Met Office

Accurate calculation of horizontal pressure forces on steeply sloping grid cells

Mike Bell & Diego Bruciaferri

Met Office

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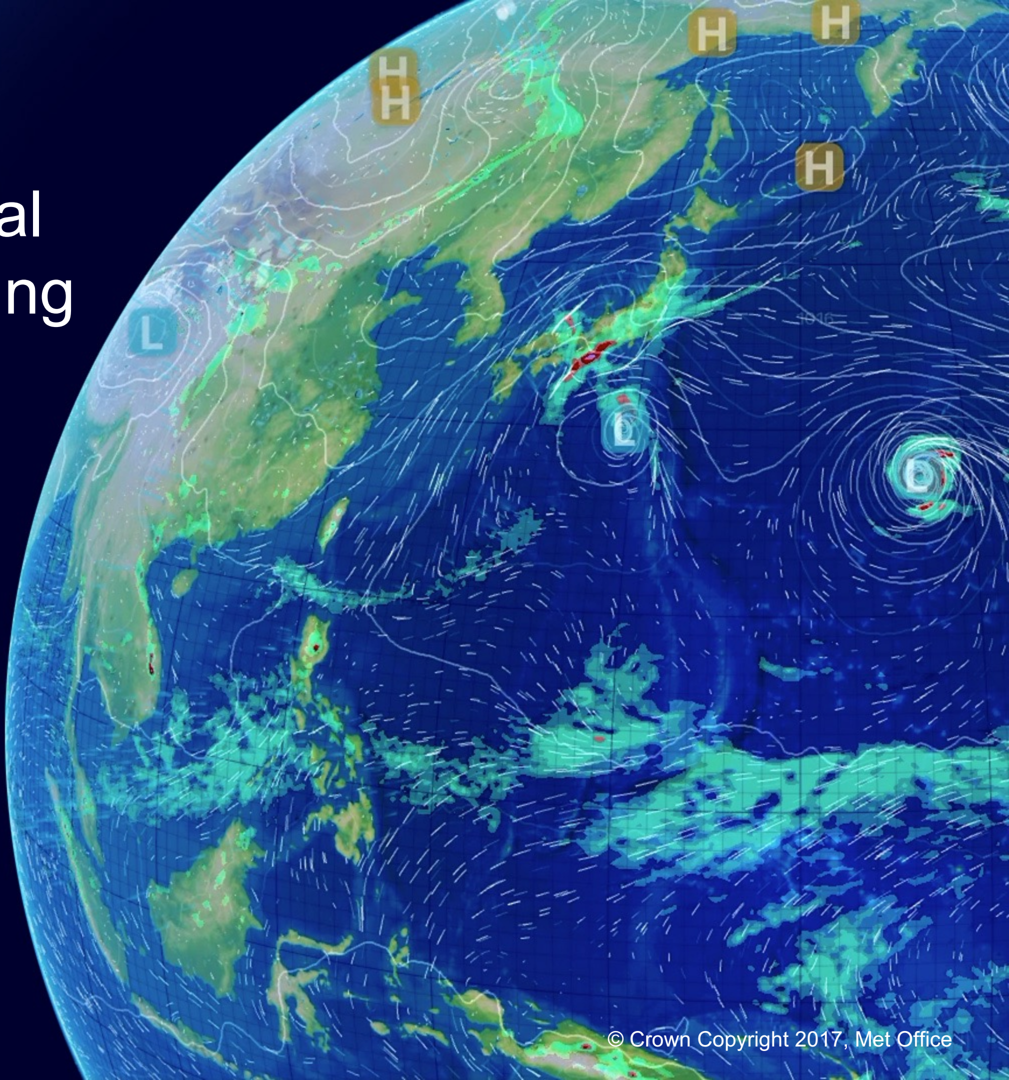

Contents

- Motivations
- Outline of methods
- Seamount test case
- Some results for the test case

- Steps toward schemes treating tracers as cell mean values
  - when density is a non-linear function of T, S and z
  - do we need to use limiters?
  - does it really matter?

- Summary

- References

Schemes subtracting a reference profile (mentioned in the abstract) will not be discussed (last slide)

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Some Motivations

- With stepped bathymetry there are well known issues with flow over sills (Colomobo et al 2020), unphysical side-walls (Adcroft & Marshall 1998) and the representation of bottom torques (Styles et al 2021)

- Regional and global configurations at the Met Office have started to use multi-envelope bathymetry (Bruciaferri et al. 2024)

- Shchepetkin & McWilliams (2003) showed that higher-order schemes with limiters can perform well

- That scheme is a bit ad hoc and treats tracers as point values; it’s attractive to aim for higher order schemes (as in Roullet & Carluer 2026) treating tracers as cell mean values

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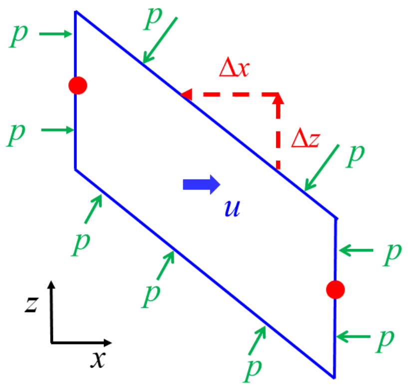
Outline of methods



- The net horizontal pressure force on a velocity cell can be calculated as the sum of the forces on the faces (ff) of the cell (figure top right - Lin 1997)

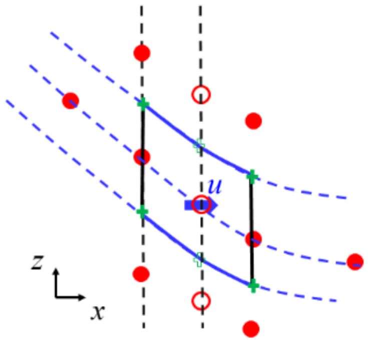
- The horizontal force on the upper face segment Δx is - $p\Delta z$
- The total horizontal force on the cell is

$$F_x = -\oint_C \left(p\frac{\partial z}{\partial s}\right) ds$$

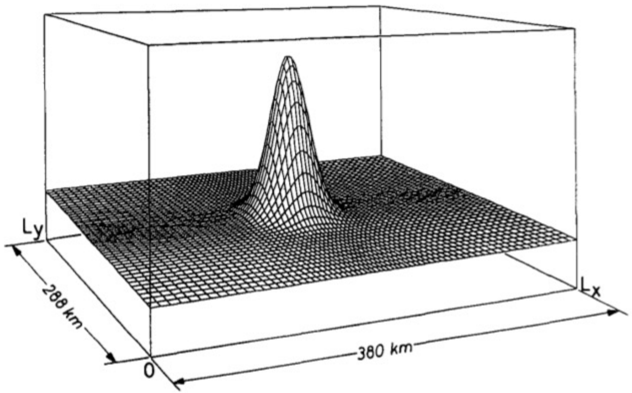
- The density Jacobian (djc) method of Shchepetkin & McWilliams (2003) involves a similar line integral of the density

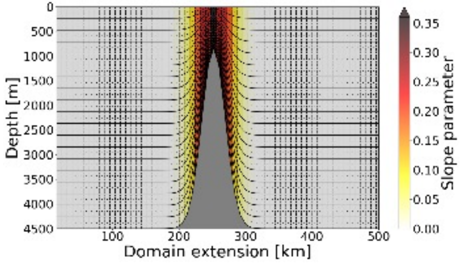
$$G_x = -\oint_C \left(\rho\frac{\partial z}{\partial s}\right) ds$$

- Second order methods to interpolate ρ, p and calculate the line integrals are not accurate enough. One needs to use higher-order methods (figure bottom right)
- The djc scheme uses constrained cubic splines; ffq_ccs does too; ffq_cub uses unconstrained cubics (on upper & lower faces)

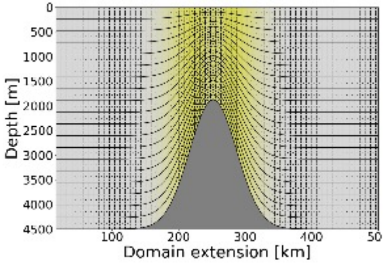


“Standard” test bed





“steep”



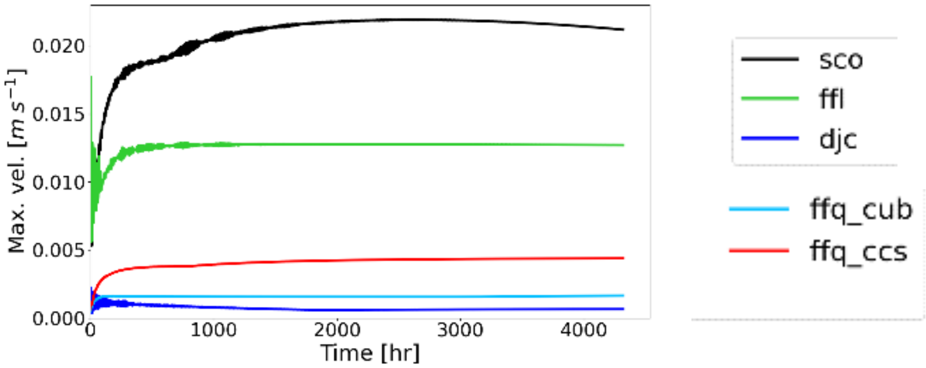
“moderate”

- Classic HPG test case of Beckmann and Haidvogel (1993)
- Isolated 2D Gaussian seamount in an E-W periodic channel (above left)
- Ocean initialised at rest with exponentially decaying density $\rho(z)$
- Velocities should be identically zero
- Details follow the two configurations of Ezer et al. (2002) closely
- No thermal diffusivity, EOS80; Centred advection; A_M = 500 m^2/s
- Seamounts have different steepness (figures above on centre & right)
- Maximum of slope parameter $r_{max} = \Delta H / 2H$ is 0.36 for “steep” and 0.07 for “moderate” configurations
- Greater standardisation of this test case is desirable

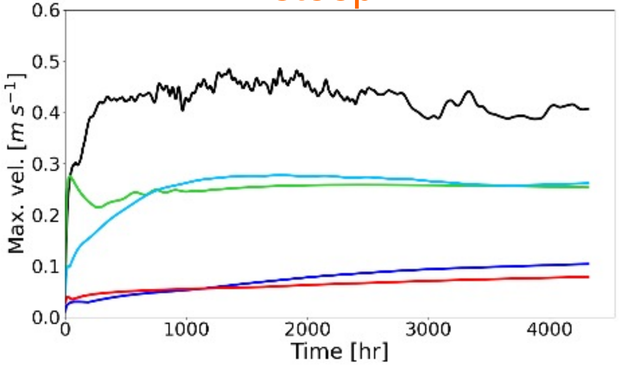
Some results for the test case

- Integrations for 180 days
- **sco** is the standard NEMO scheme
- **ffl** is forces on faces with simple linear interpolation (Lin 1997)
- **djc** and **ffq_cub** give the smallest maximum currents after spin-up for a "moderate" slope
- They are stable even though they do not conserve an analogue of energy
- For the "steep" case the **ffq_ccs** and **djc** schemes give the smallest maximum currents. The **ffq_cub** scheme is no better than **ffl**

"moderate"



"steep"



Use of cell means when density is a non-linear function

- When density is a linear function of $\theta, S,$ the cell mean values give the pressure change in the vertical exactly
- When density is a non-linear function of θ one can perform a Taylor expansion for the density about the cell mean value, θ_m

$$\rho(z) = \rho(\theta_m, z_0) + \left.\frac{\partial \rho}{\partial \theta}\right|_m (\theta - \theta_m) + \left.\frac{\partial \rho}{\partial z}\right|_0 (z - z_0) + \left.\frac{\partial^2 \rho}{\partial \theta^2}\right|_m \frac{(\theta - \theta_m)^2}{2} + \cdots$$

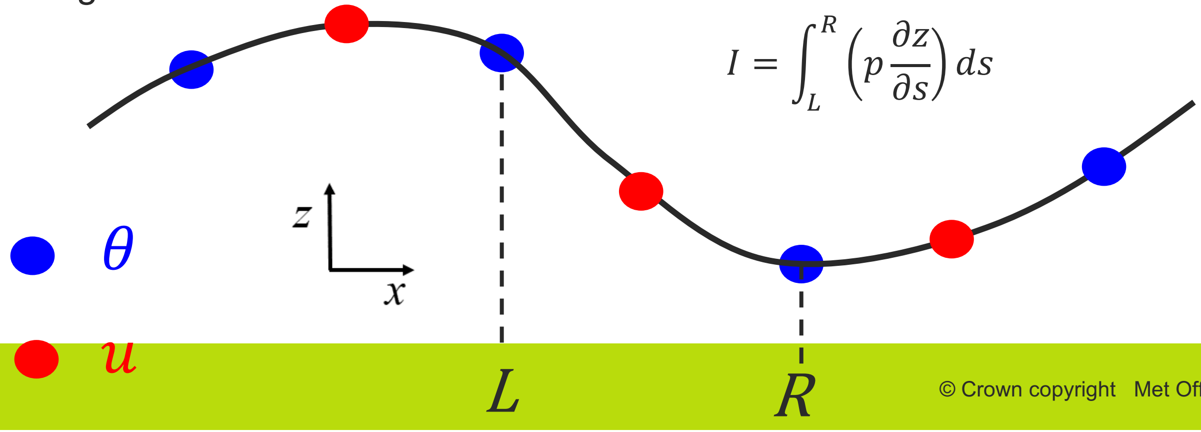
- It turns out that one can obtain fourth order accuracy for the pressure change when terms like $\left.\frac{\partial^2 \rho}{\partial \theta^2}\right|_m \left.\frac{\partial \theta}{\partial z}\right|_m^2$ are calculated with $\left.\frac{\partial \theta}{\partial z}\right|_m$ only second order accurate
- One can obtain fourth order accuracy even for the top and bottom grid cells and with stretched vertical coordinates $\mathrm{z}(s)$
- Calculation of the density derivatives is likely to be expensive; so one might use this approach only for the grid cells near the top and bottom

- Calculation of centred 4th order accurate point values at cell centres and edges is easy. Such calculations are needed in both x- and y-directions.

Do we need to use limiters?

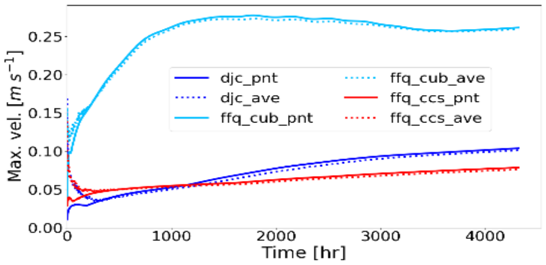
- For advection of tracers one needs to ensure the tracers interpolated to cell boundaries do not overshoot their cell mean values. Is this similarly important for calculation of forces?
- Constrained cubic splines (ccs) flatten slopes at cell boundaries and steepen them within the cell
- We can show that the integral to calculate the horizontal "force" on the cell is nonetheless fourth order accurate (when the slopes on the two sides of an interface are of the same sign)
- We tested the accuracy of calculations of the force on the upper face for the sea-mount bathymetry with the pressure varying like $\exp -kz$ or $\cos kz$
- In some calculations we used the cell height at u and θ points to calculate the slope of the face with smaller grid-spacing and 4th order accuracy, expecting the accuracy to be improved
- The ccs integral (not using the extra cell heights) was the most accurate for $\exp -kz$ despite there being no overshoot for the unconstrained integrals

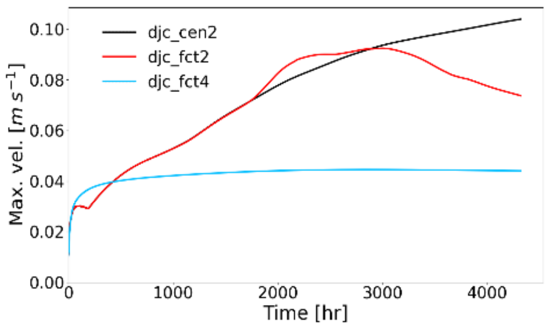
- The accuracy of the ccs scheme for the standard test case may be somewhat fortuitous?



Effects of cell mean initialisation and advection schemes

- Our **djc, ffq_ccs** and ffq_cub schemes treat the cell densities as point values.
- Marsaleix et al 2009 note that this issue affects schemes differently
- The upper figure to the right shows results for the steep configuration initialising cells using point values (pnt; solid lines) and cell average values (ave; dotted lines)
- The initial velocity errors are larger for ave (as expected) but the velocity errors after spin-up are very similar
- Mellor et al (1998) argue that after spin-up the velocity errors depend on the curl of the error in the horizontal pressure forces
- The lower figure to the right shows that the tracer scheme has a significant impact on the velocity errors after spin-up
- The fct schemes reduce the velocity errors; the fct4 scheme reduces them more than the fct2 scheme!





Summary and questions

- Shchepetkin & McWilliams' density Jacobian (djc) scheme gives robust results with smaller velocity errors than second order schemes; it is pragmatic rather than purist

- Schemes to calculate the forces on the faces of cells treating tracers as point values (ff_cub and ff_ccs) are stable and competitive but not better.

- It's more consistent with NEMO's representation of heat to treat tracers as cell mean values. But the impact on simulations might be relatively small (initialisation with grid cell means rather than point values has little impact on the results)

- Uniformly 4th order accurate calculation of the horizontal pressure force from cell mean tracers with a non-linear equation of state is relatively straightforward

- Are monotone interpolation schemes really necessary? The accuracy of the density Jacobian scheme for the Gaussian seamount seems to have another source

A first version of these results was published as IMMERSE Deliverable 3.3: Accurate calculation of pressure forces Dec 2021. That project received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 821926

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Subtraction of a reference profile

- We tried a "pre-processing" step for the djc and ff schemes that subtracted a "reference" density profile $\rho_{\mathrm{ref}}(z)$ from the density at every point used in the calculation of the horizontal force on a column of cells
- This reduced the magnitude of the pressure imbalances but not the velocities in the first 10 days of spin-up. Later in the integrations with very steep bathymetry, large velocities were obtained
- This "pre-processing" led to action not equalling reaction on the side-faces
- A revised "pre-processing" step calculated the density as two components at just the upper and lower faces; one component calculated using the "reference" profile and the other with the differences from it
- For this scheme action equals reaction on the side-faces. But large velocities were again obtained later in integrations with very steep bathymetry