Eddies & waves at the grid-scale

Split-form, anisotropic viscosities for ocean models

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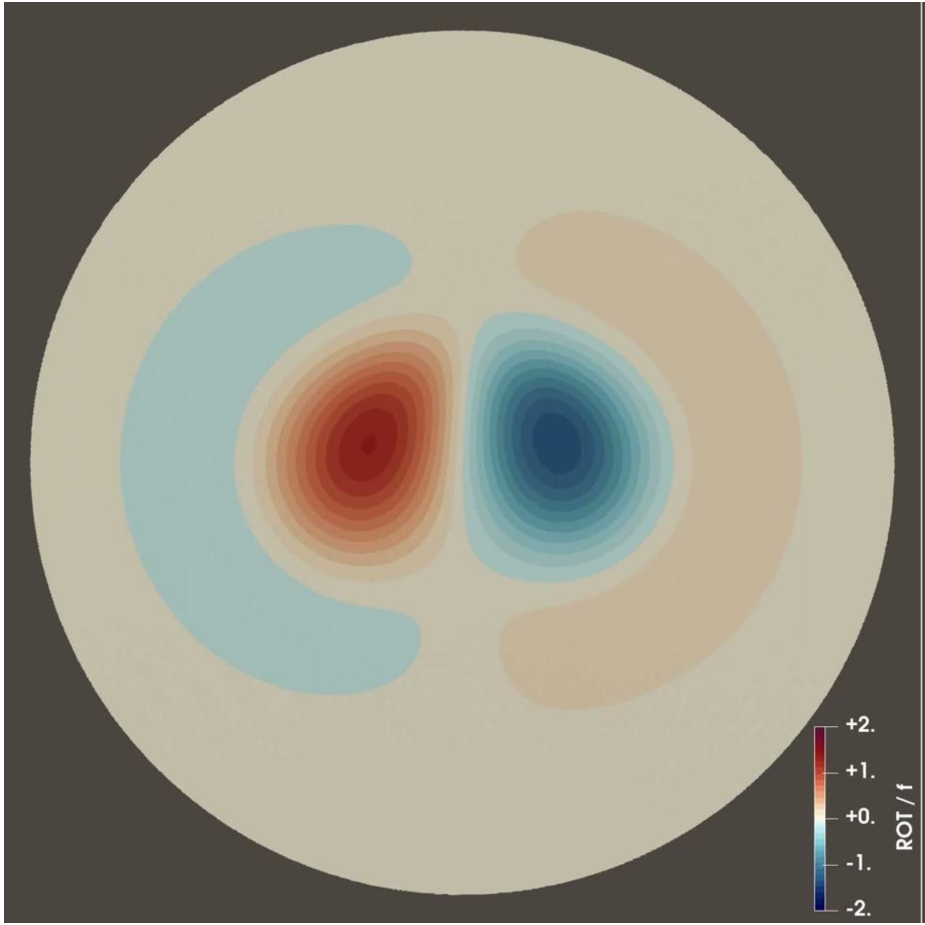
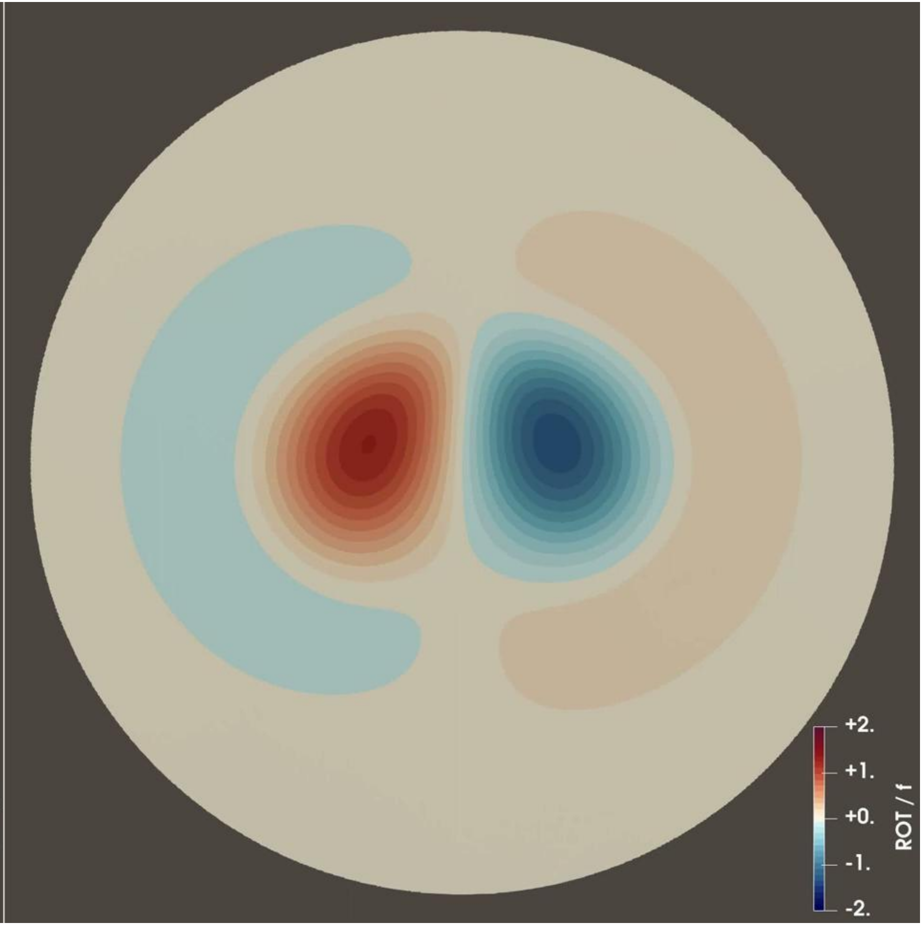
COMMODORE, September 2026

The ‘Turbulence Closure’ (Viscosity) Controls What’s Resolved…



‘Fancy’ Closure (same mesh)



‘Standard’ Viscosity

Focus on Turbulence Closures — Physics-Splitting

Focus on momentum dissipation terms — viscosity (Laplacian & Biharmonic)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + f\mathbf{u}^{\perp} = \frac{1}{\rho}\nabla p + \underline{\nu_k \nabla^k \mathbf{u}}$$

Split real parameterisation of physical turbulence from control of numerical artefacts at grid-scale (due to advection / nonlinear operators, etc)

Parameterise unresolved eddies

$$\frac{\partial \mathbf{u}}{\partial t} + \underline{(\mathbf{u}\cdot\nabla)\mathbf{u}} + f\mathbf{u}^{\perp} = \frac{1}{\rho}\nabla p + \boxed{\nu_k \nabla^k \mathbf{u}} + \boxed{\nu_k \nabla^k \mathbf{u}}$$

Physically motivated

Generation & control of grid-scale noise

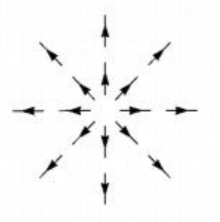
Replace the closure (espec. the numerical part) with something more appropriate than viscosity?

Focus on Turbulence Closures — Physics-Splitting

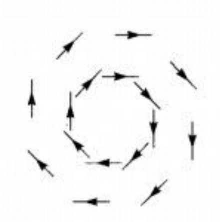
Split viscosity to target physical modes and processes

$$\nu\,\nabla^2\mathbf{u} = \nu\Big(\nabla\,\underbrace{\nabla\cdot\mathbf{u}}_{\text{vertical}} - \nabla^{\mathrm{T}}\,\underbrace{\nabla\times\mathbf{u}}_{\text{rotation}}\Big)$$

i.e. control wave-like vertical dynamics separate to eddy-like rotational motions (Helmholtz decomposition)



$$w = \int_{z_b}^{z_t} \underline{\nabla\cdot\mathbf{u}}\,\mathrm{d}z$$



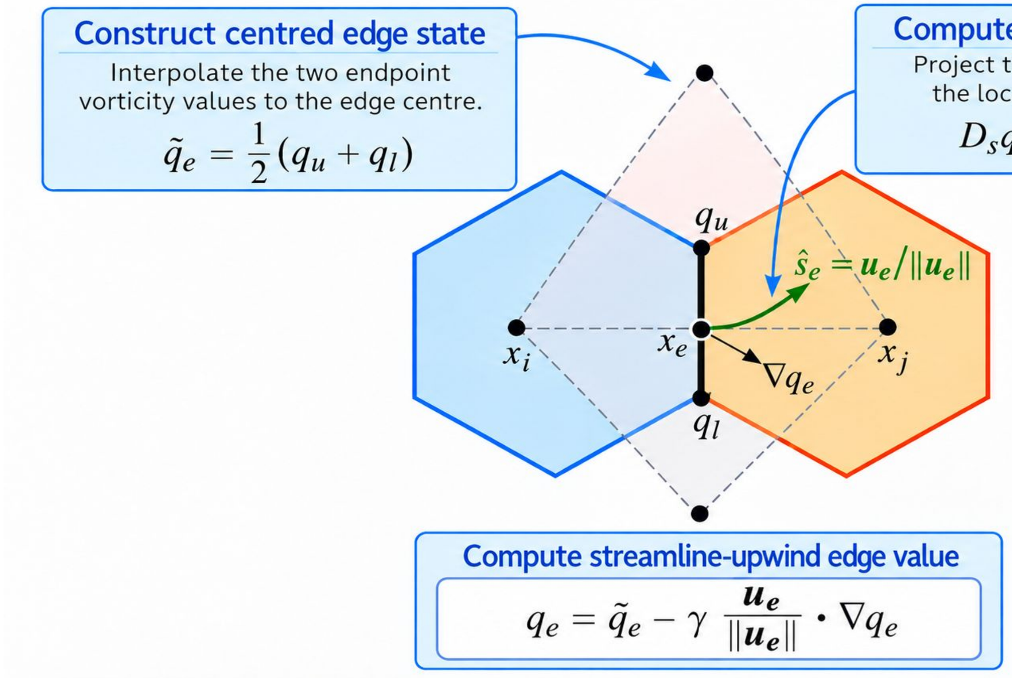
$$\mathrm{PV} = \tfrac{1}{h}\big(\underline{\nabla\times\mathbf{u}} + f\big)$$

Leads to a split-form, anisotropic turbulence closure, allowing ‘physics-appropriate’ viscosity for each mode

$$\nu\,\nabla^2\mathbf{u} \rightarrow \underline{\nu_{\text{waves}}}\,\nabla\,\underbrace{\nabla\cdot\mathbf{u}}_{\text{vertical}} - \underline{\nu_{\text{eddies}}}\,\nabla^{\mathrm{T}}\,\underbrace{\nabla\times\mathbf{u}}_{\text{rotation}} \qquad \nu_{\text{waves}} \gg \nu_{\text{eddies}}$$

Rotational term — Eddies and PV upwinding

Actually, replace the rotational viscosity with a ‘better’ FCT-like momentum advection scheme



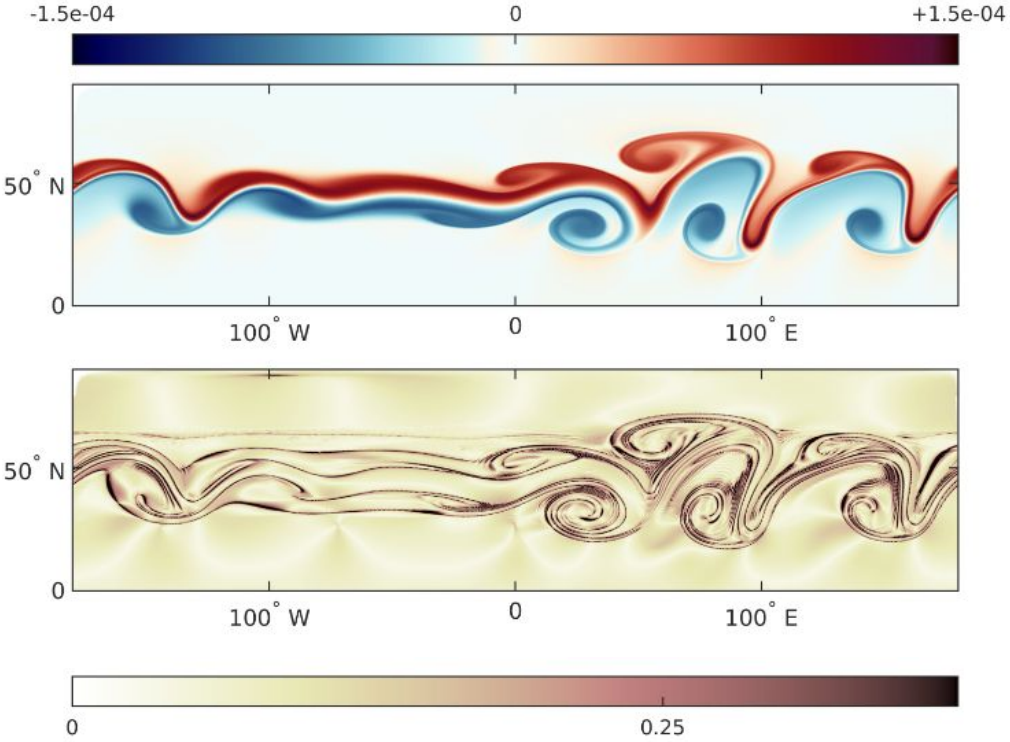
$$\frac{\partial \mathbf{u}}{\partial t} + q h \mathbf{u}^{\perp} = -g\nabla(h+b) - \nabla K,$$

$$q = \frac{\xi + f}{h}, \quad \xi = \nabla \times \mathbf{u},$$

Similar, but different to WENO PV ideas — only 2nd/1st-order accurate here, but compact & exactly energy conserving

Rotational term — Eddies and PV upwinding

Upwind PV advection controls grid-scale noise rather than viscosity — similar to (quasi-monotone) tracer advection



$$q_e = \tilde{q}_e - \gamma l_e \frac{\mathbf{u}_e}{|\mathbf{u}_e|} \cdot \nabla_e q\,.$$

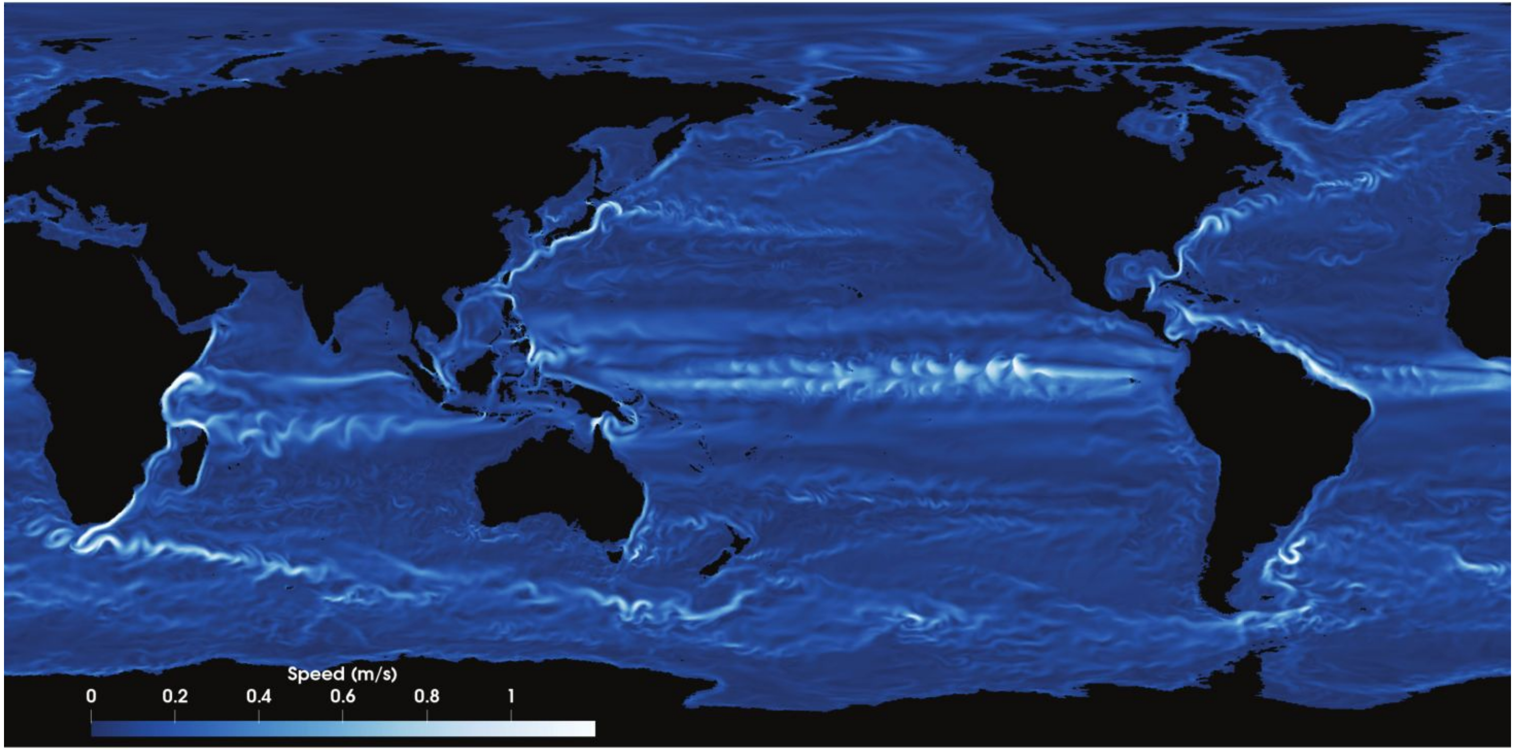
Flow-adaptive upwind coefficient

$$\Delta = \frac{|q_{\mathrm{hi}} - q_{\mathrm{lo}}|}{|\nabla q|}, \qquad \gamma_e(\mathbf{u}) = \frac{\Delta^2}{\Delta^2 + 1}$$

Ensures minimal (enstrophy) dissipation

Impact on ¼ deg MPAS-O

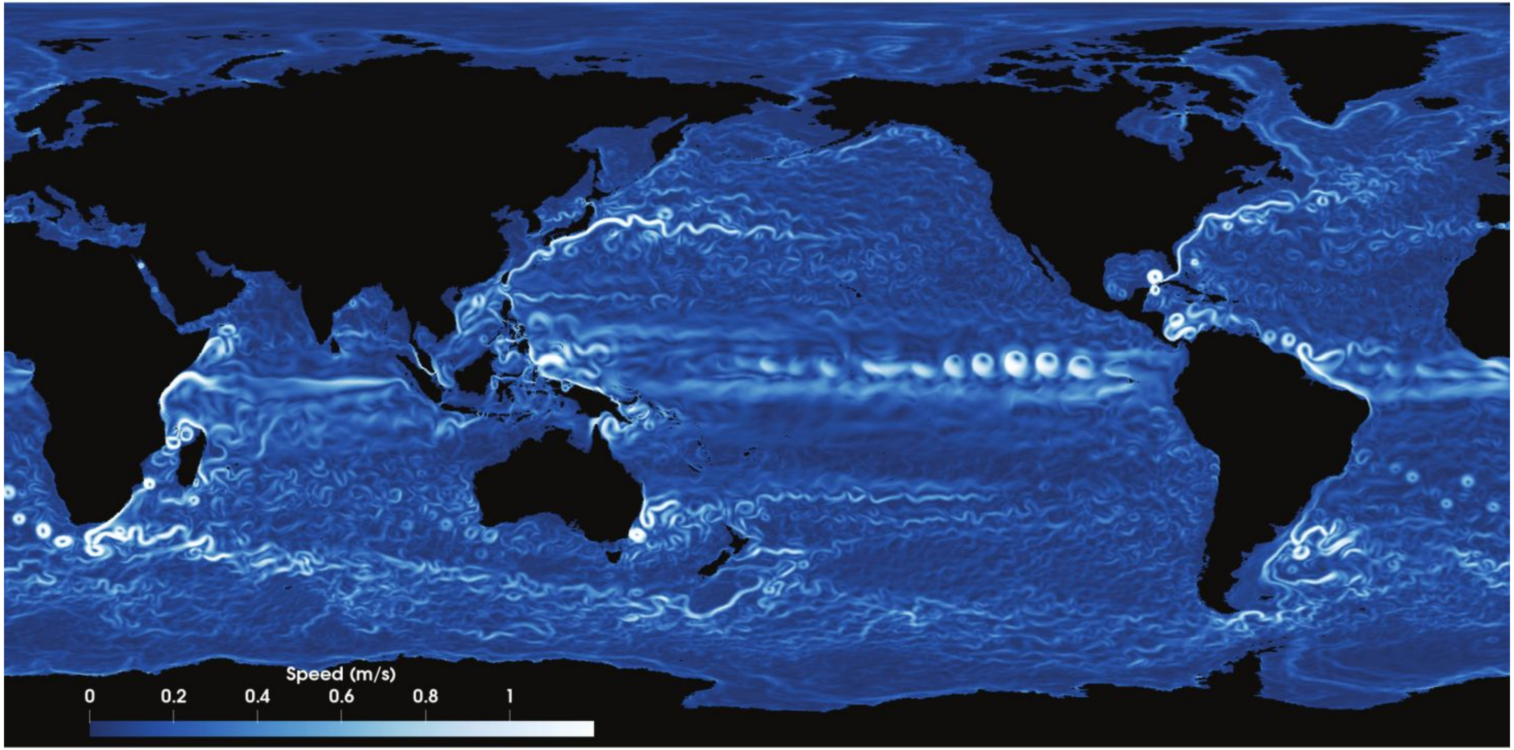
Original scheme too dissipative to resolve eddies — 'old' $\nu_k \nabla^k \mathbf{u}$ closure:



Global circulation not well represented at O(30km) resolution.

Impact on ¼ deg MPAS-O

New, quasi energy conserving ideas — $q_{\text{up}} + \nu_k^{\delta} \nabla \delta$ upwind method:



Improvement in intensity + structure of circulation; quasi eddy-permitting, correct boundary current separation, resolved Agulhas rings, etc...

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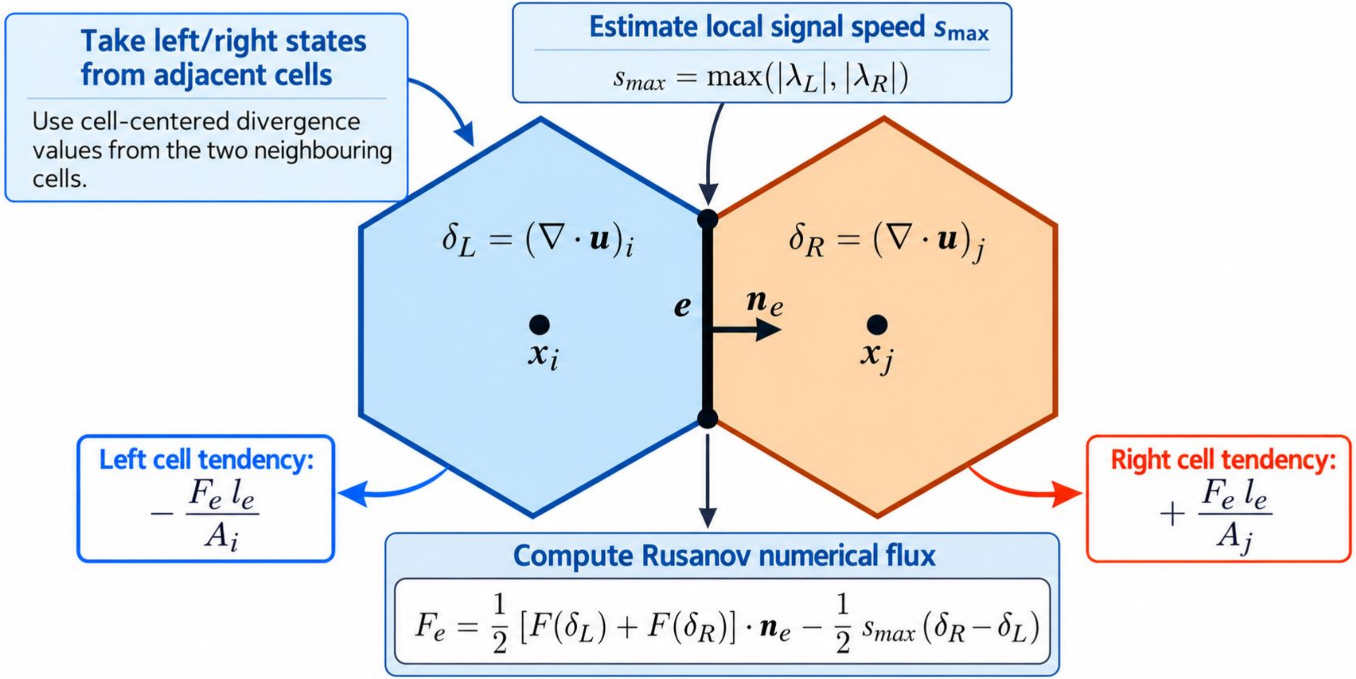
Divergent Term — Use a Riemann Solver to Control Diapycnal Fluxes

$$\nu \nabla^2 \mathbf{u} \rightarrow \nu_{\text{waves}} \nabla \underbrace{\nabla \cdot \mathbf{u}}_{\text{vertical}} - \nu_{\text{eddies}} \nabla^T \underbrace{\nabla \times \mathbf{u}}_{\text{rotation}}$$



Divergent Term — Use a Riemann Solver to Control Diapycnal Fluxes

A Riemann-Solver approach sets dissipation via the physics of the flow

$$\nu_{\mathrm{wav2d}} = \gamma\Big(|\mathbf{U}| + \sqrt{gH}\Big)$$

$$\nu_{\mathrm{wav3d}} = \gamma\left(|\mathbf{u}| + \int_{z_b}^{z_t} \frac{N(z)}{m\pi}\,\mathrm{d}z\right), \quad N(z) = \left(-\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{\frac{1}{2}}$$

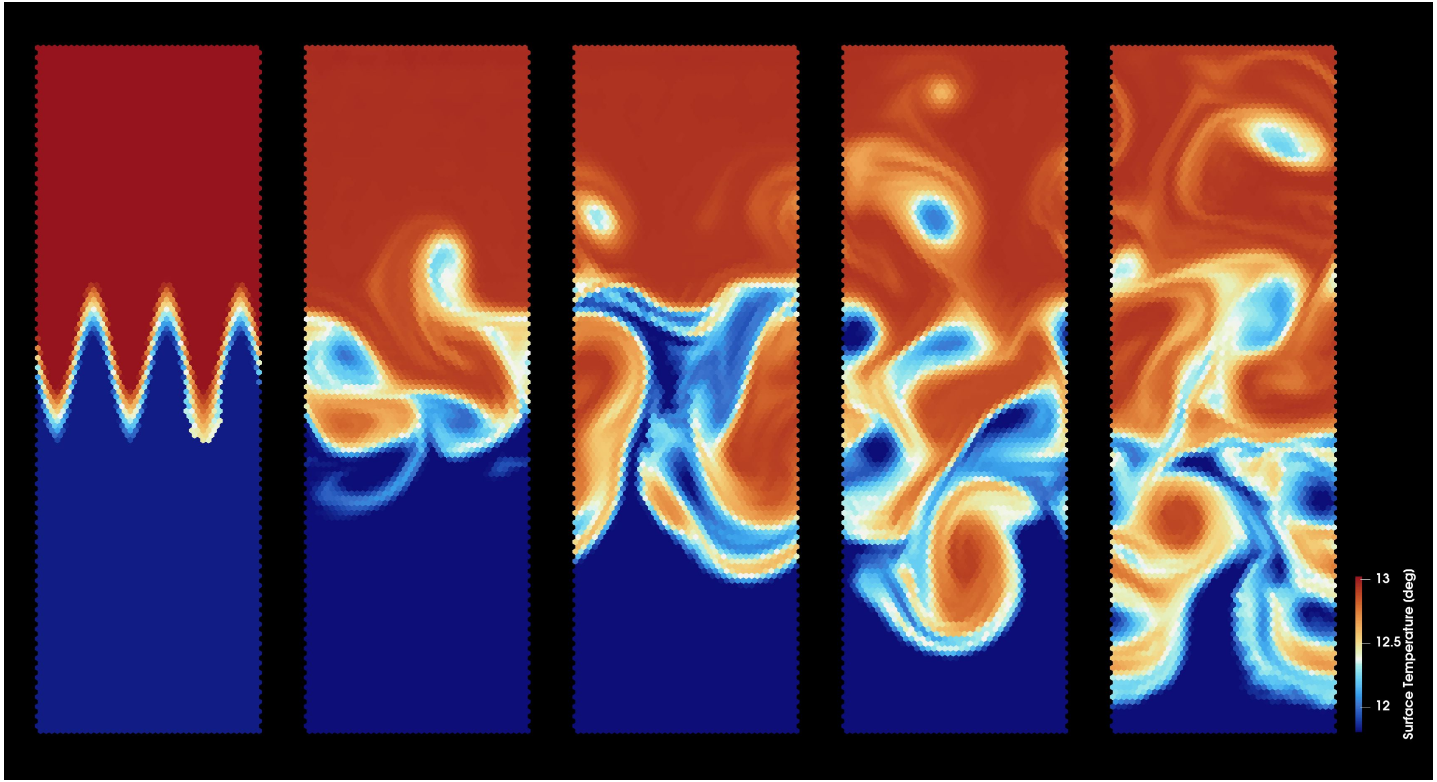
This borrows from methods common in e.g. aerospace applications[a], where it's been shown that **the 'correct' dissipation rate should scale with the wave-speed of the physics**.

Considering 2d and 3d dynamics separately, we introduce two viscosities scaled with estimates of the **external & internal gravity waves speeds**.

[a] Jameson et al (2015): The Origins and Further Development of the Jameson-Schmidt-Turkel (JST) Scheme..., AIAA Journal, 55(5).

Impact in Idealised Cases: Baroclinic Spin-down

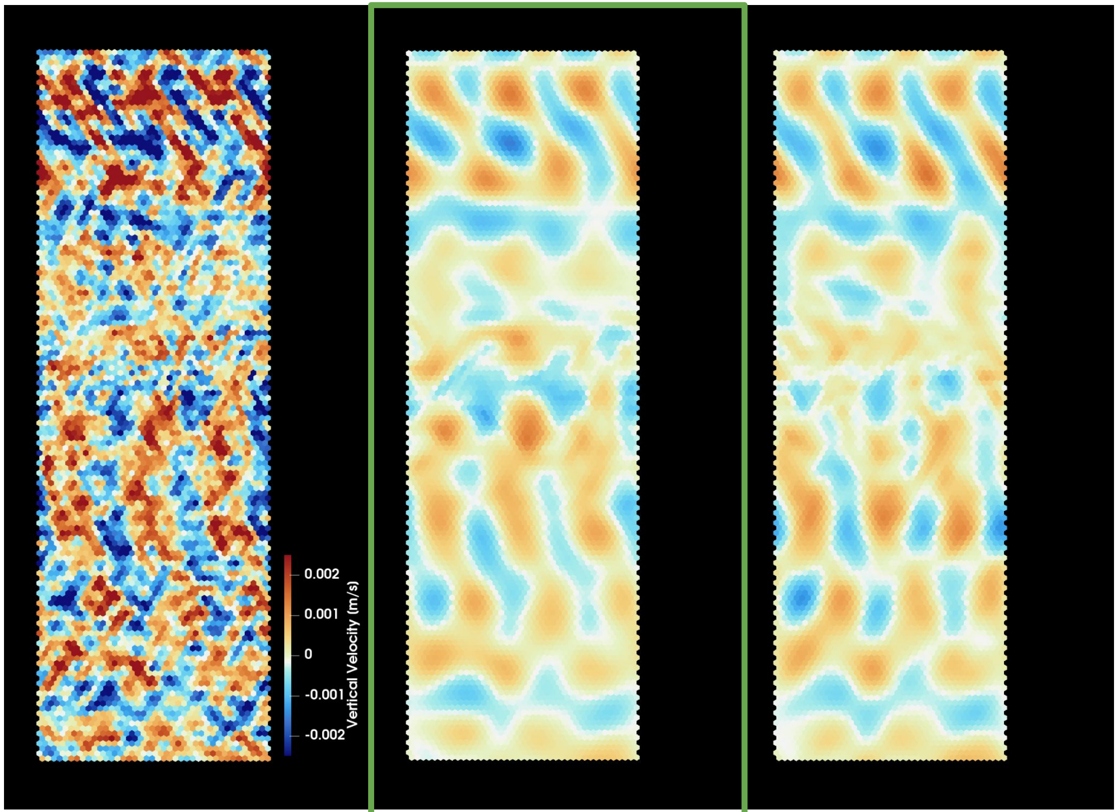
Implement new ‘wave-dissipation’ closure in MPAS-O, assess impact in idealised test case



0-100 days

Impact in Idealised Cases: Baroclinic Spin-down

Dramatic impact on vertical velocities (t=25 days)



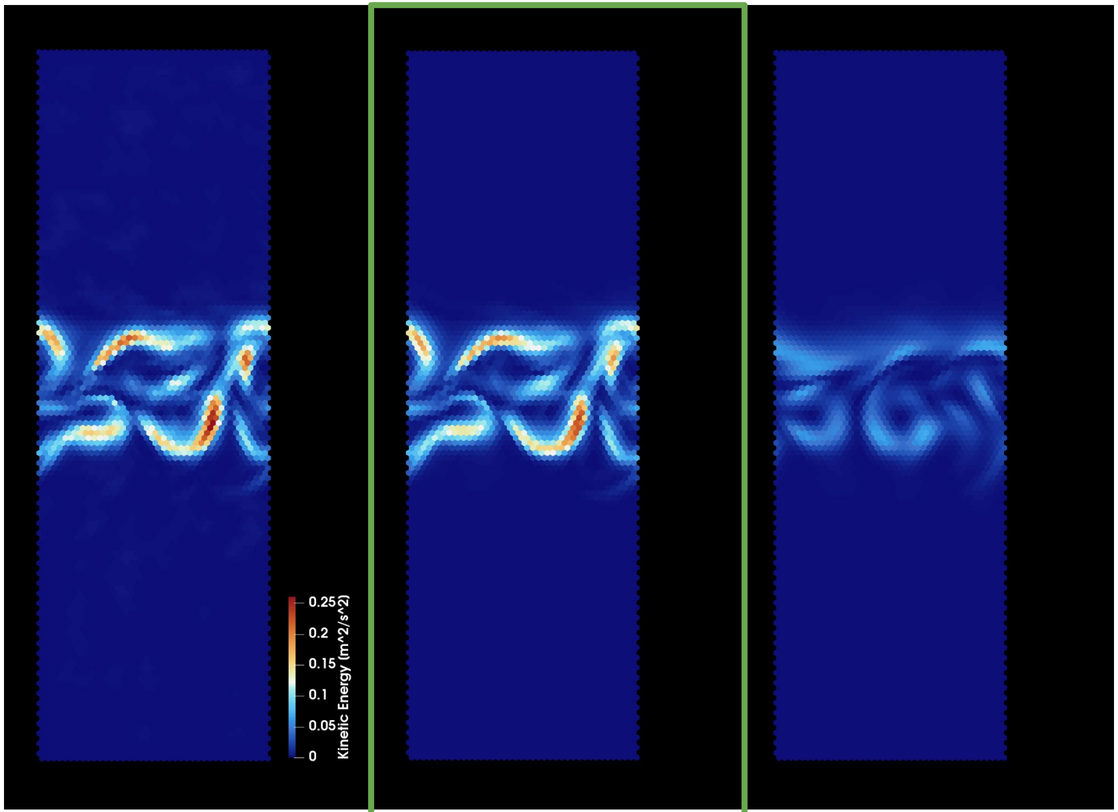
nu=10 | **nu=10, wd=1/40** | **nu=100**

Too little viscosity (left) = spurious vertical velocities + mixing

New closure (middle) = similar (quiescent) vertical transport as **10x viscosity**

Impact in Idealised Cases: Baroclinic Spin-down

Kinetic energy (relatively) unaffected (t=25 days)



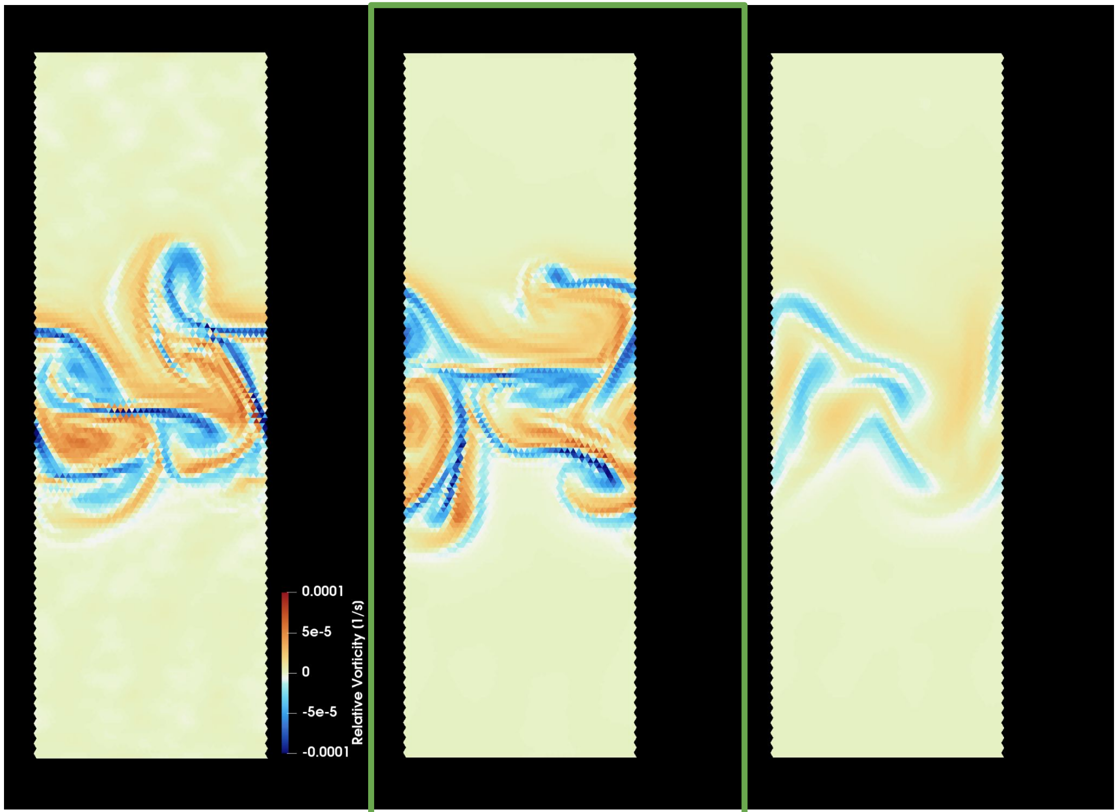
nu=10 | **nu=10, wd=1/40** | **nu=100**

Too high viscosities (right) = excessive damping of eddy energy

New closure (middle) = similar high KE, despite removal of vertical noise

Impact in Idealised Cases: Baroclinic Spin-down

Relative vorticity (relatively) unaffected (t=50 days)



nu=10 **nu=10, wd=1/40** **nu=100**

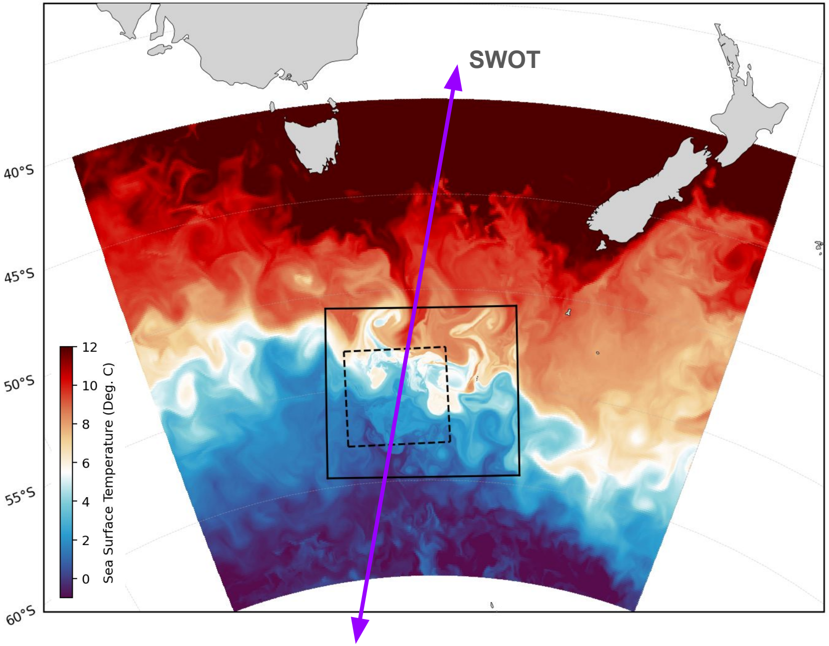
Too high viscosities (right) = excessive damping of eddy vorticity

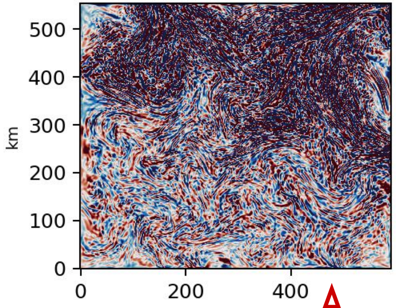
New closure (middle) = similar vorticity, despite removal of vertical noise

Eddy-Rich Flows and (Spurious?) Diapycnal Motion

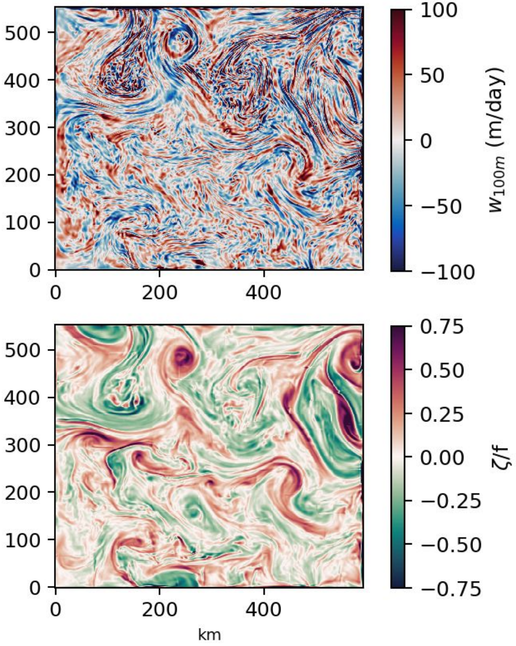
O(1-2 km) model of ACC dynamics adj. Macquarie Island highlights difficulties in representation of vertical motion — **spurious, grid-scale modes**…

Physical submesoscale fronts and eddies





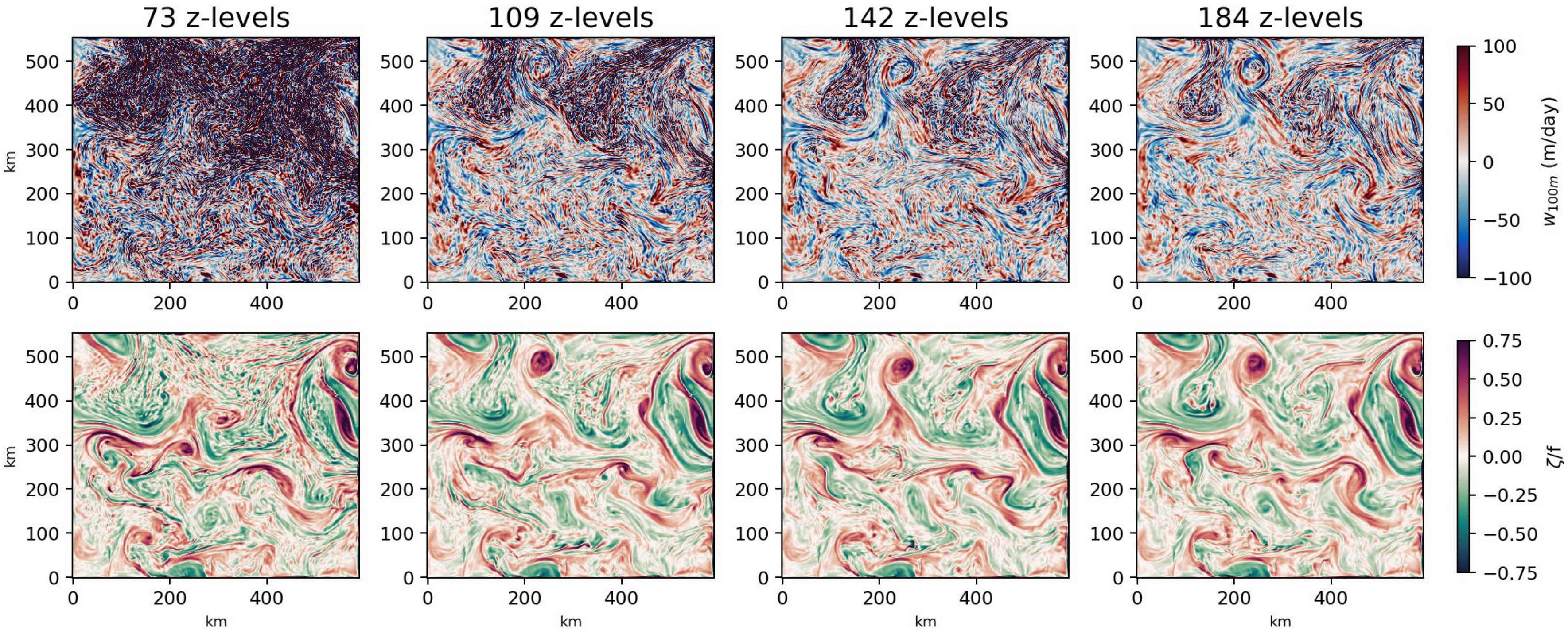
Overly energetic vertical motion = noise, numerical mixing



How to control (spurious) vertical fluxes, without overly damping submesoscale eddies?

Impact in Operational Cases: ACC Meanders

(Very) spurious vertical transport at coarse vertical resolution, as per recent "Baroclinic Instability of the Computational Kind" (BICK[a]) results.



[a]*Ménesguen, Ducousso, Vic, Le Gentil, 2025. Exploring baroclinic instability of the computational kind (BICK) in numerical simulations of the ocean. JAMES*

Where Do the (Spurious) Diapycnal Modes Come From?

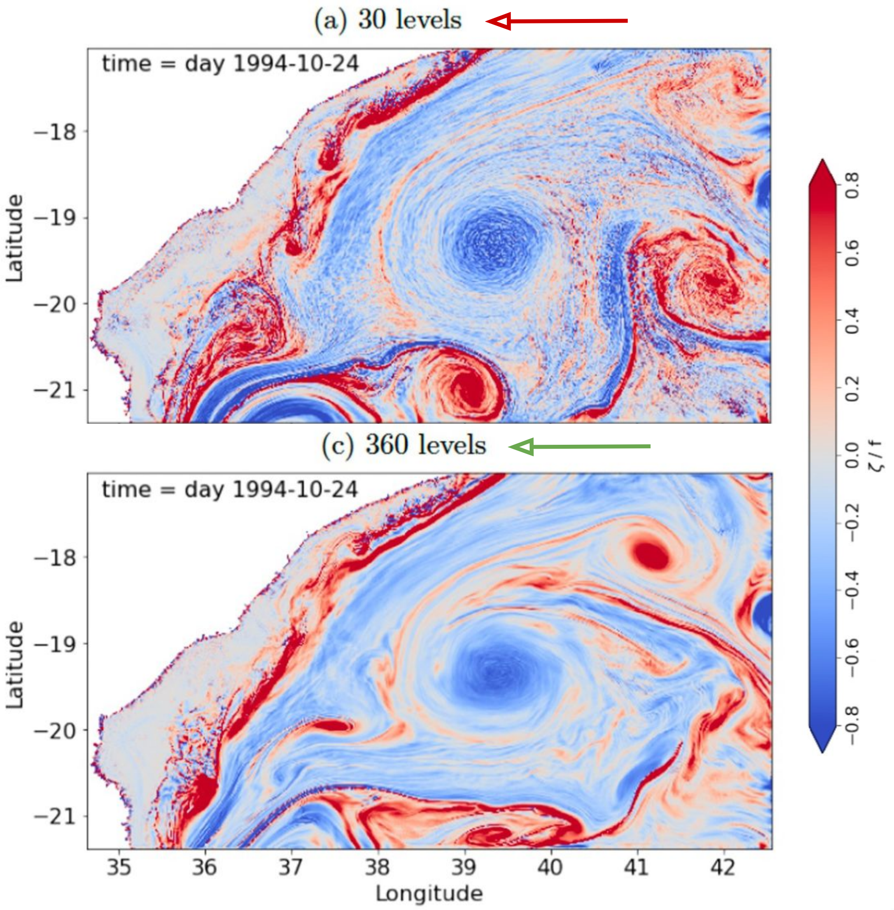
JAMES | Journal of Advances in Modeling Earth Systems®

Research Article | Open Access

Exploring Baroclinic Instability of the Computational Kind (BICK) in Numerical Simulations of the Ocean

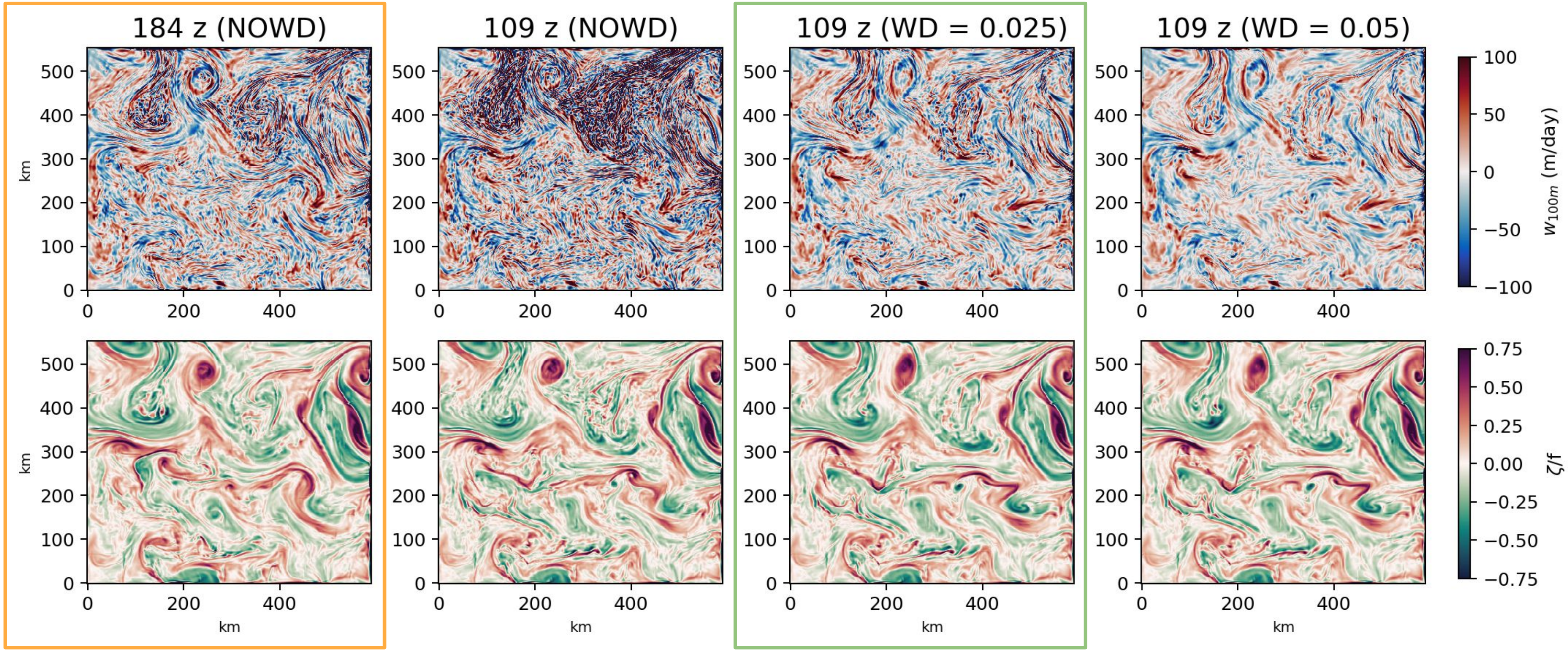
Claire Ménesguen, Nicolas Ducousso, Clément Vic, Sylvie Le Gentil

Recent work with CROCO has highlighted insufficient vertical resolution as a driver of spurious numerical modes in energetic, submesoscale permitting configurations

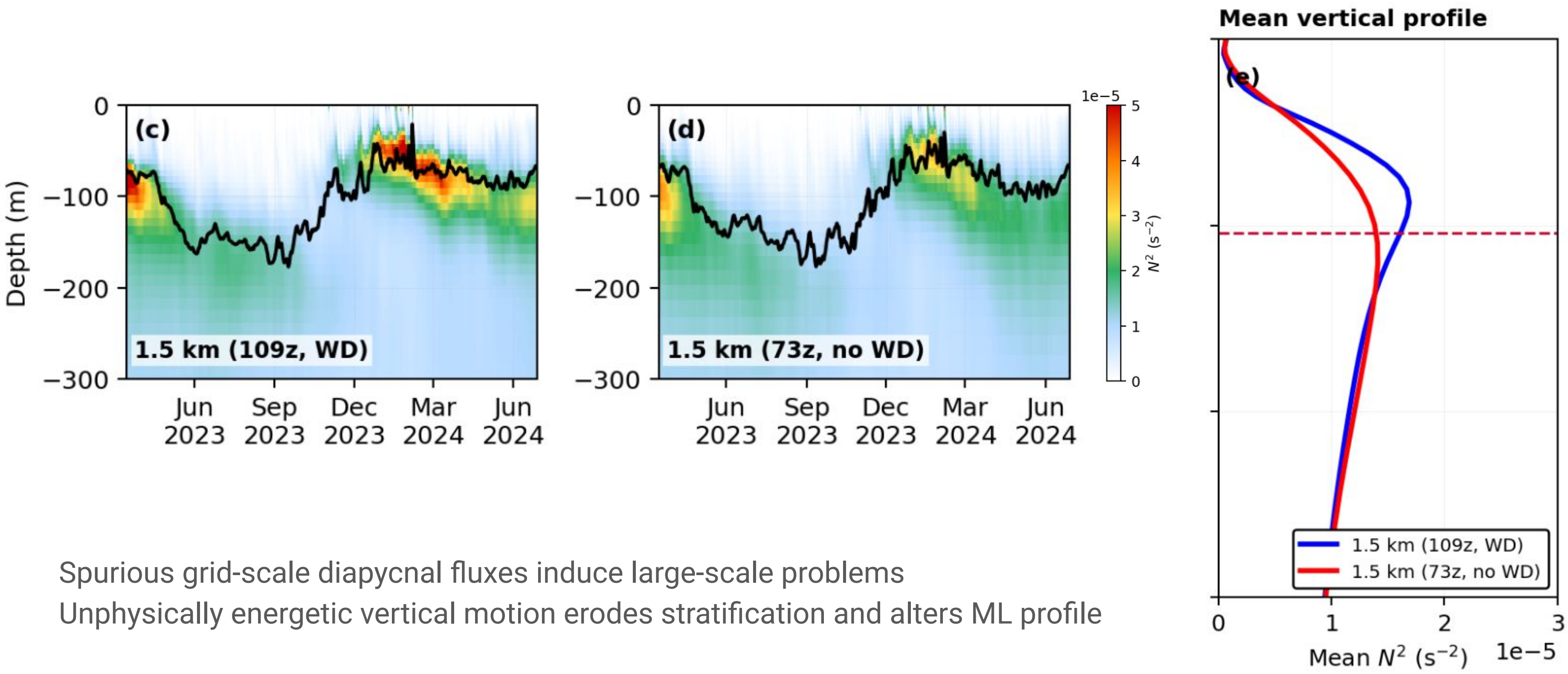


Impact in Operational Cases: ACC Meanders

New turbulence closure filters spurious vertical noise **at ~ ½ the #layers & without damping submesoscale eddy features**



Is Spurious Motion Really that Destructive?



Spurious grid-scale diapycnal fluxes induce large-scale problems

Unphysically energetic vertical motion erodes stratification and alters ML profile

tl;dr

- Don't do $\nu_k \nabla^k \mathbf{u}$ (this unphysically treats dissipation of eddy & wave modes isotropically)

- Instead $\nu \nabla^2 \mathbf{u} \rightarrow \nu_{\text{waves}} \nabla \underbrace{\nabla \cdot \mathbf{u}}_{\text{vertical}} - \nu_{\text{eddies}} \nabla^{\mathrm{T}} \underbrace{\nabla \times \mathbf{u}}_{\text{rotation}}$

- In fact, just use a better (upwind) PV advection scheme & eliminate rotational viscosity altogether

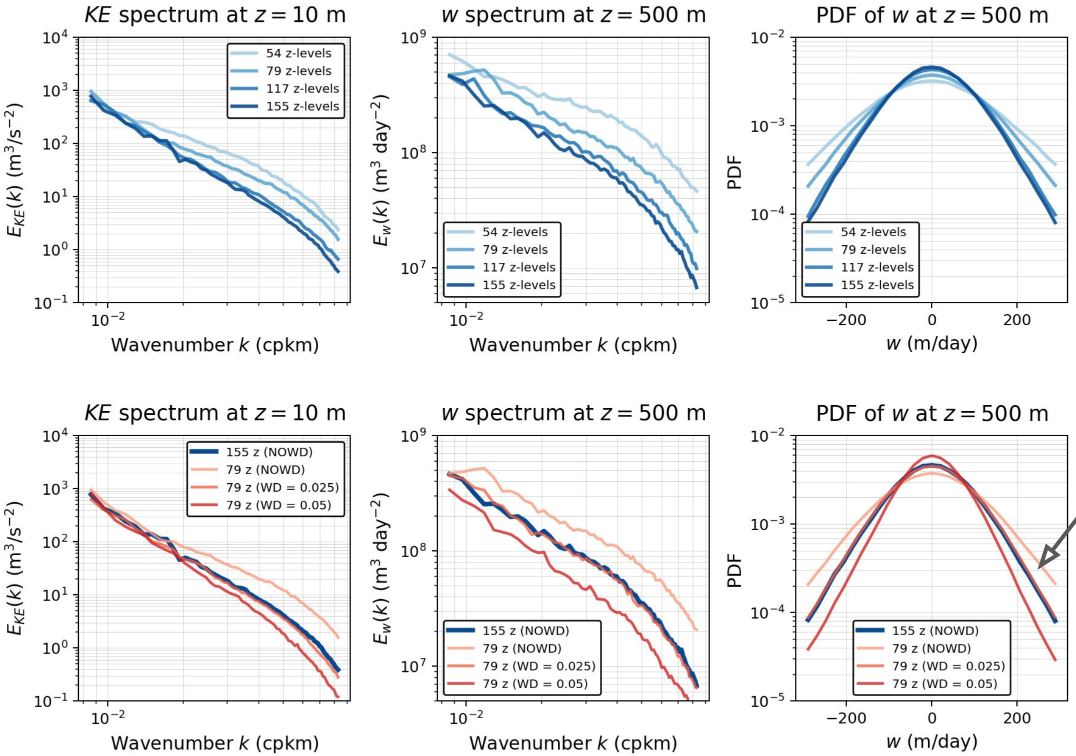
$$q_e = \tilde{q}_e - \gamma l_e \frac{\mathbf{u}_e}{|\mathbf{u}_e|} \cdot \nabla_e q$$

- And don't try to guess the correct diapycnal flux dissipation, use 'Riemann-based' closures

$$\nu_{\text{wav2d}} = \gamma\left(|\mathbf{U}| + \sqrt{gH}\right) \qquad \nu_{\text{wav3d}} = \gamma\left(|\mathbf{u}| + \int_{z_b}^{z_t} \frac{N(z)}{m\pi}\,\mathrm{d}z\right), \qquad N(z) = \left(-\frac{g}{\rho}\frac{\partial \rho}{\partial z}\right)^{\frac{1}{2}}$$

Impact in Operational Cases: ACC Meanders

Behaviour confirmed in spectral space — **new closure leads to ~same turbulent cascade as very high (vertical) resolution config., but using ~ ½ the #layers**.



New closure (WD=1/40; 80 layers) overlies spectra of 160 layer config.