

ICTP School on Quantum Thermodynamics
meets Ultrafast Dynamics

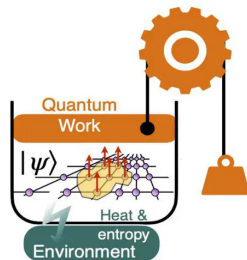
Open Quantum Systems

A brief journey

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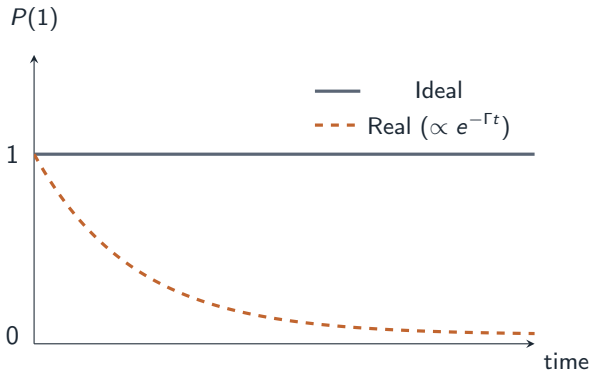


Density matrices, decoherence, and

- I. **Foundations** — ideal vs. real quantum systems, the density matrix, composite systems, the partial trace, entanglement as decoherence, and the system+bath picture.
- II. **Phenomenological master equations** — coherent (Liouville–von Neumann) evolution, the relaxation and dephasing master equations, and their unification into the general Lindblad form.
- III. **Microscopic derivation** — the same Lindblad equation obtained from $H_T = H_S + H_I + H_E$ via the Born, Markov, and secular approximations.
- IV. **Synthesis** — Experimental context, applications, and open frontiers.

Foundations: Density Matrices & Composite Systems

1.1 — Ideal vs. Real Quantum Systems: Relaxation

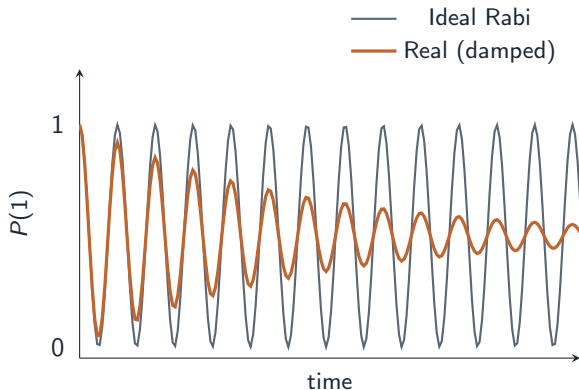


● $|1\rangle$

— $|0\rangle$

Let us consider an excited qubit. Without the environment it should stay excited forever under unitary (Schrödinger) evolution. However, a real qubit instead **relaxes** back to $|0\rangle$. Population leaks irreversibly into the environment.

1.2 — Ideal vs. Real Quantum Systems: Dephasing



Under resonant driving, an ideal qubit performs perfect, undamped **Rabi oscillations** between $|0\rangle$ and $|1\rangle$. A real qubit's oscillations **dephase**. Random phase noise from the environment washes out the coherence and the oscillation amplitude decays even though no energy is necessarily exchanged.

Relaxation (T_1) and dephasing (T_2) are the two elementary ways a real quantum system disagrees with unitary evolution.

Why the numbers matter: coherence times across platforms

Year	Platform	Coherence	Source
1999	Superconducting Cooper-pair box	first oscillations	Nakamura et al., <i>Nature</i> 398 , 786
2009	NV centers (diamond)	$T_2 \approx 1.8$ ms	Balasubramanian et al., <i>Nat. Mater.</i> 8 , 383
2021	Trapped ion (single qubit)	$T_2 > 1$ hour	<i>Nat. Commun.</i> 12 , 233
2021	Superconducting transmon	$T_1 > 0.3$ ms	<i>Nat. Commun.</i> 12 , 1779
2025	Transmon (Ta, 2D)	$T_1 > 1$ ms	<i>Nature</i> (2025), Princeton (Houck lab)

There are order of magnitude of differences in coherence times — All described by the *same* theoretical language: the quantum master equation for ρ .

1.3— The Density Matrix: Why We Need It

The appropriate description of a qubit prepared as “either $|0\rangle$ or $|1\rangle$, each with probability $1/2$ ” is *not* the coherent superposition

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}},$$

but a genuinely different object: a **statistical mixture** of the two states, represented by the **density matrix**

$$\rho = \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2}.$$

A superposition and a 50/50 mixture are physically distinct. The density matrix is the only object general enough to describe a system that is entangled with, or statistically uncertain about, the rest of the world.

1.4 — Populations and Coherences

With $|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

Full mixture

$$\rho = \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Diagonal entries only $\Rightarrow$ **populations**, no coherence.

Superposition

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow \rho' = |+\rangle\langle +| = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Off-diagonal entries $\Rightarrow$ **coherences** (phase information).

Density matrix — general definition and defining properties

$$\rho = \sum_i p_i |\Psi_i\rangle\langle \Psi_i|, \quad \text{mixture of } |\Psi_i\rangle \text{ each with probability } p_i.$$

1. Hermitian: $\rho = \rho^\dagger$
2. Unit trace: $\text{Tr}\{\rho\} = 1$
3. Positive semi-definite: $\langle \Psi | \rho | \Psi \rangle \geq 0$ for all $|\Psi\rangle$

1.5 — Composite Systems: The Partial Trace

Two subsystems A and B , each mixed with probability $1/2$ between $|0\rangle$ and $|1\rangle$ *independently*:

$$\rho = \frac{|0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0| + |1\rangle_A \langle 1| \otimes |1\rangle_B \langle 1|}{2} = \frac{|00\rangle \langle 00| + |11\rangle \langle 11|}{2}.$$

To recover the state of A alone, **trace out** B :

$$\rho_A = \text{Tr}_B \rho = \sum_i {}_B \langle i | \rho | i \rangle_B$$

$$\rho_A = {}_B \langle 0 | \rho | 0 \rangle_B + {}_B \langle 1 | \rho | 1 \rangle_B$$

$$\rho_A = \frac{|0\rangle_A \langle 0| + |1\rangle_A \langle 1|}{2}$$

Exactly the single-qubit mixture we started with.

The partial trace is the unique operation that reproduces the correct measurement statistics for every observable of A alone, whatever happens to B . It is how “the rest of the world is inaccessible” enters the mathematics.

1.6 — Entanglement Looks Like Decoherence From Inside

Now entangle A and B instead of merely correlating them classically:

$$|\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \quad \rho = \frac{|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

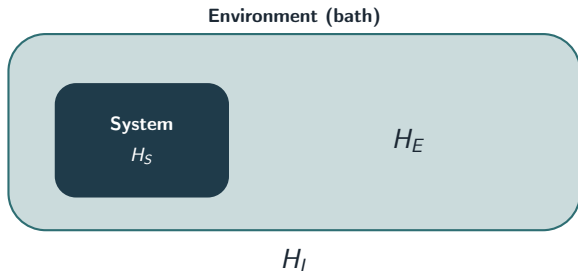
The off-diagonal $|00\rangle\langle 11|$ term is a genuine **coherence** between the two branches. Tracing out B :

$$\rho_A = {}_B\langle 0| \rho |0\rangle_B + {}_B\langle 1| \rho |1\rangle_B = \frac{|0\rangle_A\langle 0| + |1\rangle_A\langle 1|}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Interaction with additional (inaccessible) quantum systems is exactly how *real* quantum systems reproduce the loss of coherence seen at the start of this talk.

The coherence is gone from A alone **decoherence**. Not because A “chose” anything, but because the phase information now lives non-locally, in the correlations with B .

1.7 — The Open-Quantum-System Picture



$$H_T = H_S + H_I + H_E$$

Real quantum systems are modeled by embedding the system of interest in an **environment** of other quantum systems it interacts with. An environment large enough that its own state is essentially unaffected is also called a **(thermal) bath**.

Goal for the rest of this talk: an equation of motion for $\rho_S = \text{Tr}_E \rho_{SE}$ alone — the bath traced out, its effect retained through H_I .

Phenomenological Master Equations

2.1 — Coherent Evolution

Schrödinger equation

$$\frac{\partial}{\partial t} |\Psi(t)\rangle = -\frac{i}{\hbar} H |\Psi(t)\rangle, \quad |\Psi(t)\rangle = U(t) |\Psi(0)\rangle, \quad \frac{\partial}{\partial t} \langle\Psi(t)| = \frac{i}{\hbar} \langle\Psi(t)| H, \quad \langle\Psi(t)| = \langle\Psi(0)| U^\dagger(t).$$

For an ensemble $\rho = \sum_i p_i |\Psi_i\rangle \langle\Psi_i|$, each member evolves this way, so $\rho(t) = \sum_i p_i |\Psi_i(t)\rangle \langle\Psi_i(t)|$ and

$$\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} H \rho(t) + \frac{i}{\hbar} \rho(t) H.$$

Liouville–von Neumann equation $\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} [H, \rho(t)]$

This is exact, unitary, closed-system evolution.

2.2 — The Relaxation Master Equation

With $H = \frac{\hbar\omega_0}{2}\sigma_z = \frac{\hbar\omega_0}{2}(|1\rangle\langle 1| - |0\rangle\langle 0|)$ and the ladder operators

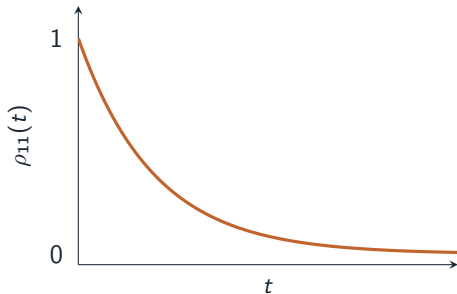
$$\sigma^- = |0\rangle\langle 1|, \quad \sigma^+ = |1\rangle\langle 0| = (\sigma^-)^\dagger,$$

postulate the simplest dissipative addition to Liouville–von Neumann that drives population from $|1\rangle$ to $|0\rangle$ at rate Γ :

$$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{\Gamma}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

Solving, with $\rho(0) = \begin{pmatrix} \rho_{11} & \rho_{10} \\ \rho_{01} & \rho_{00} \end{pmatrix}$:

$$\rho(t) = \begin{pmatrix} \rho_{11}e^{-\Gamma t} & \rho_{10}e^{-(\Gamma/2+i\omega_0)t} \\ \rho_{01}e^{-(\Gamma/2-i\omega_0)t} & \rho_{00} + \rho_{11}(1 - e^{-\Gamma t}) \end{pmatrix}$$



Populations relax exponentially at rate $\Gamma \equiv 1/T_1$; coherences decay at $\Gamma/2$ and pick up the free precession phase $e^{\mp i\omega_0 t}$.

2.3 — The Dephasing Master Equation

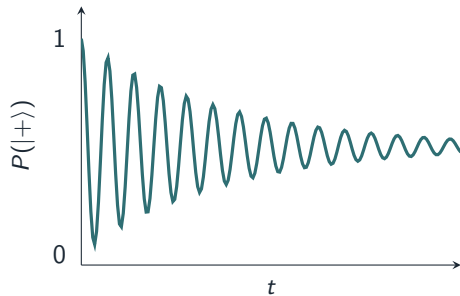
Same $H = \frac{\hbar\omega_0}{2}\sigma_z$, but now the postulated dissipator uses σ_z itself — randomizing phase without touching populations:

$$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{\Gamma}{2}(\sigma_z\rho\sigma_z - \rho)$$

Solving:

$$\rho(t) = \begin{pmatrix} \rho_{11} & \rho_{10}e^{-(\Gamma+i\omega_0)t} \\ \rho_{01}e^{-(\Gamma-i\omega_0)t} & \rho_{00} \end{pmatrix}$$

Populations ρ_{11}, ρ_{00} are exactly conserved; only the coherences decay, at rate $\Gamma \equiv 1/T_2^*$.



$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$: an equal superposition, prepared once and left to free-precess, loses its oscillation amplitude while its populations stay fixed.

2.4 — Postulating the General Lindblad Form

Liouville–von Neumann	$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)]$
Relaxation	$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{\Gamma}{2}(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-)$
Dephasing	$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \Gamma(\sigma_z\rho\sigma_z - \rho)$
Lindblad (general)	$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \sum_i \frac{\gamma_i}{2}(2L_i\rho L_i^\dagger - L_i^\dagger L_i\rho - \rho L_i^\dagger L_i), \quad \gamma_i > 0$

Using $\sigma_z\sigma_z = \mathbb{I}$, the dephasing equation is recovered from the relaxation-type pattern with $L = \sigma_z$, $\gamma = \Gamma$ (up to the factor-of-2 convention), both relaxation and dephasing are special cases of *one* general structure. Pick a set of **jump operators** L_i and non-negative **rates** γ_i .

2.5— The Lindblad Equation Is Trace-Preserving

$$\frac{\partial}{\partial t} \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] + \sum_i \frac{\gamma_i}{2} (2L_i \rho L_i^\dagger - L_i^\dagger L_i \rho - \rho L_i^\dagger L_i)$$

Take the trace term by term, using cyclicity $\text{Tr}\{ABC\} = \text{Tr}\{CAB\} = \text{Tr}\{BCA\}$ and linearity:

$$\text{Tr}\{[H, \rho(t)]\} = \text{Tr}\{H\rho(t)\} - \text{Tr}\{\rho(t)H\} = 0,$$

$$\text{Tr}\{2L_i \rho L_i^\dagger - L_i^\dagger L_i \rho - \rho L_i^\dagger L_i\} = 2\text{Tr}\{L_i^\dagger L_i \rho\} - \text{Tr}\{L_i^\dagger L_i \rho\} - \text{Tr}\{L_i^\dagger L_i \rho\} = 0.$$

$$\text{Tr}\left\{\frac{\partial}{\partial t} \rho(t)\right\} = -\frac{i}{\hbar} \text{Tr}\{[H, \rho(t)]\} + \sum_i \frac{\gamma_i}{2} \text{Tr}\{2L_i \rho L_i^\dagger - L_i^\dagger L_i \rho - \rho L_i^\dagger L_i\} = 0$$

$\text{Tr}\{\rho(t)\} = 1$ for all t the probability is conserved automatically, term by term, for *any* choice of jump operators L_i and rates $\gamma_i > 0$. This is a structural property of the Lindblad form.

2.6— Dynamical Maps and Complete Positivity

Writing the Lindblad equation as a linear superoperator acting on ρ :

$$\frac{\partial}{\partial t}\rho(t) = \mathcal{L}\rho(t) \implies \rho(t) = e^{\mathcal{L}t}\rho(0) \equiv \mathcal{E}(t)\rho(0) \quad (\text{a dynamical map}).$$

The Lindblad master equation is **trace-preserving**
 $\Rightarrow$ it guarantees $\text{Tr}\{\rho(t)\} = 1$ stays true.

The Lindblad master equation generates **completely positive** dynamical maps $\Rightarrow$ it guarantees $\rho(t) \geq 0$ stays true, *even* if the system is entangled with an untouched spectator.

$\rho(t) = \mathcal{E}(t)\rho(0)$ is guaranteed to remain a valid density matrix at all times.

But where do L_i and γ_i actually come from, physically? So far they were guessed from the phenomenology we wanted to reproduce (T_1 , T_2). Next, we derive them from the microscopic system+bath Hamiltonian.

Microscopic Derivation: Born, Markov, Secular

3.1 — Setting Up the Microscopic Problem

Total (system + environment) Hamiltonian, evolving *unitarily* under Liouville–von Neumann — exactly, with no dissipation yet:

$$H_T = \underbrace{H_S}_{\text{system}} + \underbrace{H_E}_{\text{bath}} + \underbrace{H_I}_{\text{coupling}}, \quad \frac{\partial}{\partial t} \rho_{SE}(t) = -\frac{i}{\hbar} [H_T, \rho_{SE}(t)]$$

The reduced system state is obtained by tracing out the bath:

$$\rho_S(t) = \text{Tr}_E \{ \rho_{SE}(t) \}.$$

Goal: to achieve a closed equation for $\rho_S(t)$ alone.

Write the coupling in the generic factorized form $H_I = \sum_{\alpha} A_{\alpha} \otimes B_{\alpha}$, with A_{α} system operators and B_{α} bath operators.

3.2 — Interaction Picture and Formal Integration

Move to the interaction picture w.r.t. $H_0 = H_S + H_E$: $\tilde{\rho}_{SE}(t) = U_0^\dagger(t)\rho_{SE}(t)U_0(t)$, $U_0(t) = e^{-iH_0 t/\hbar}$, and likewise $\tilde{H}_I(t) = U_0^\dagger(t)H_I U_0(t)$. Liouville–von Neumann becomes

$$\frac{\partial}{\partial t}\tilde{\rho}_{SE}(t) = -\frac{i}{\hbar}[\tilde{H}_I(t), \tilde{\rho}_{SE}(t)].$$

Formally integrating once (*exact, no approximation yet*):

$$\tilde{\rho}_{SE}(t) = \tilde{\rho}_{SE}(0) - \frac{i}{\hbar} \int_0^t ds [\tilde{H}_I(s), \tilde{\rho}_{SE}(s)]$$

Substituting this back into the right-hand side of the original equation gives an exact, closed integro-differential equation — the starting point for every approximation that follows:

$$\frac{\partial}{\partial t}\tilde{\rho}_{SE}(t) = -\frac{i}{\hbar}[\tilde{H}_I(t), \tilde{\rho}_{SE}(0)] - \frac{1}{\hbar^2} \int_0^t ds [\tilde{H}_I(t), [\tilde{H}_I(s), \tilde{\rho}_{SE}(s)]] .$$

3.3 — The Born Approximation

Trace out the bath and assume (i) system and bath start uncorrelated, $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E$, and $\text{Tr}_E\{\tilde{H}_I(t), \rho_S(0) \otimes \rho_E\} = 0$ from the previous slide's assumption, so the first term drops. This leaves

$$\frac{\partial}{\partial t} \tilde{\rho}_S(t) = -\frac{1}{\hbar^2} \int_0^t ds \text{Tr}_E \left\{ [\tilde{H}_I(t), [\tilde{H}_I(s), \tilde{\rho}_{SE}(s)]] \right\}.$$

Born approximation. For weak coupling, the bath is huge and effectively unaffected by the system: it stays in its (stationary) state ρ_E at all times, and correlations that build up between S and E remain small and never feed back appreciably. Formally, $\tilde{\rho}_{SE}(s) \approx \tilde{\rho}_S(s) \otimes \rho_E$ for all $s \leq t$.

$$\frac{\partial}{\partial t} \tilde{\rho}_S(t) \approx -\frac{1}{\hbar^2} \int_0^t ds \text{Tr}_E \left\{ [\tilde{H}_I(t), [\tilde{H}_I(s), \tilde{\rho}_S(s) \otimes \rho_E]] \right\} \quad (\text{Redfield equation}).$$

3.4 — The Two Markov Approximations

The Redfield equation still has memory: $\tilde{\rho}_S(s)$ under the integral depends on the *entire past*. Two successive steps remove it.

Step 1 — substitute $s \rightarrow t - \tau$

Change variables $\tau = t - s$. The integral runs over how far into the past the bath “remembers” via its correlation functions $\langle \tilde{B}_\alpha(t) \tilde{B}_\beta(t - \tau) \rangle$.

Step 2 — Markov #1

Replace $\tilde{\rho}_S(t - \tau) \rightarrow \tilde{\rho}_S(t)$ inside the integral. The bath changes much faster (τ_B) than the system ρ_S changes on τ_R , i.e., $\tau_B \ll \tau_R$. The bath forgets rapidly about the system.

Step 3 — Markov #2

Extend the upper integration limit $t \rightarrow \infty$: valid once $t \gg \tau_B$, since the bath-correlation integrand contributes negligibly beyond τ_B . The environment only remember the recent past, the system remains almost the same.

Together, these give the **Markov approximation**: $\rho_S(t)$ now depends only on its *present* value, not its history.

$$\frac{\partial}{\partial t} \tilde{\rho}_S(t) \approx -\frac{1}{\hbar^2} \int_0^\infty d\tau \operatorname{Tr}_E \left\{ [\tilde{H}_I(t), [\tilde{H}_I(t - \tau), \tilde{\rho}_S(t) \otimes \rho_E]] \right\}$$

3.5 — Eigenoperator Decomposition of the Coupling

Decompose each system coupling operator A_α into H_S -eigenoperators. With $H_S = \sum_\varepsilon \varepsilon \Pi_\varepsilon$ (spectral projectors Π_ε) and Bohr frequencies $\omega = \varepsilon' - \varepsilon$, define

$$A_\alpha(\omega) = \sum_{\varepsilon' - \varepsilon = \omega} \Pi_\varepsilon A_\alpha \Pi_{\varepsilon'}, \quad \text{so} \quad A_\alpha = \sum_\omega A_\alpha(\omega) = \sum_\omega A_\alpha^\dagger(\omega).$$

These satisfy the key interaction-picture property (each $A_\alpha(\omega)$ rotates at a single, pure frequency ω):

$$U_0^\dagger(t) A_\alpha(\omega) U_0(t) = e^{-i\omega t} A_\alpha(\omega) \implies \tilde{H}_I(t) = \sum_{\alpha, \omega} e^{-i\omega t} A_\alpha(\omega) \otimes \tilde{B}_\alpha(t)$$

This turns the double commutator into an explicit double sum over frequencies ω, ω' , each term oscillating in time as $e^{i(\omega' - \omega)t}$ — exactly the structure needed for the next (secular) approximation.

3.6 — Bath Correlation Functions

Substituting the decomposition and performing the bath trace produces one-sided Fourier transforms of the (stationary) bath correlation functions:

$$\Gamma_{\alpha\beta}(\omega) = \int_0^\infty d\tau e^{i\omega\tau} \langle \tilde{B}_\alpha^\dagger(t) \tilde{B}_\beta(t-\tau) \rangle_{\rho_E}$$

Splitting into Hermitian and anti-Hermitian parts defines two physically distinct real quantities:

$$\Gamma_{\alpha\beta}(\omega) = \frac{1}{2} \gamma_{\alpha\beta}(\omega) + i S_{\alpha\beta}(\omega), \quad \gamma_{\alpha\beta}(\omega) = \int_{-\infty}^\infty d\tau e^{i\omega\tau} \langle \tilde{B}_\alpha^\dagger(t) \tilde{B}_\beta(t-\tau) \rangle_{\rho_E}.$$

$\gamma_{\alpha\beta}(\omega)$: the **full** Fourier transform — real, and (by Bochner's theorem) forms a positive-semidefinite matrix in α, β . These become the physical **decay rates**.

$S_{\alpha\beta}(\omega)$: the antisymmetric part — generates a Hermitian correction to the system Hamiltonian, the **Lamb shift** H_{LS} , a coherent renormalization from the system-bath coupling.

3.7 — The Secular (Rotating-Wave) Approximation

The master equation now contains cross terms oscillating as $e^{i(\omega' - \omega)t}$ for every pair of Bohr frequencies ω, ω' .
Two regimes:

Non-degenerate spectrum

If the relaxation timescale $\tau_R \sim 1/\gamma$ is much longer than the inverse level-spacing $1/|\omega - \omega'|$, terms with $\omega \neq \omega'$ average to zero over the system's own dynamical timescale and may be dropped.

Secular approximation: keep only $\omega = \omega'$ terms. Physically. The phases due $(\omega - \omega')t$ average to zero faster than the bath can couple them.

What survives is diagonal in ω : a single frequency sum, with $\gamma_{\alpha\beta}(\omega)$ and $S_{\alpha\beta}(\omega)$.

3.8 — The Derived Lindblad Master Equation

Collecting every term, the Redfield equation — after Born, Markov, and secular approximations:

$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}[H_S + H_{LS}, \rho_S(t)] + \sum_{\omega} \sum_{\alpha, \beta} \gamma_{\alpha\beta}(\omega) \left(A_{\beta}(\omega) \rho_S(t) A_{\alpha}^{\dagger}(\omega) - \frac{1}{2} \{ A_{\alpha}^{\dagger}(\omega) A_{\beta}(\omega), \rho_S(t) \} \right)$$

with the Lamb-shift Hamiltonian $H_{LS} = \sum_{\omega} \sum_{\alpha, \beta} S_{\alpha\beta}(\omega) A_{\alpha}^{\dagger}(\omega) A_{\beta}(\omega)$, which commutes with H_S and only renormalizes system energy levels.

Every object here — jump operators $A_{\alpha}(\omega)$, rates $\gamma_{\alpha\beta}(\omega)$, and the Lamb shift H_{LS} — is computed directly from H_S , H_E , H_I , and the bath state ρ_E .

3.9(a) — Diagonal Form and Positivity

The rate matrix $\gamma_{\alpha\beta}(\omega)$ is Hermitian and positive-semidefinite for each fixed ω . It can therefore always be diagonalized by a unitary $u(\omega)$:

$$\sum_{\alpha,\beta} u_{k\alpha}(\omega) \gamma_{\alpha\beta}(\omega) u_{k'\beta}^*(\omega) = \gamma_k(\omega) \delta_{kk'}, \quad \gamma_k(\omega) \geq 0.$$

Defining new jump operators $A_k(\omega) = \sum_{\alpha} u_{k\alpha}(\omega) A_{\alpha}(\omega)$ brings the master equation to the manifestly diagonal:

$$\frac{\partial}{\partial t} \rho_S(t) = -\frac{i}{\hbar} [H_S + H_{LS}, \rho_S(t)] + \sum_{k,\omega} \gamma_k(\omega) \left(A_k(\omega) \rho_S A_k^\dagger(\omega) - \frac{1}{2} \{ A_k^\dagger(\omega) A_k(\omega), \rho_S \} \right)$$

Because every $\gamma_k(\omega) \geq 0$, this is *automatically* a valid Lindblad generator — trace-preserving and completely positive, now guaranteed by construction rather than assumed.

3.9(b) — Lindblad Form: The Jump Operators

These $A_k(\omega)$ are the microscopic jump operators: each one describes one bath-induced transition of the system at Bohr frequency ω .

$$\frac{\partial}{\partial t}\rho_S = -\frac{i}{\hbar}[H_S + H_{LS}, \rho_S] + \sum_{k,\omega} \gamma_k(\omega) \left(A_k(\omega)\rho_S A_k^\dagger(\omega) - \frac{1}{2} \left\{ A_k^\dagger(\omega)A_k(\omega), \rho_S \right\} \right)$$

Now combine the pair of labels (k, ω) into one label j and define

$$L_j \equiv A_k(\omega), \quad \gamma_j \equiv \gamma_k(\omega).$$

$$\frac{\partial}{\partial t}\rho_S = -\frac{i}{\hbar}[H_S + H_{LS}, \rho_S] + \sum_j \gamma_j \left(\underbrace{L_j \rho_S L_j^\dagger}_{\text{jump}} - \frac{1}{2} \underbrace{\left\{ L_j^\dagger L_j, \rho_S \right\}}_{\text{normalization and positivity}} \right)$$

This is exactly the Lindblad form introduced earlier.

3.10 — Consistency Check: Recovering the Postulated Equations

Take $H_S = \frac{\hbar\omega_0}{2}\sigma_z$ coupled linearly to a bosonic bath, $H_I = \sigma_x \otimes B$ with $B = \sum_k g_k(b_k + b_k^\dagger)$ (the standard spin-boson model). The eigenoperators of σ_x w.r.t. H_S are $A(\pm\omega_0) = \sigma^\mp$, and the machinery above gives rates $\gamma(\omega_0) \propto \bar{n}(\omega_0) + 1$ (emission) and $\gamma(-\omega_0) \propto \bar{n}(\omega_0)$ (absorption), where $\bar{n}(\omega_0)$ is the bath's thermal occupation.

At zero temperature ($\bar{n} \rightarrow 0$) only emission survives, $\gamma(\omega_0) \equiv \Gamma$, $\gamma(-\omega_0) \rightarrow 0$: this is *exactly* the relaxation master equation postulated before, with Γ now an explicit function of the coupling strengths g_k and the bath spectral density..

An analogous coupling through σ_z (dephasing-type) reproduces the dephasing equation.

Synthesis: Postulate, Derivation, Experiment

	Part II — Postulated	Part III — Derived
Starting point	Physical intuition: population decay at rate Γ , dephasing at rate Γ	Microscopic $H_T = H_S + H_I + H_E$, exact Liouville–von Neumann
Jump operators L_i	Guessed (σ^- for relaxation, σ_z for dephasing)	Computed: H_S -eigenoperators $A_\alpha(\omega)$ of the coupling H_I
Rates γ_i	Free parameters, fit to observed T_1, T_2	Fourier transforms $\gamma_{\alpha\beta}(\omega)$ of bath correlation functions — computable from H_I and ρ_E
Energy shifts	Not present	Lamb shift H_{LS} , an automatic by-product
Structural guarantees	Assumed (trace-preserving, completely positive <i>by choice of form</i>)	<i>Proven</i> : automatic once $\gamma_k(\omega) \geq 0$, itself guaranteed by Bochner's theorem

When Does the Lindblad Equation Actually Apply?

- **Born:** coupling H_I weak compared to H_S, H_E energy scales;
- **Markov:** bath correlation time τ_B (set by the bath's spectral width) much shorter than the system's relaxation time $\tau_R \sim 1/\Gamma$;
- **Secular:** level splittings $|\omega - \omega'|$ resolved on the timescale τ_R , i.e. $\Gamma \ll |\omega - \omega'|$.

Breakdown examples. Strong coupling (many solid-state qubits at low T), structured/non-Markovian baths (photonic-crystal or engineered environments), or near-degenerate levels (as in some multi-qubit decoherence-free subspaces) all violate one assumption — and require non-Markovian or strong-coupling master equations.

Rule of thumb

If measured T_1, T_2 are exponential (single-rate) decays, Lindblad is almost always an excellent effective description.

Experimental Grounding: Lindblad Dynamics Across Platforms

Platform	What was measured	Source
Trapped Hg^+ ion	First observation of quantum jumps: discrete, random telegraph switching of fluorescence	Bergquist et al., <i>Phys. Rev. Lett.</i> 57 , 1699 (1986)
Superconducting Cooper-pair box	First demonstration of coherent qubit control and its decoherence (Rabi oscillations, T_2)	Nakamura, Pashkin & Tsai, <i>Nature</i> 398 , 786 (1999)
Rydberg atom + microwave cavity	Real-time monitoring of individual photon quantum jumps in a cavity	Gleyzes et al., <i>Nature</i> 446 , 297 (2007)
NV center in diamond	Room-temperature T_2 and Lindblad-type spin-bath decoherence	Balasubramanian et al., <i>Nat. Mater.</i> 8 , 383 (2009)
Modern transmon qubits	GKSL-based T_1/T_2 benchmarking of state-of-the-art tantalum devices	Place et al., <i>Nat. Commun.</i> 12 , 1779 (2021); Wang et al., <i>Nature</i> (2025)

Across every platform, the same structure — a Lindblad equation with a handful of jump operators and rates — reproduces measured decay curves quantitatively.

Why This Matters: Applications

Quantum computing

Lindblad rates $\gamma_k(\omega)$ set gate fidelities and coherence budgets; error-correction thresholds are derived assuming (approximately) Markovian.

Quantum sensing

Decoherence rates $\gamma_k(\omega)$ measured via T_1 , T_2 directly quantify a sensor's coupling to its environment — the basis of NV-center and trapped-ion magnetometry.

Quantum thermodynamics

Heat and work are defined via the two terms of the Lindblad equation (unitary = work-like, dissipative = heat-like); e.g. Levy & Kosloff, *Europhys. Lett.* **107**, 20004 (2014).

Open quantum biology & chemistry

Excitation transport in light-harvesting complexes is modeled with secular and non-secular Lindblad equations; see e.g. Cattaneo et al., *New J. Phys.* **21**, 113045 (2019).

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