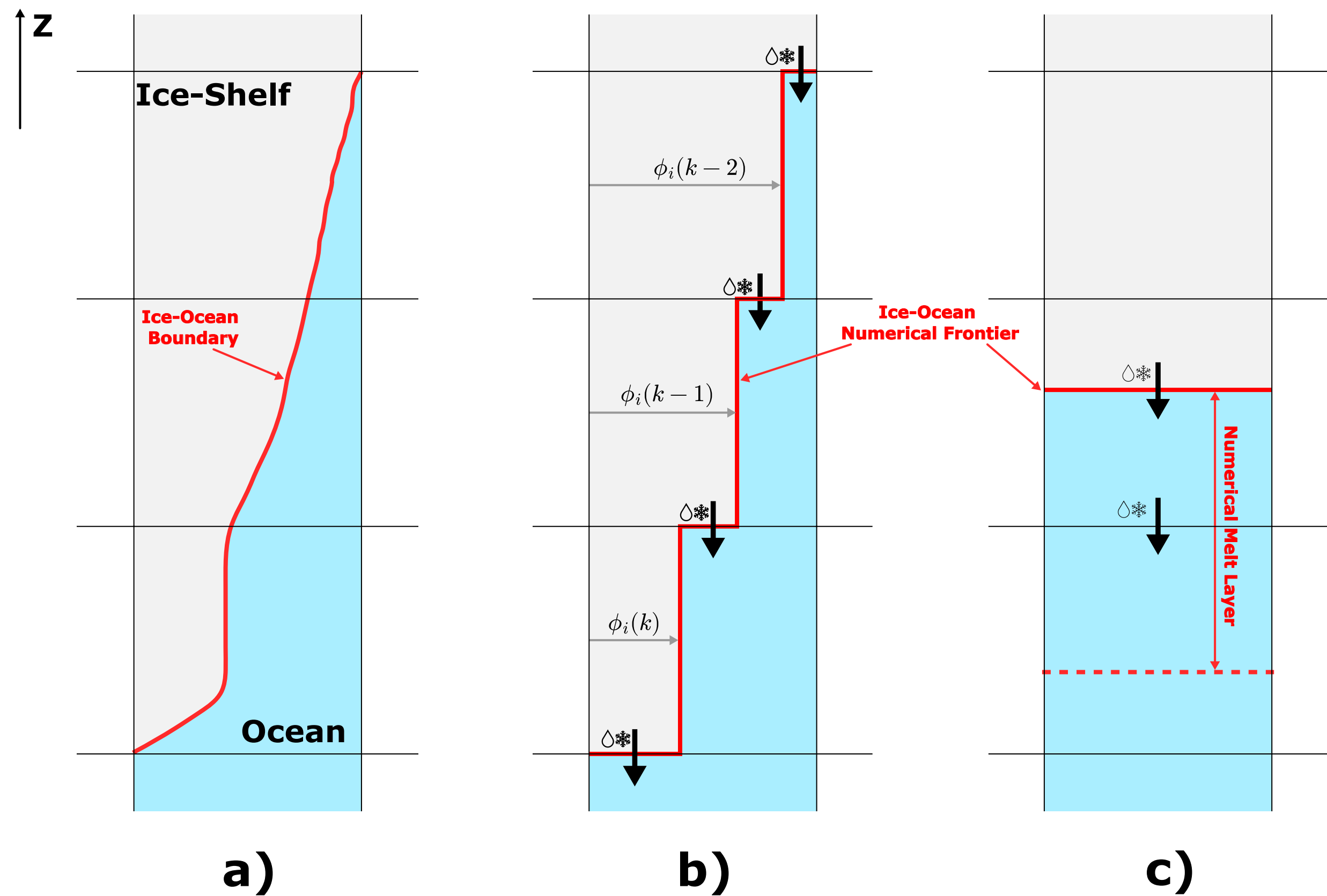


Figure: Illustration of the mixing of meltwaters with vertical resolution

Cavity representations lead to large discrepancies in basal melt rates



Investigating the potential of the BVP method for ice-ocean interaction



Penalized icedraft

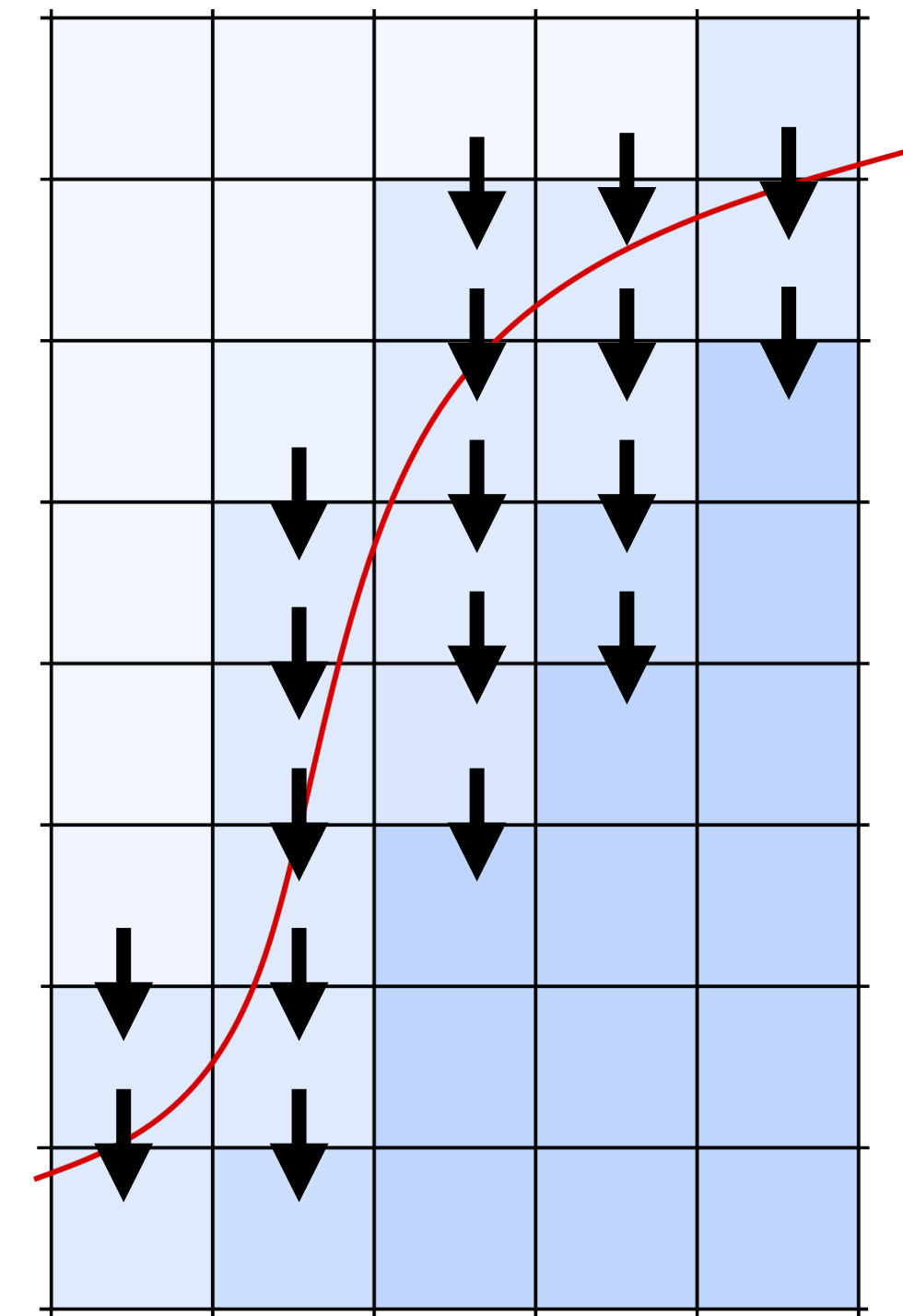


Figure: Schematic of basal melt computation under partial cells and penalization

Idealised configuration of an ice-shelf: ISOMIP+



S  N

Ice-sheet Grounding line Freeboard T-S restoring region

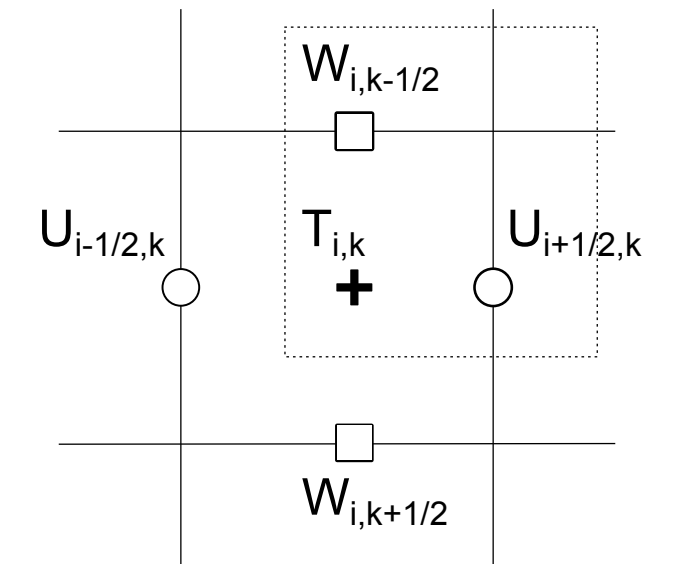
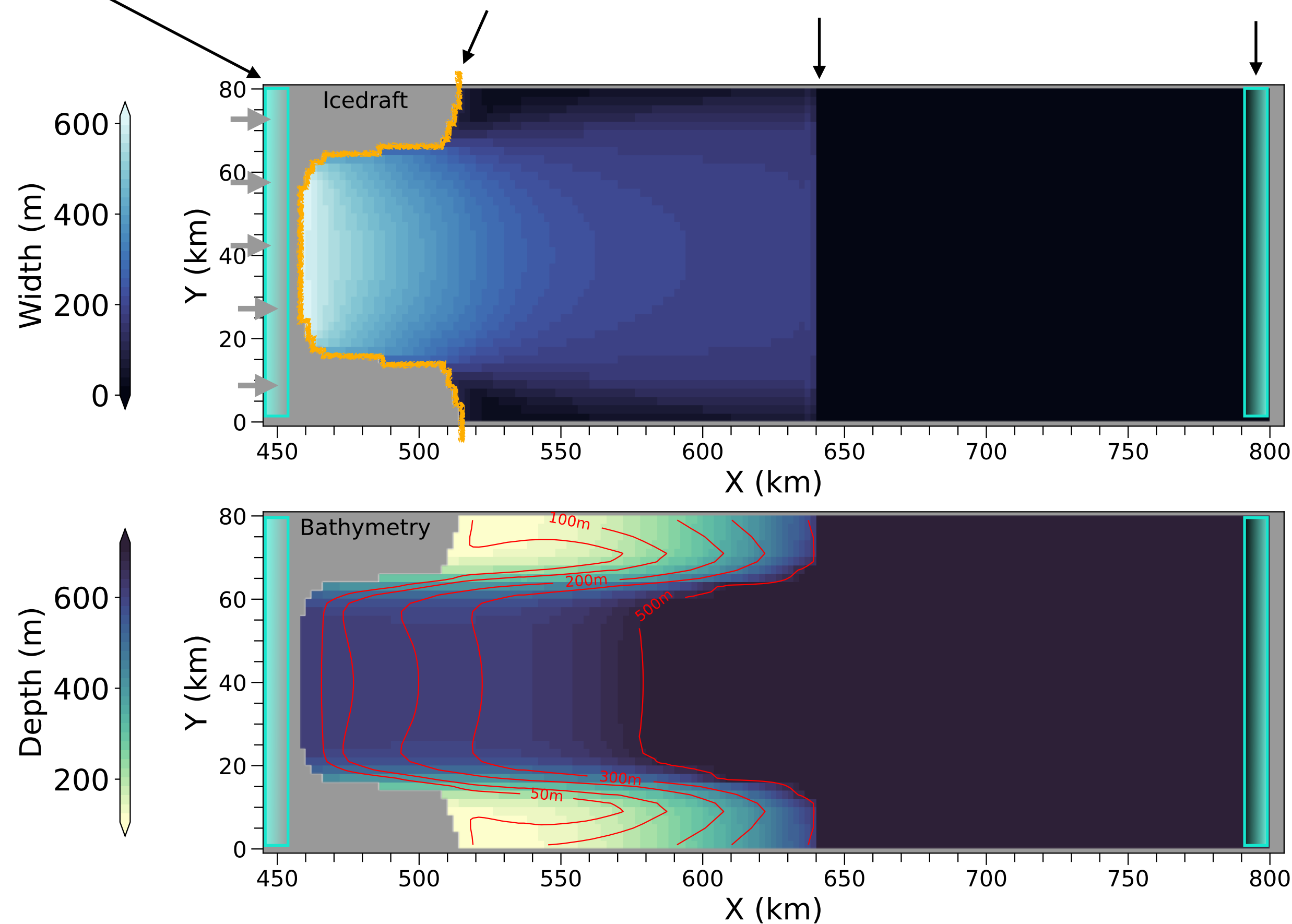


Figure: C-grid

- From Asay-Davis et al. (2016)
 - Enhanced Vertical Diffusivity (EVD) parametrisation activated
- ocean0 case with WARM initialisation and restoring
 - Fixed ice topography
 - Integrated for 5 years
 - Comparison at temporal equilibrium
- **We'll compare the BVP with a partial-cells configuration using a numerical melt layer (Losch (2008))**

Porosities are defined as the cell-average of ice, land and ocean

Ice porosity

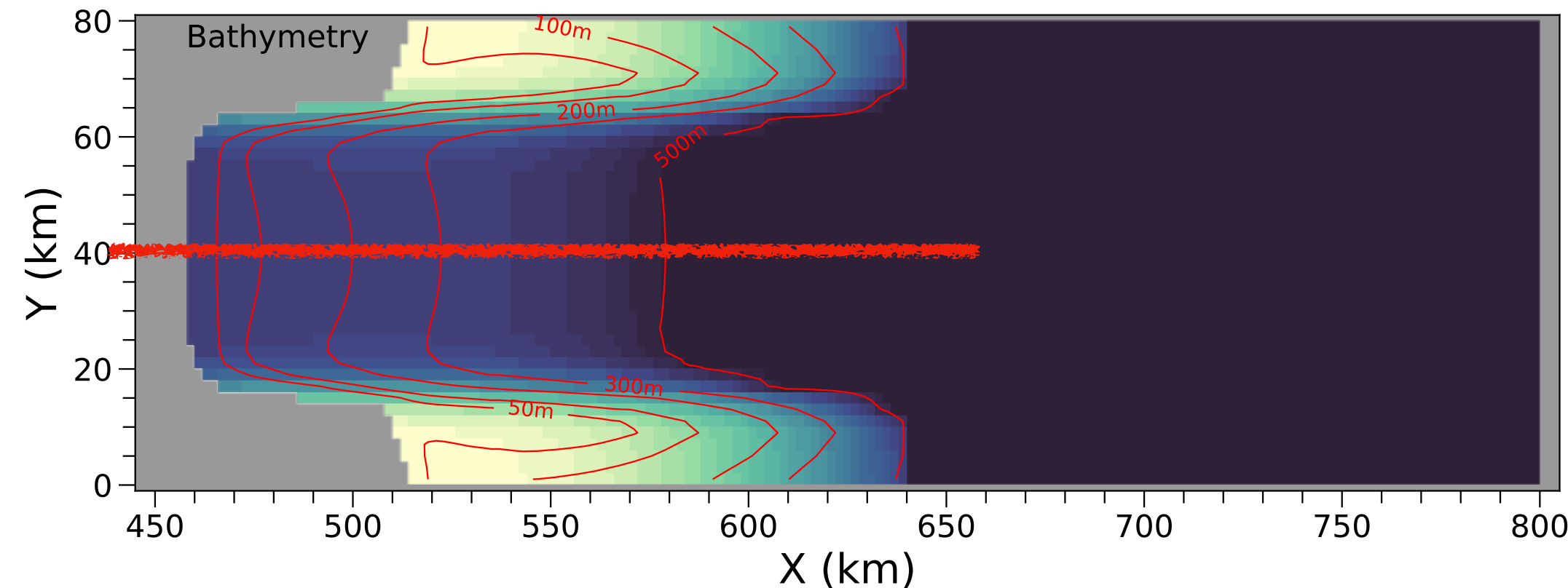
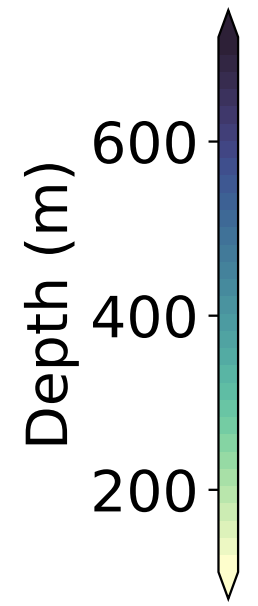
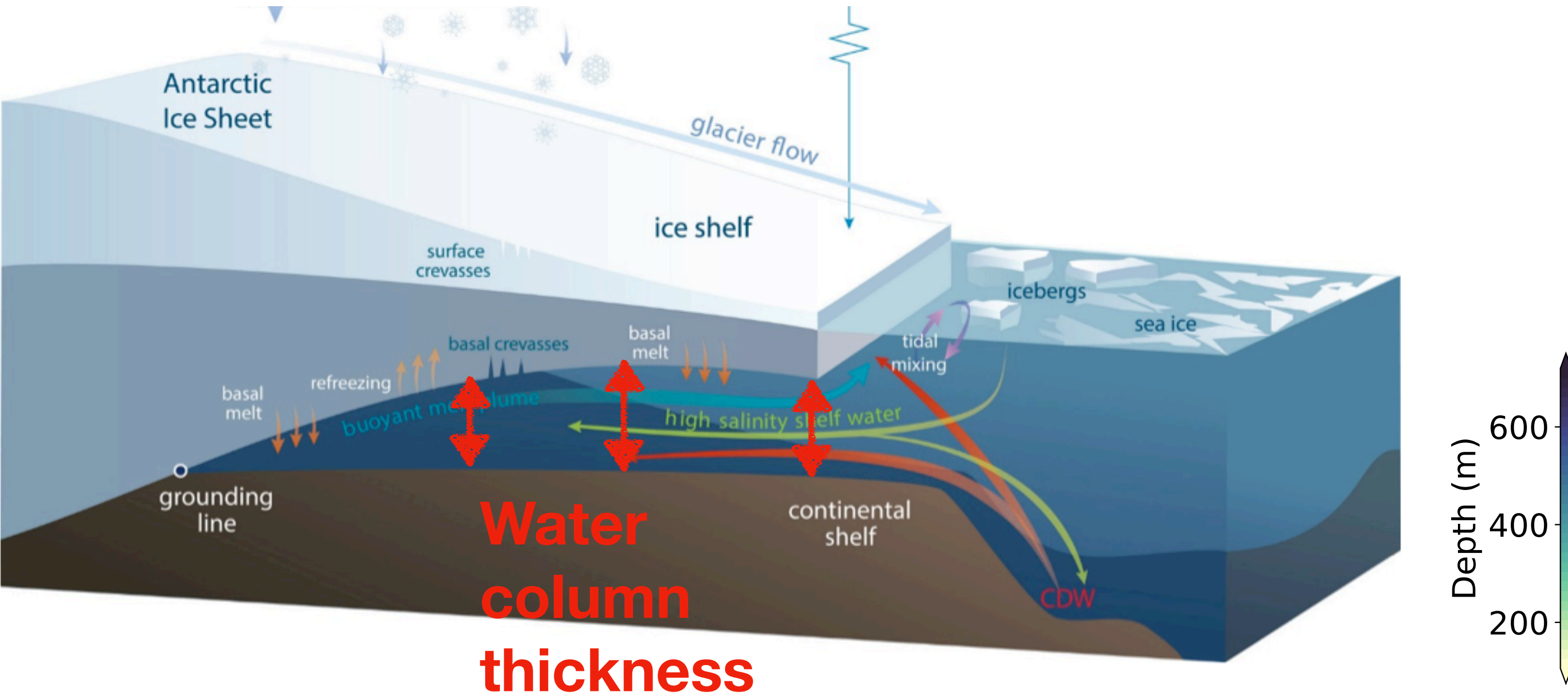
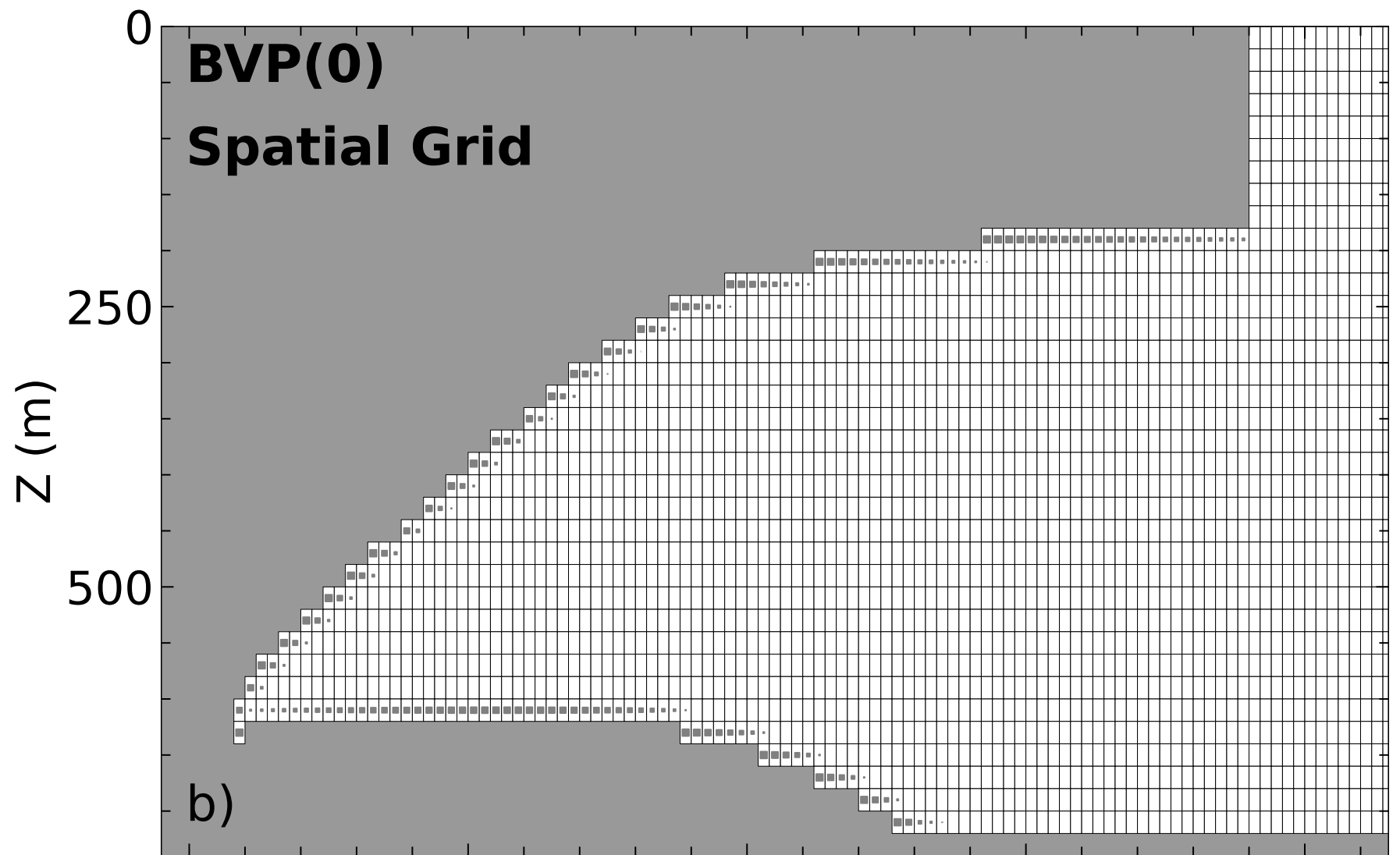
$$(\phi_i)_{i,j,k} = \frac{1}{V_{i,j,k}} \iint_{A_{i,j}} \max(\min(d(x,y) - z_{top}^k, 0), \Delta z), dxdy$$

Land porosity

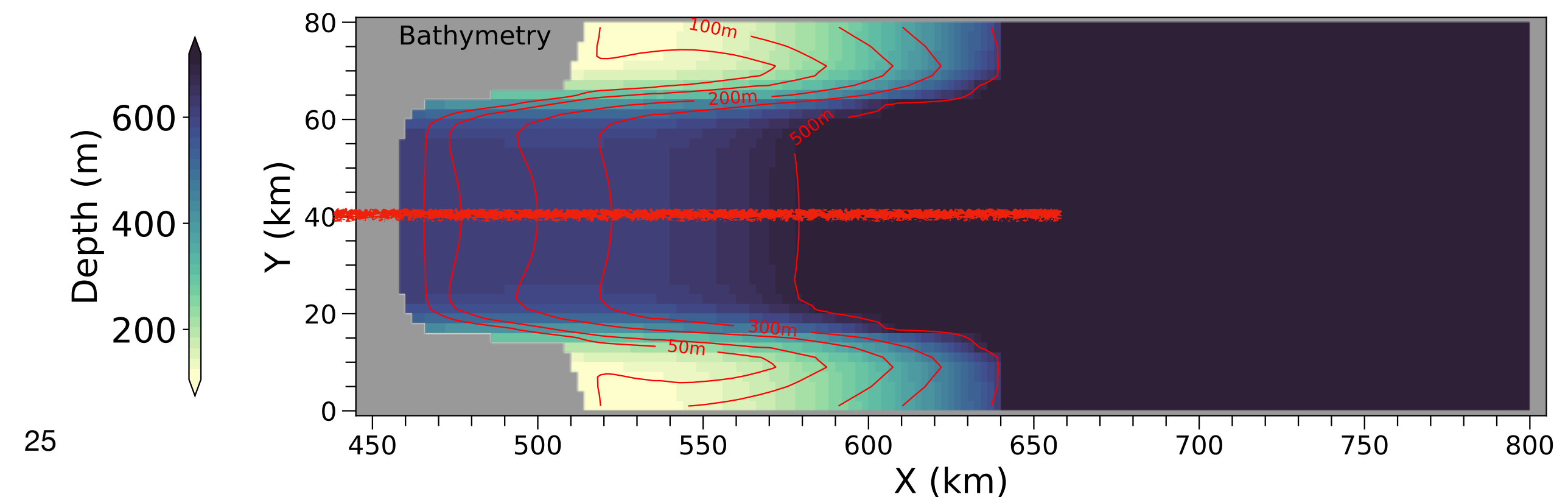
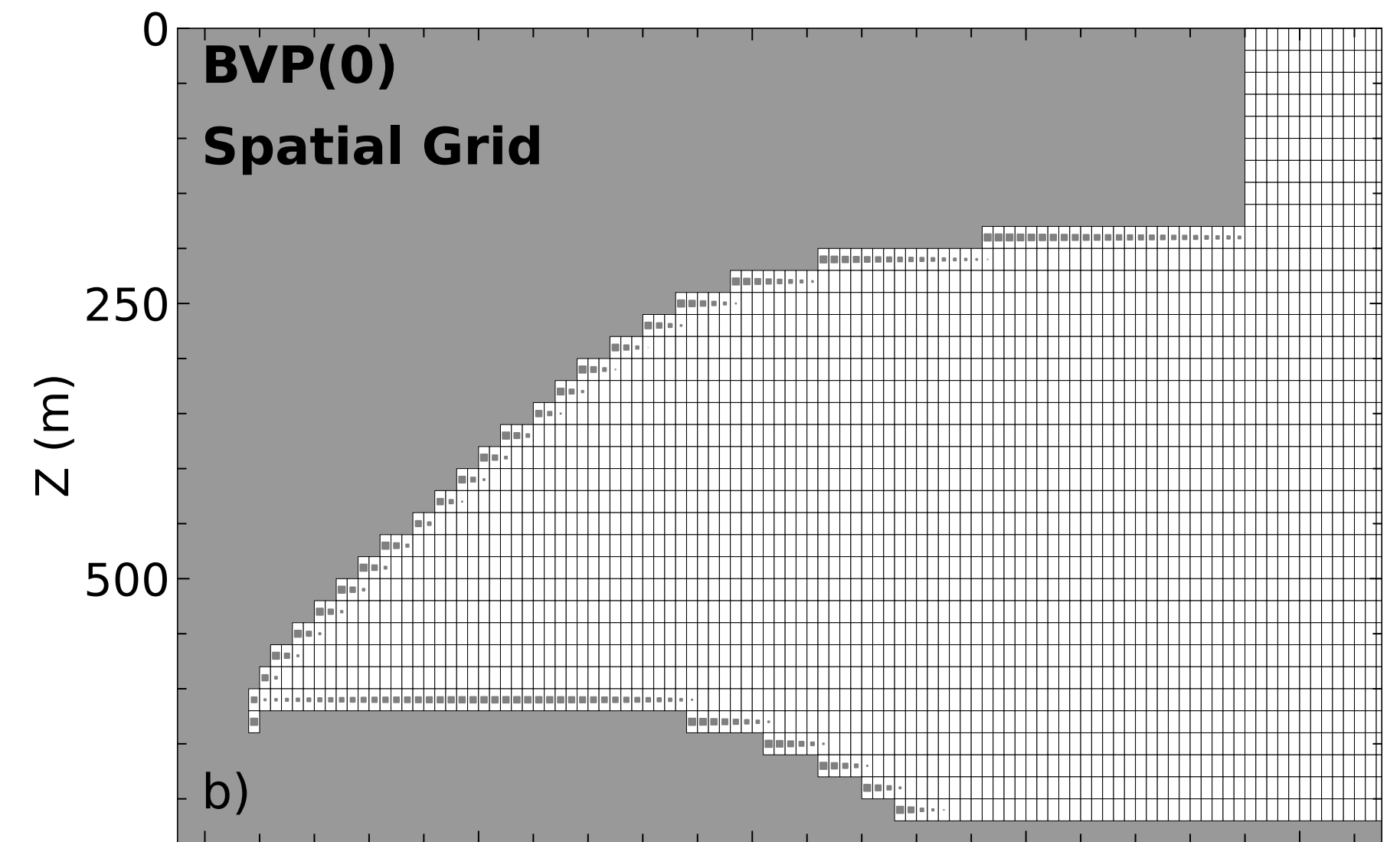
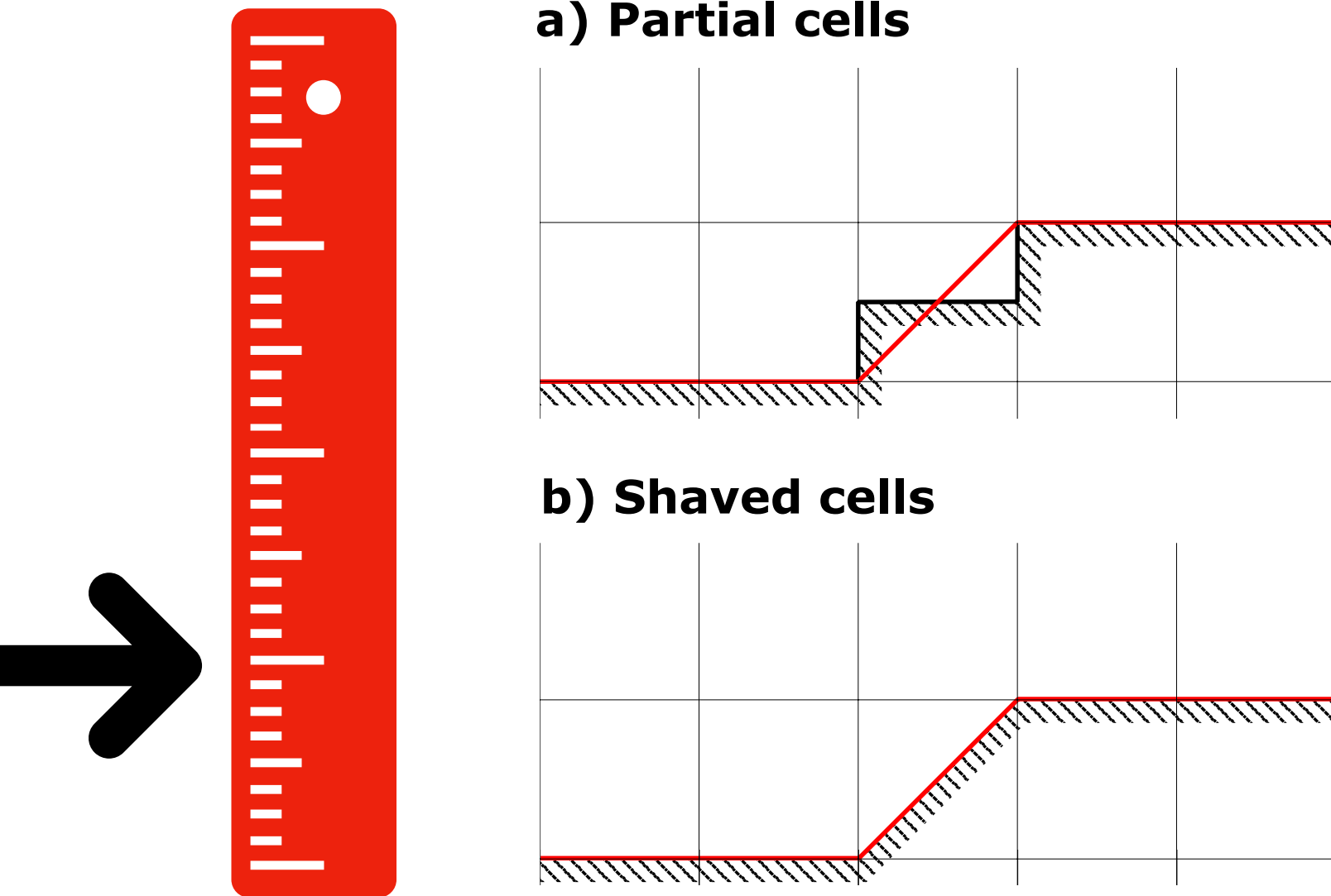
$$(\phi_l)_{i,j,k} = \frac{1}{V_{i,j,k}} \iint_{A_{i,j}} \max(\min(z_{bot}^k - b(x,y), 0), \Delta z), dxdy \rightarrow$$

Porosity

$$(\phi)_{i,j,k} = 1 - (\phi_l)_{i,j,k} - (\phi_i)_{i,j,k}$$



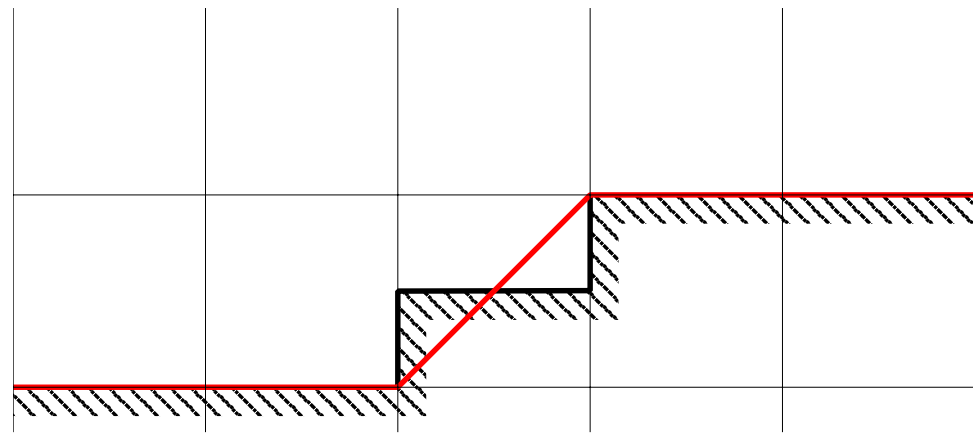
Direct penalization is equivalent to shaved cells approach



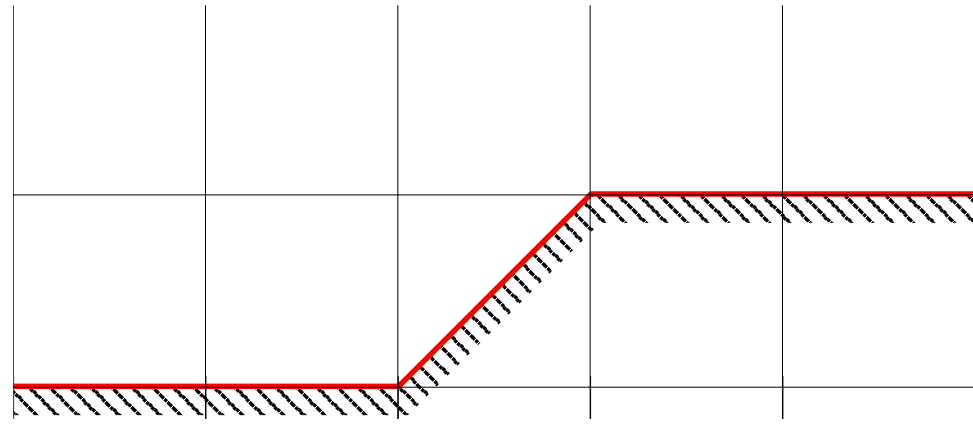
Smoothing with penalization spreads porous interface



a) Partial cells



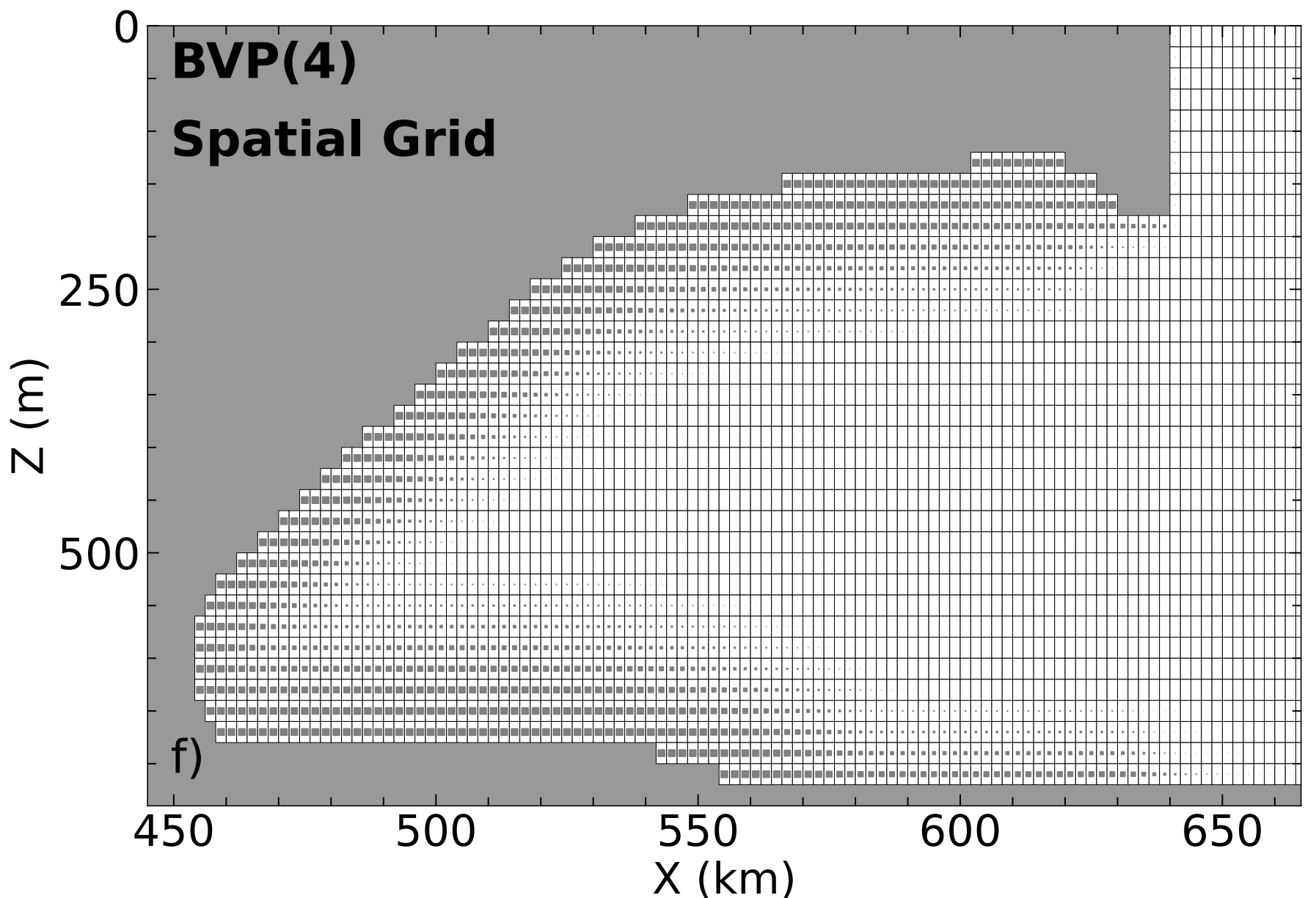
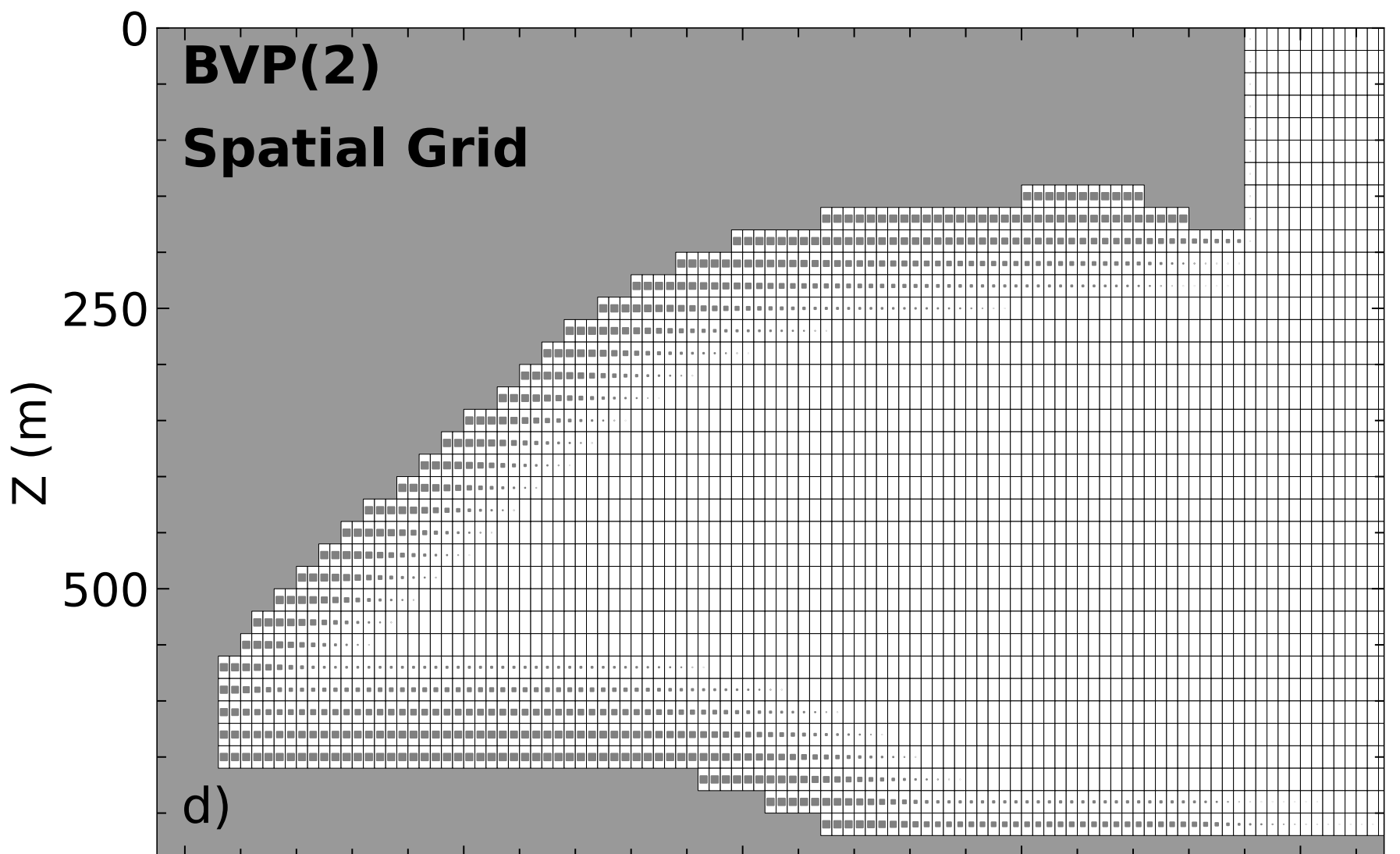
b) Shaved cells



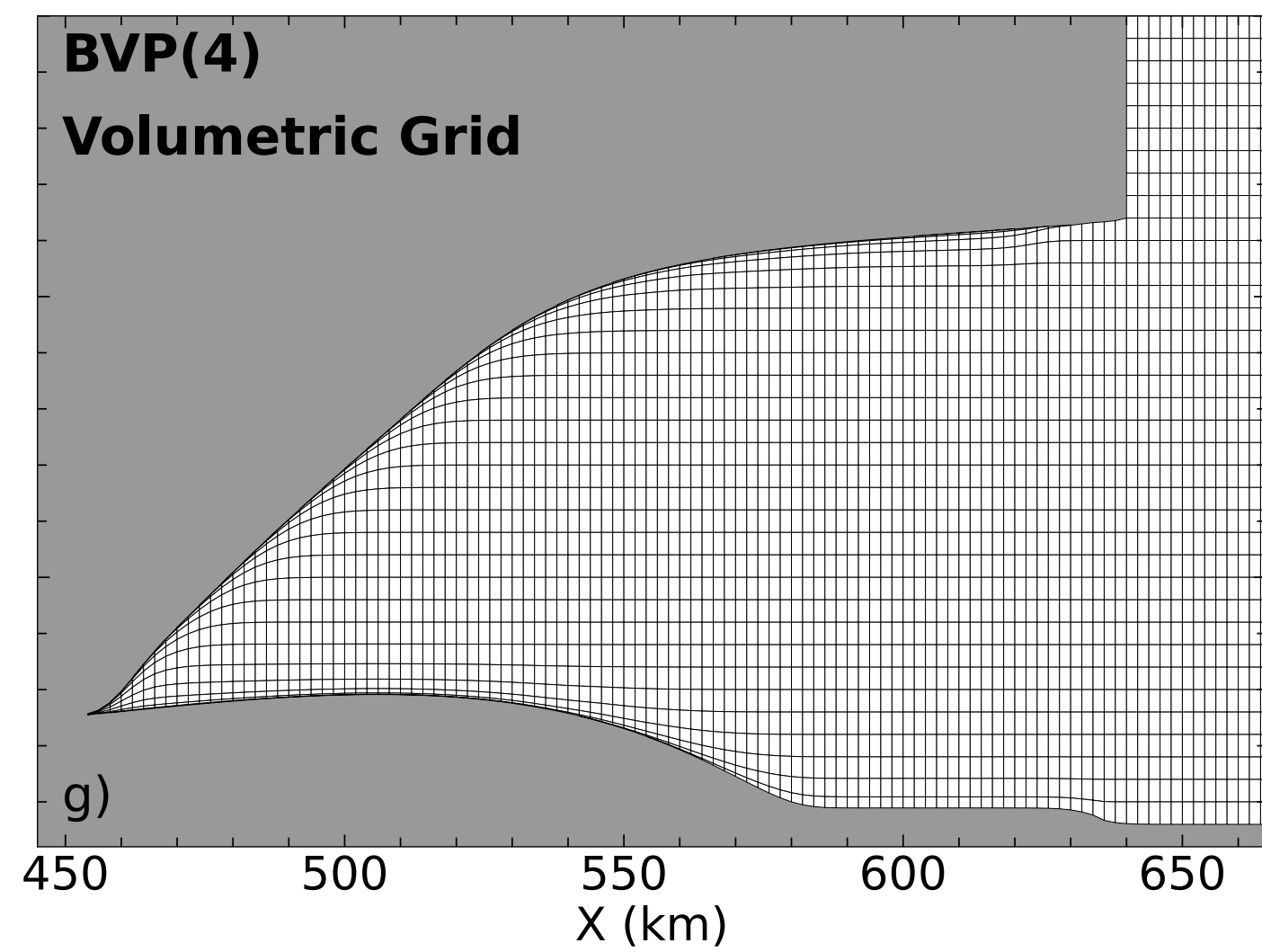
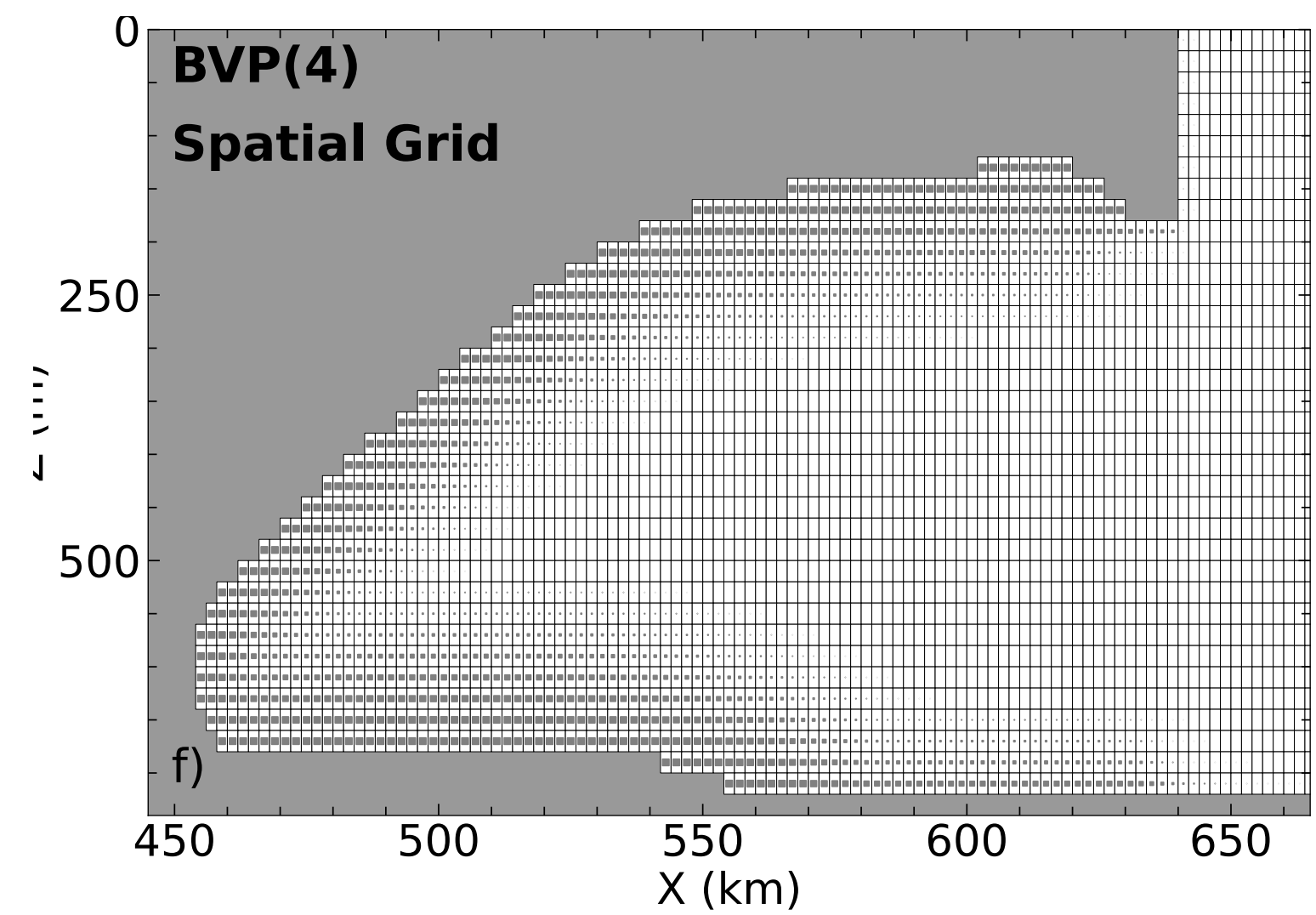
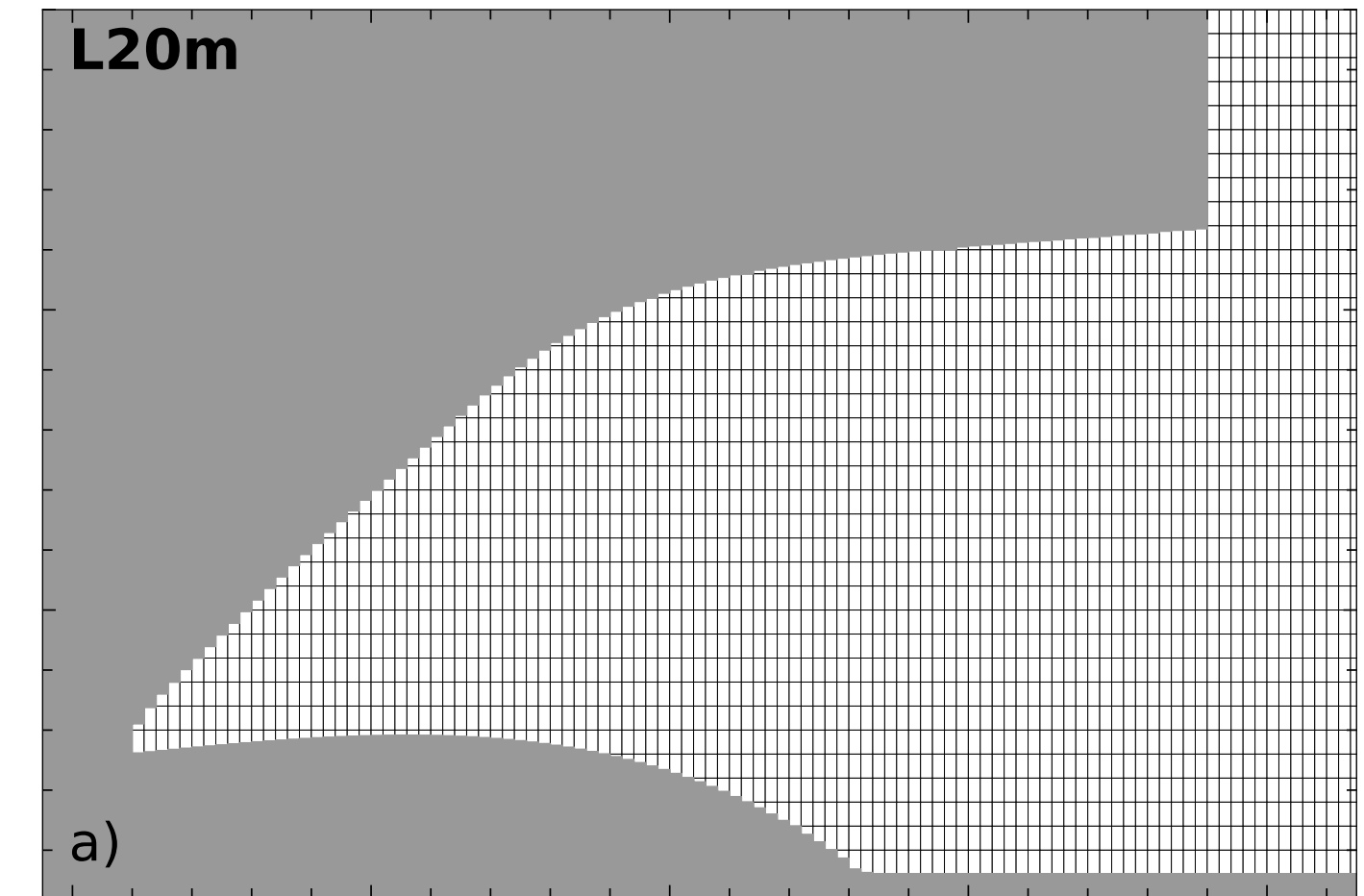
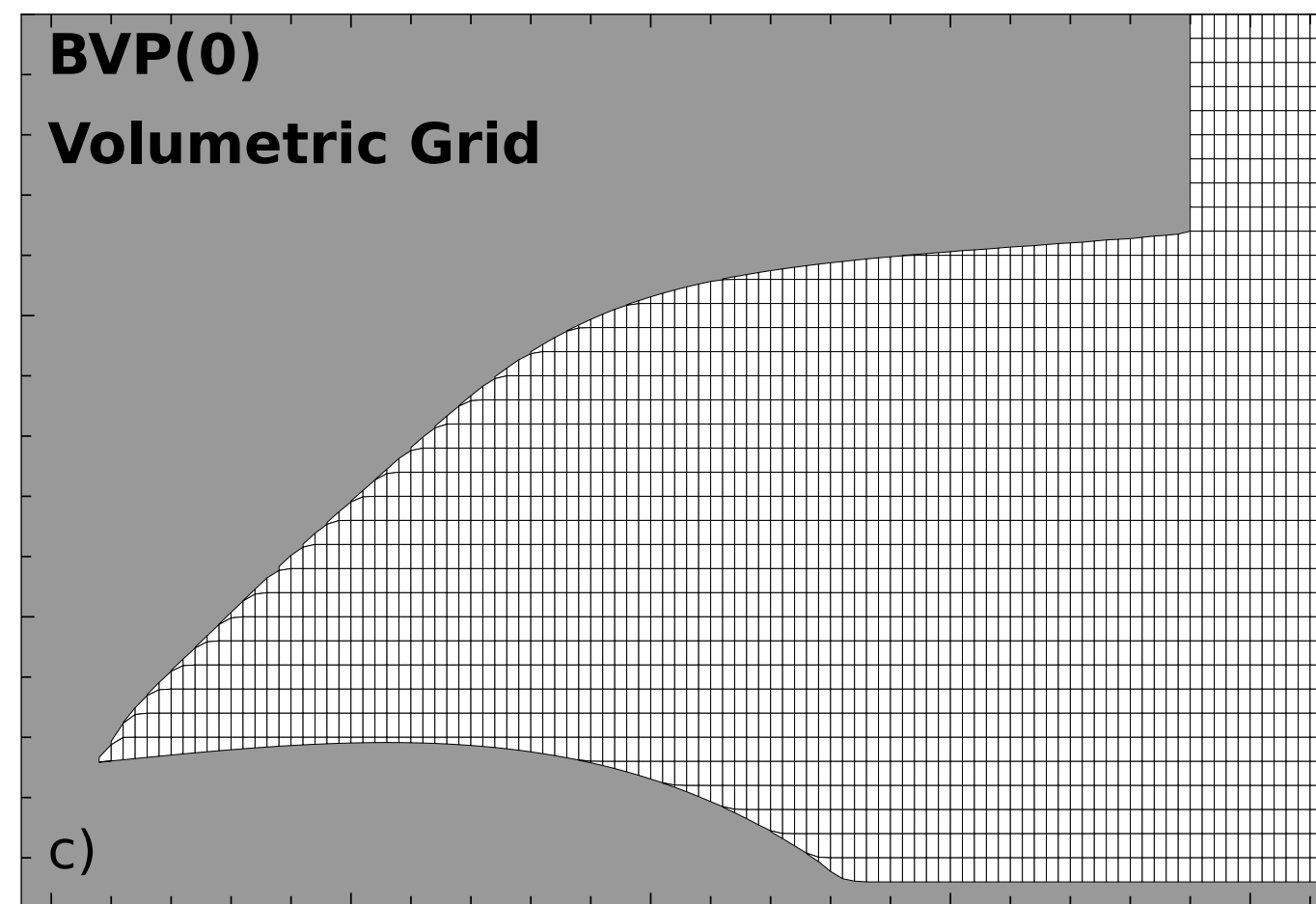
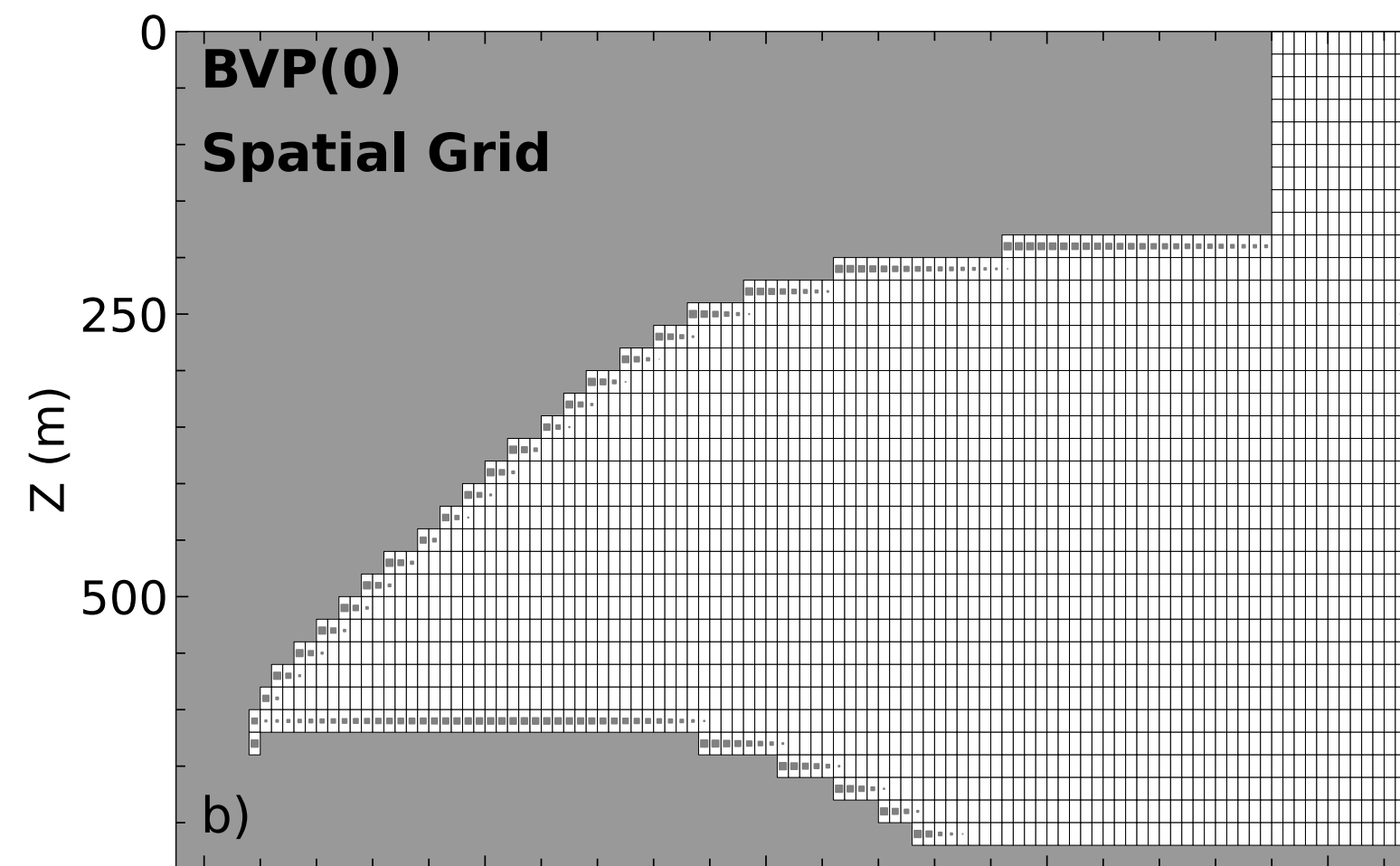
c) Penalization



With smoothing

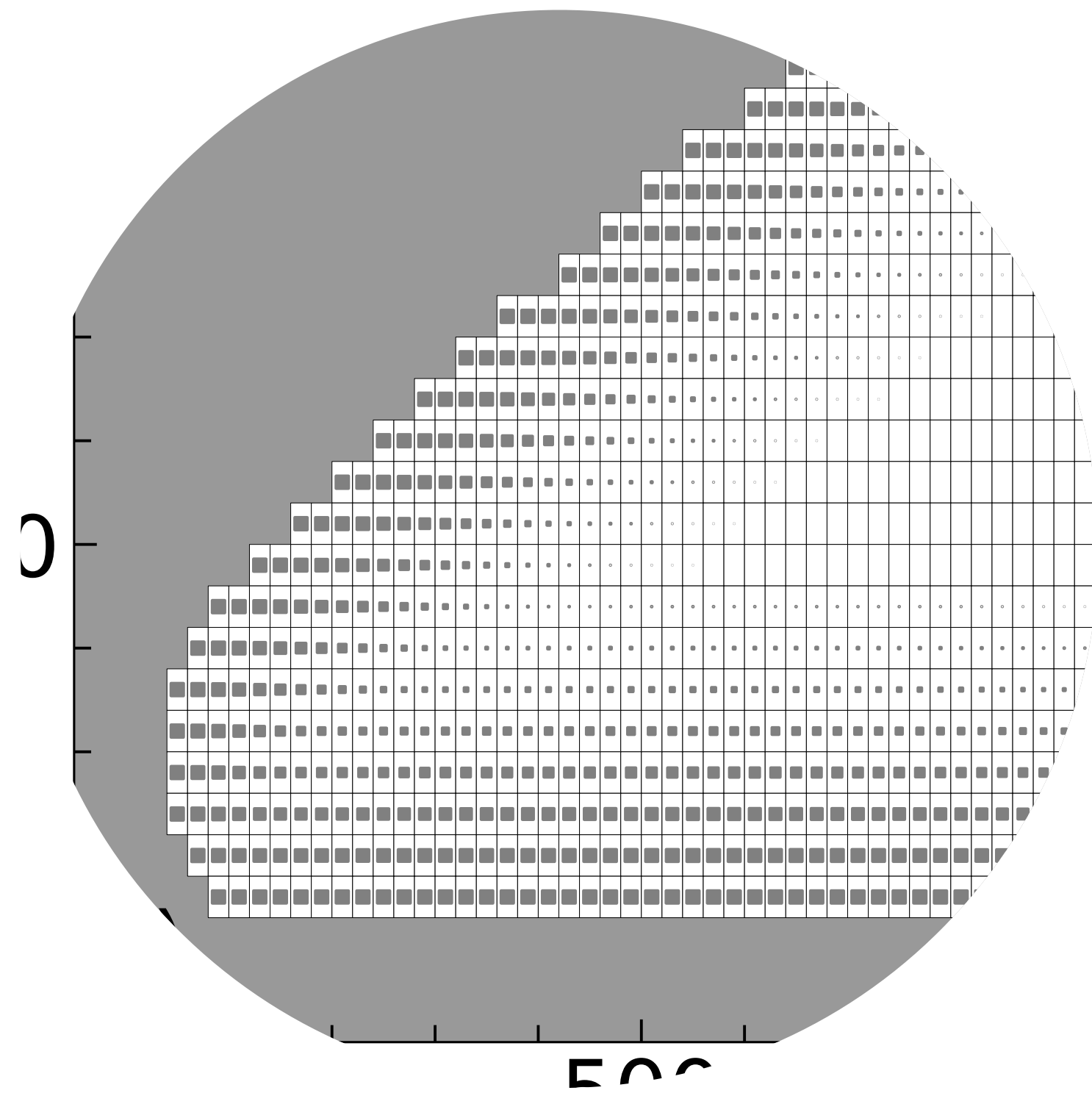


Volumetric representation is the grid smoothed by currents

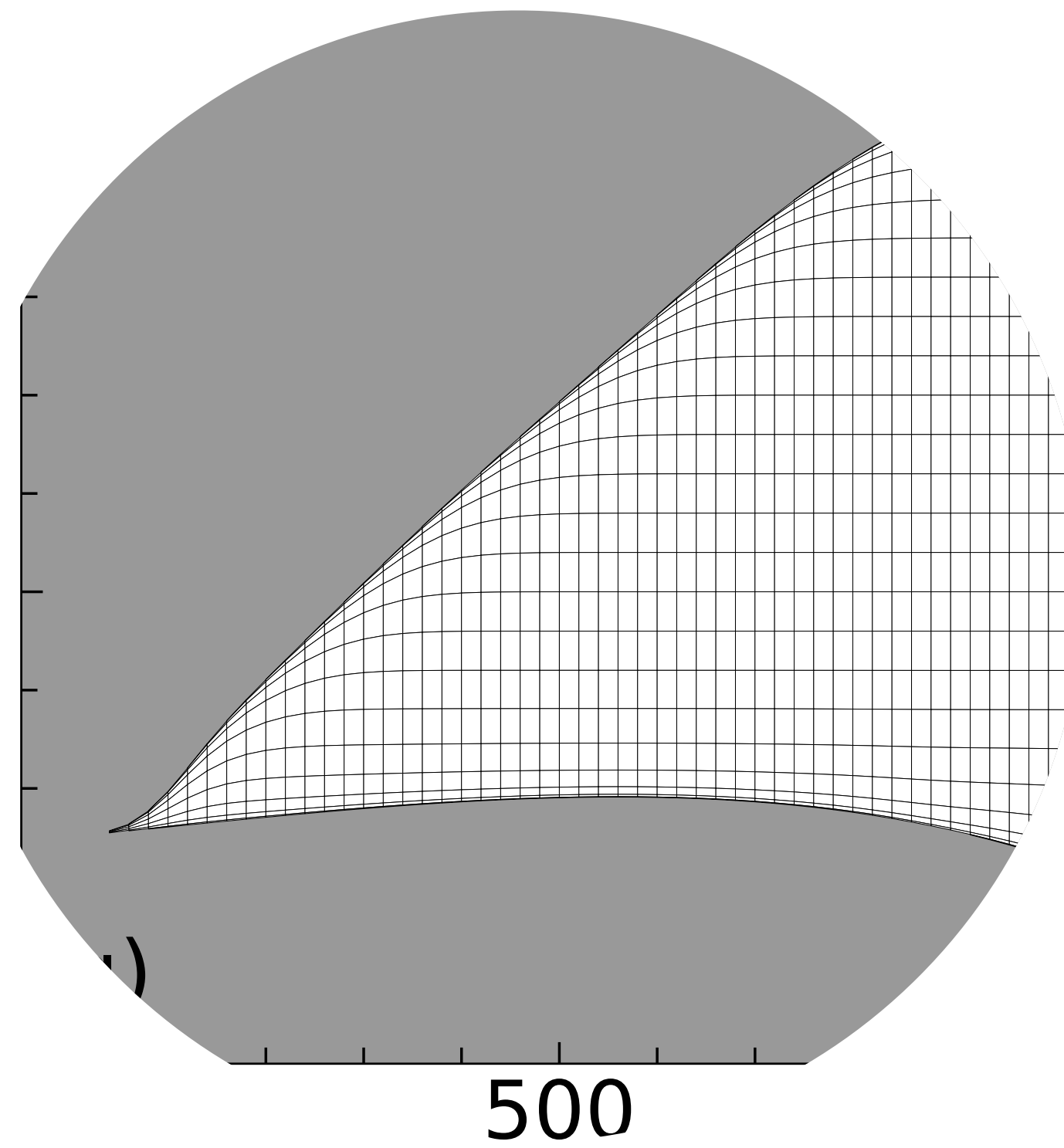


Smoothing with Penalization allows a nearly-continuous topography

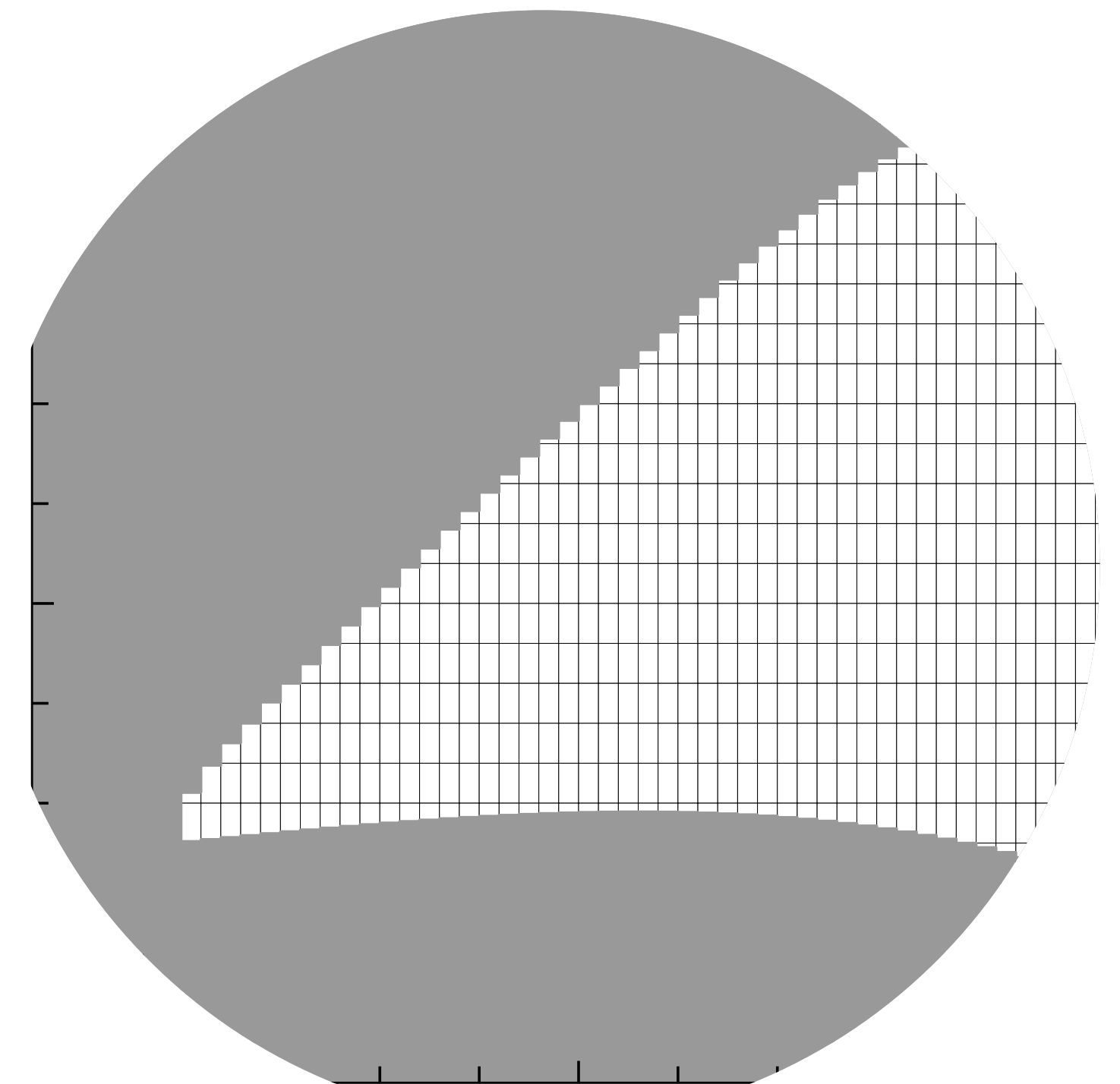
f) BVP(4)
Spatial grid



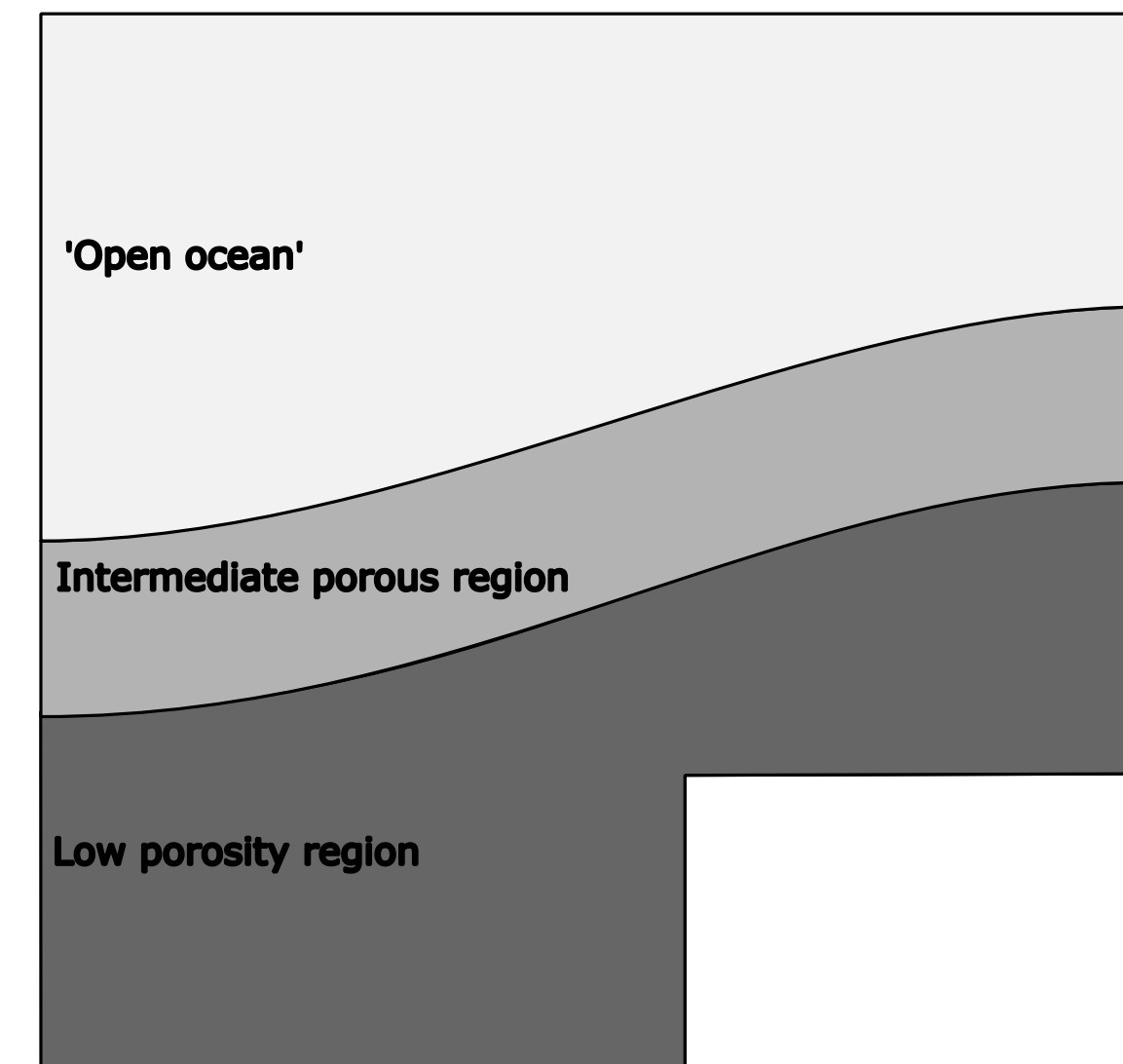
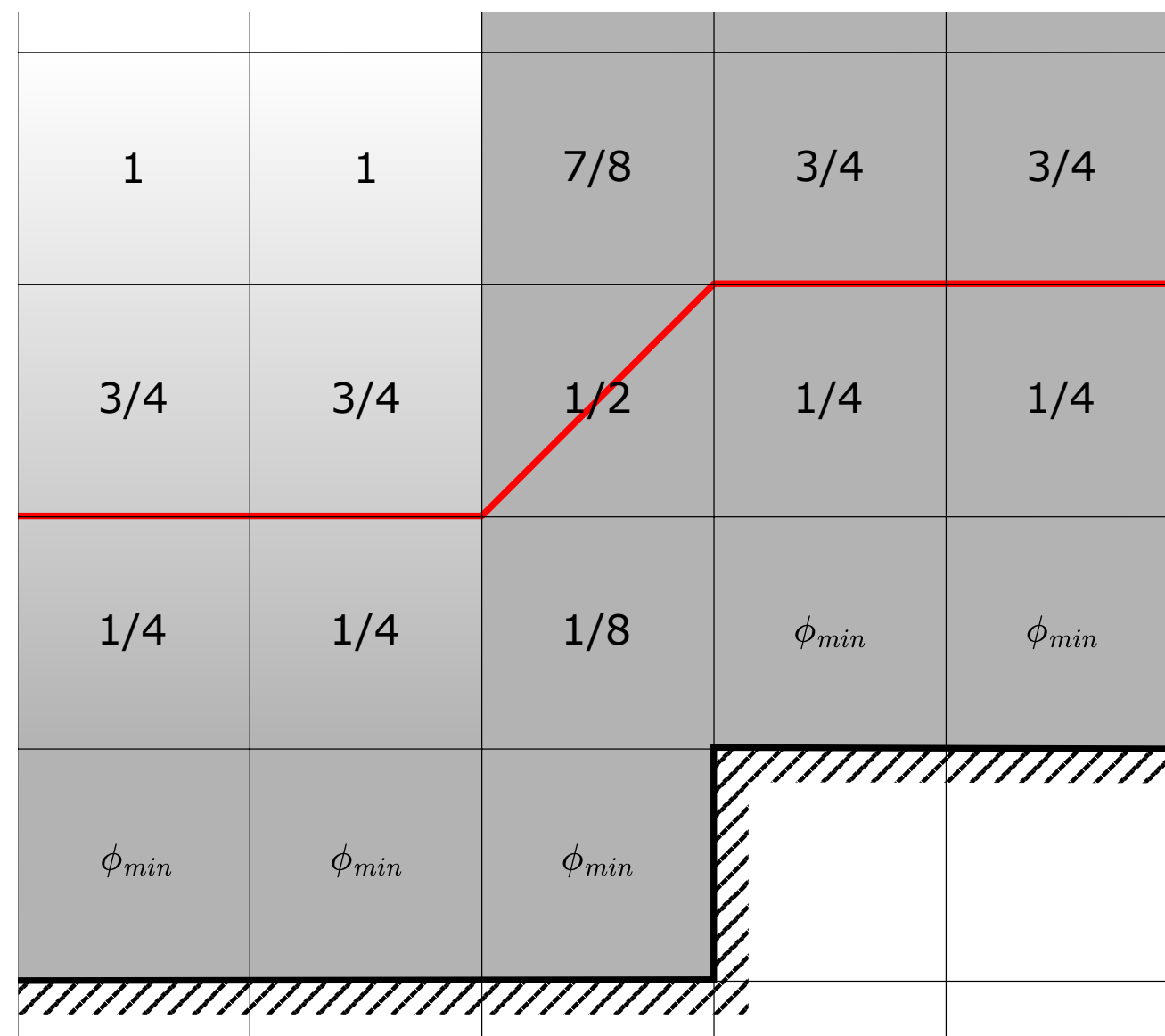
g) BVP(4)
Volumetric grid



c) Partial cells
L20m



Permeability enforces no-slip in the low porosity region



$$\sigma = 0$$

$$\sigma \text{ high} \Rightarrow \mathbf{u} \rightarrow 0$$

- CFL-preserving permeability:

$$(\sigma_u)_{i,j,k} = \max \left(f(\alpha_{i-1,j,k}), f(\alpha_{i,j,k}), f(\alpha_{i+1,j,k}) \right),$$

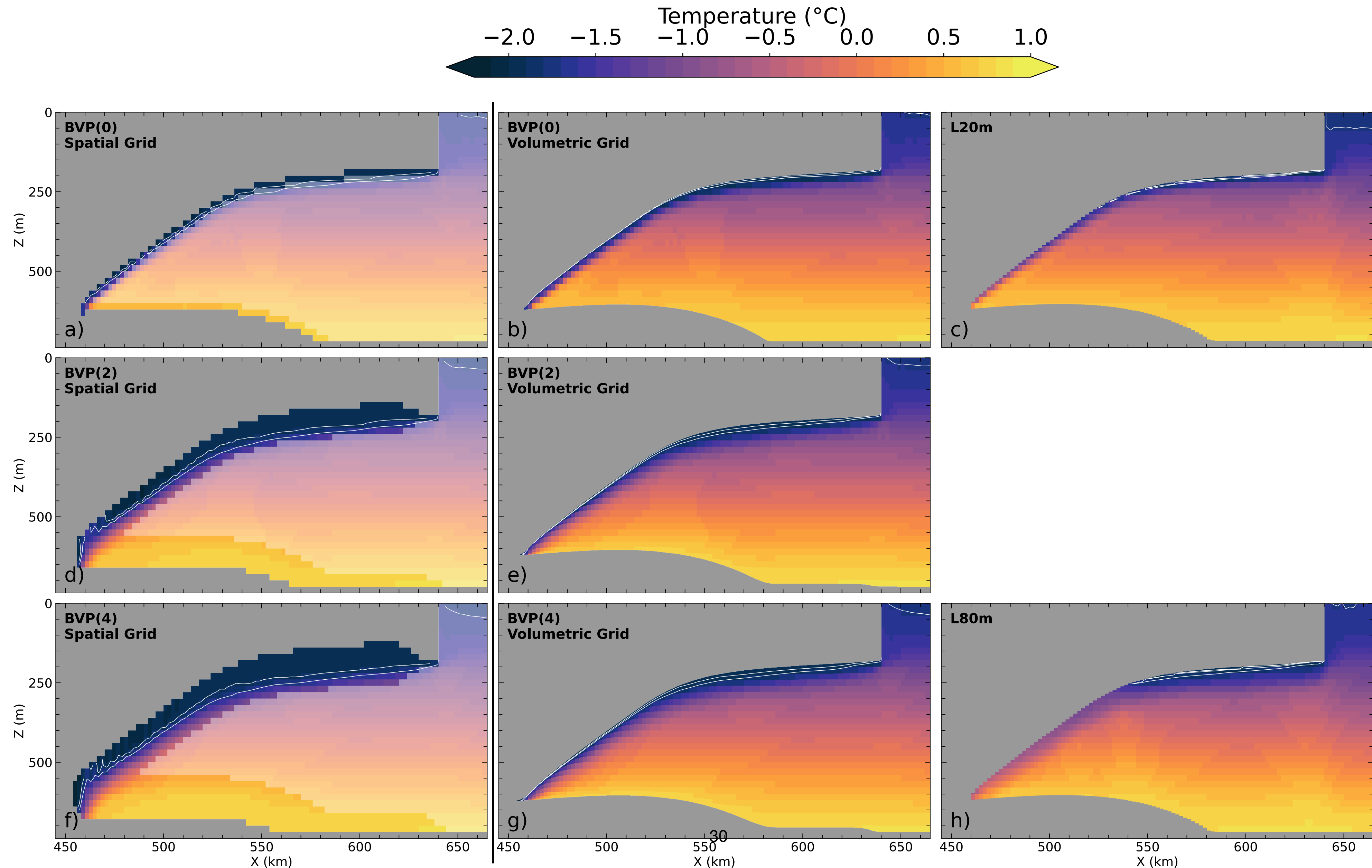
$$(\sigma_v)_{i,j,k} = \max \left(f(\alpha_{i,j-1,k}), f(\alpha_{i,j,k}), f(\alpha_{i,j+1,k}) \right),$$

$$\text{with } f(\alpha) = \frac{1}{\Delta t} \max \left(0, \frac{\alpha}{\gamma} - 1 \right)$$

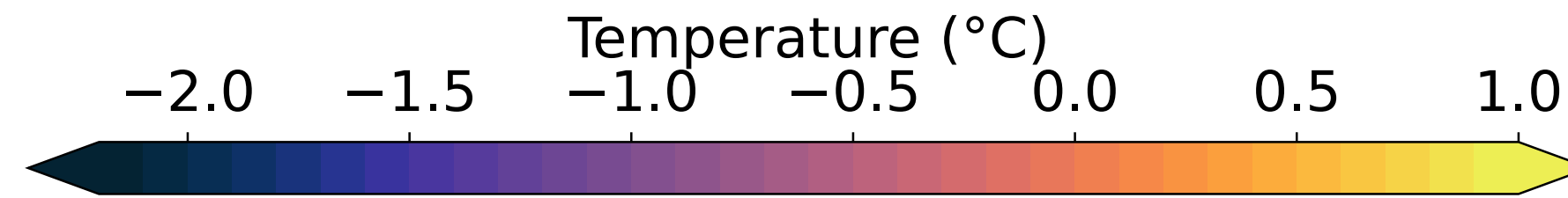
- Barotropic-baroclinic splitting:

$$\frac{1}{\tilde{H}} \sum_k \left(\sigma_k u_k \tilde{h}_k \right) = \left(\frac{1}{\tilde{H}} \sum_k \sigma_k \tilde{h}_k \right) u_b + \frac{1}{\tilde{H}} \sum_k \left(\sigma_k u'_k \tilde{h}_k \right)$$

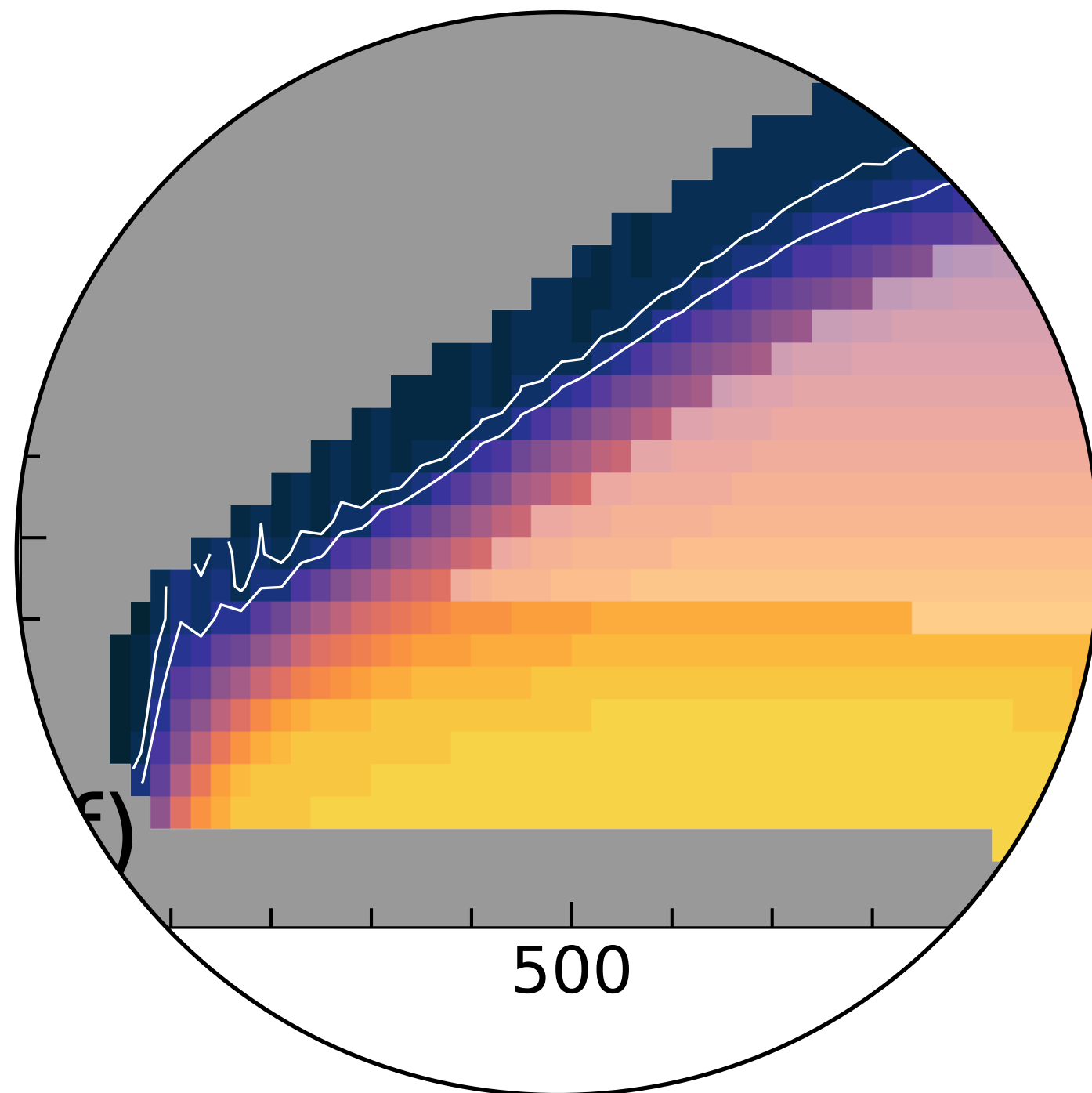
Smoothing with penalization reduces spurious mixing under ice-shelf



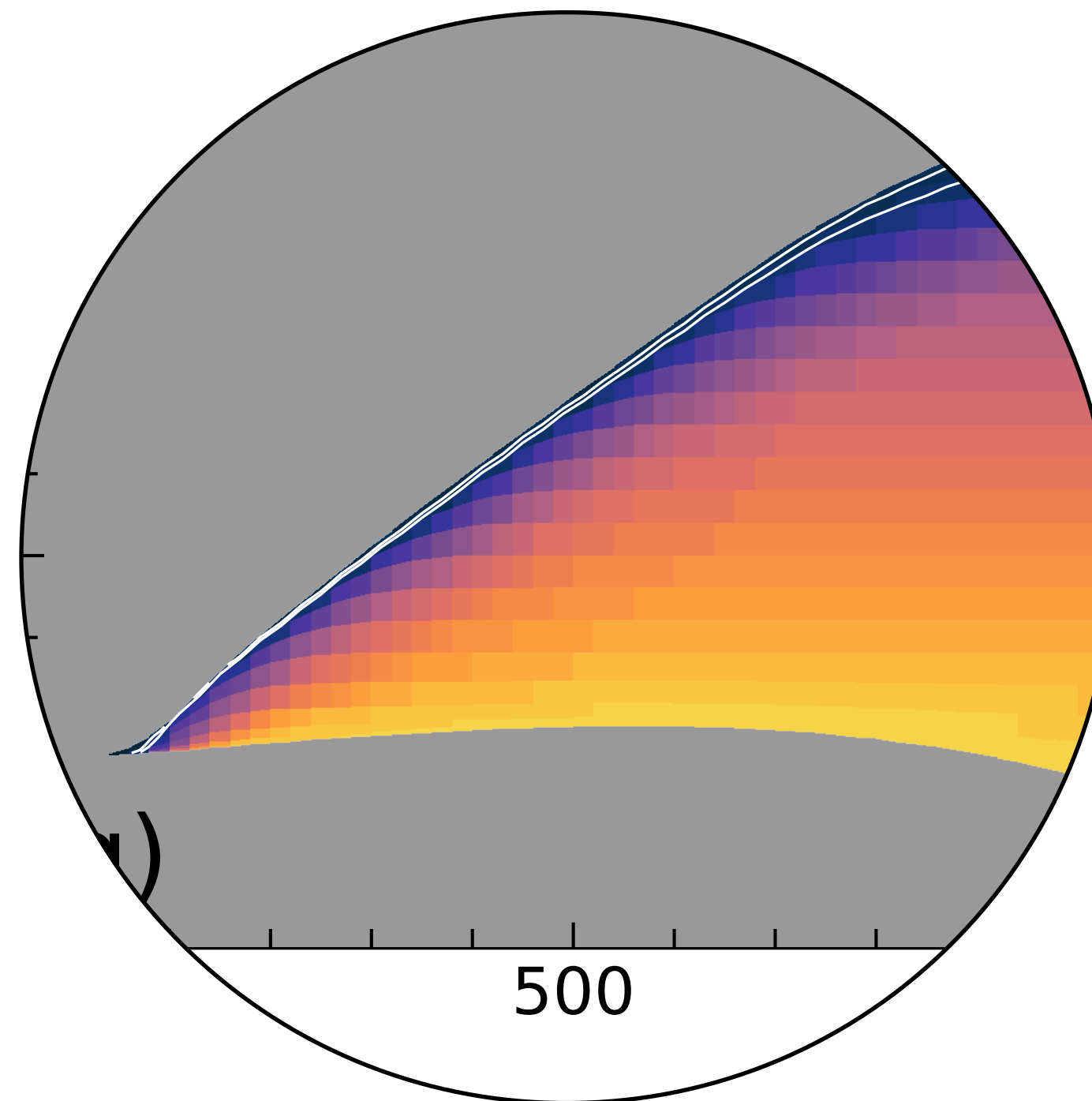
Smoothing with penalization reduces spurious mixing under ice-shelf



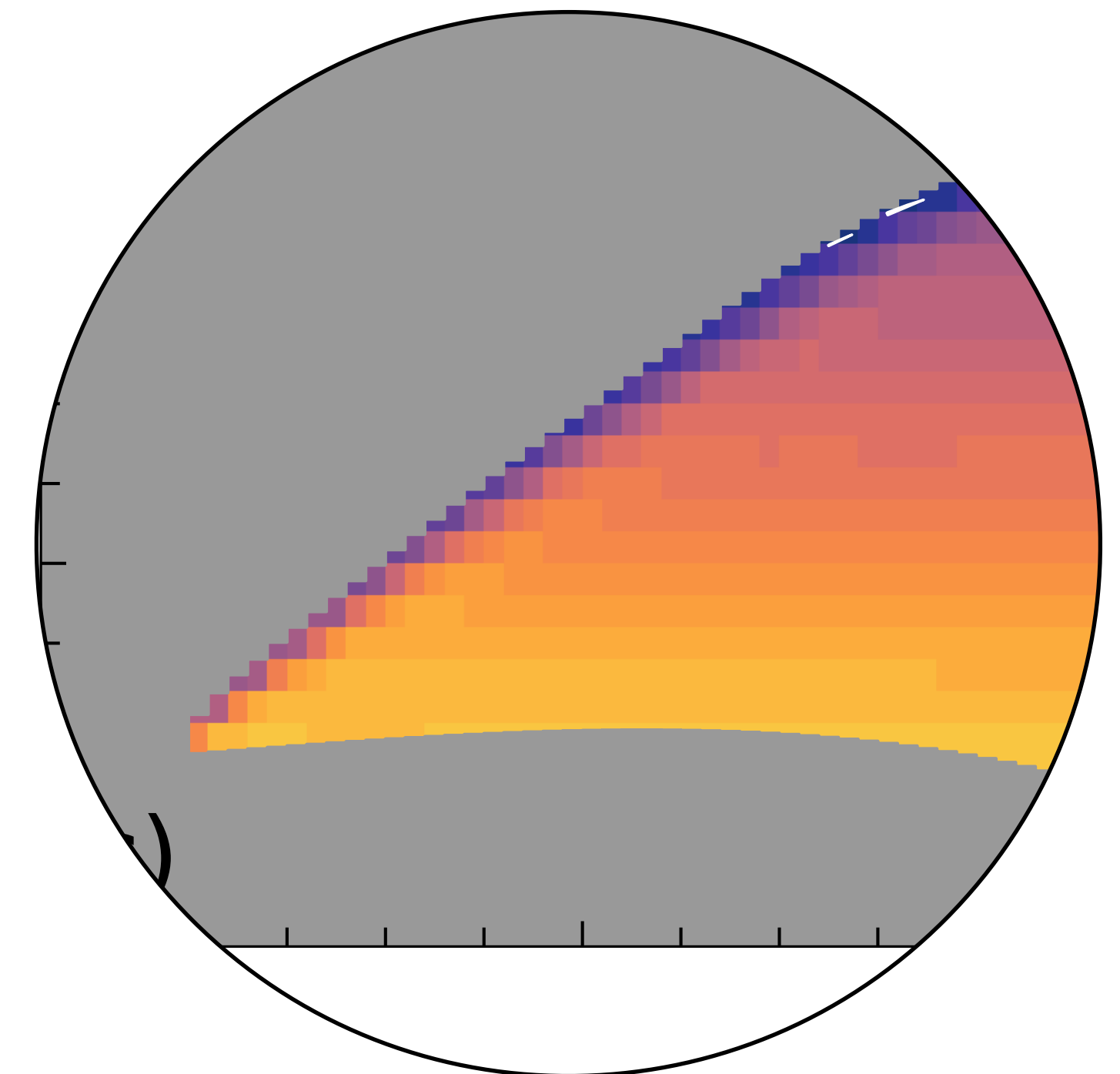
f) BVP(4)
Spatial grid



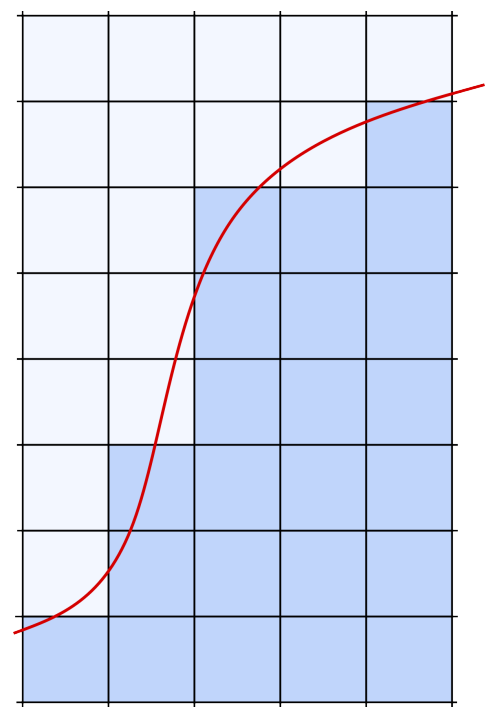
g) BVP(4)
Volumetric grid



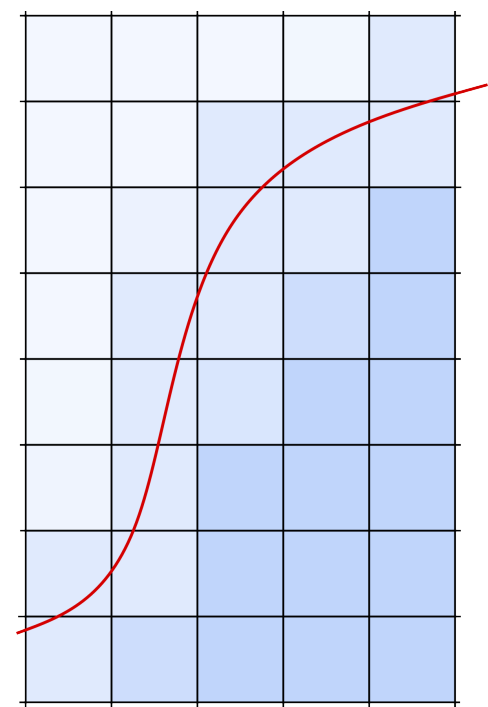
c) Partial cells
L20m



... and leads to smoother ocean currents and basal melt rates

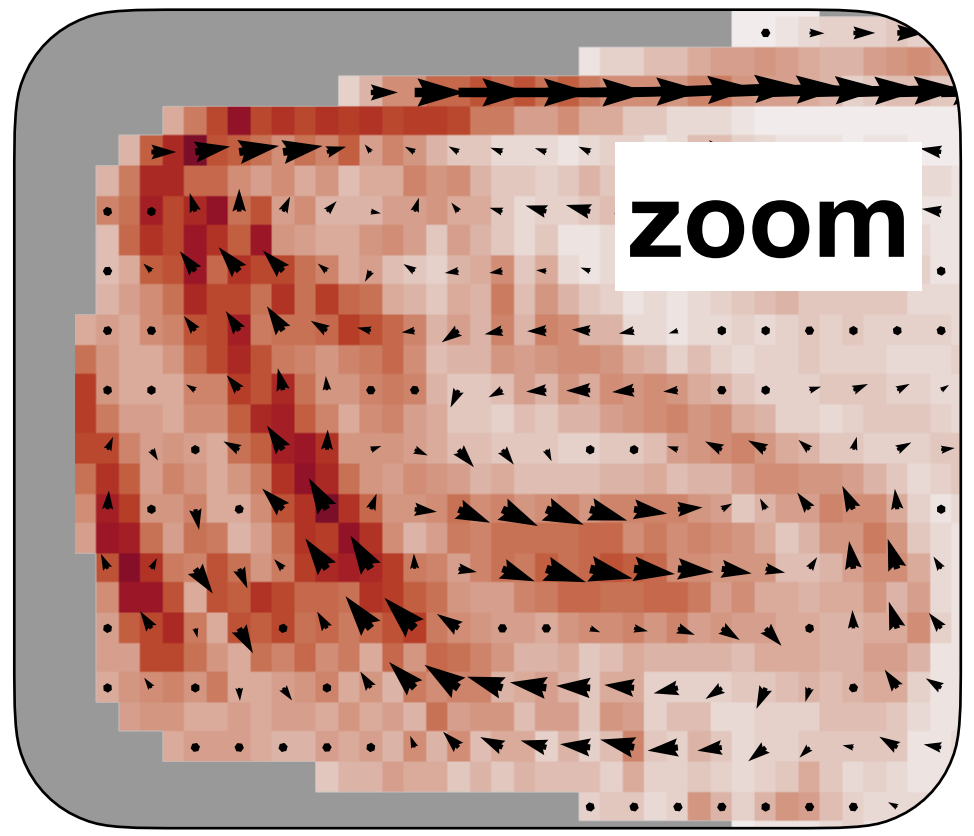
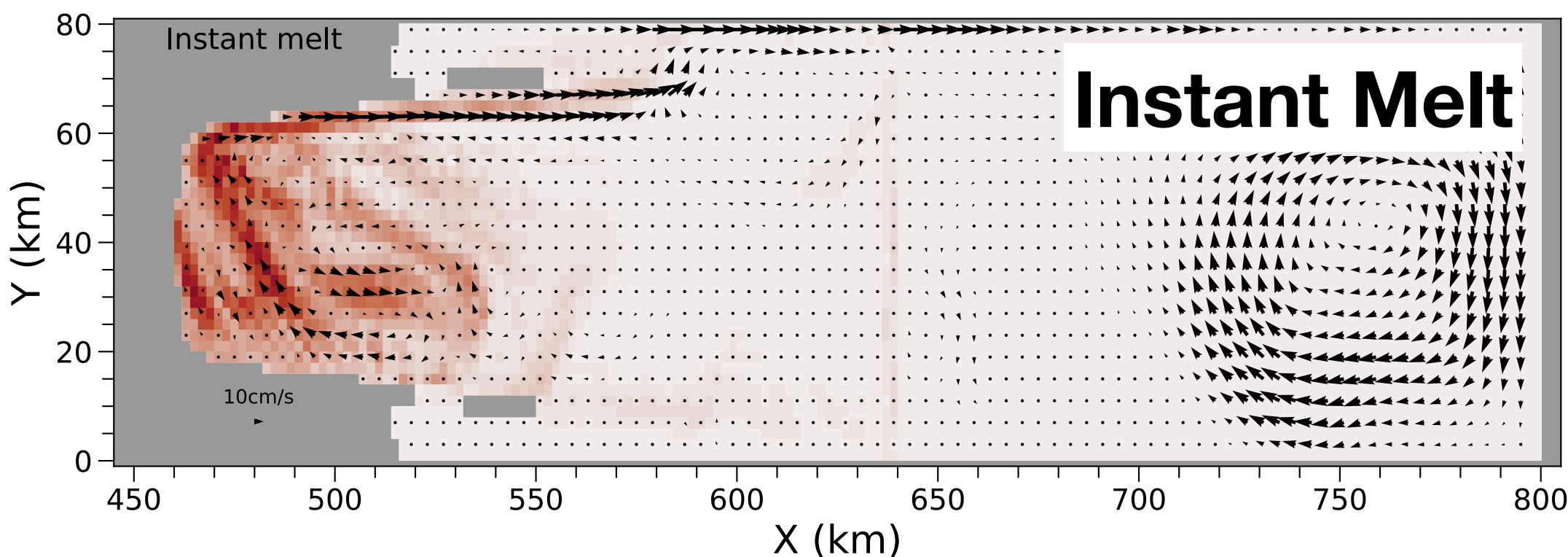
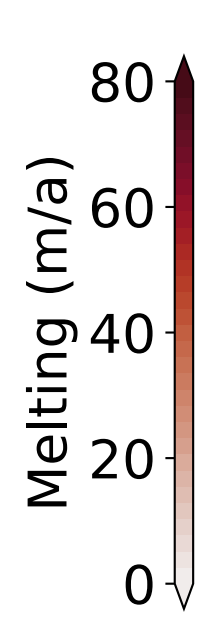


**Traditional
« lego-like »
topography**

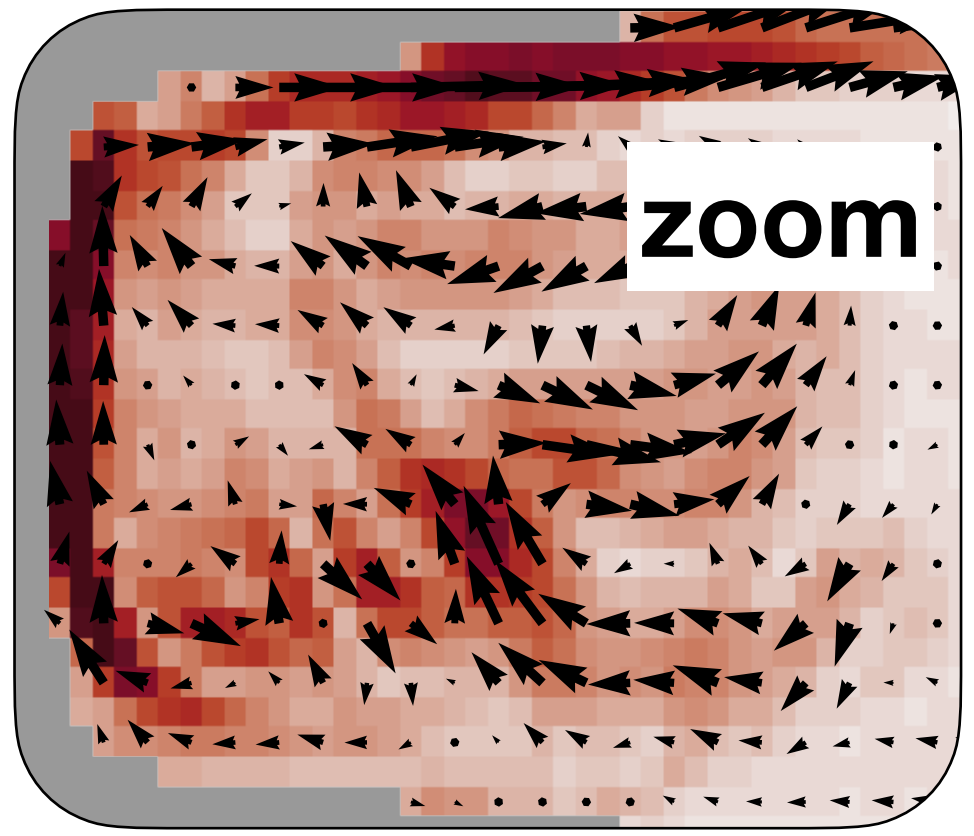
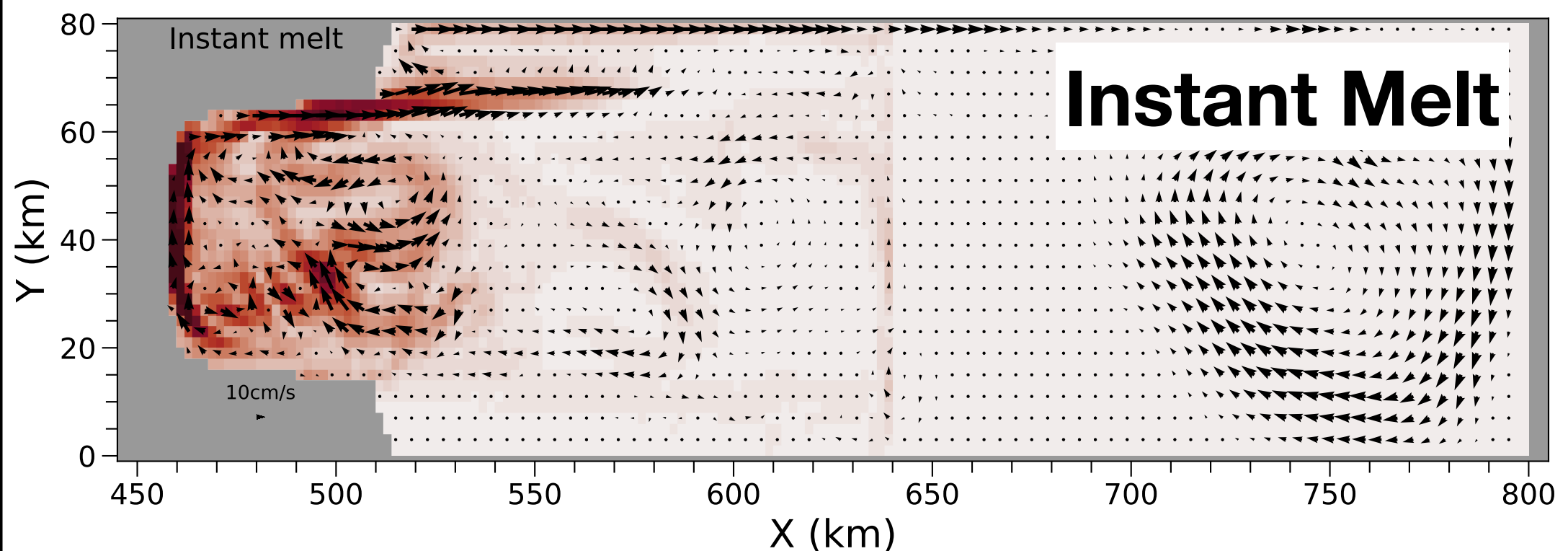


**Smoothed
Penalised
topography**

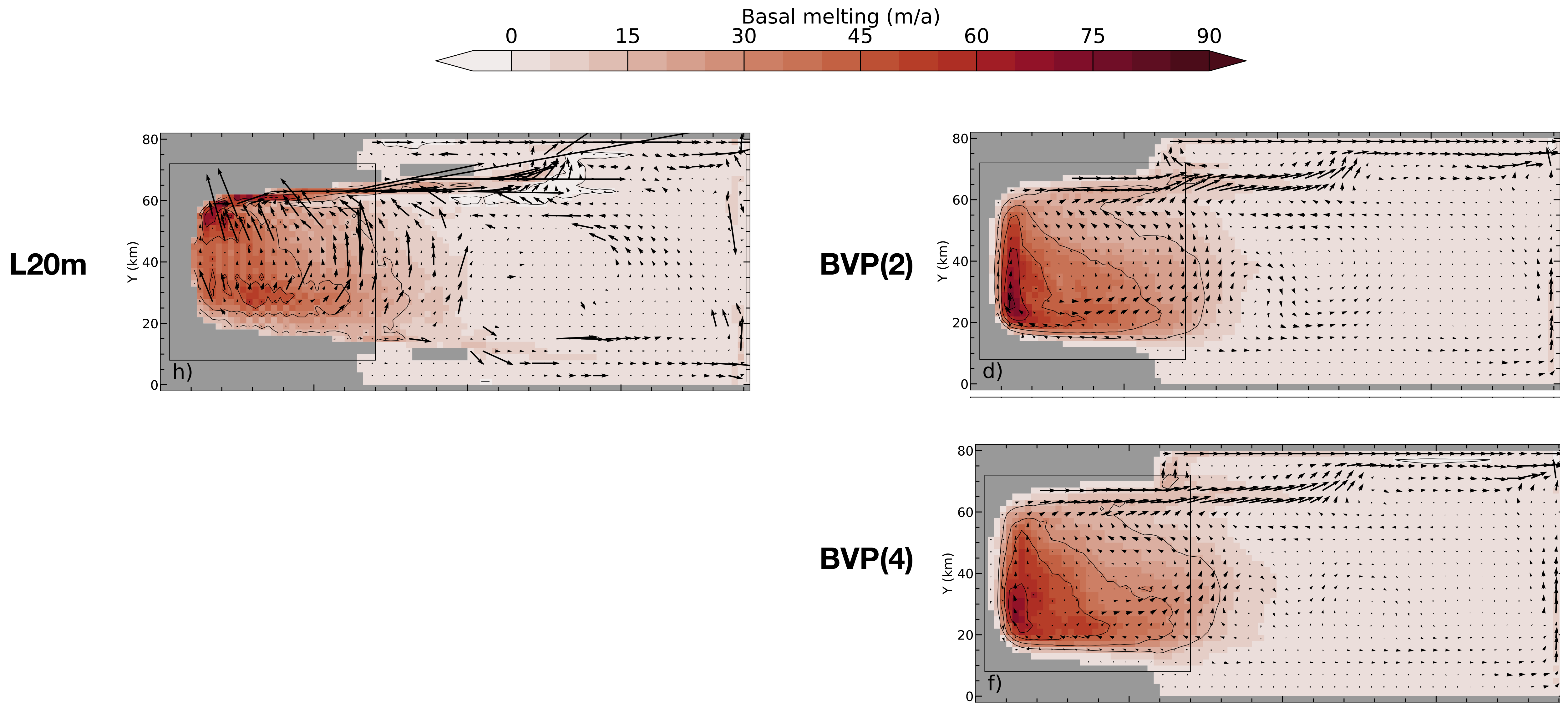
L20m



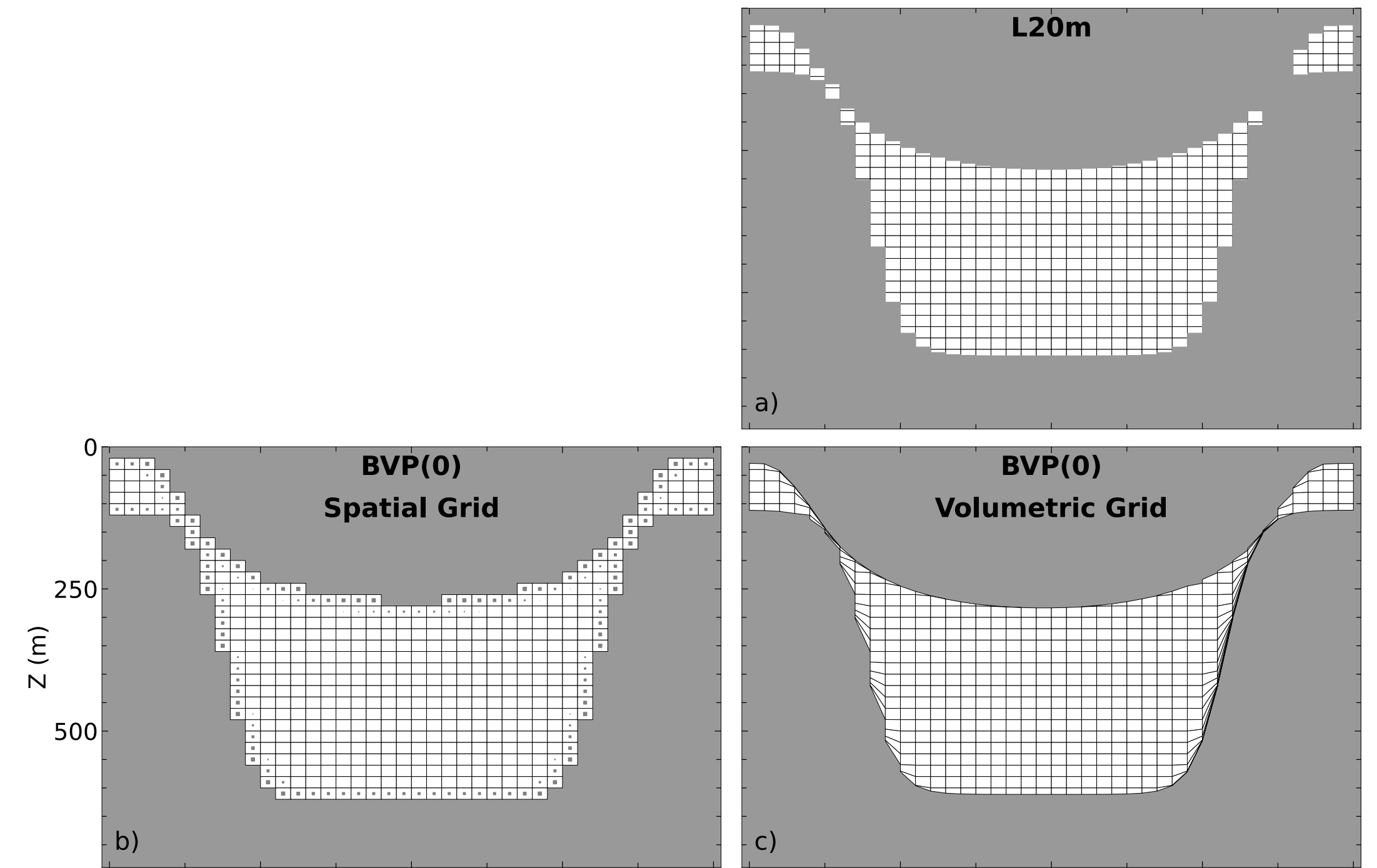
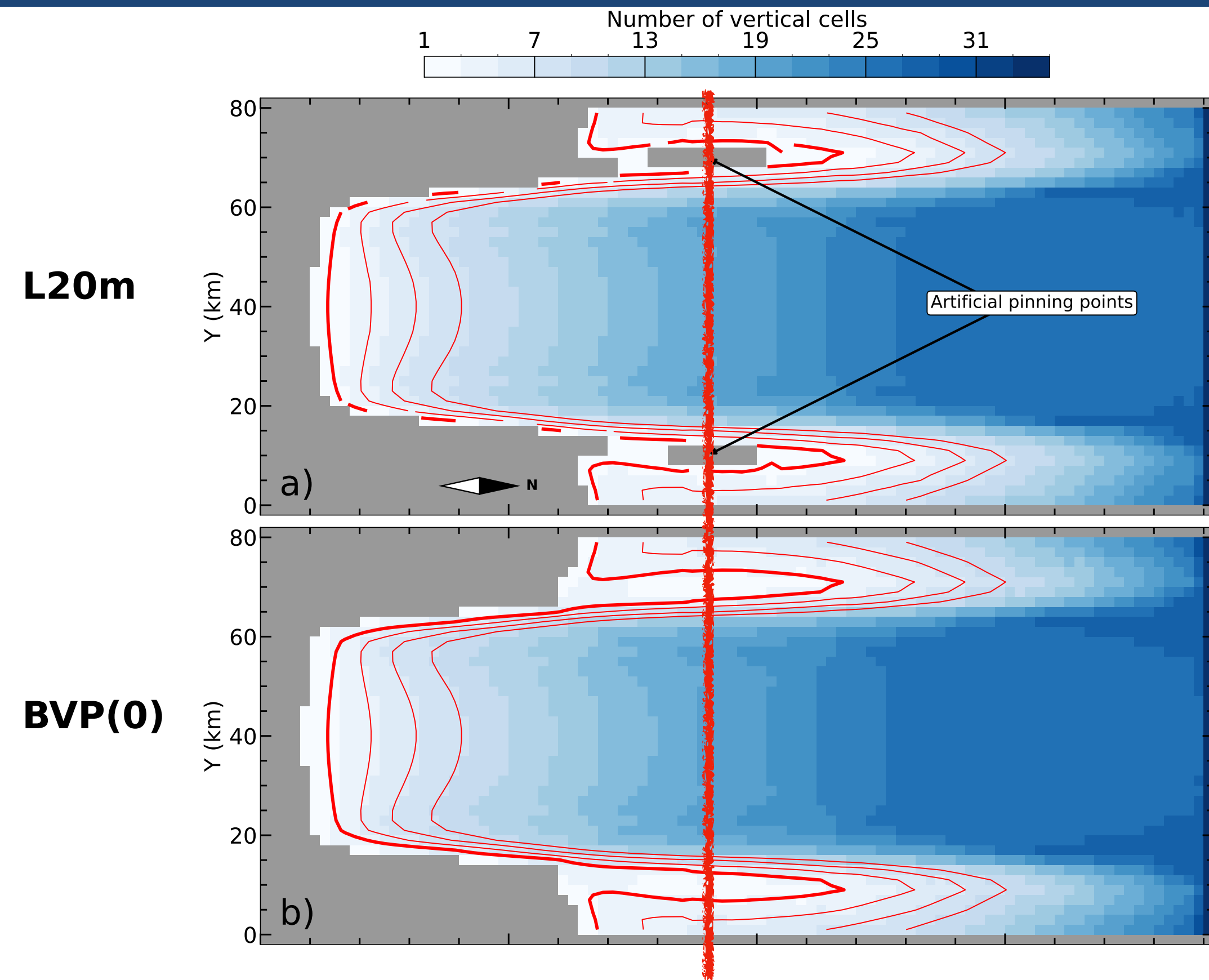
BVP(4)



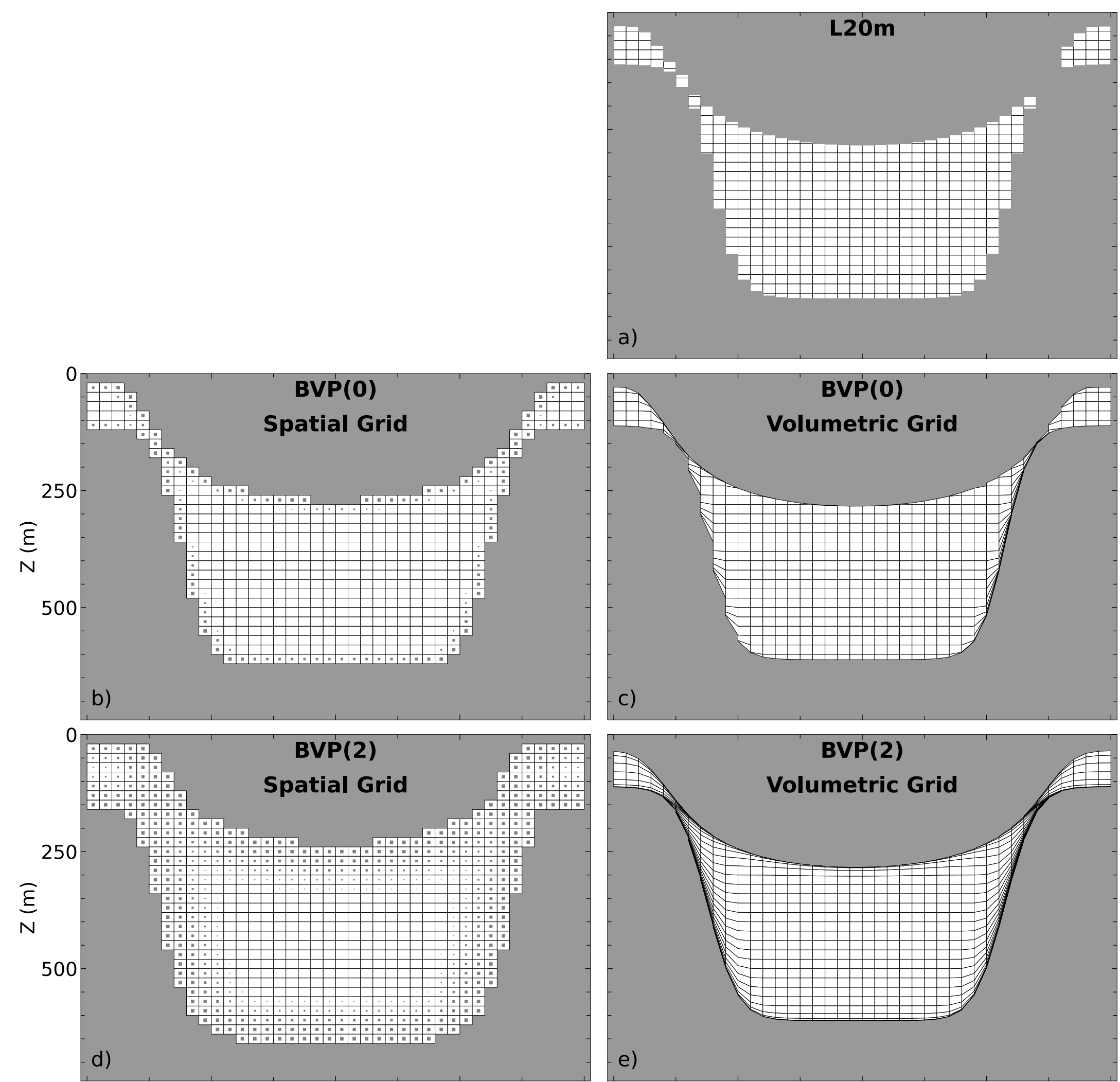
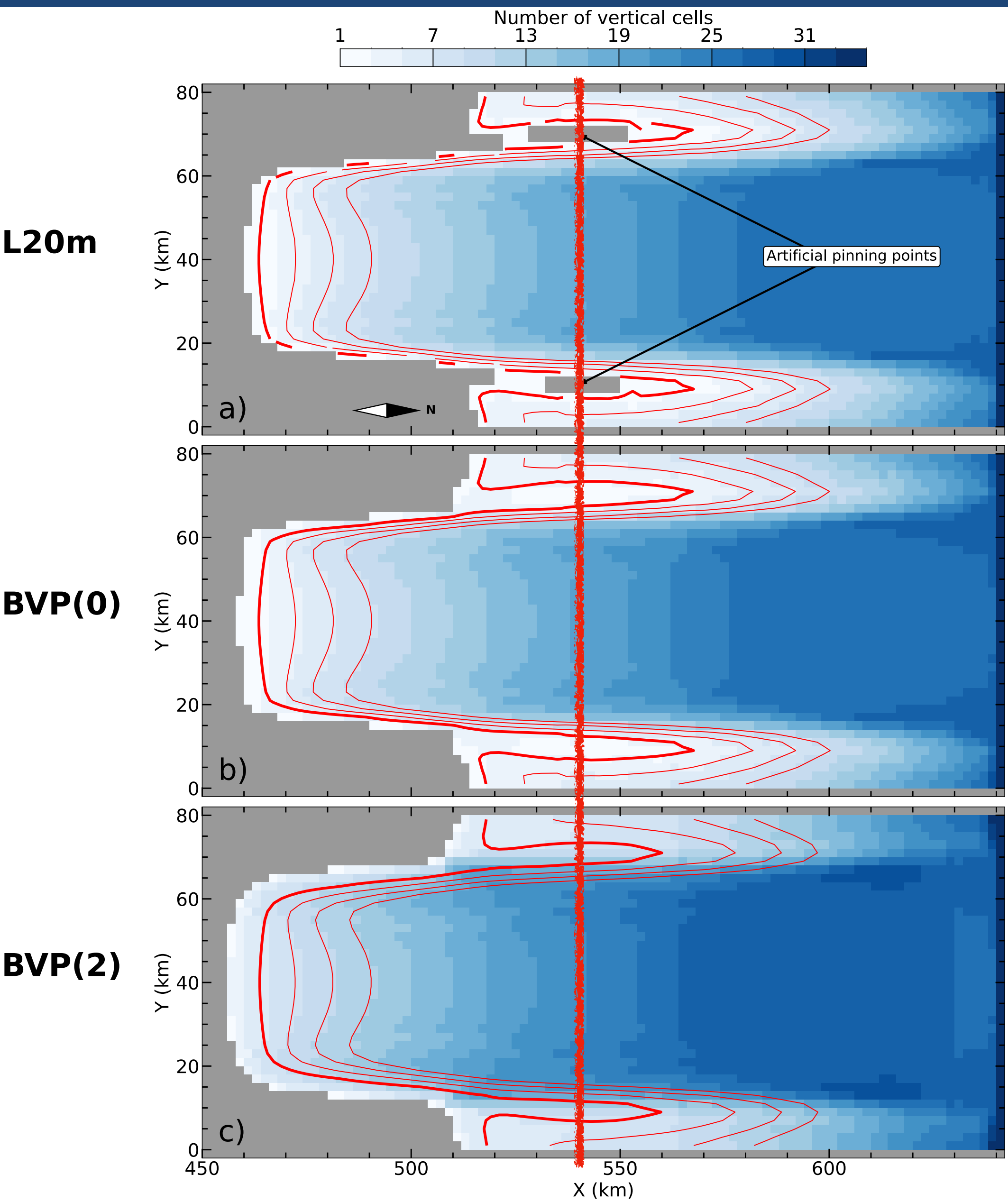
However, mean meltpatterns differs with more melt close to the GZ



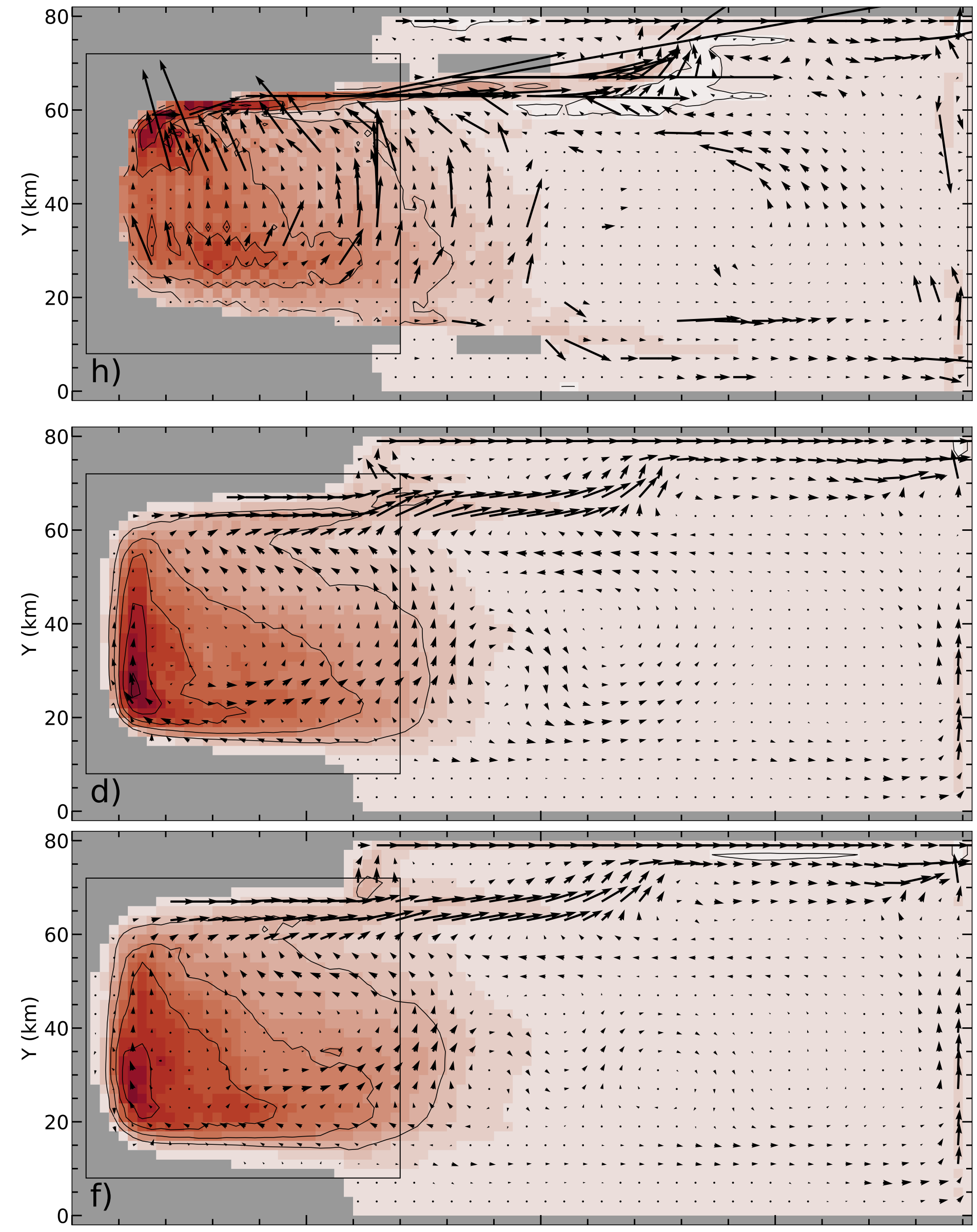
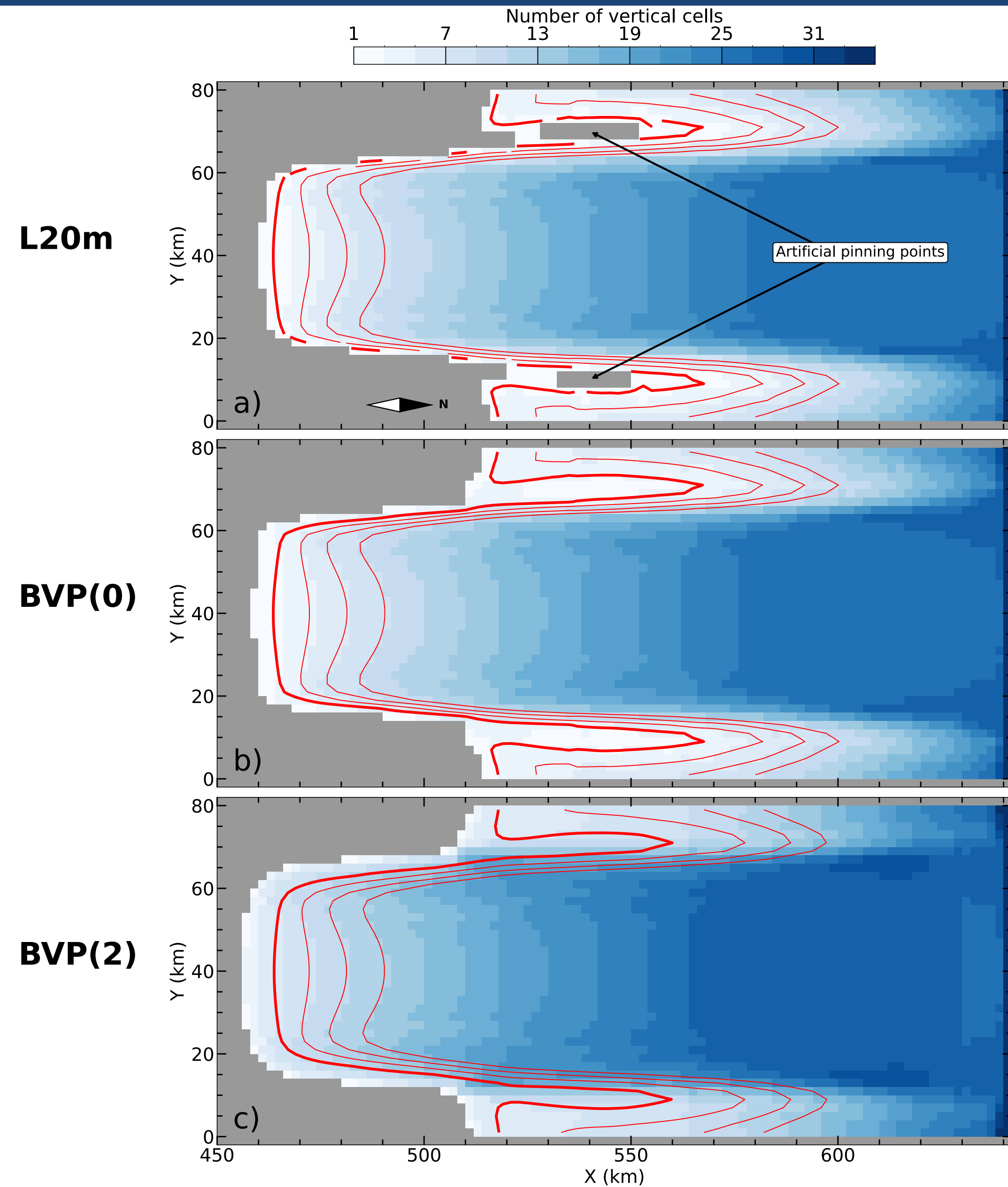
Smoothing with BVP increases the number of vertical levels at the GZ



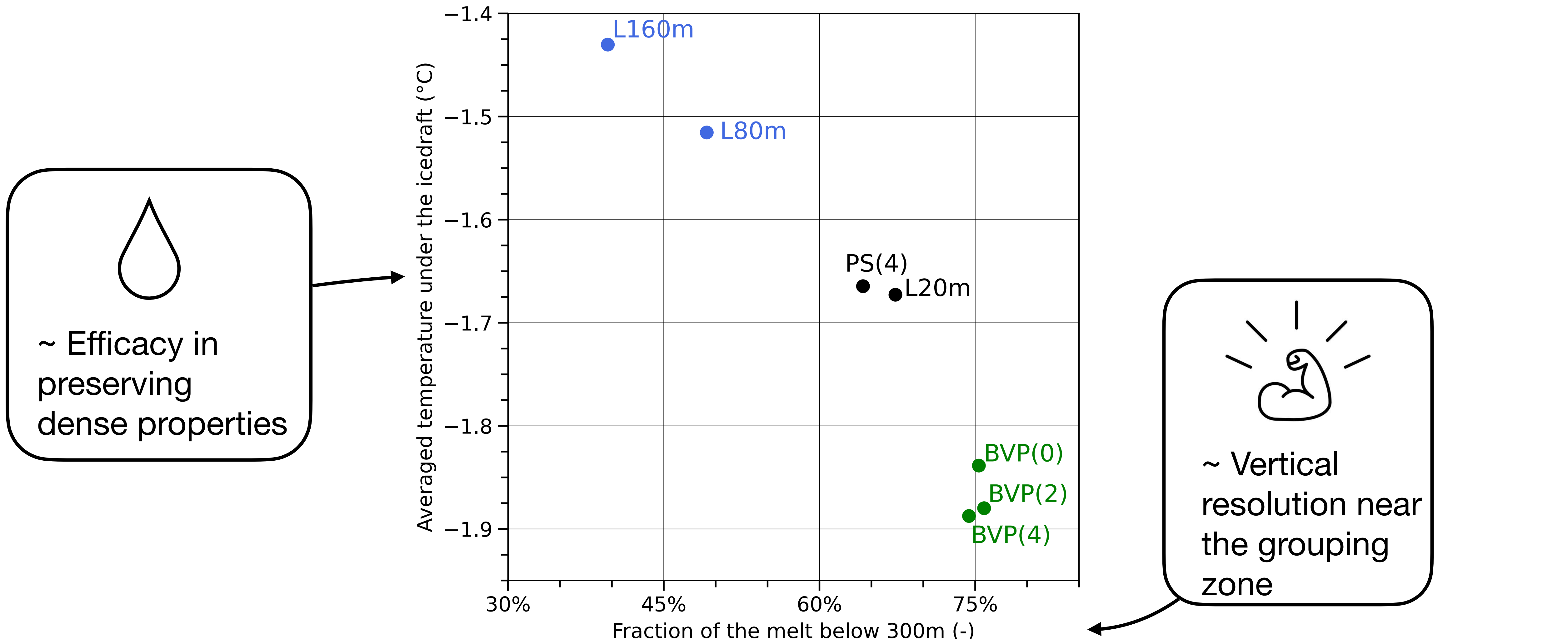
Smoothing with BVP increases the number of vertical levels at the GZ



Smoothing with BVP increases the number of vertical levels at the GZ



Smoothing with BVP reduces mixing and yields smoother currents

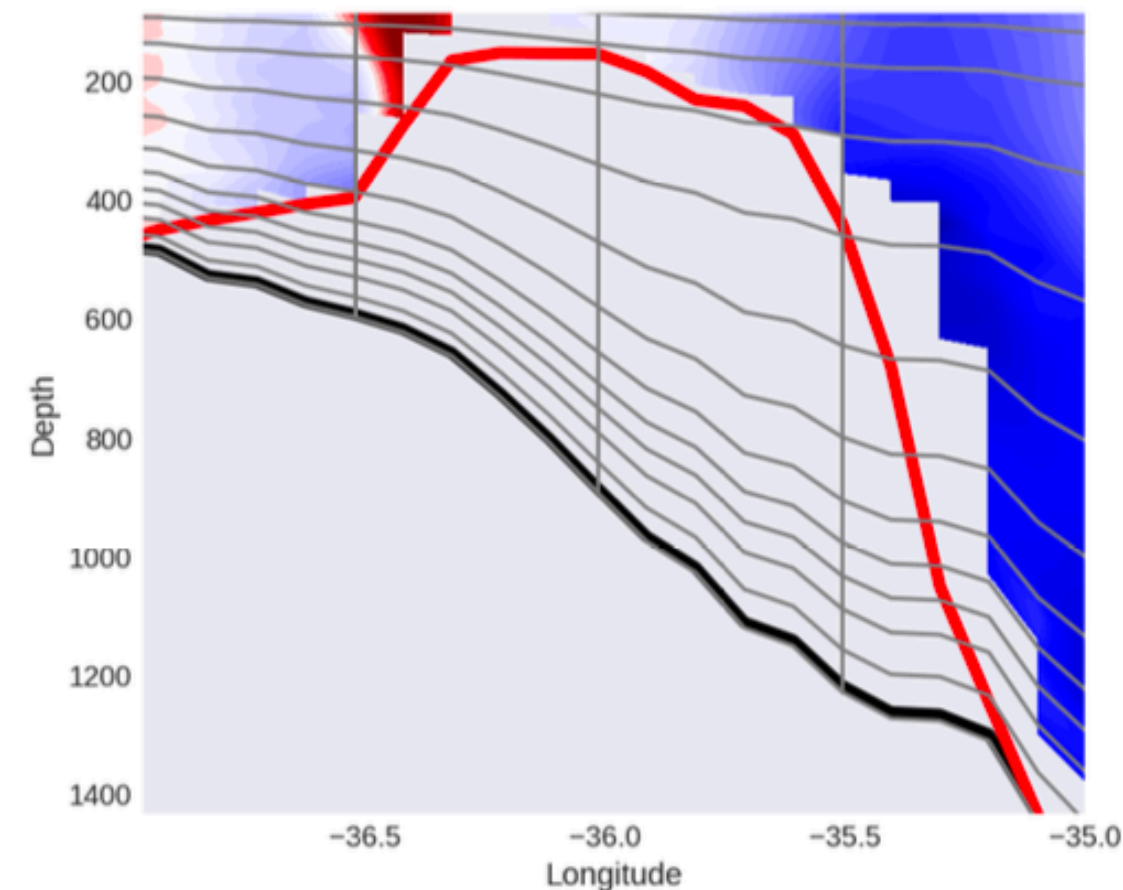


Conclusions

- We developed the BVP methods for ice-ocean interactions
- A few passes of smoothing of the topography **reduces spurious mixing** and yields **smoother basal melt rates and ocean currents** under the cavity.
- Averaged melt patterns and circulation under the cavity are sensitive to the vertical resolution near the grounding zone
- At least 4 vertical levels are needed to resolve complex grounding line

Perspective: Combining the benefits of terrain-following approach with BVP

Coarser horizontal resolution



Finer horizontal resolution

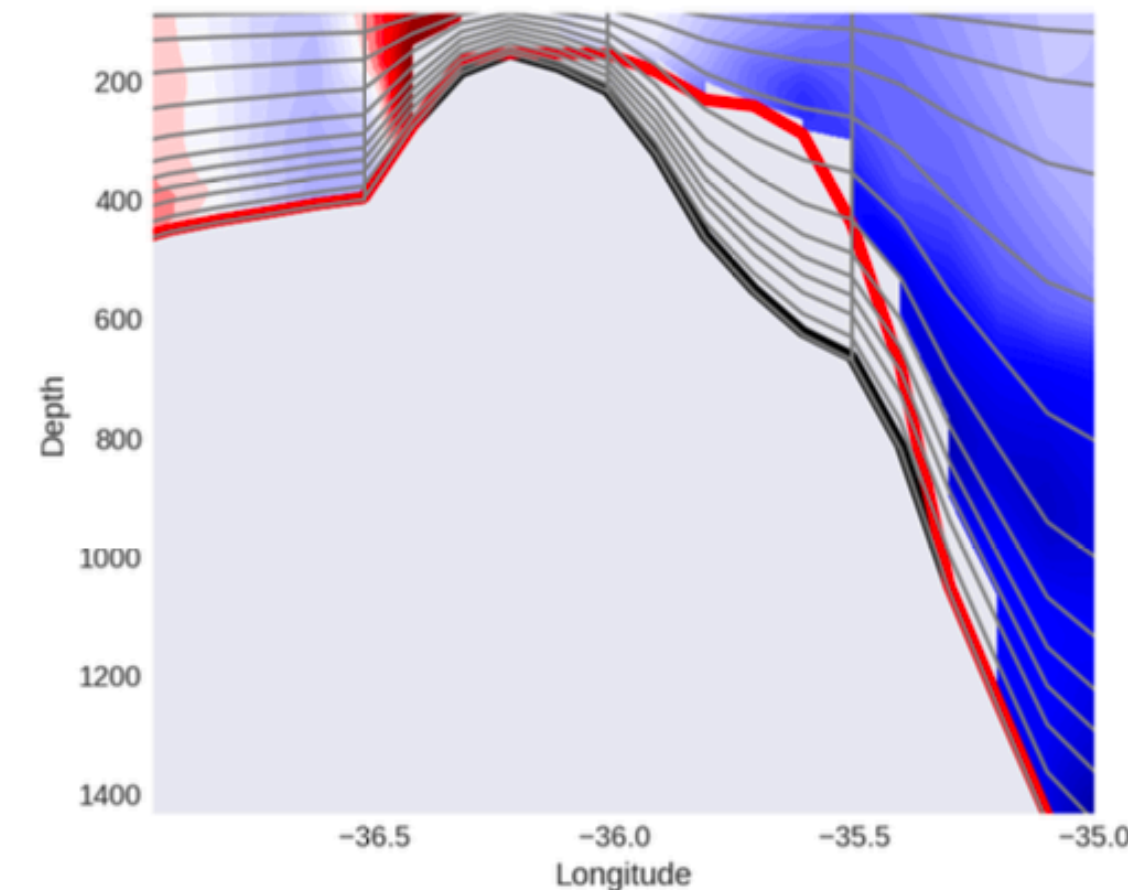


Figure: « z on top of s » coordinate
From Colombo (2018).

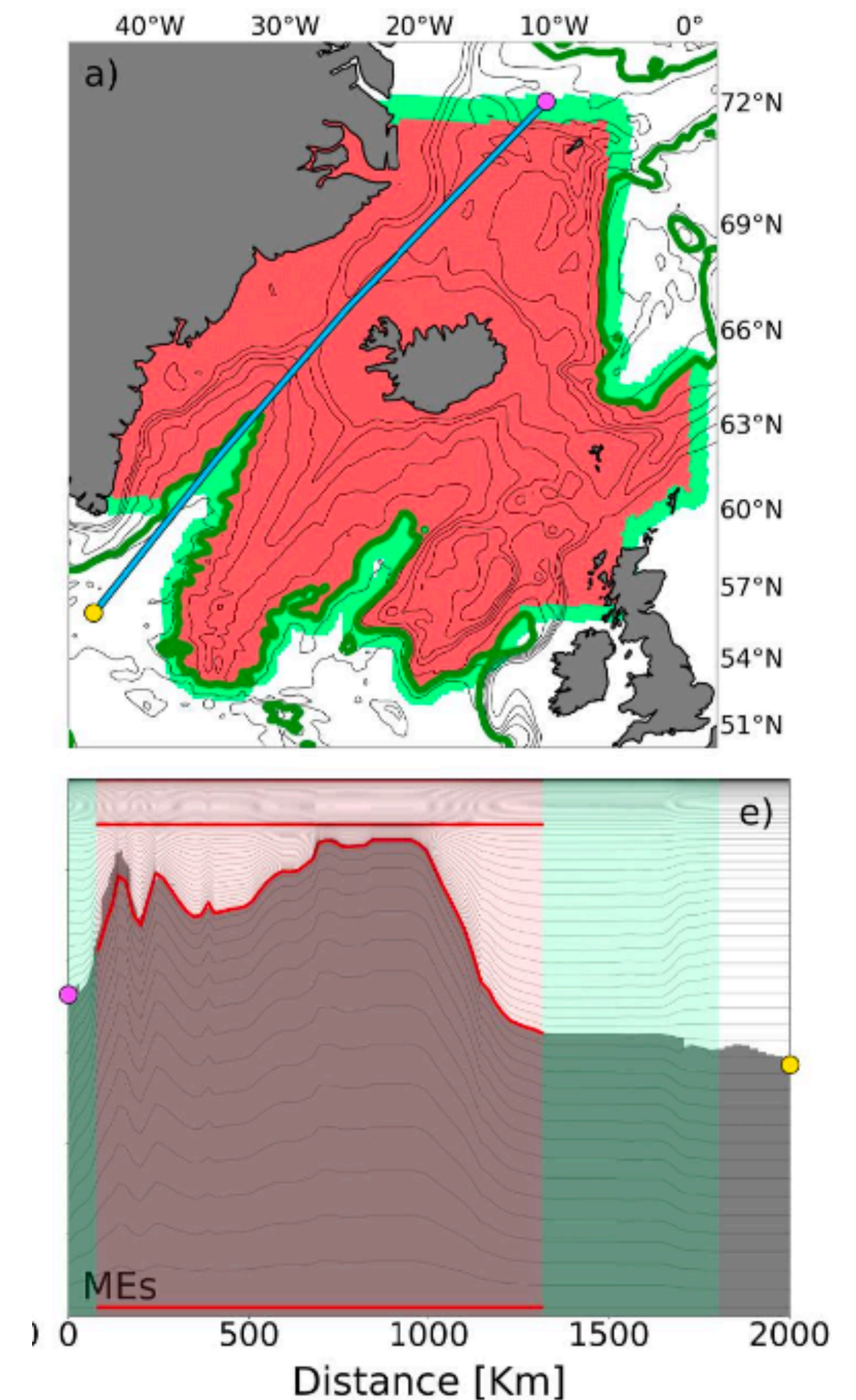
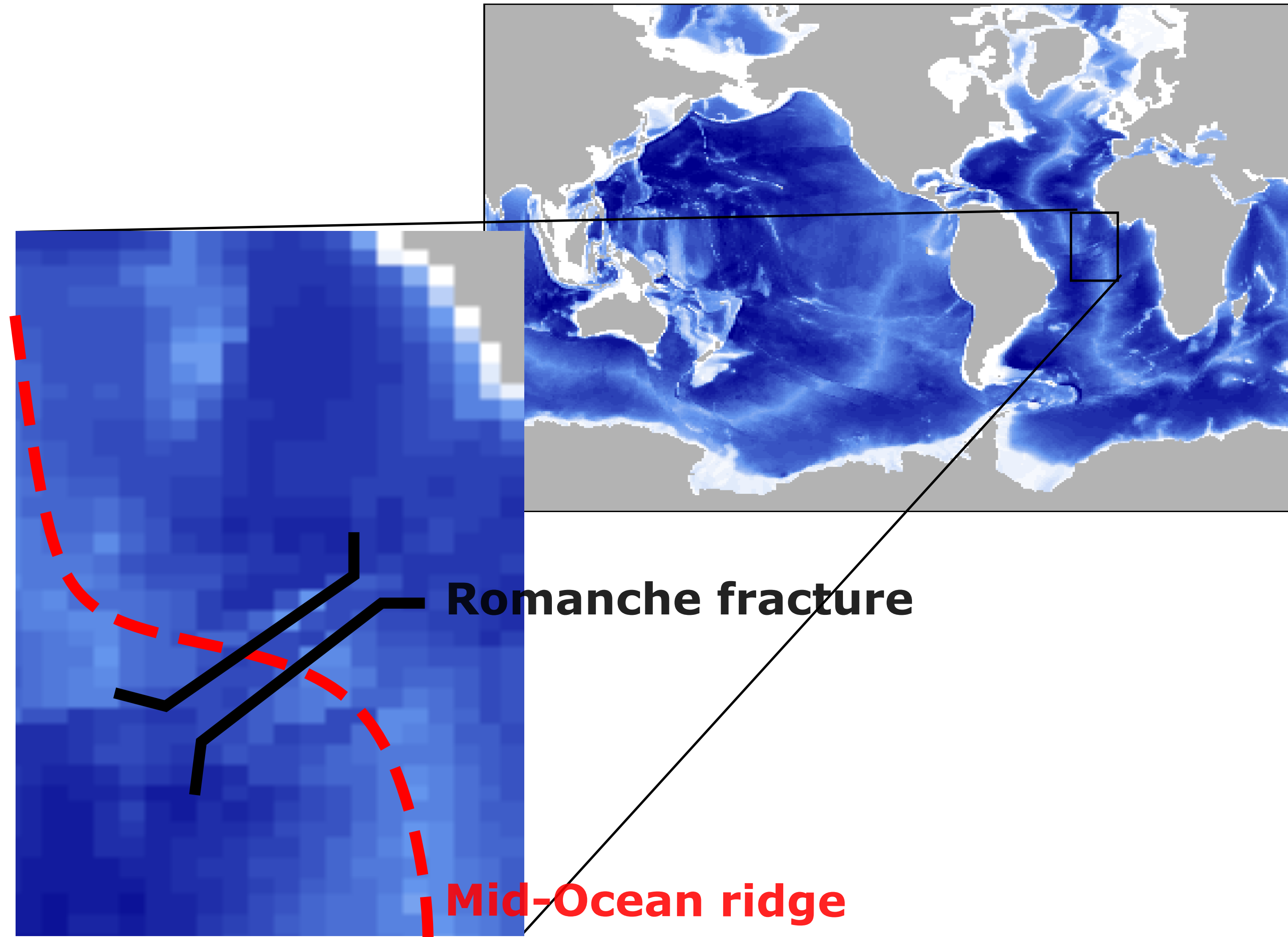


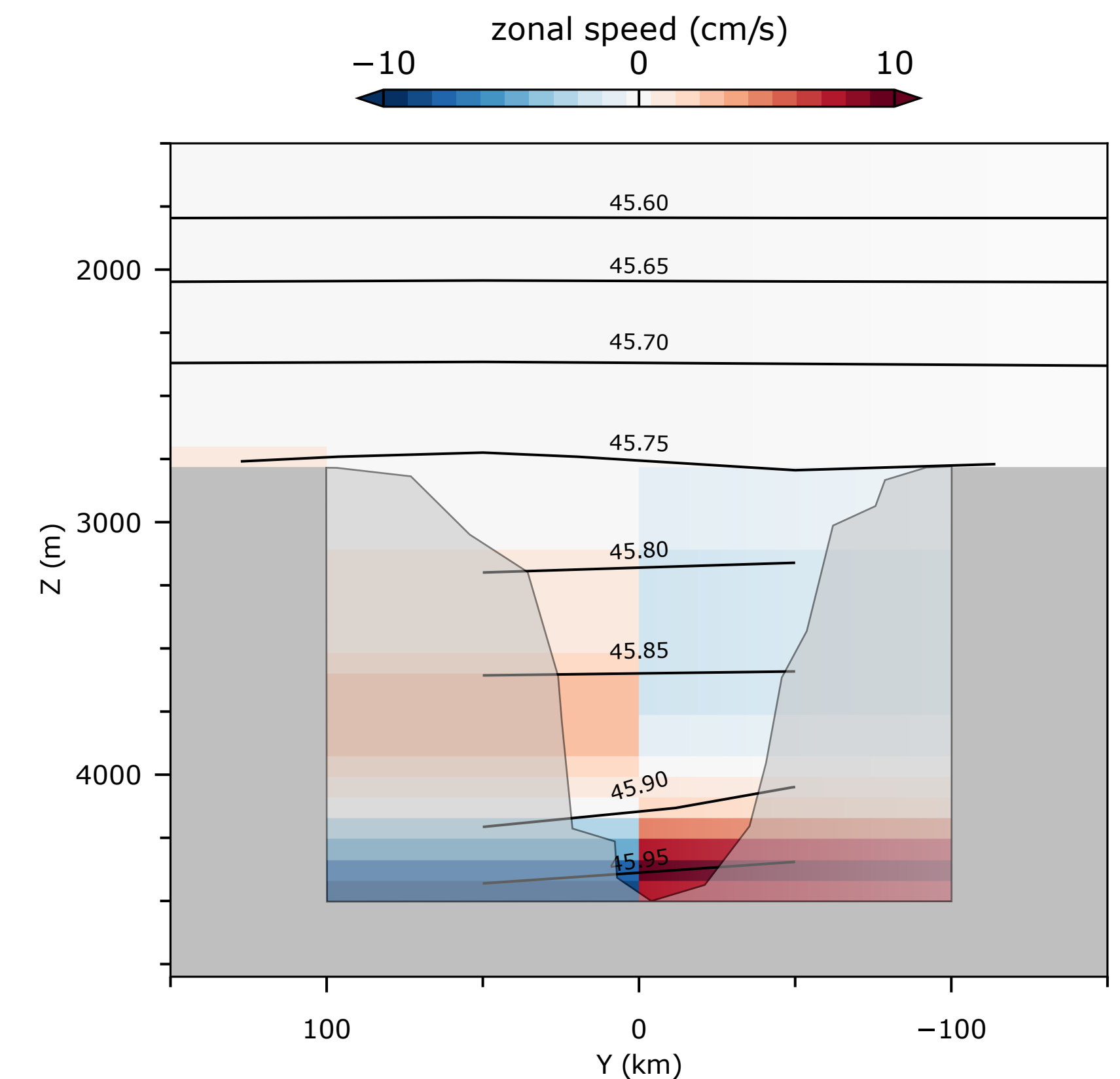
Figure: Multi-Enveloppes approach
From Bruciaferri et al. (2024).

Perspective: Capturing bottom throughs with the Penalisation



Physical width of Romanche fracture $\sim 20km$

Usual spatial resolution of climate models near tropics $\sim 100km$



Perspective: Using of BVP to represent moving geometries of the cavity

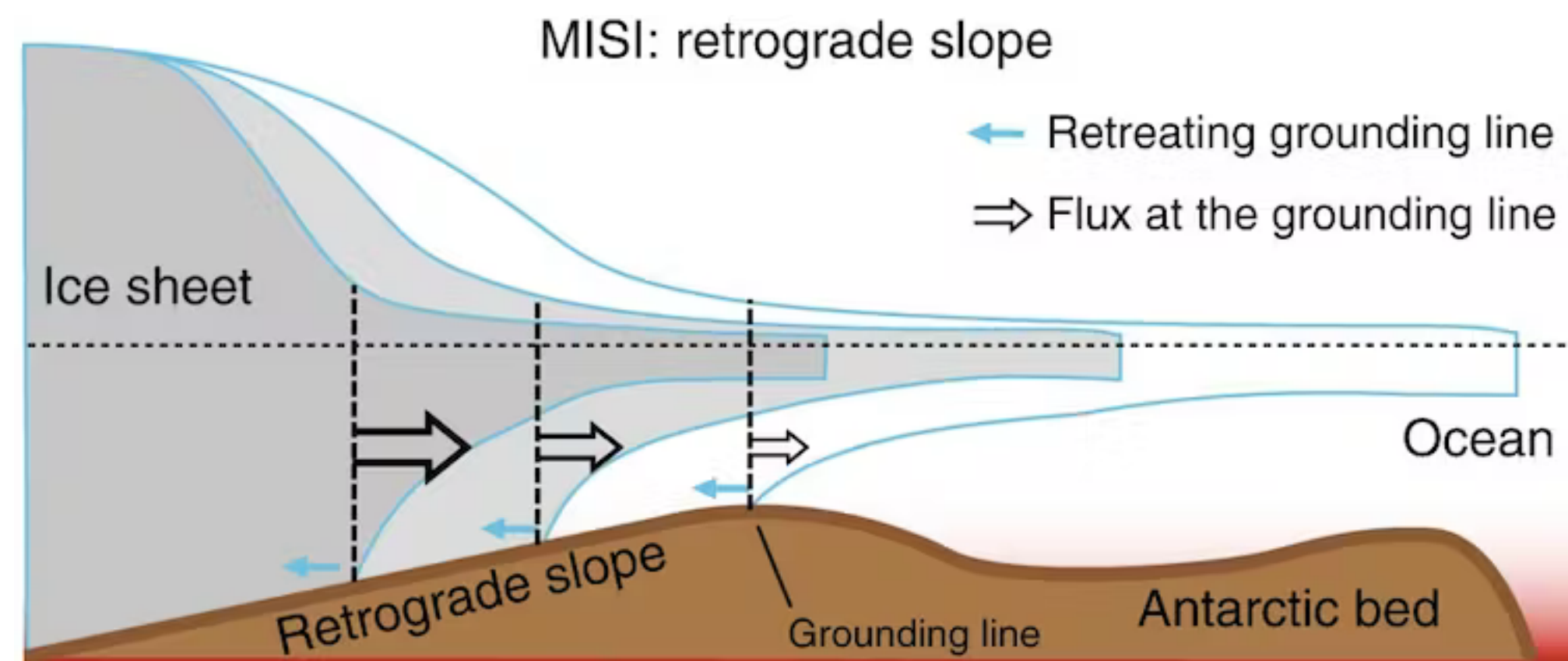


Figure: Marine Ice Sheet Instability (MISI).
From Pattyn (2018)

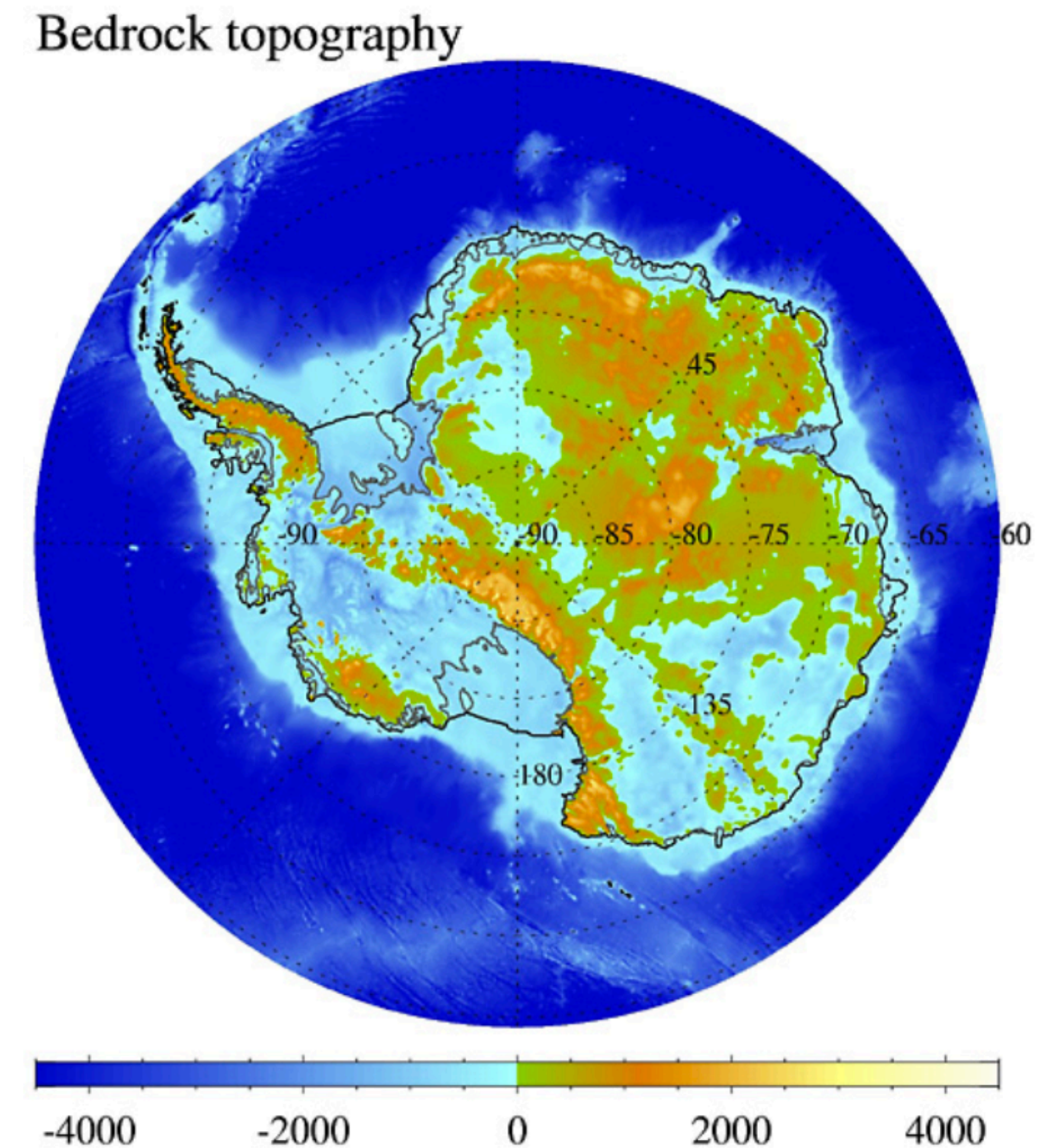


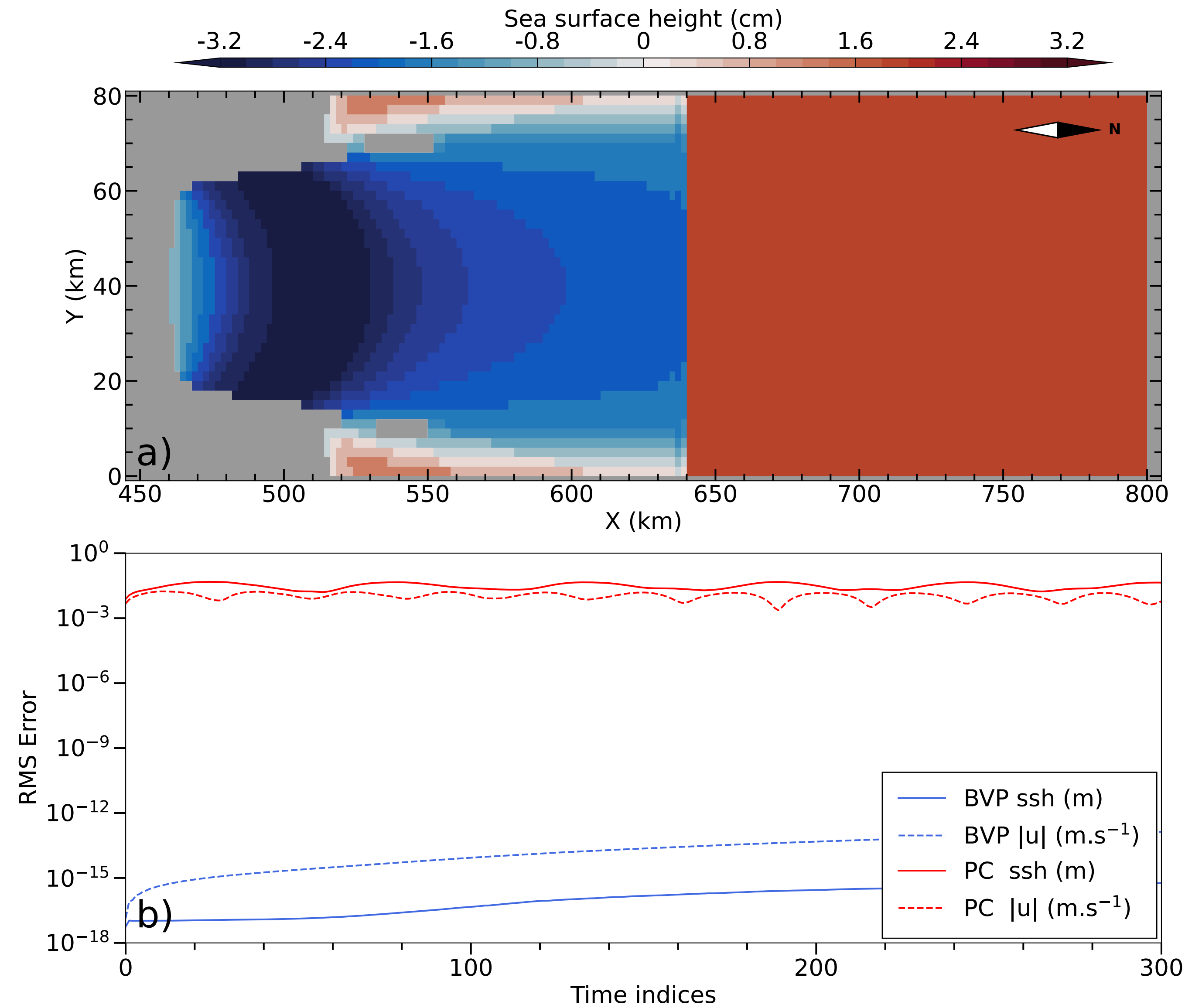
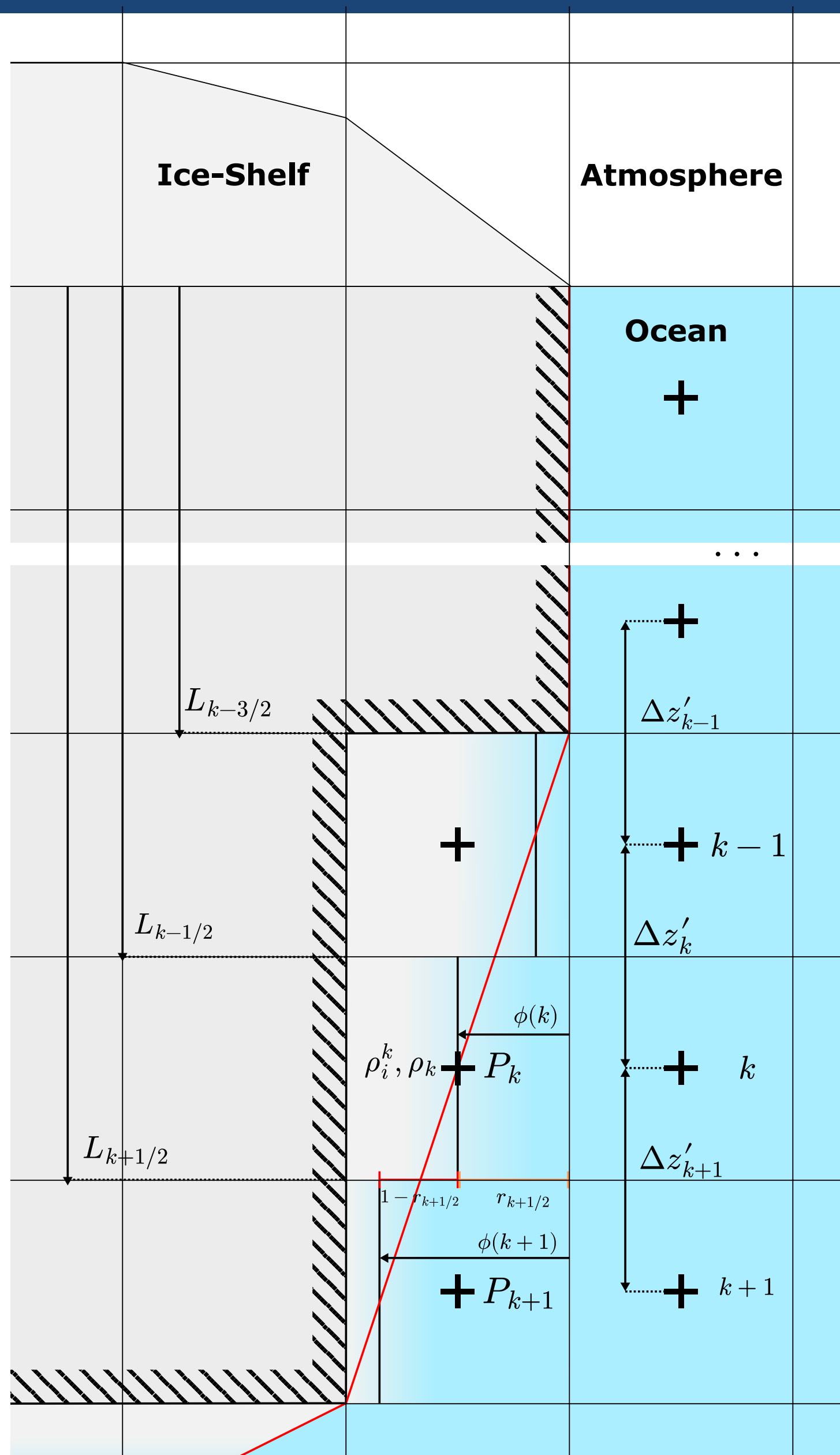
Figure: Antarctic subset of the bedrock topography.
From Timmermann et al. (2010)

Thank you!
antoine.nasser@princeton.edu

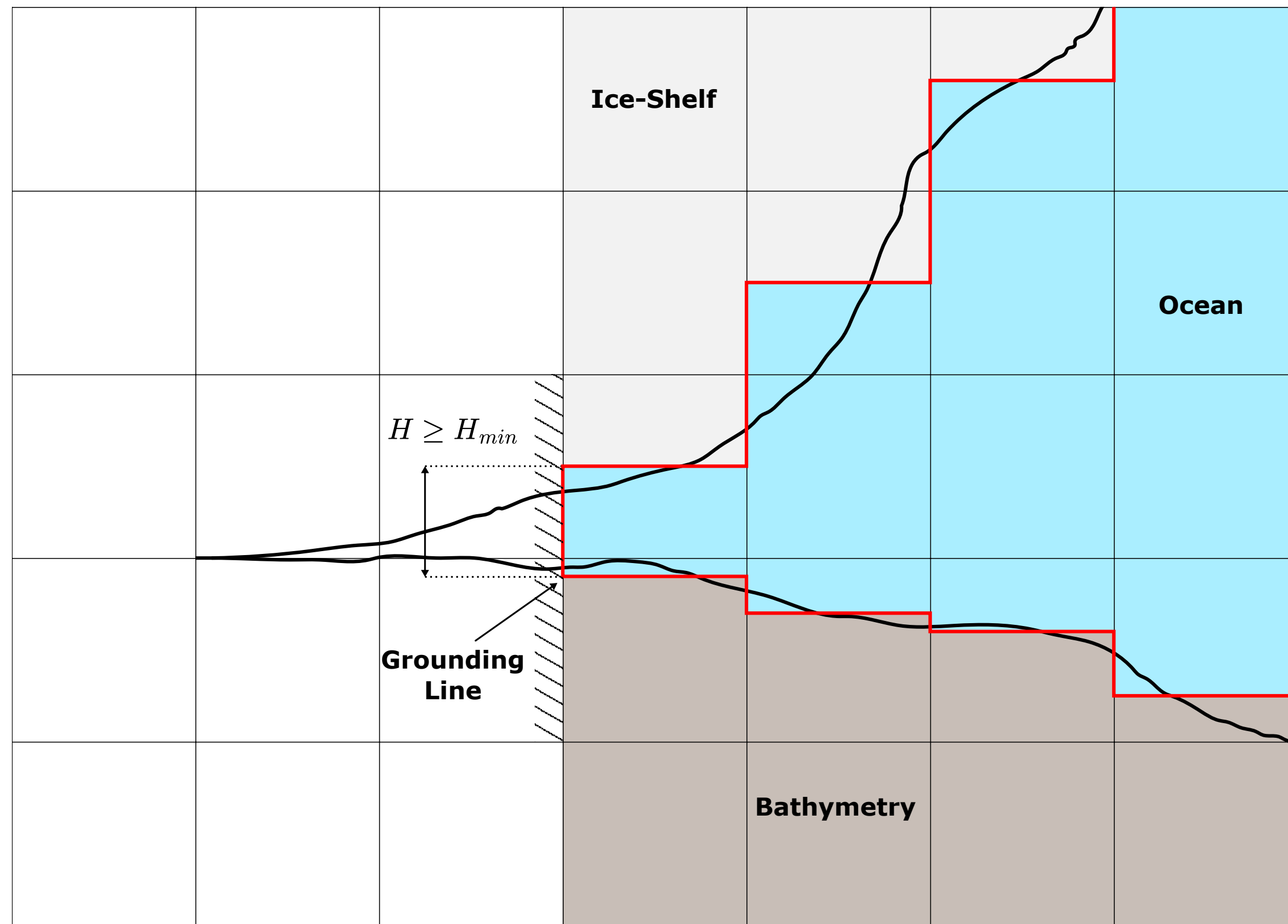


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N° 101003536.

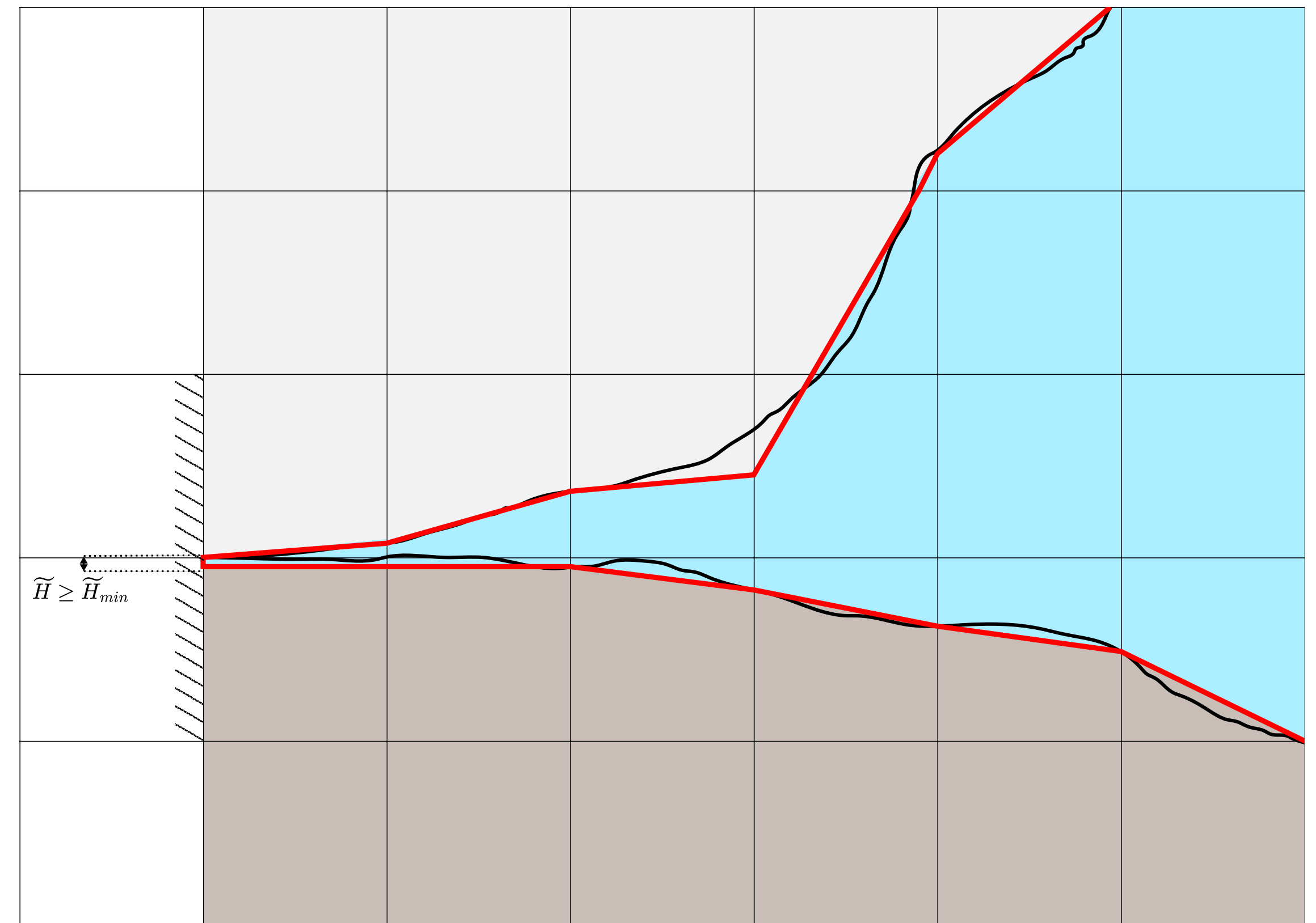
Appendix: HPG errors removed with BVP



Appendix: 2dz poorly resolves grounding line

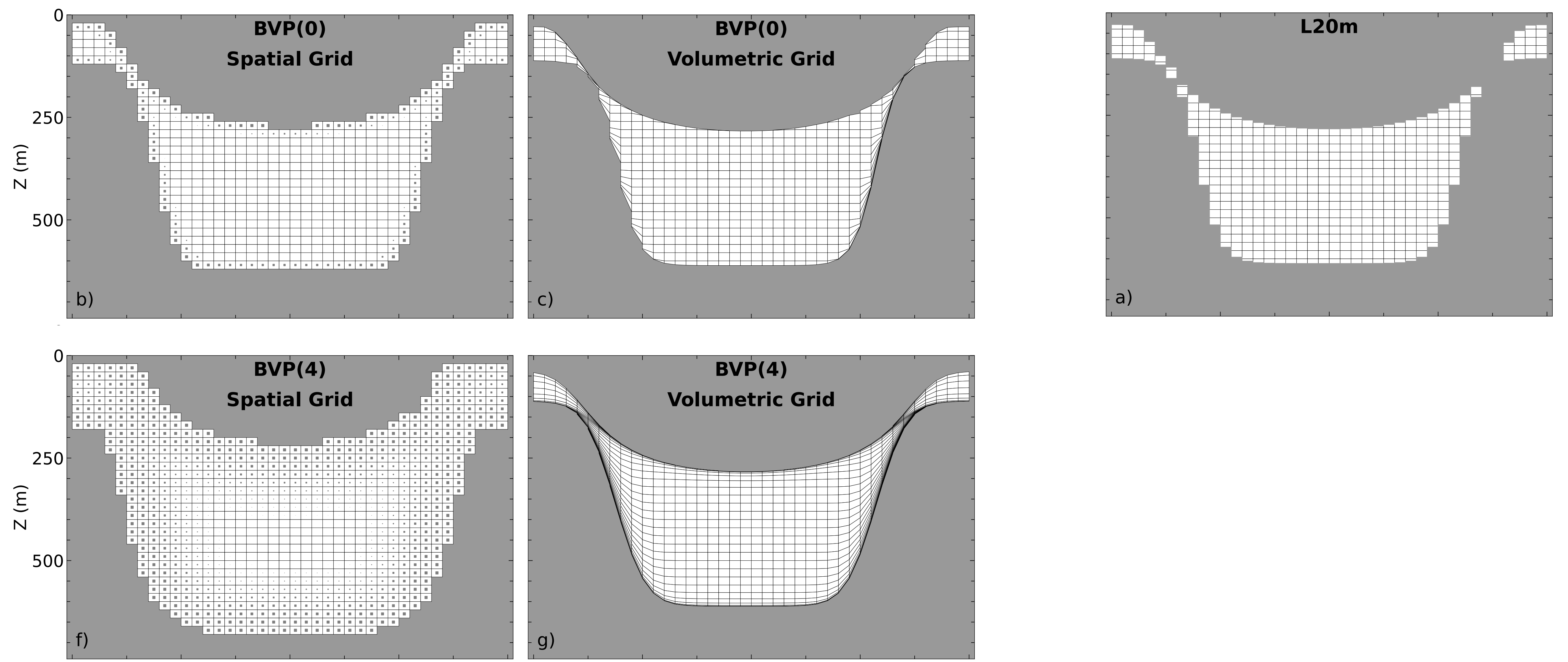


a)

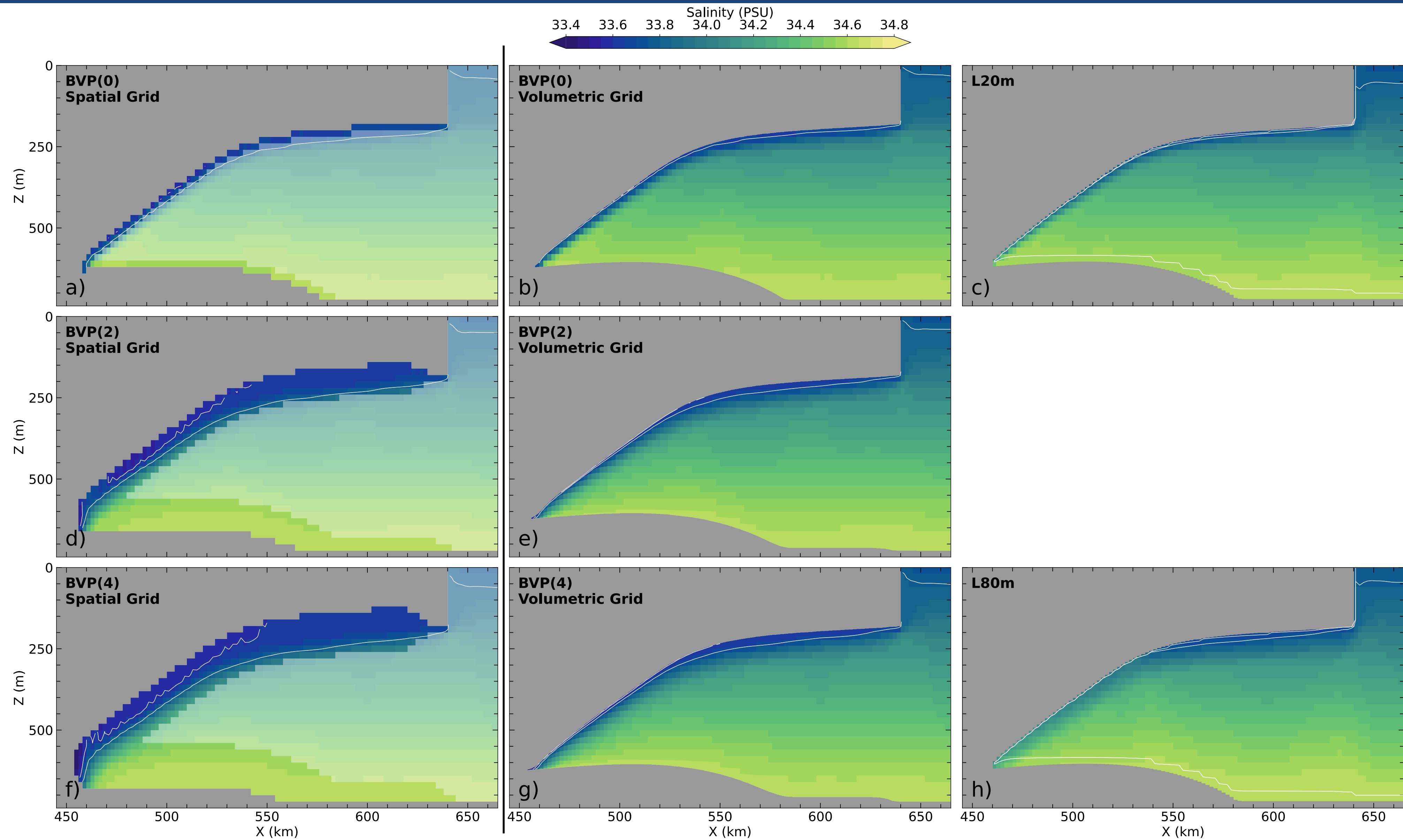


b)

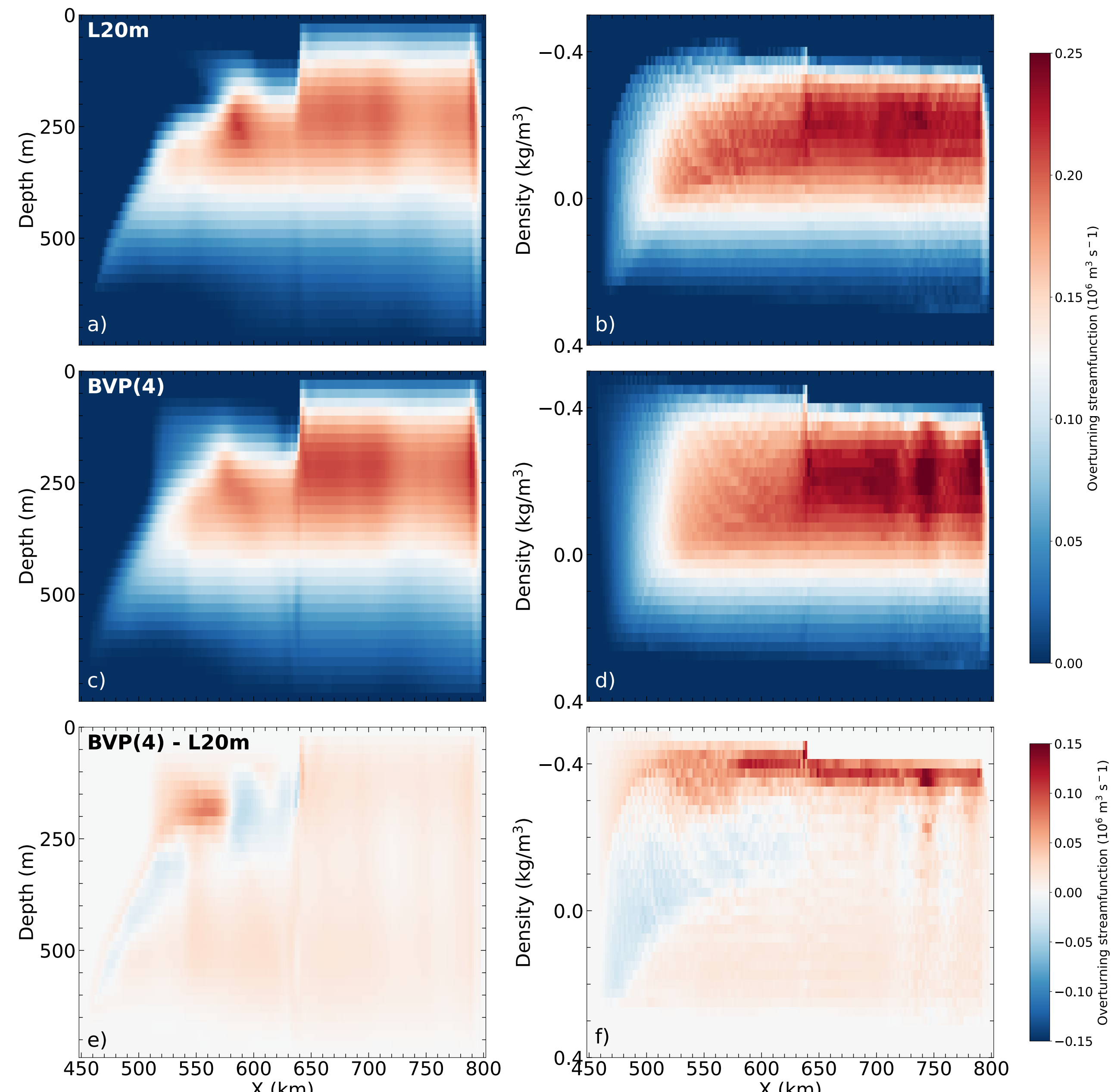
Appendix: Meridional transect



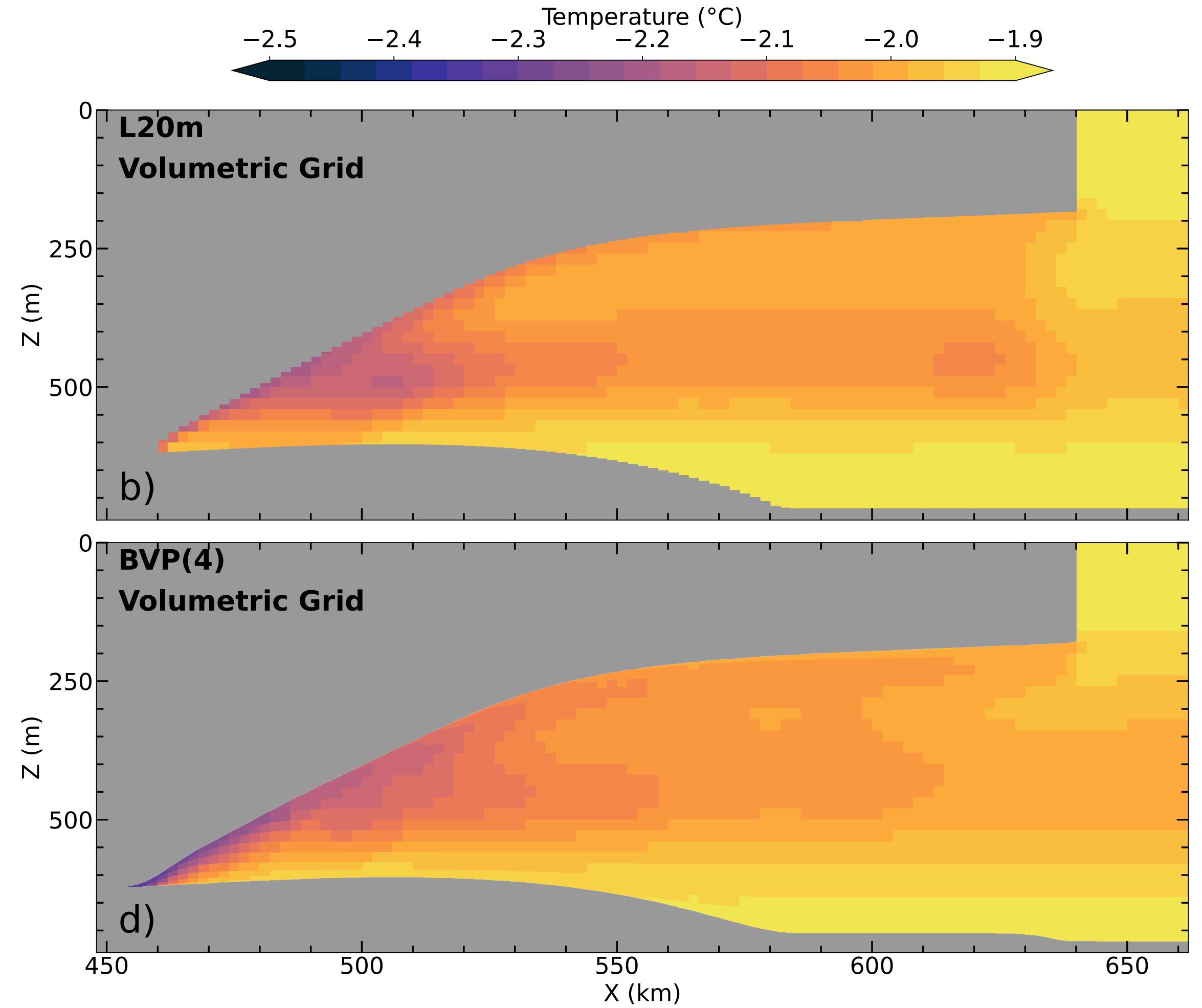
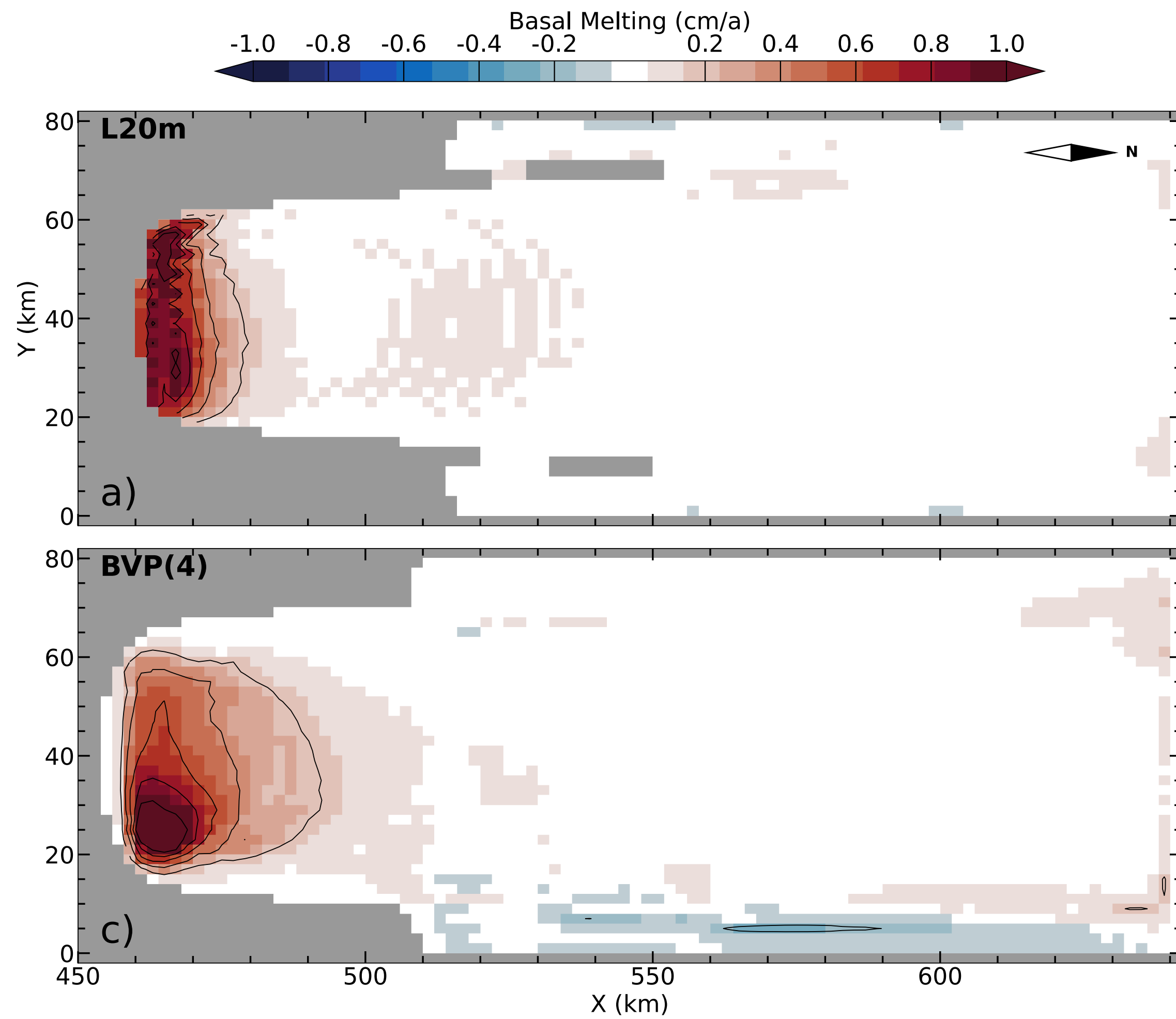
Appendix: Salinity transects



Appendix: MOC WARM case

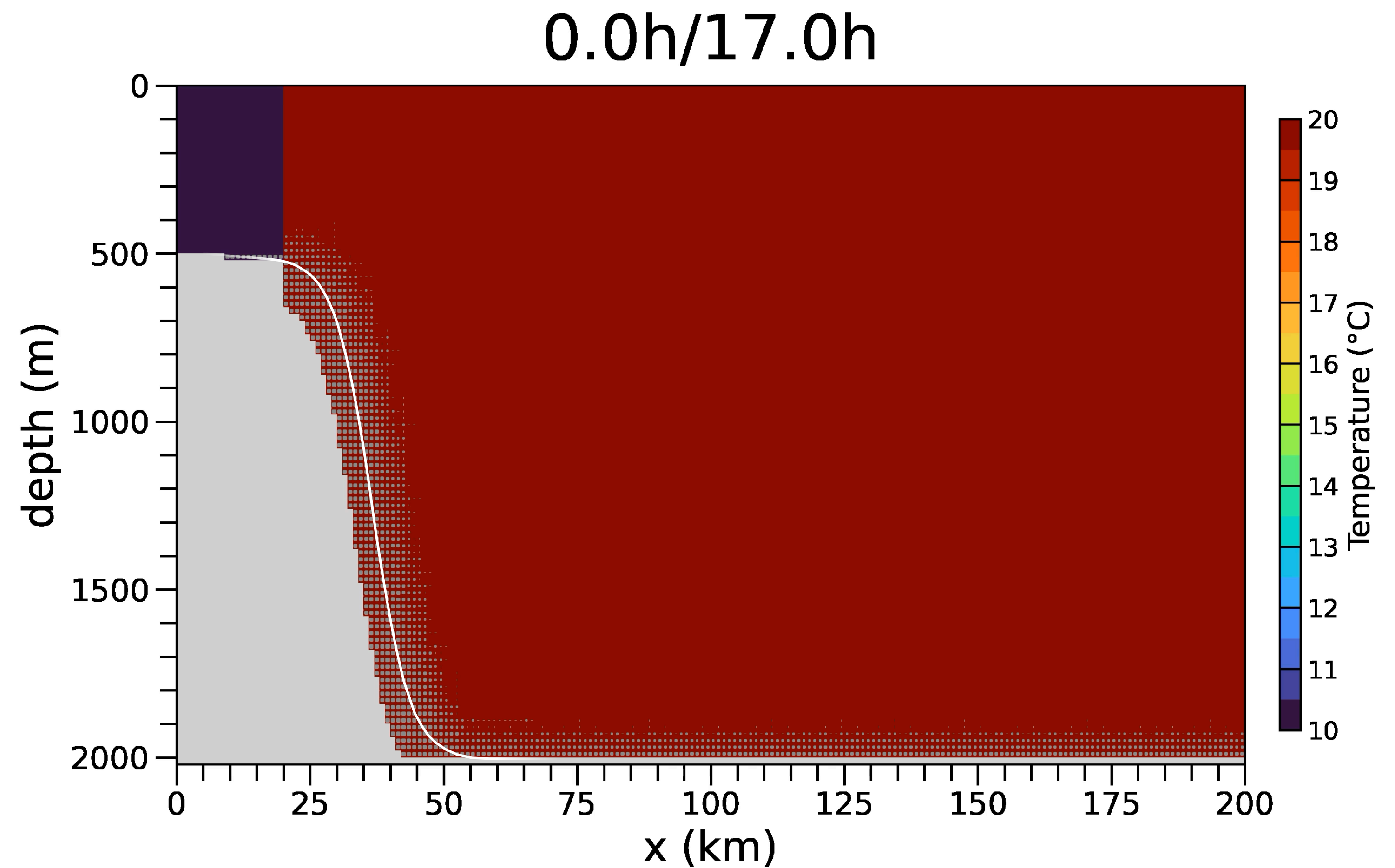
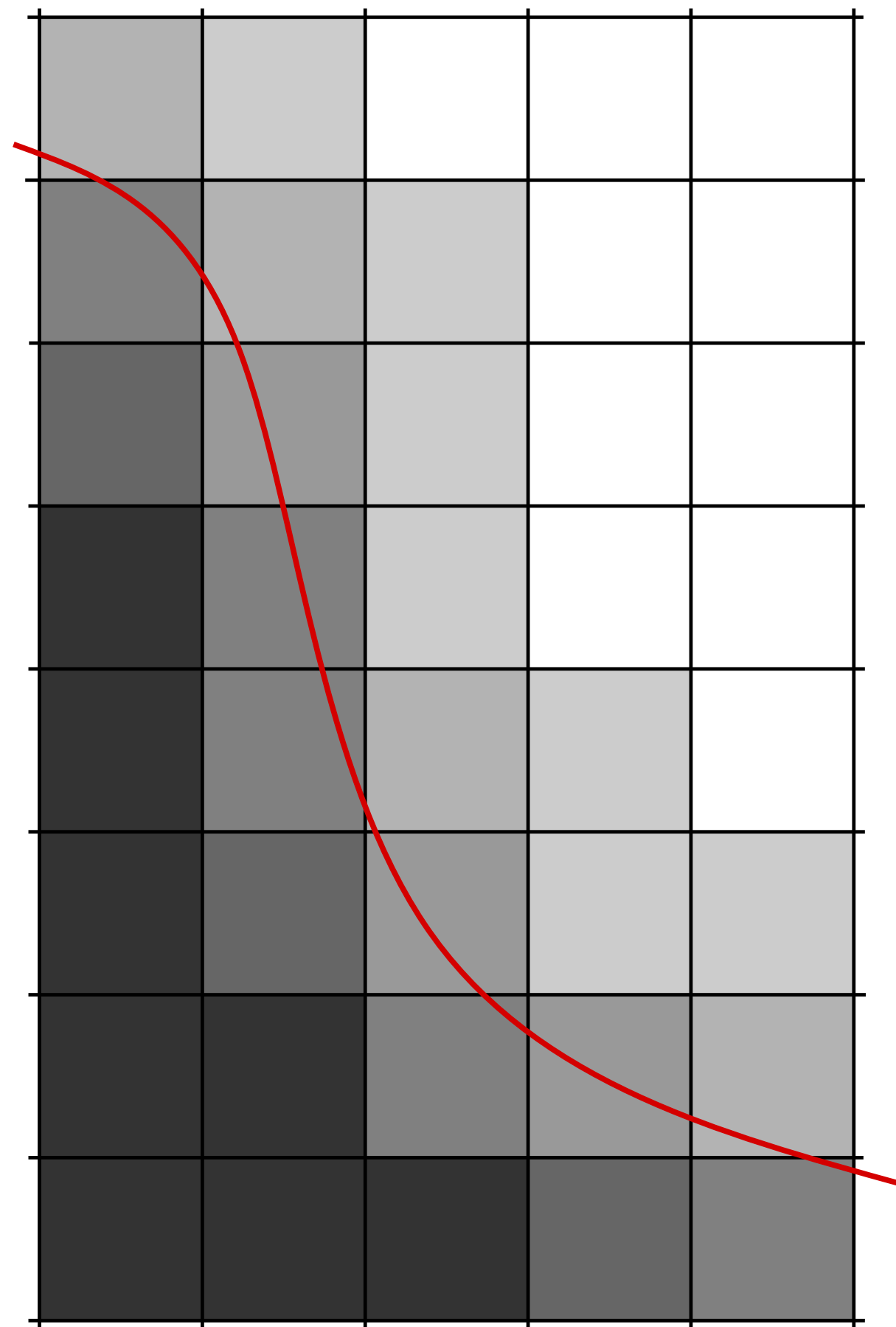


Appendix: COLD scenario



Appendices : Video I overflow resorting to the BVP method

Penalisation



The BVP method: a closer look at advective stability conditions

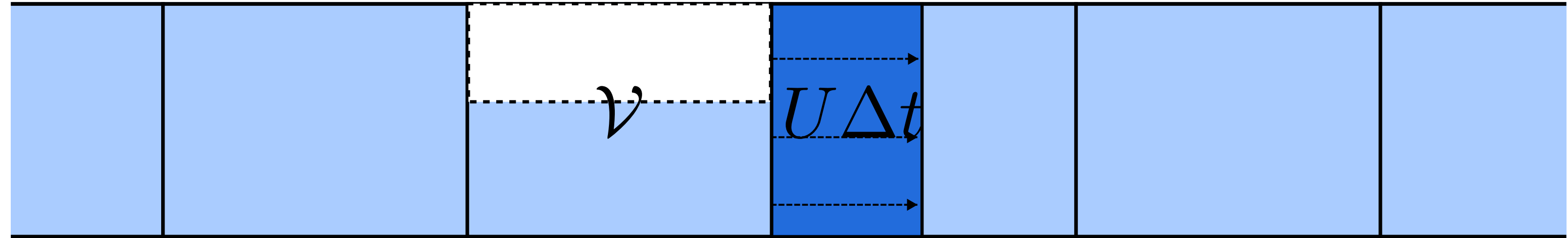
Nasser, A.-A., Debreu, L., de Lavergne, C., Madec, G



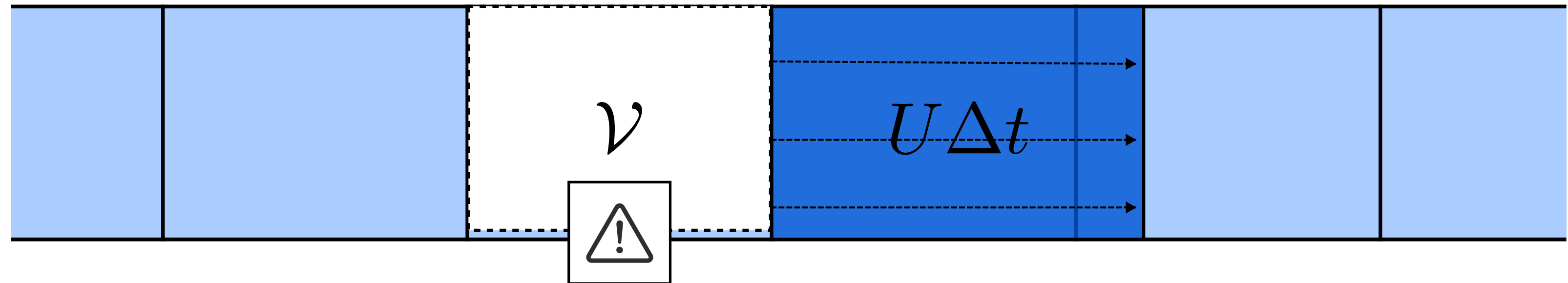
Nasser, A.-A. (2023). "Advancing the representation of flows along topography in z-coordinate ocean models". PhD thesis, Sorbonne université. <https://www.theses.fr/2023SORUS446>.

Assessing advective stability in numerical simulations

Stable



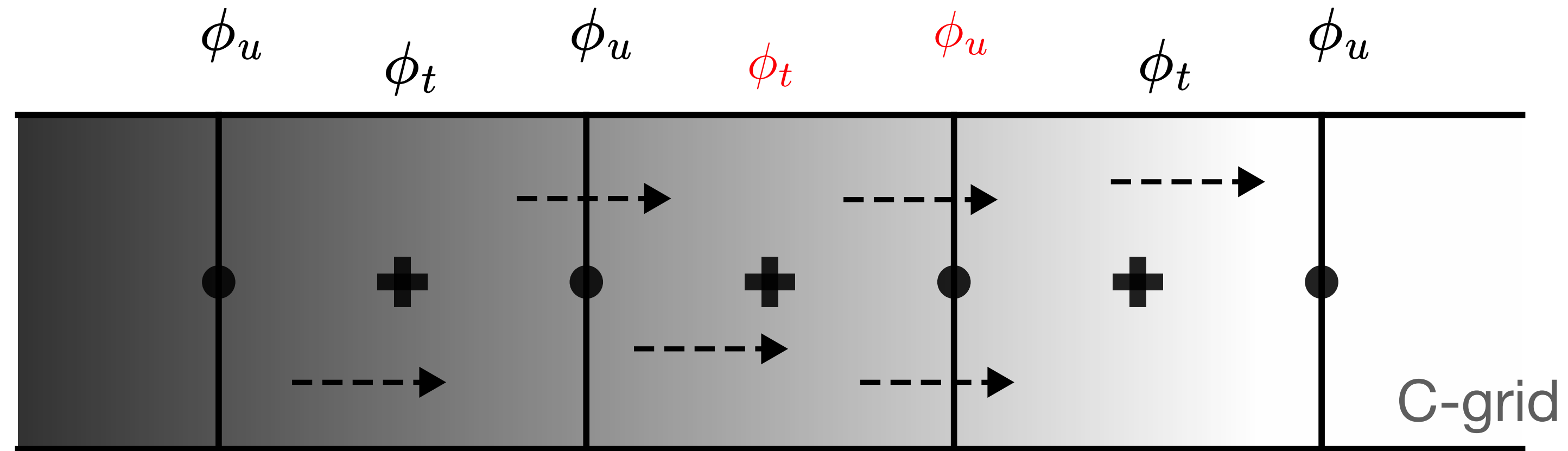
Not Stable



- Courant–Friedrichs–Lewy (CFL) condition:

$$\alpha_x = \frac{\text{Volume leaving the cell}}{\text{Liquid volume of the cell}} = \frac{U \Delta t}{\mathcal{V}} < \alpha^* \quad \text{Courant number}$$

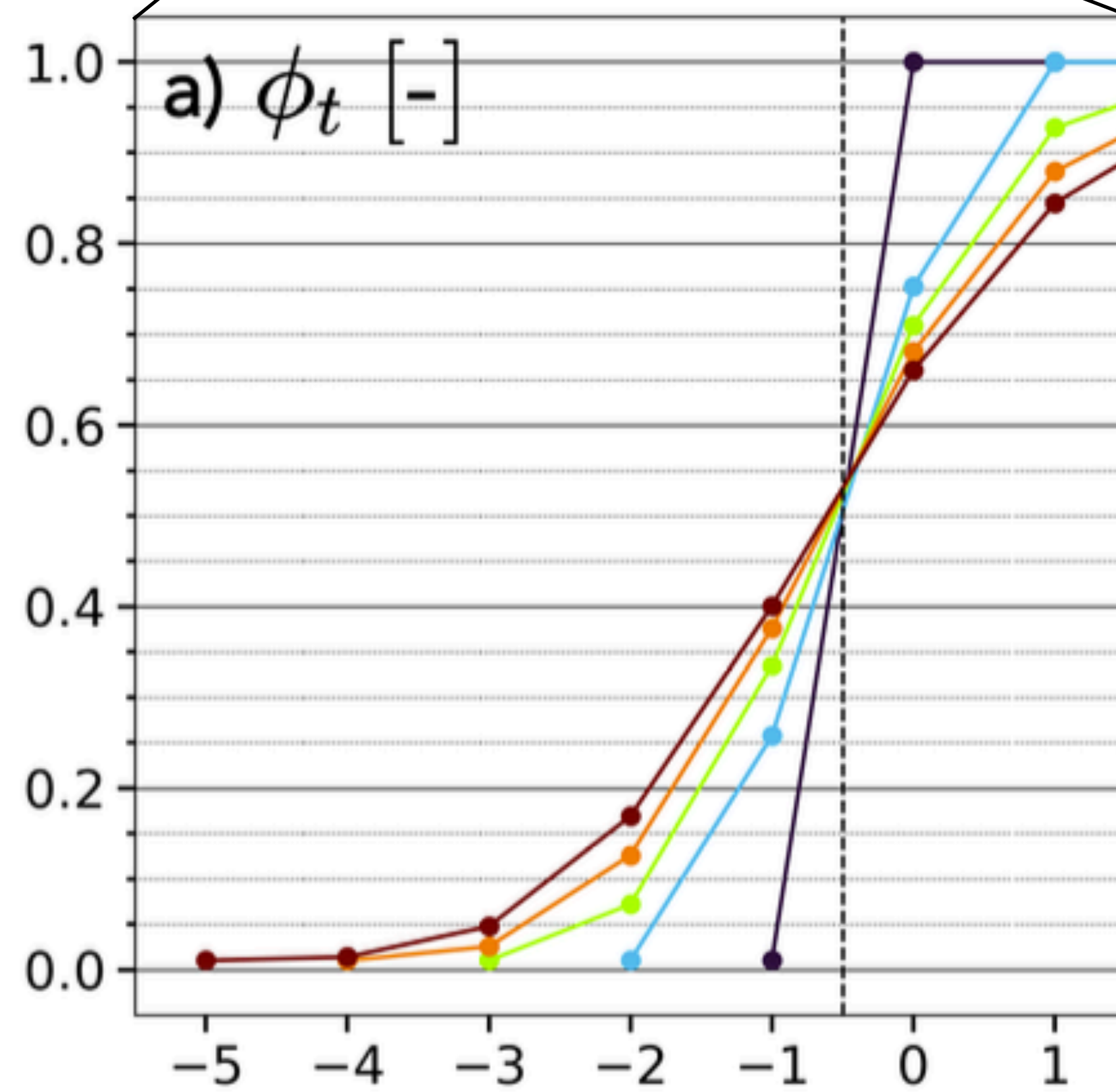
BVP method jeopardizes the advective stability condition



$$\widetilde{\alpha}_x = \frac{\text{Effective volume leaving the cell}}{\text{Effective liquid volume of the cell}} = \frac{\phi_u U \Delta t}{\phi_t \mathcal{V}} < \alpha^*$$

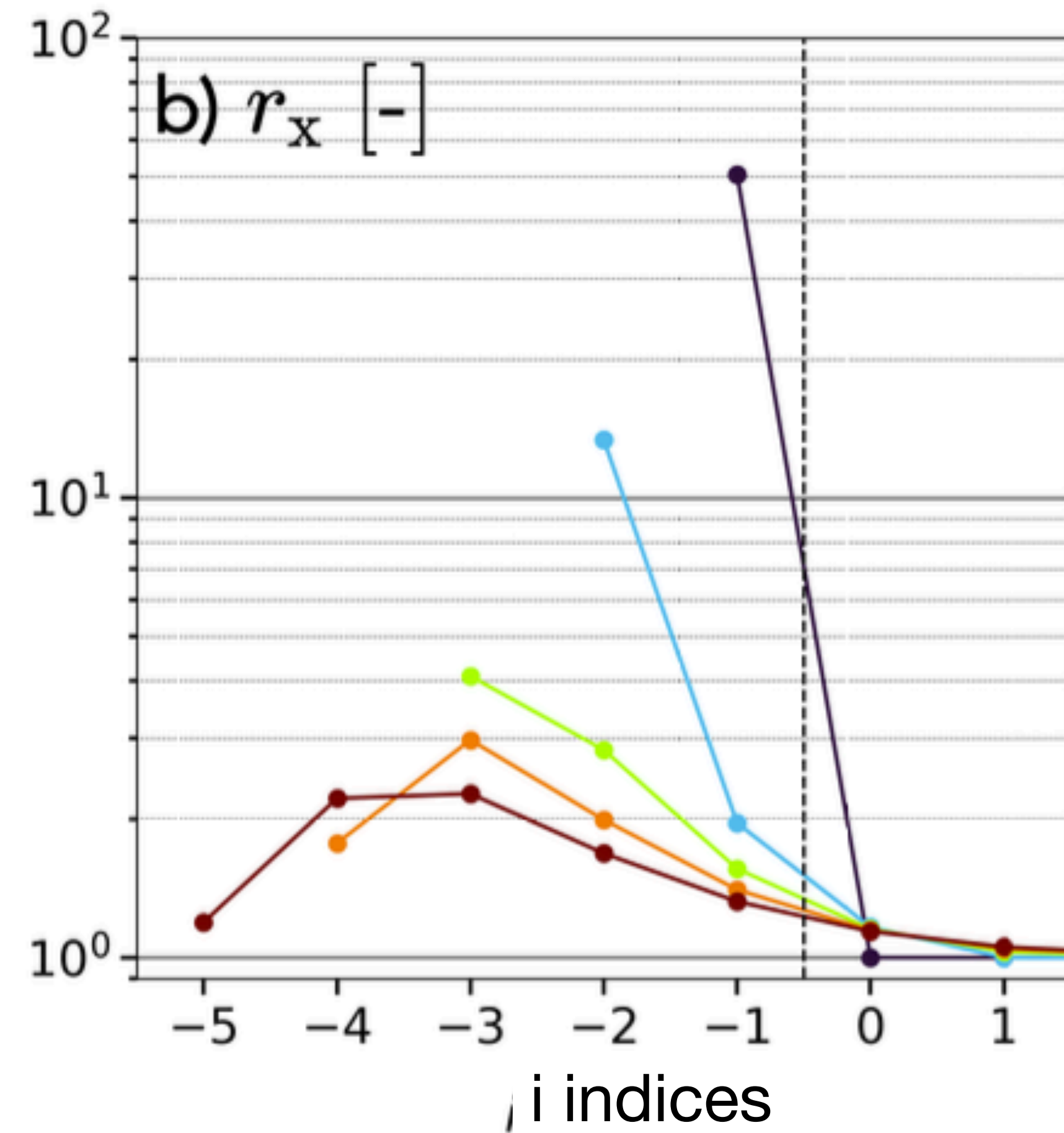
$$\widetilde{\alpha}_x = r_x \alpha_x \text{ with } r_x = \frac{\phi_u}{\phi_t}$$

Spreading the porous region mitigates stability constraint



$$\phi_0 = 0.01$$

$$(\widetilde{a}_x)_i = (r_x)_i a_x$$

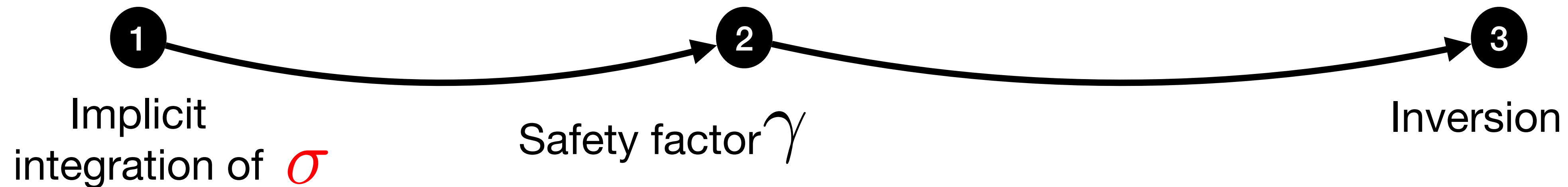


Minimal friction that preserves numerical stability of the BVP method

- Add σ to decelerate the flow

$$1) \quad \tilde{U}^{n+1,*} = \tilde{U}^n + \Delta t \text{ RHS}$$

$$2) \quad \tilde{U}^{n+1} = \frac{\tilde{U}^{n+1,*}}{1 + \sigma \Delta t} \quad \alpha_x(\tilde{U}^{n+1}) = \frac{\alpha_x(\tilde{U}^{n+1,*})}{1 + \sigma \Delta t} \leq \gamma < \alpha^* \quad (\sigma_0)_i \geq \frac{1}{\Delta t} \left(\frac{(\tilde{\alpha}_x(\tilde{U}^{n+1,*}))_i}{\gamma} - 1 \right)$$



Advective stability condition of the BVP method

Key points

- Jumps in porous medium jeopardizes advective stability.
- Numerical stability can be guaranteed by spreading porosity region and defining friction in a consistent way.
- **BVP method can be used without changing the reference timestep!**