

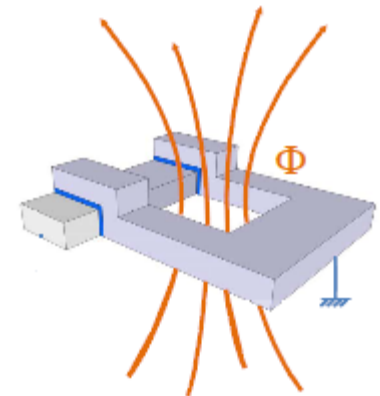
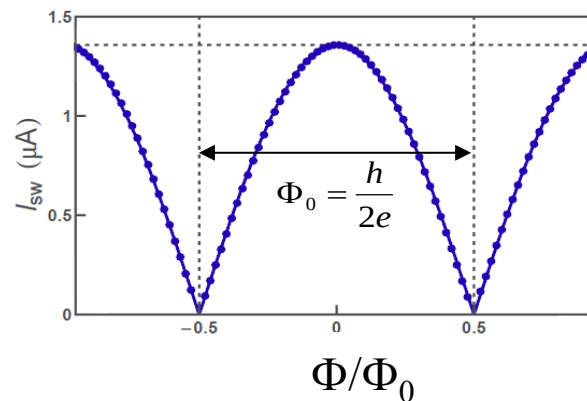


Superconductivity: macroscopic manifestation of quantum physics

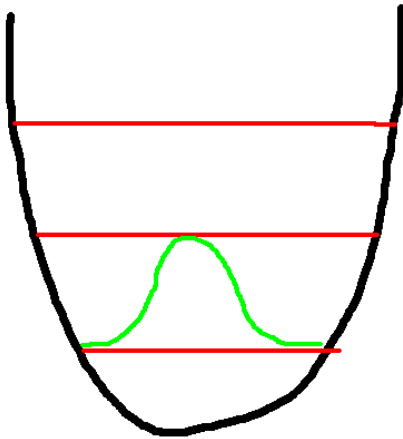
Maciej Zgirski

Institute of Physics of the Polish Academy of Sciences, **MagTop**, **CoolPhon Group**

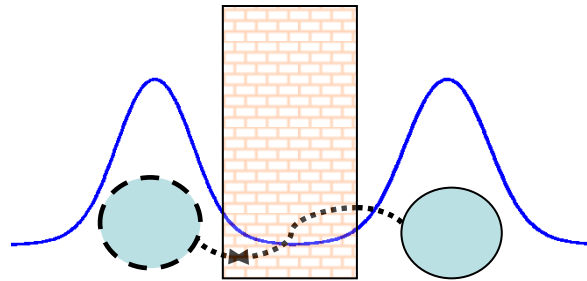
Summer School: Ultrafast electronics meets quantum thermodynamics, ICTP Trieste, 24/09/2026



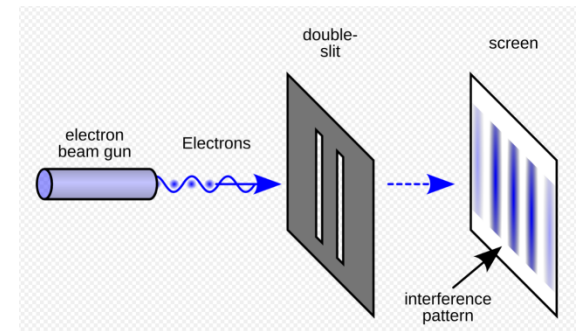
Quantum world



Discrete energy levels



Tunnelling



https://en.wikipedia.org/wiki/Double-slit_experiment#/media/File:Double-slit.svg

Interference/diffraction

Phenomena associated with single atoms/molecules

Can macroscopic objects (ensembles of particles) behave in a quantum way, i.e. as they were a single particle?

Engineered *artificial atoms* consisting of many particles?

A collective state needed...

Well known example: LASER

Macroscopic wavefunction

$$\Psi(r) = \sqrt{\rho} e^{i\theta(r)}$$

Derivation in
Feynman
Lectures (vol. III)

$$\vec{j}_s = \frac{-\hbar}{2m_e} (\nabla\theta - \frac{2e}{\hbar} A) \rho (2e)$$

$\vec{j}_s$ -supercurrent density

ρ – density of Cooper pairs

θ - macroscopic superconducting phase

The same for large number of Cooper pairs!!!

$$\vec{j} = \frac{\hbar}{m} (\nabla\theta - \frac{q}{\hbar} A) \cdot \zeta$$

Formula in Feynman treats ρ as a density of charge and m and q as a mass and charge of the Cooper pair respectively

Can we see the phase?

Fluxoid quantization

$$-\nabla\theta = \frac{-m_e j_s}{e\rho\hbar} + \frac{2eA}{\hbar}$$

$$\oint -\nabla\theta dl = \oint \frac{-m_e j_s}{e\rho\hbar} dl + \oint \frac{2eA}{\hbar} dl$$

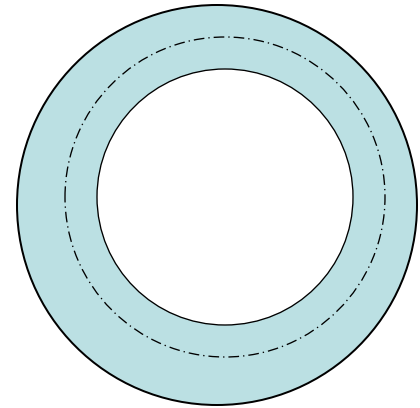
$$2\pi n = \oint \frac{-m_e j_s}{e\rho\hbar} dl + \frac{2e}{\hbar} \Phi_{tot}$$

$$\underbrace{\Phi_{tot} - \frac{m_e}{2e^2\rho} \oint j_s dl}_{\text{fluxoid}} \equiv \Phi' = n \frac{h}{2e} = n \Phi_0$$

fluxoid

LONDON, 1950

$$\Phi_0 = \frac{h}{2e} \quad \text{fluxon (flux quantum)}$$



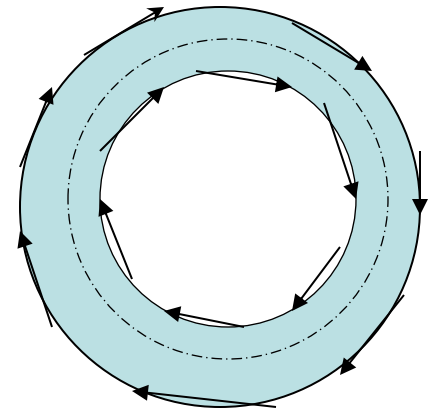
Superconducting Ring

$j = 0$ inside the ring due to the Meissner effect

screening current I_s flows only on the surface of the ring

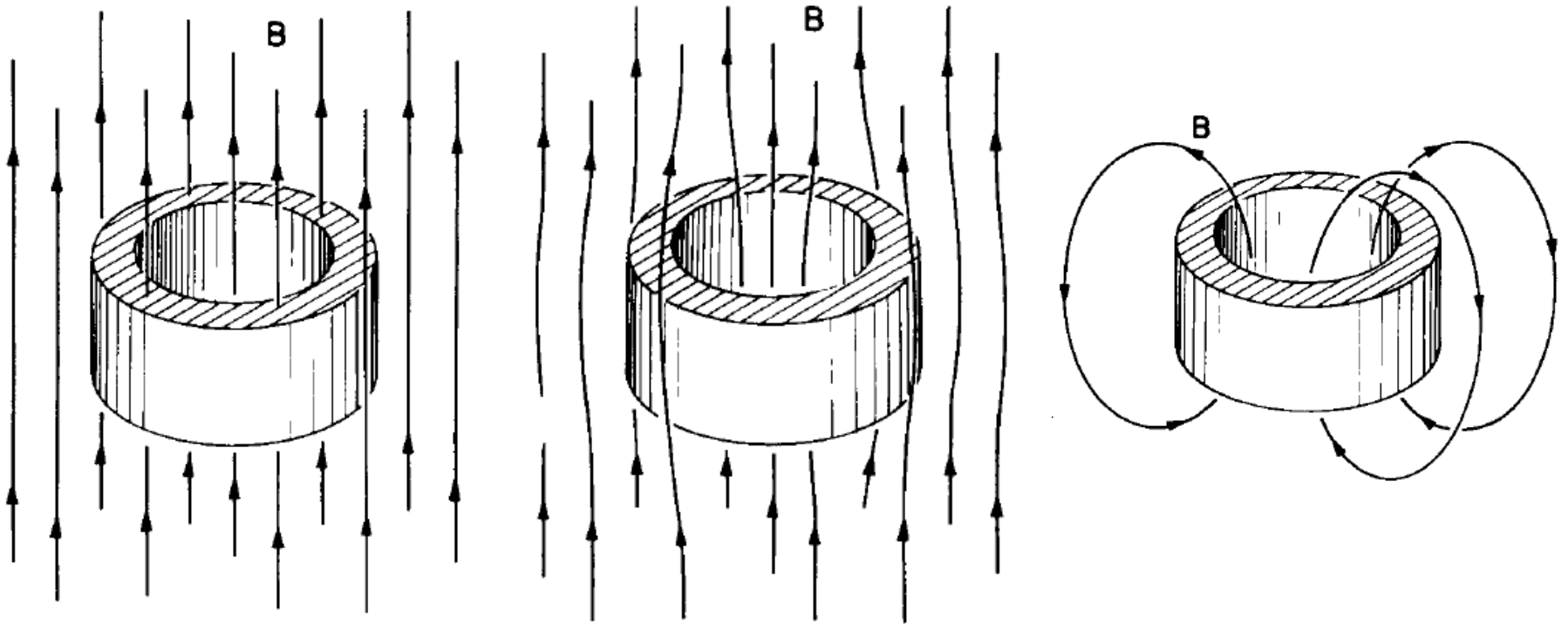
$$\Phi_{tot} = n \Phi_0$$

Fluxoid quantization becomes flux quantization !!!



$$\Phi_{tot} = \Phi_{ext} - L_{geom} I_s$$

Superconducting Ring – Meissner effect



$$\Phi_{tot} = n \Phi_0$$

$$\Phi_0 = \frac{h}{2e} \text{ fluxon (flux quantum)}$$

What happens if $j \neq 0$?

$$-\nabla\theta = \frac{-m_e j_s}{e\rho\hbar} + \frac{2eA}{\hbar}$$

$$\oint -\nabla\theta dl = \oint \frac{-m_e j_s}{e\rho\hbar} dl + \oint \frac{2eA}{\hbar} dl$$

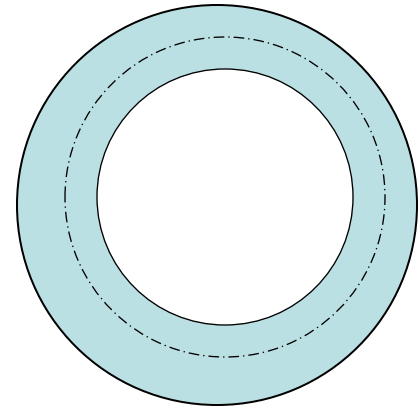
$$2\pi n = \oint \frac{-m_e j_s}{e\rho\hbar} dl + \frac{2e}{\hbar} \Phi_{tot}$$

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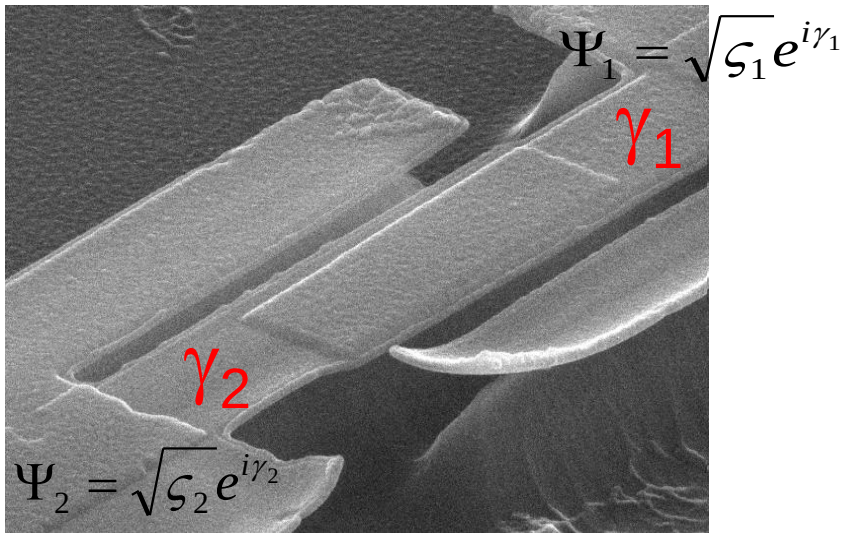


$$\Phi_{tot} - L_K I = n \Phi_0$$

$$\Phi_{tot} = \Phi_{ext} - L_{geom} I$$

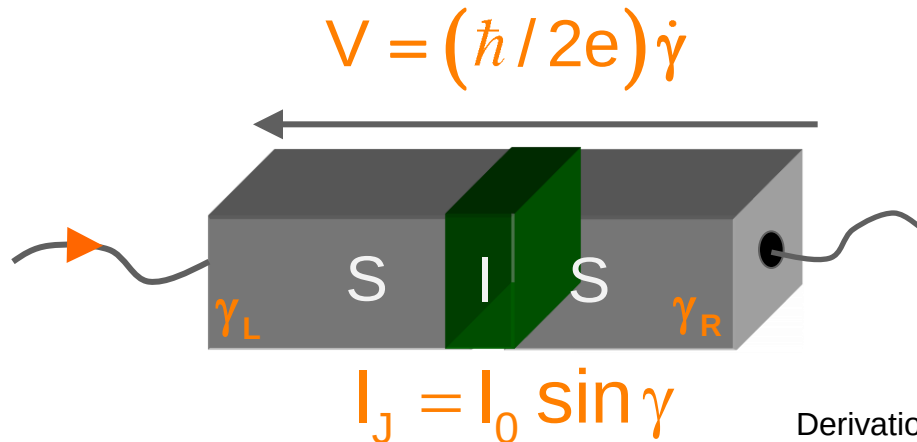
$$\Phi_{ext} - (L_K + L_{geom}) I = n \Phi_0$$

Josephson relations



$$I = I_0 \cdot \sin(\gamma_2 - \gamma_1)$$

$$\frac{d(\gamma_2 - \gamma_1)}{dt} = \frac{2 \cdot e}{\hbar} \cdot V$$



Derivation of Feynman relations:

Feynman, vol. III, seminar on superconductivity

E-beam lithography

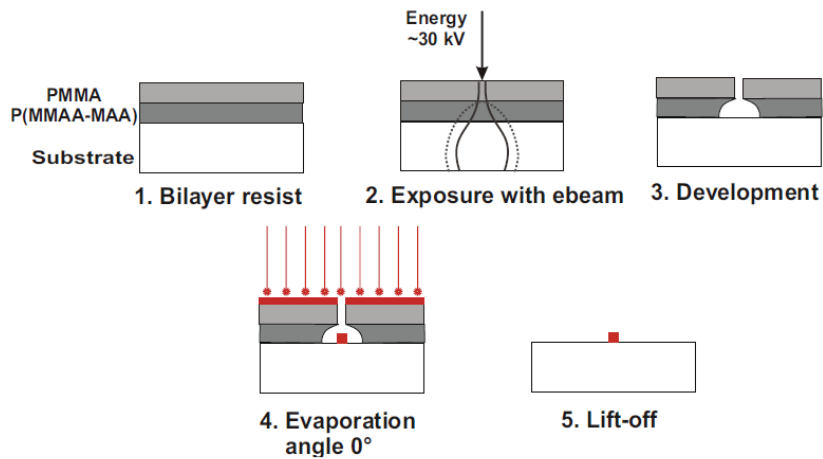


Figure 2.1 Fabrication of metallic structures by using positive electron beam lithography and evaporation techniques. Bilayer resist is used to achieve the undercut structure shown in step 3.

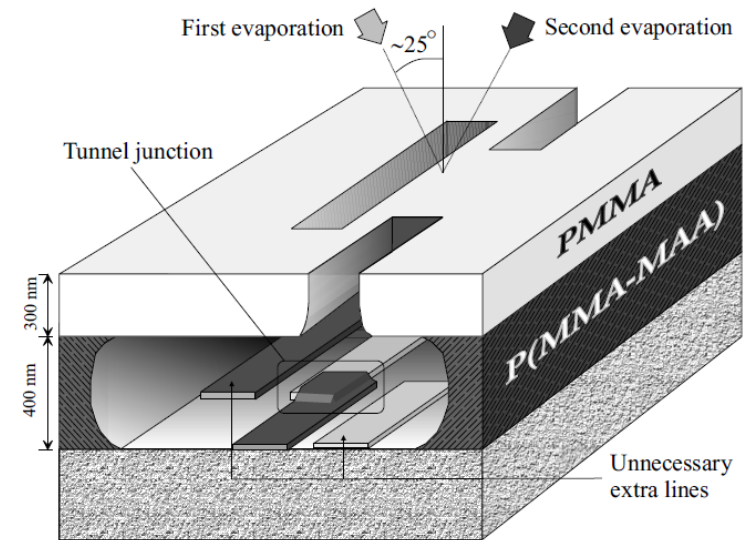
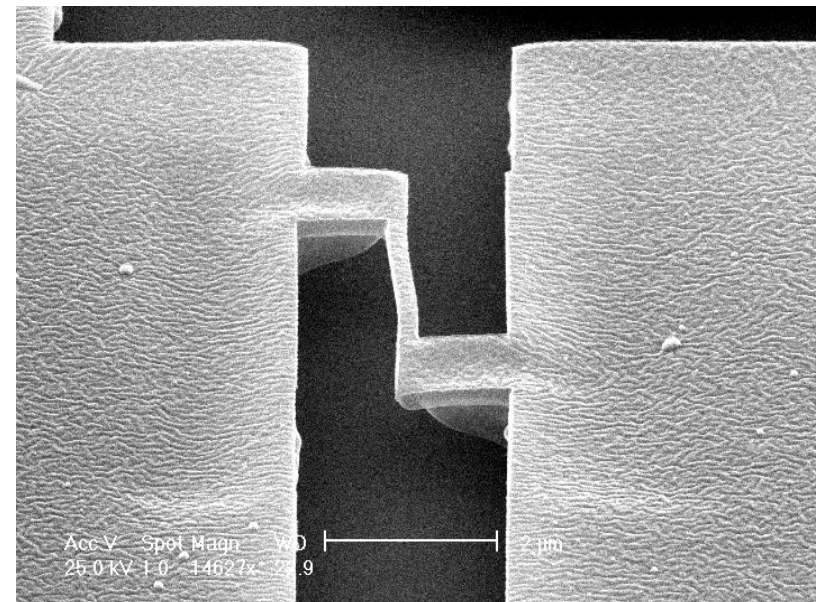
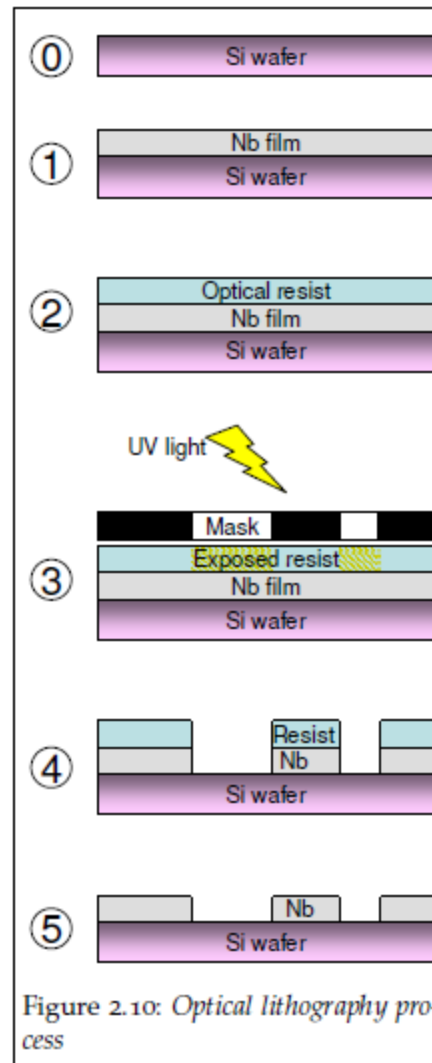


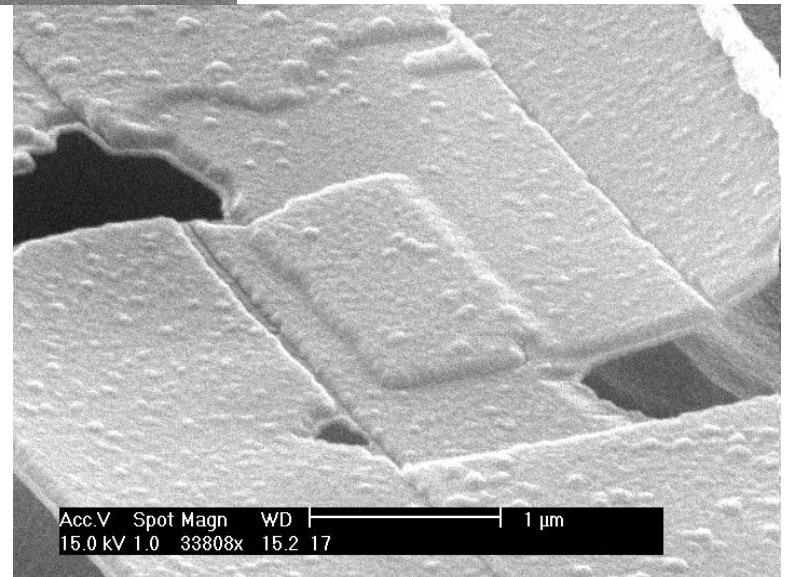
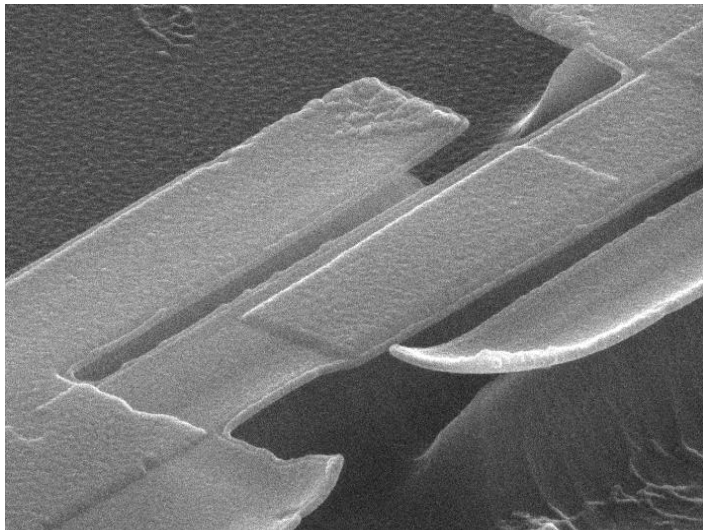
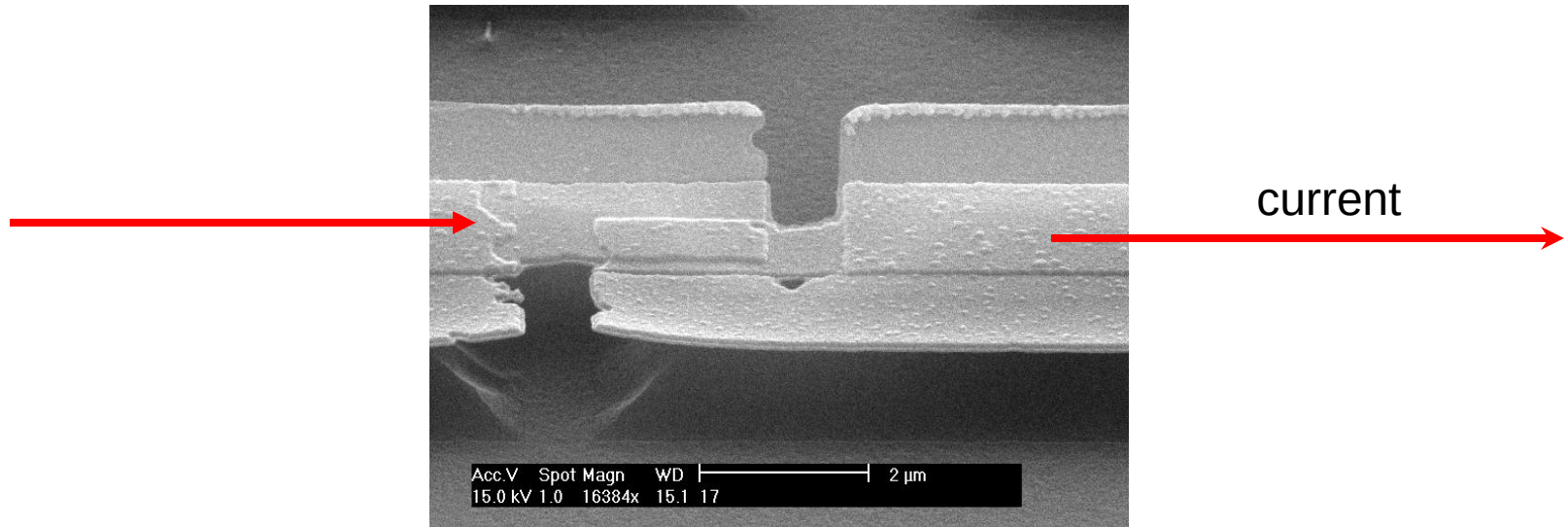
FIGURE 2.1 Principle of the self-aligning shadow evaporation technique. The tunnel junction is formed between the two metallic layers (usually Al) evaporated at different angles so that they overlap slightly. The tunnelling barrier is formed by oxidising the first layer before evaporating the second one. As a side effect of the technique there will be some extra lines, which are not used in measurements.



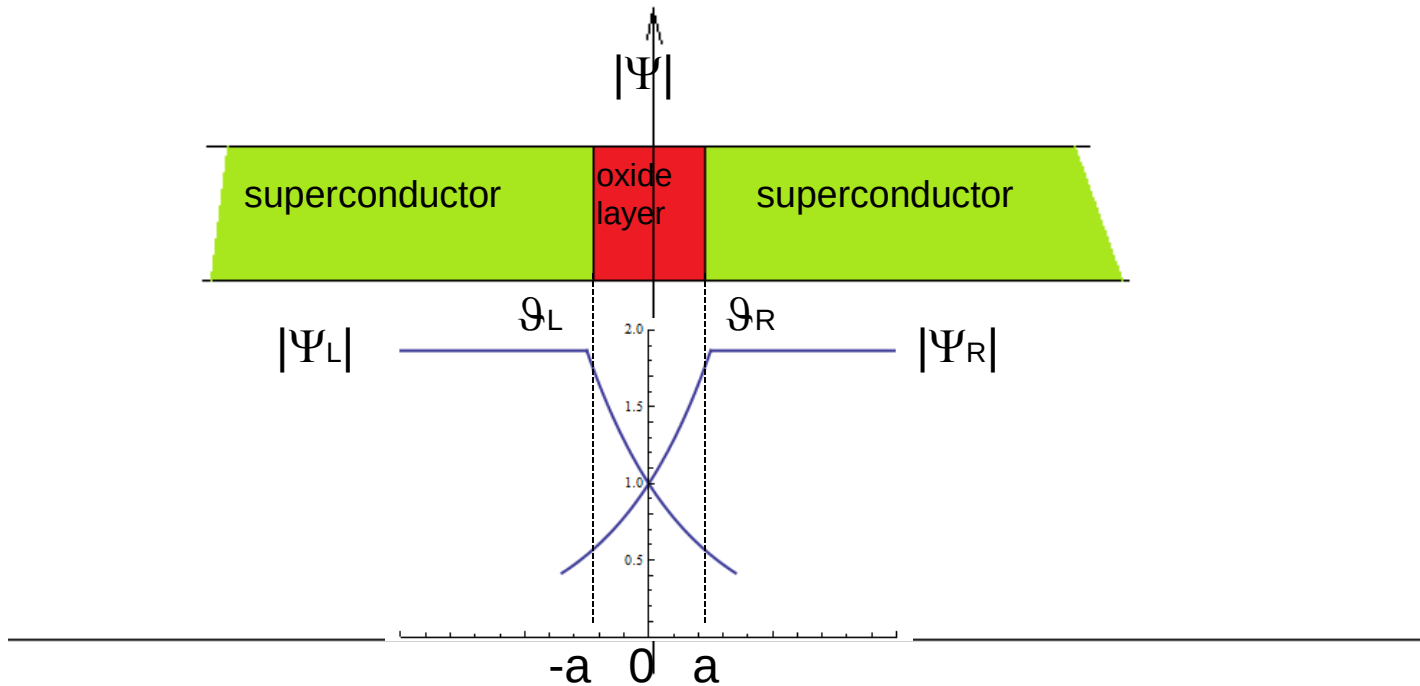
Photolithography



Tunnelling Josephson junction (JJ)



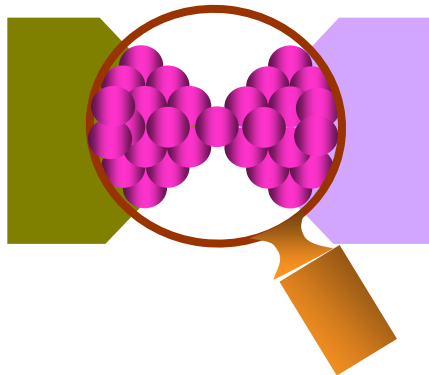
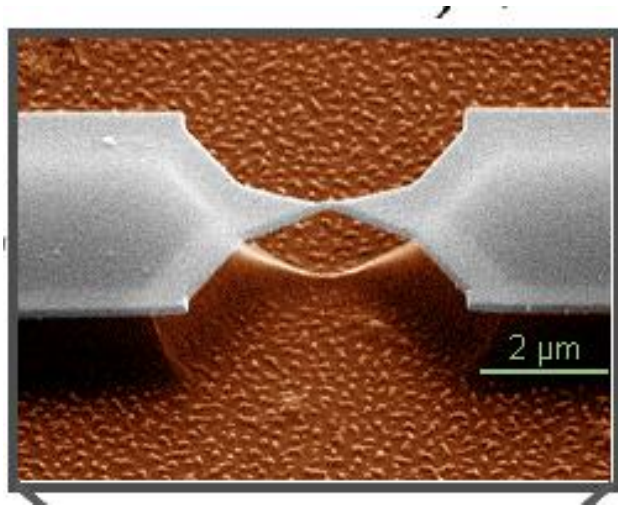
Tunnelling Josephson junction (JJ)



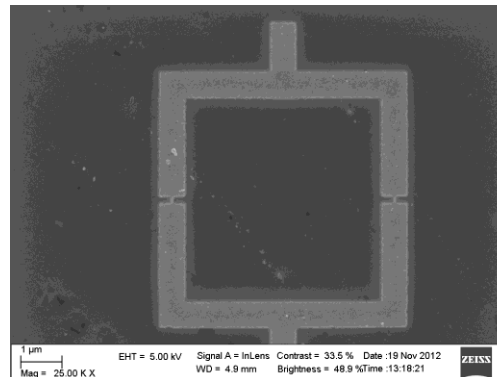
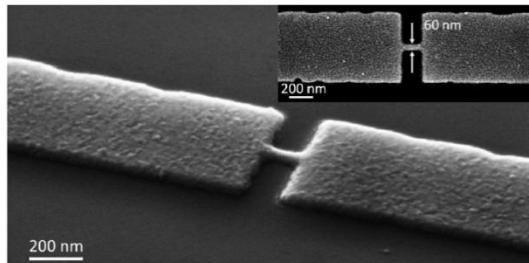
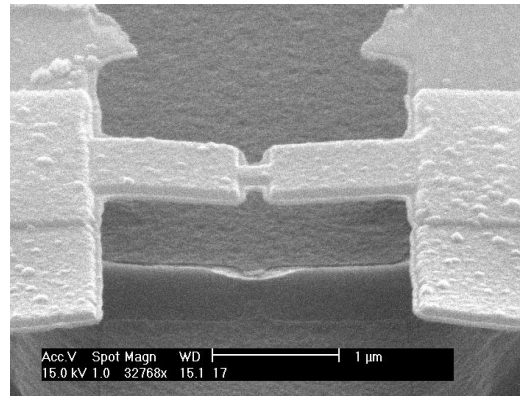
$$\psi = \sqrt{\zeta} \exp(i \vartheta_L) \exp\left(-\frac{l+a}{\lambda}\right) + \sqrt{\zeta} \exp(i \vartheta_R) \exp\left(\frac{l-a}{\lambda}\right)$$

ZOO of Josephson junctions

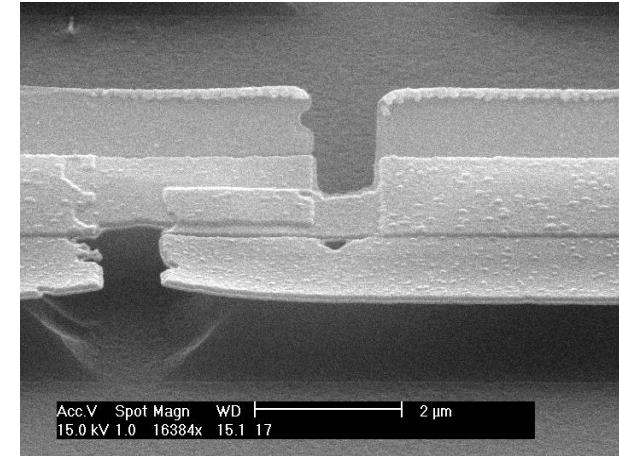
Atomic point contact



Diffusive weak link



Tunnel junction



Note: The famous 1st Josephson relation holds only for tunnelling junctions.

But it can be extended to treat all types of JJ !!!

⇒ ANDREEV BOUND STATES give a generalized picture of Josephson effect

Ring + 1 Josephson junction

$$2\pi n = \oint \frac{-m_e j_s}{e\rho\hbar} dl + \frac{2e}{\hbar} \Phi_{tot}$$

$$2\pi n = - \underbrace{\oint_a^b \frac{m_e j_s}{e\rho\hbar} dl - \oint_b^a \frac{m_e j_s}{e\rho\hbar} dl}_{\gamma} + \frac{2e}{\hbar} \Phi_{tot}$$

γ - superconducting phase across a junction

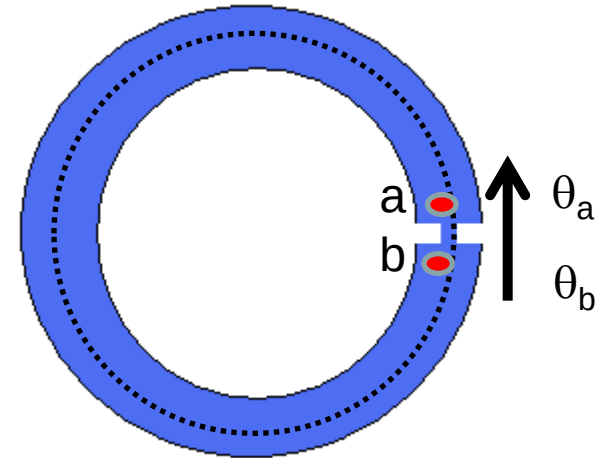
$$2\pi n = \frac{2e}{\hbar} \Phi_{tot} - \gamma - \oint_b^a \frac{m_e j_s}{e\rho\hbar} dl$$

$$(\gamma + 2\pi n) \bmod(2\pi) = \gamma = \frac{2\pi}{\Phi_0} (\Phi_{tot} - L_K I_s)$$

Phase relation for a ring with a single junction

$$n\Phi_0 = \Phi_{tot} - L_K I_s(\gamma) - \frac{\Phi_0}{2\pi} \gamma$$

Fluxoid quantization for a ring interrupted with a junction



$$\gamma = \theta_a - \theta_b$$

The total flux threading the ring is not quantized any more !!!

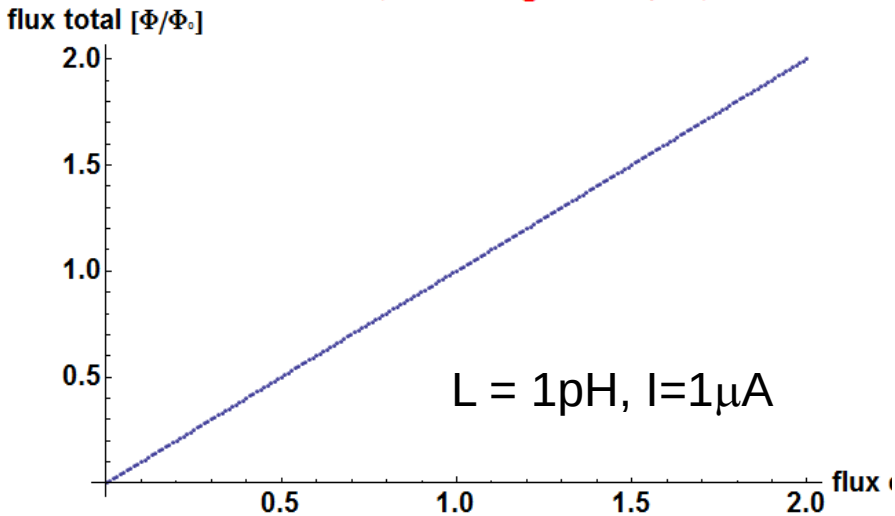
Screening current

$$\Phi = \Phi_{ext} - (L_{geom} + L_K)I_s = \Phi_{tot} - L_K I_s$$

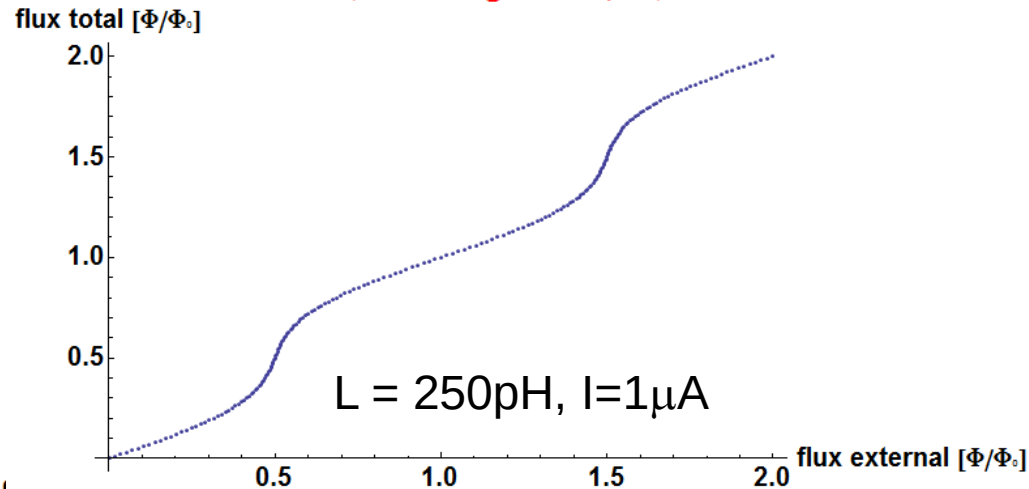
$$\Phi = \Phi_{ext} - LI_c \sin\left(\frac{2\pi}{\Phi_0} \Phi\right)$$

From weak to strong link...

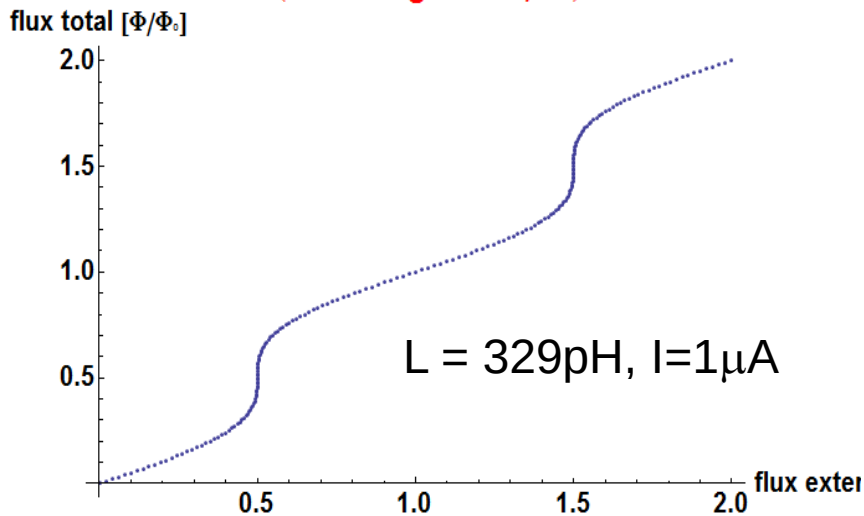
0.00303719 (screening $2\pi L \cdot i_c / \Phi_0$)



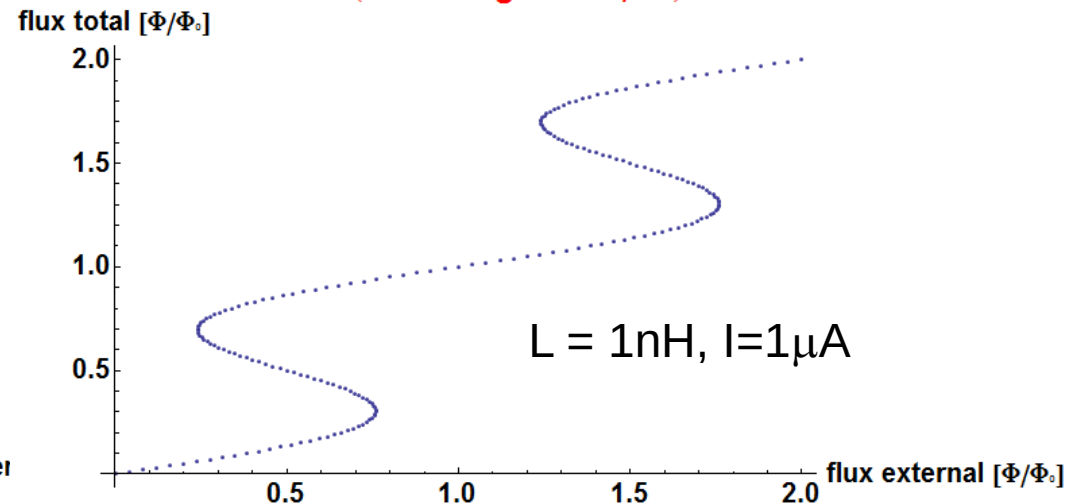
0.759297 (screening $2\pi L \cdot i_c / \Phi_0$)



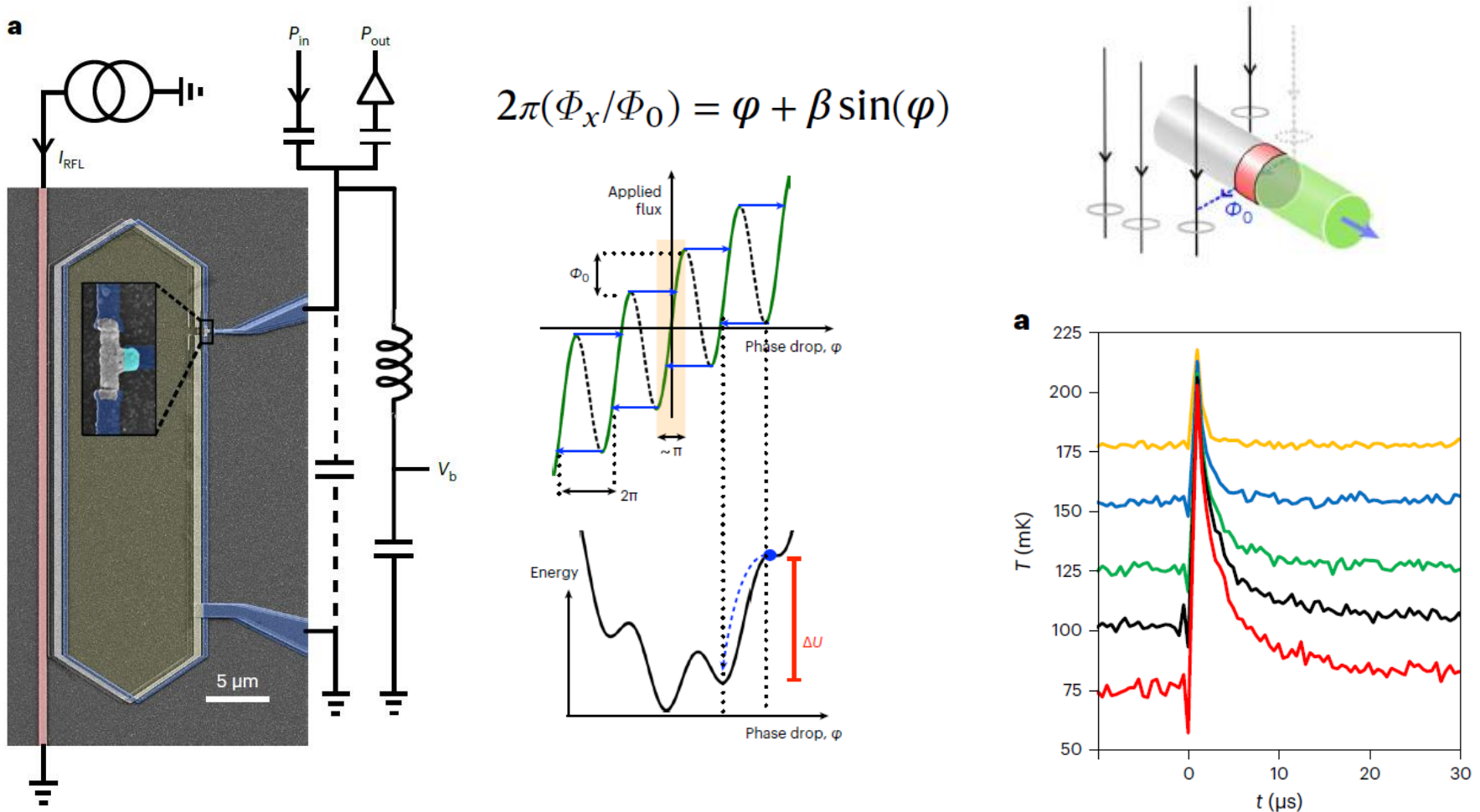
1. (screening $2\pi L \cdot i_c / \Phi_0$)



3.03719 (screening $2\pi L \cdot i_c / \Phi_0$)



Heating due to a phase slip



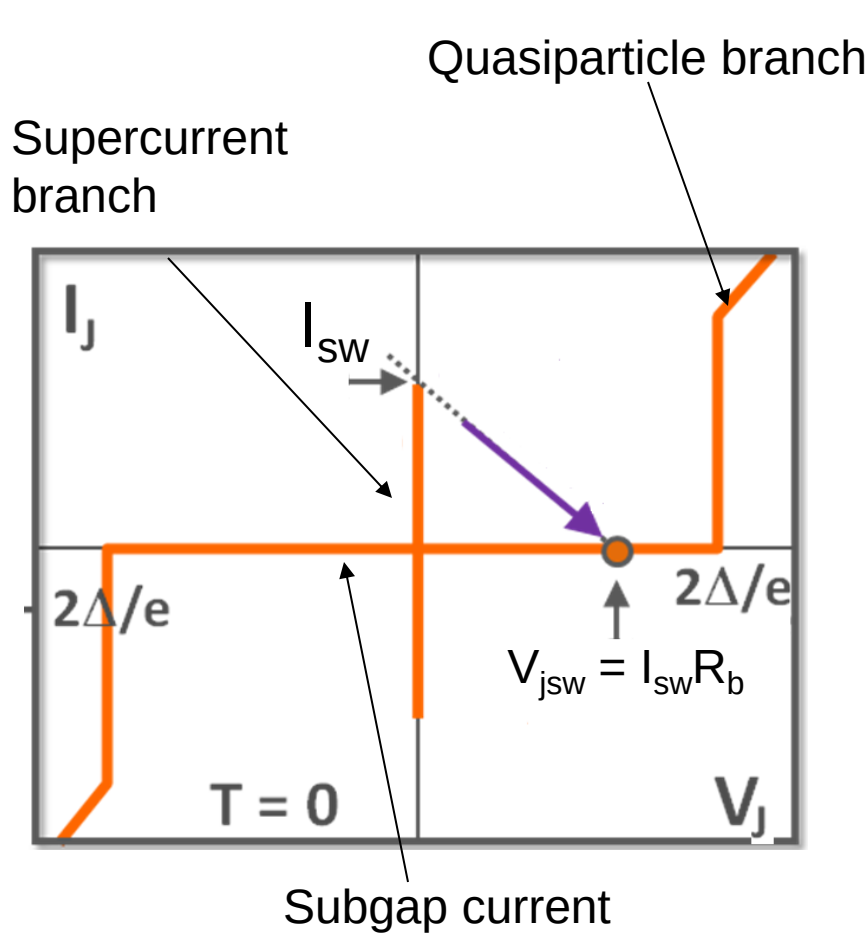
E. Gumus, et al., *Calorimetry of a phase slip in a Josephson junction*, Nature Physics 19, 196 (2023)

Conservation of probability

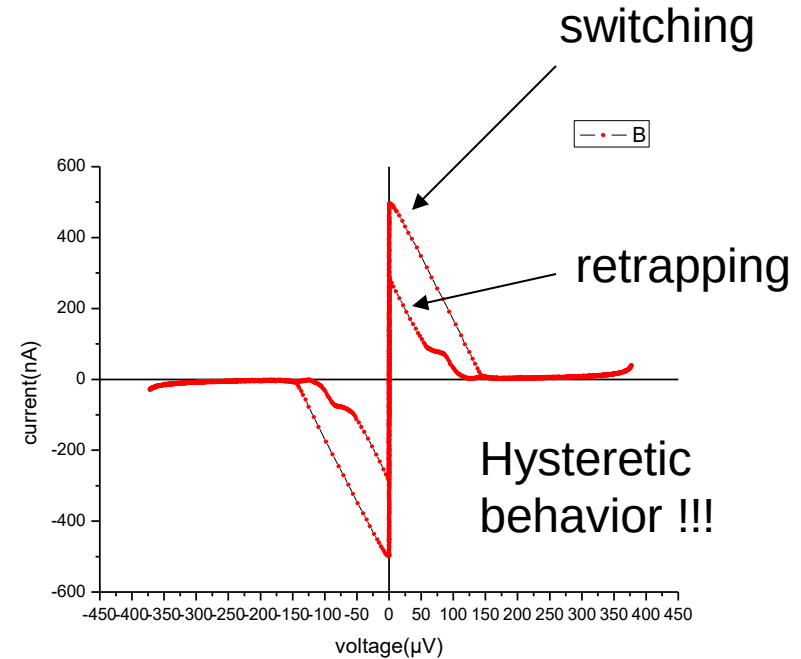
$$\frac{\partial P}{\partial t} = -\nabla \cdot \vec{j} = \psi^* \frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi^*}{\partial t}$$

$$\vec{j} = \frac{1}{2} \left\{ \left[\frac{p - qA}{m} \psi \right]^* \psi + \psi^* \left[\frac{p - qA}{m} \right] \psi \right\}$$

IV curve



I-V characteristics of JJ biased through R_B bias resistor. JJ supports supercurrent only to certain level. On crossing the threshold value I_0 finite voltage develops across JJ.



$$I_j = I_j(V_j)$$

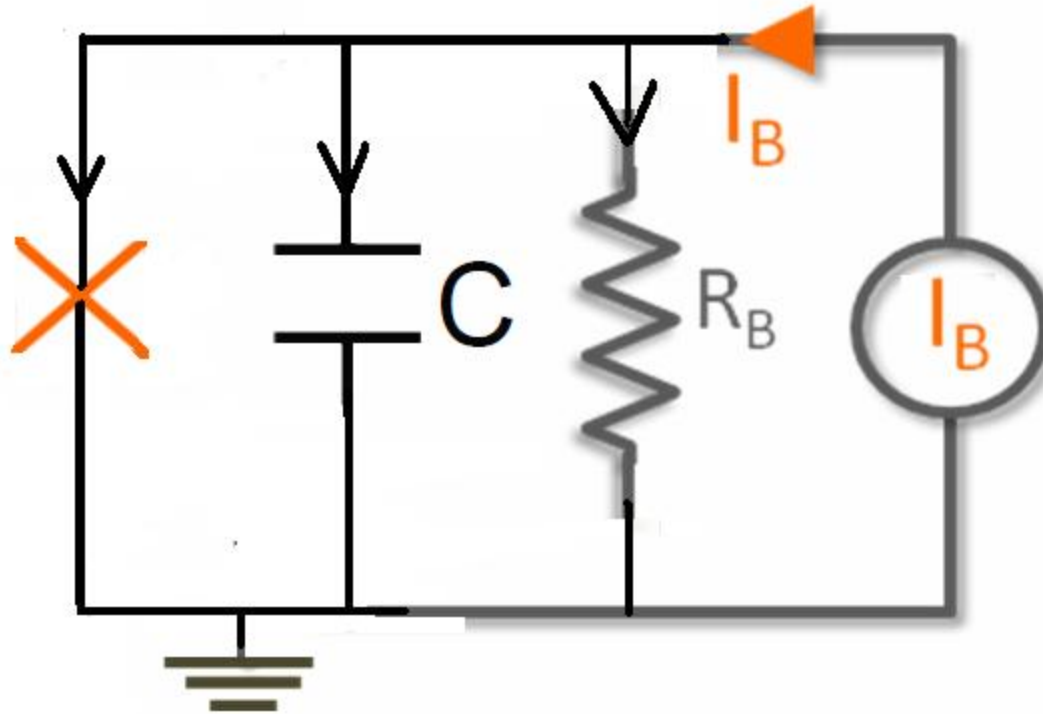
$$V_B = I_j \cdot R_b + V_j \Rightarrow I_j = -\frac{1}{R_b} \cdot V_j + \frac{1}{R_b} \cdot V_b \text{ (load line)}$$

$$\text{At switching } V_j = 0, V_b = V_{bsw} \text{ and } I_j = I_{sw} = \frac{V_{bsw}}{R_b}$$

$$I_j = -\frac{1}{R_b} \cdot V_j + I_{sw} \text{ (switching line)}$$

RCSJ model

(Resistively and Capacitively Shunted Junction)



$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C \frac{dV}{dt} + I_0 \cdot \sin \gamma$$

RCSJ model

$$I_b = I_R + I_C + I_{JJ} = \frac{V}{R} + C \frac{dV}{dt} + I_0 \cdot \sin \gamma$$

$$\dot{\gamma} = \frac{1}{\varphi_0} \cdot V$$

$$I_b = \frac{\varphi_0}{R} \dot{\gamma} + C \varphi_0 \ddot{\gamma} + I_0 \sin \gamma$$

First, consider $\gamma \rightarrow 0$, $I_b = 0$ and map it into harmonic oscillator:

$$C \varphi_0^2 \ddot{\gamma} + \frac{\varphi_0^2}{R} \dot{\gamma} + \varphi_0 I_0 \gamma = 0$$

$$m = C \varphi_0^2, \quad \omega_0 = \left(\frac{I_0}{C \cdot \varphi_0} \right)^{1/2}, \quad b = \frac{\varphi_0^2}{R}, \quad Q_0 = \frac{\omega_0}{b/m} = RC \omega_0, \quad k = \varphi_0 I_0 = E_J$$

Back to full equation:

$$\ddot{\gamma} + \frac{\omega_0}{Q_0} \dot{\gamma} + \omega_0^2 \left(\sin \gamma - \frac{I_b}{I_0} \right) = 0$$

Harmonic oscillator:

$$m \ddot{x} + b \dot{x} + kx = 0$$

$$E(t) = E_0 e^{-\gamma t} \text{ (energy of damped harmonic oscillator)}$$

$$\gamma = \frac{b}{m}, \quad Q = \frac{\omega_0}{\gamma}, \quad -kx = -\nabla E_p$$

$$\ddot{x} + \frac{\omega_0}{Q} \dot{x} + \omega_0^2 x = 0$$

Q - quality factor, amplitude of harmonic oscillator falls by a factor of e in Q/π cycles of free oscillations

$$\ddot{\gamma} + \frac{\omega_0}{Q_0} \dot{\gamma} + \omega_0^2 \left(\sin \gamma - \frac{I_b}{I_0} \right) = 0$$

Wygląda jak harmonic oscillator, ale teraz restoring force wynosi nie $k\gamma$ (jak w prawie Hooke'a) tylko:

$$F = -k \left(\sin \gamma - \frac{I_b}{I_0} \right) = -\nabla E_p$$

$$E_p = +k \int \left(\sin \gamma - \frac{I_b}{I_0} \right) d\gamma = -E_J \left(\cos \gamma + \frac{I_b}{I_0} \gamma \right) \quad \textit{tilted washboard potential}$$

Tilted washboard potential

$$E_p = -E_J \left(\cos \gamma + \frac{I_b}{I_0} \gamma \right)$$

$$\gamma \leftrightarrow x$$

$$V/\phi_0 \text{ (napięcie)} \leftrightarrow v \text{ (prędkość)}$$

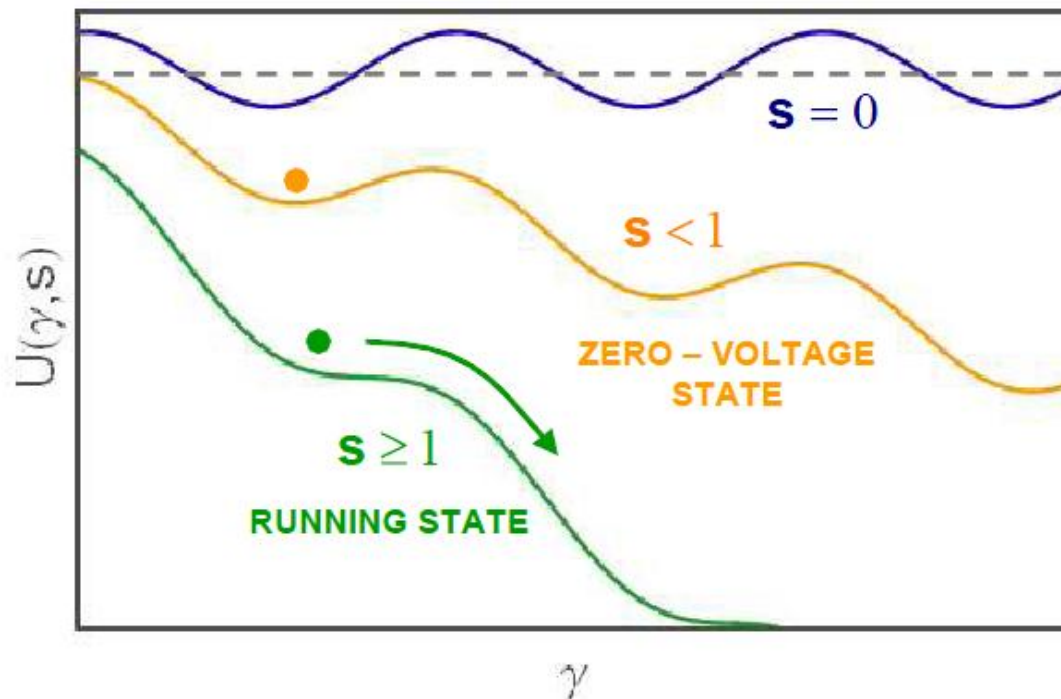


Fig. 2.3. Tilted washboard potential associated to the dynamics of the phase in absence of fluctuations (zero temperature in this classical model). For $s \leq 1$, the potential presents wells in which the particle is trapped. For $s \geq 1$, the wells disappear and the particle gets into to a running state.

Q (quality factor) $\leftrightarrow$ hysteresis

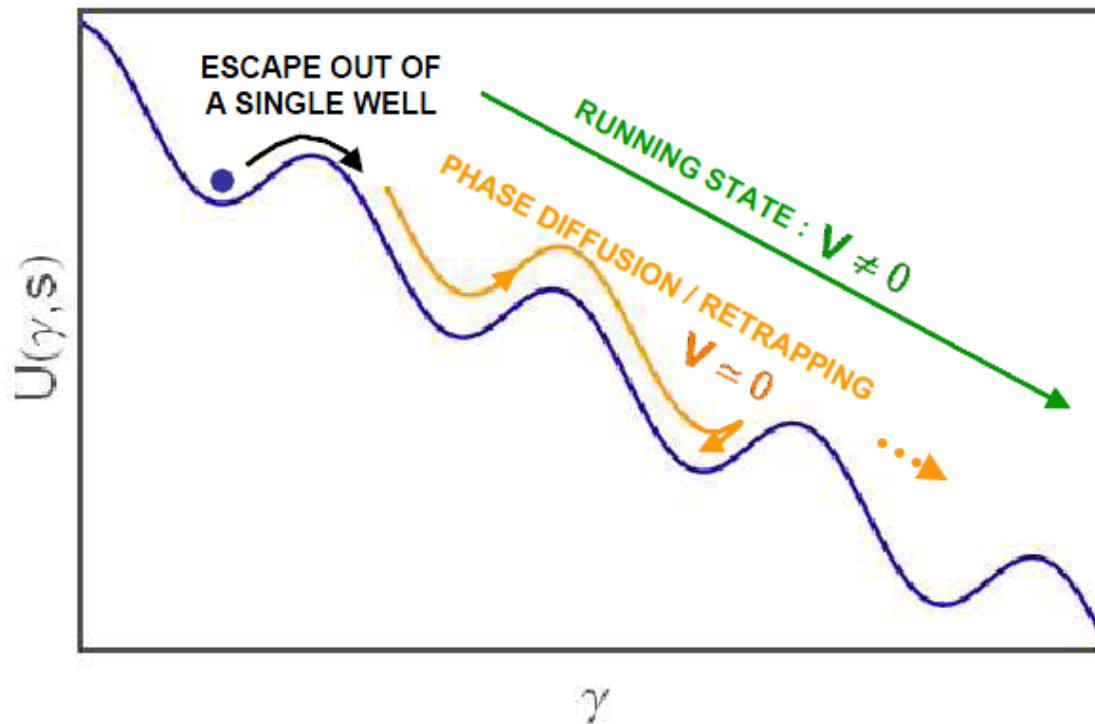


Fig. 2.4. Dynamics of the particle in presence of fluctuations (finite temperature in this classical model). The particle can overcome the barrier and escape out of the single well. The dynamics of the particle depends on the damping. If the damping is small, the particle gains enough energy to reach a running state, leading to a finite voltage. Otherwise, the particle enters a diffusion regime, escaping from one well to be retrapped in a following one, with a very small average velocity.