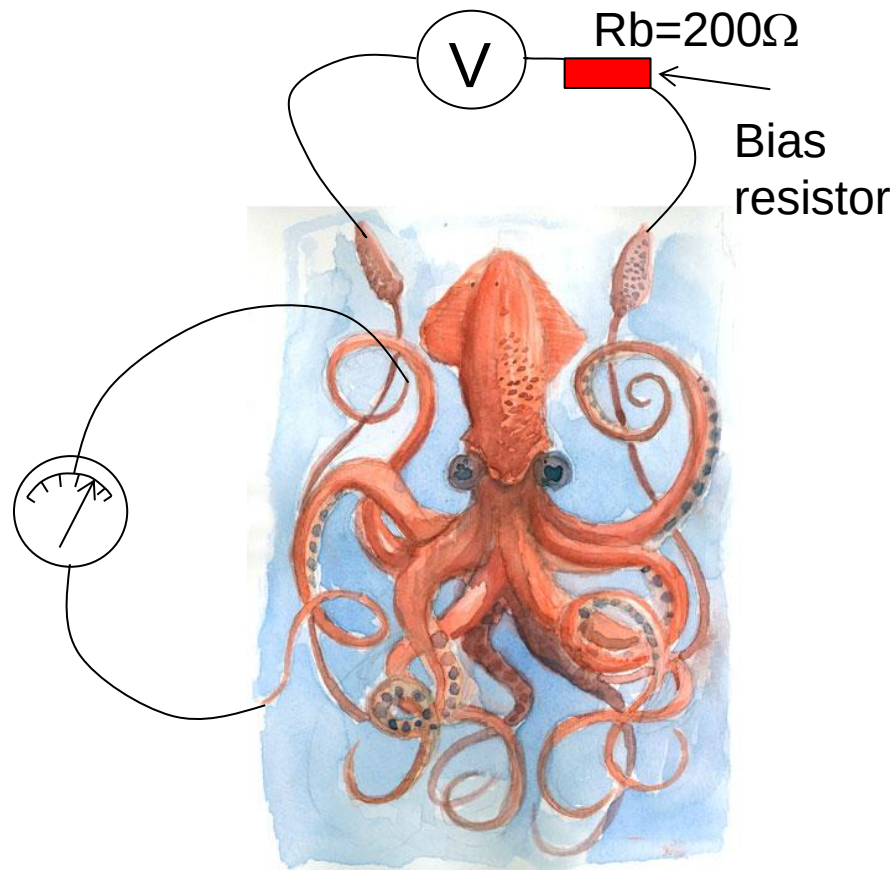




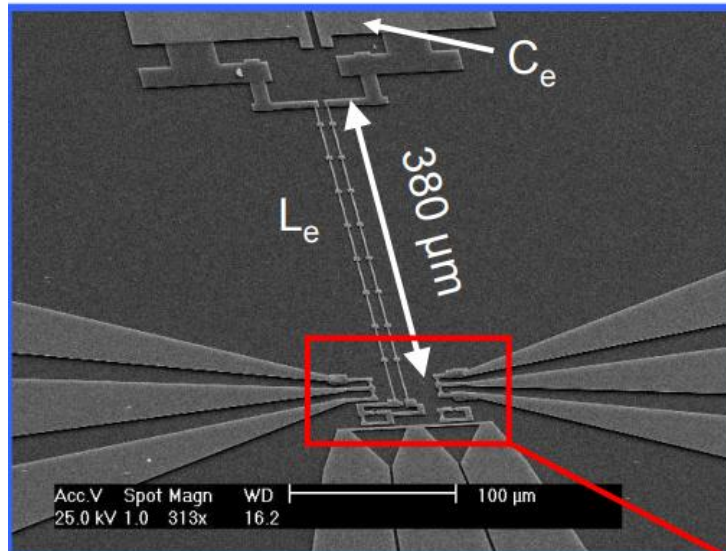
Tutorial: Superconducting Quantum Interference Device (SQUID)



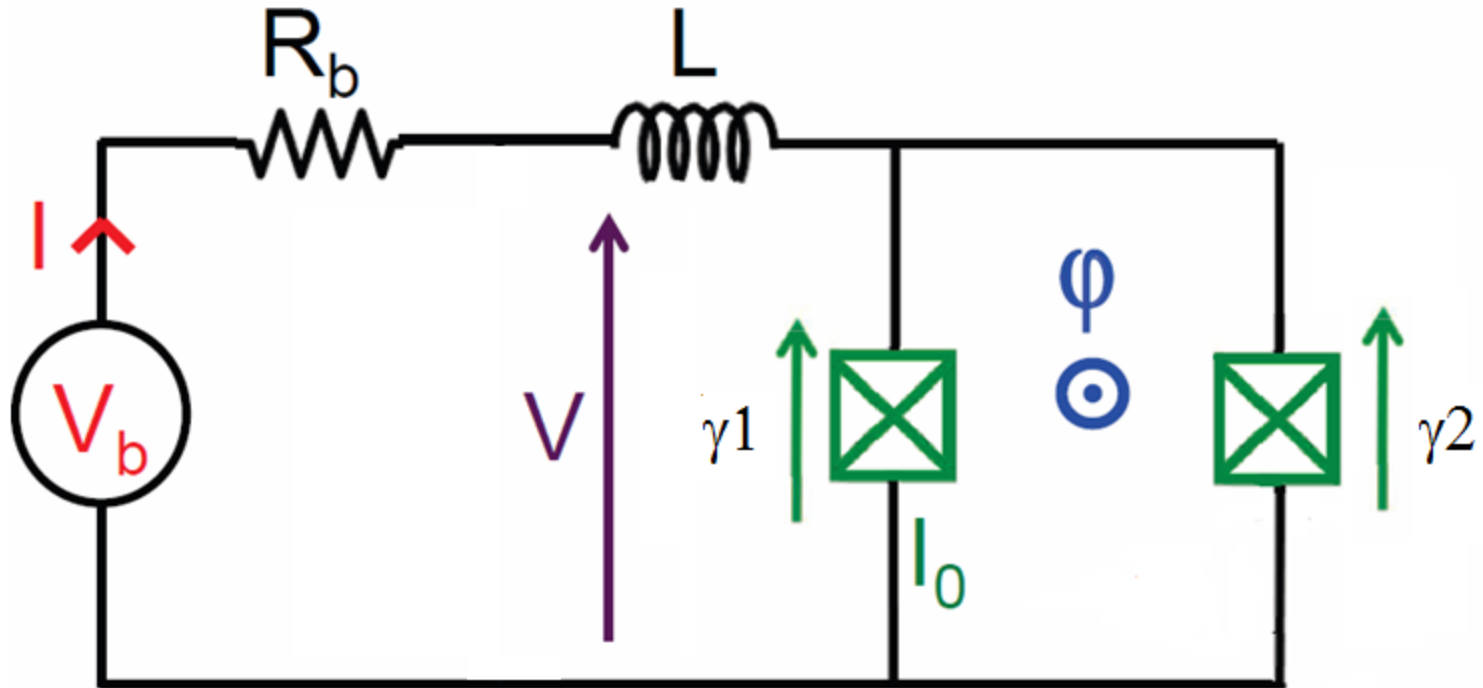
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Summer School: Ultrafast electronics meets quantum thermodynamics, ICTP Trieste, 24/09/2026



SQUID circuit



$$\gamma_1 - \gamma_2 = 2\pi \frac{\Phi}{\Phi_0} = \varphi$$

Φ – magnetic flux through the SQUID loop +
kinetic inductance term

φ - „magnetic” phase

$$\Phi = \Phi_{ext} - (L_{geom} + L_K)I_s = \Phi_{tot} - L_K I_s$$

Critical current of the SQUID

$$I_{J2} = I_b / 2 - I_p(\varphi)$$

$$I_{J1} = I_b / 2 + I_p(\varphi)$$

$$I_b = I_{J1} + I_{J2} = I_0(\sin \gamma_1 + \sin \gamma_2) = 2I_0 \sin \frac{\gamma_1 + \gamma_2}{2} \cos \frac{\gamma_1 - \gamma_2}{2}$$

$$I_b = 2I_0 \cos \frac{\varphi}{2} \sin \frac{\gamma_1 + \gamma_2}{2}$$

$$I_c = \max I_b \Rightarrow \frac{\gamma_1 + \gamma_2}{2} = \frac{\pi}{2} \quad \& \quad \gamma_1 - \gamma_2 = \varphi = 2\pi \frac{\Phi}{\Phi_0}$$

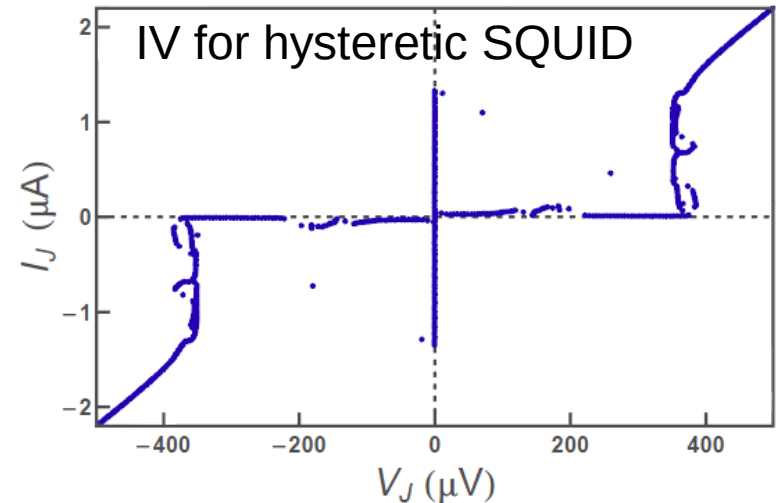
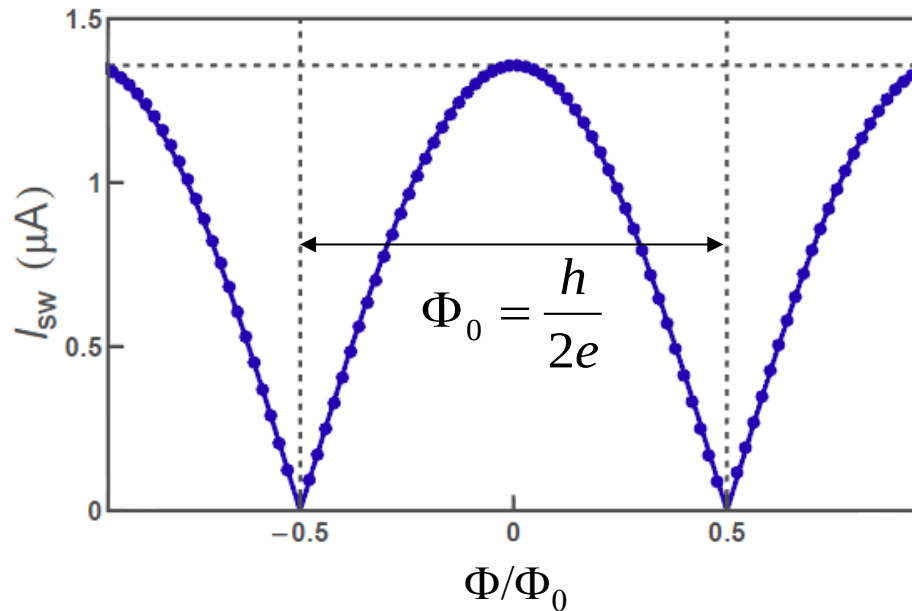
$$\Rightarrow I_c = 2I_0 \left| \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \right| \quad \text{if } \gamma_1 = \frac{\pi}{2} - \frac{\varphi}{2} \quad \& \quad \gamma_2 = \frac{\pi}{2} + \frac{\varphi}{2}$$

$$\Phi_0 = \frac{h}{2e} = 2 \cdot 10^{-15} \text{ Wb} \quad - \quad \text{flux quantum}$$

For $\Phi = \Phi_0/2 \Rightarrow \gamma_1 - \gamma_2 = \pi \Rightarrow$ perfect cancellation of currents flowing through two arms of the SQUID

SQUID is a JJ with the magnetic field-regulated critical current
(tunable JJ !!!)

Interference pattern for SQUID



Symmetric SQUID is superconducting analog of double slit optical (or electron) interferometer:

applied flux – $\Phi \Leftrightarrow d \cdot \sin\theta$ - path difference

Flux quantum – $\Phi_0 \Leftrightarrow \lambda$ - wavelength

For symmetric SQUID (2 x JJ):
$$I_c = 2 \cdot I_{JJ}^0 \cdot \left| \cos\left(\pi \cdot \frac{\Phi}{\Phi_0}\right) \right|$$

What will happen for $\Phi = \Phi_0/2$?

Analyze sinusoid graphically to show evolution of two junction phases both with magnetic flux and bias current:

Magnetic flux alone shifts the phases symmetrically around zero:
 $\gamma_1 = -\gamma_2$, $\gamma_1 - \gamma_2 = 2\pi \Phi/\Phi_0$

Bias current shifts two phases as a unit preserving distance between them
 $\gamma_1 - \gamma_2 = 2\pi I/I_0$

Special case:

$\gamma_1 - \gamma_2 = \pi$, maximum screening current circulating in the SQUID,
Equal to the critical current of the single junction I_0 .

For $\Phi = \Phi_0/2 \Rightarrow |\gamma_1 - \gamma_2| = \pi \Rightarrow I_{j1} = -I_{j2} = I_{JJ} \Rightarrow$ perfect
cancellation of currents flowing through two arms of the SQUID

SQUID can not support superconducting current!!!

Link for SQUID in Wolfram Cloud



<https://www.wolframcloud.com/obj/7cc22595-86dd-4096-86ca-333fdcb4a955>

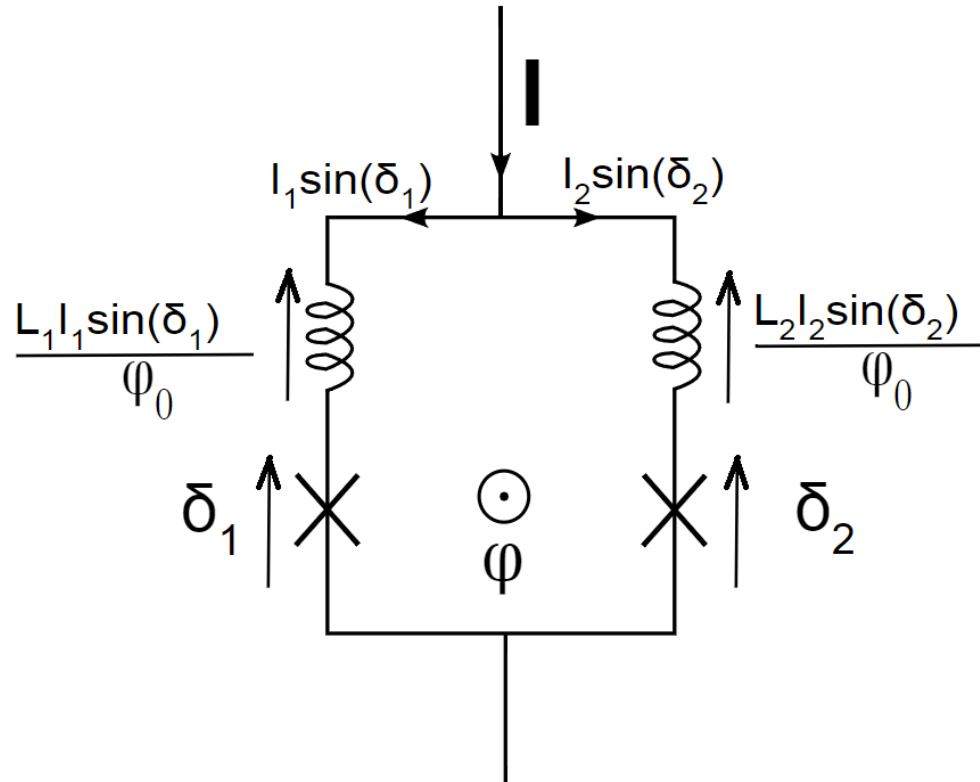
SQUID – numerical study of a general case

$$L_1 = L(1 - \eta)$$

$$L_2 = L(1 + \eta)$$

$$I_1 = I_0(1 - \alpha)$$

$$I_2 = I_0(1 + \alpha)$$

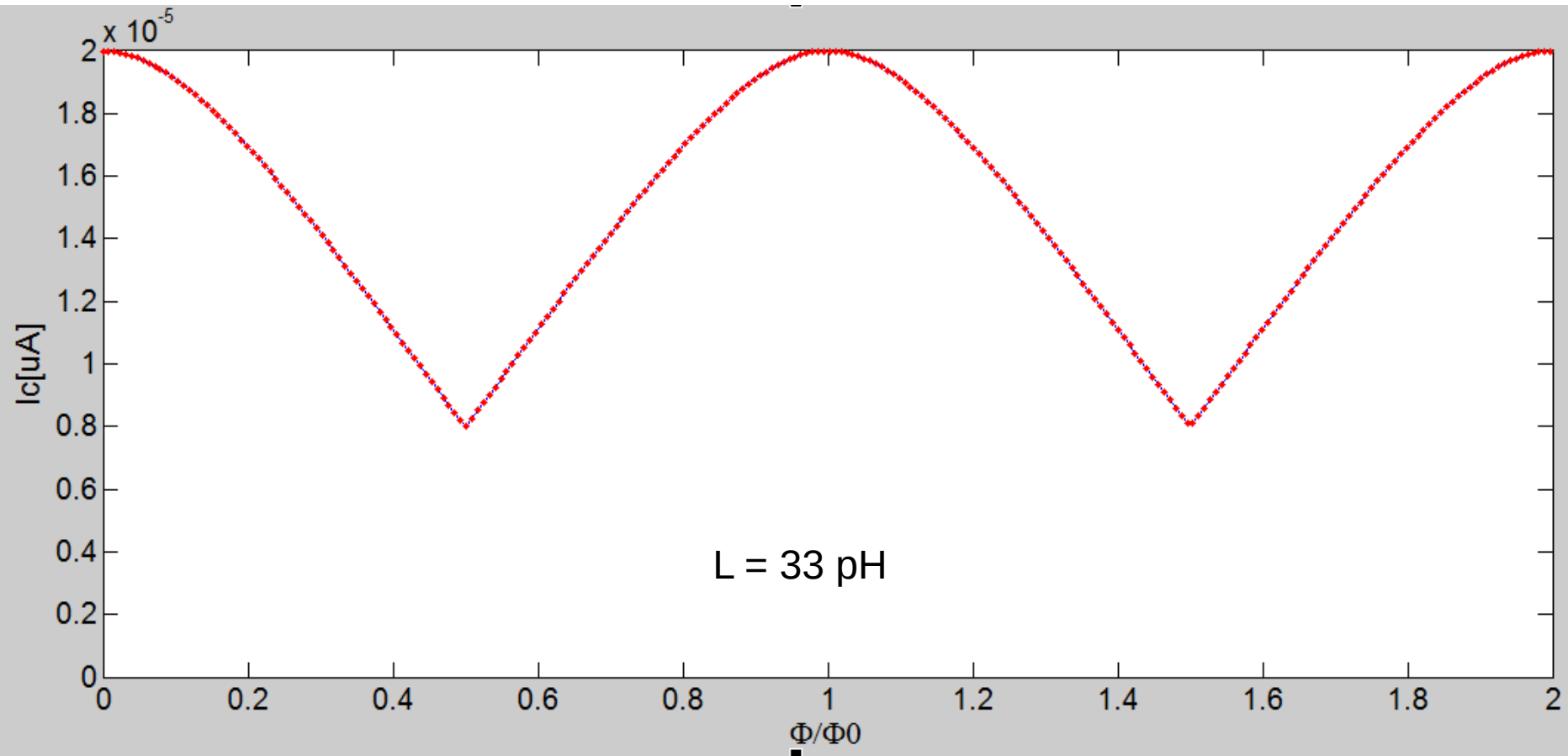


Phase equation:

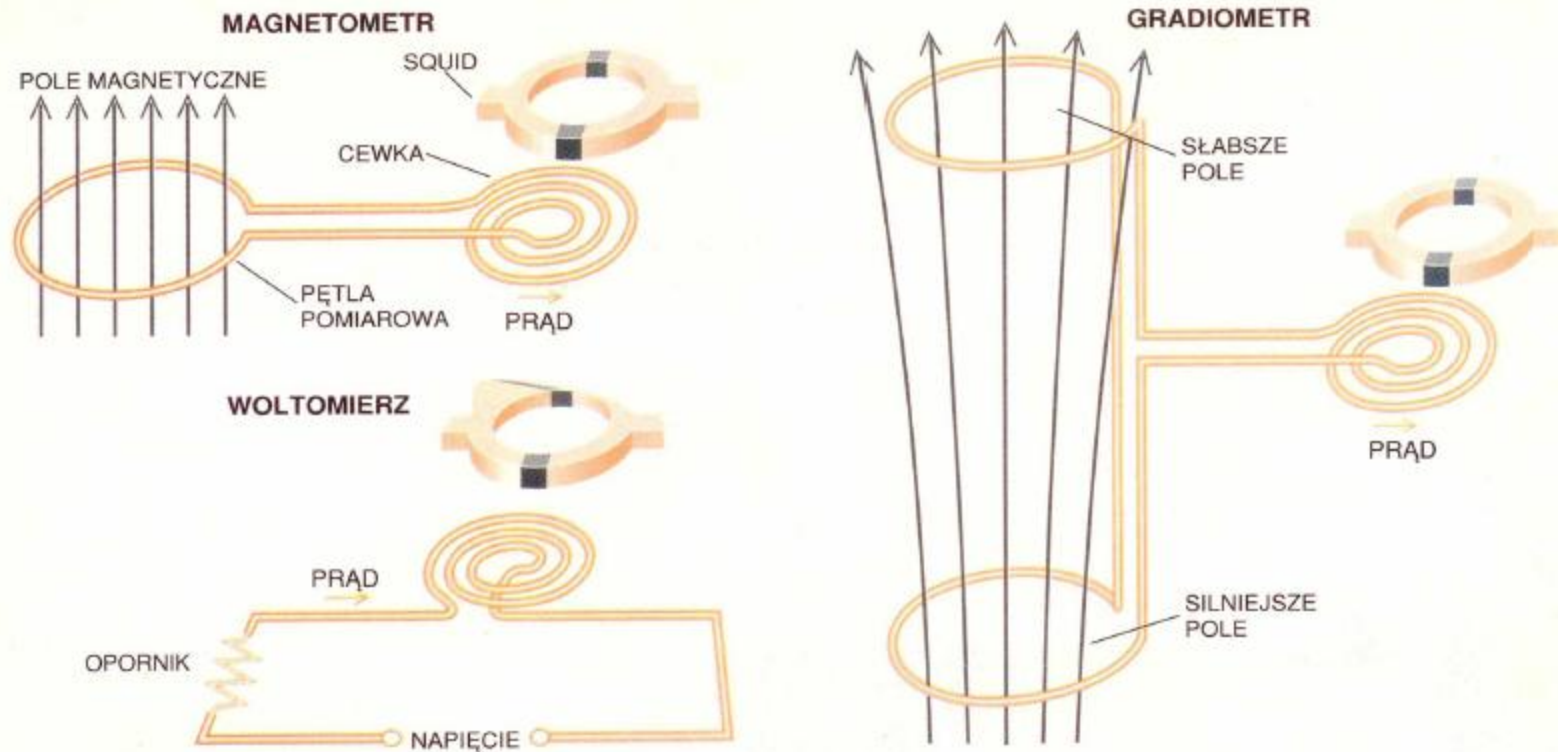
$$\delta_1 + \frac{L_1 I_1 \sin(\delta_1)}{\varphi_0} - \delta_2 - \frac{L_2 I_2 \sin(\delta_2)}{\varphi_0} = 2\pi \frac{\Phi_{ext}}{\Phi_0}$$

SQUID with self-flux

$$\Phi = \Phi_{ext} - (L_{geom} + L_K)I_s = \Phi_{tot} - L_K I_s$$



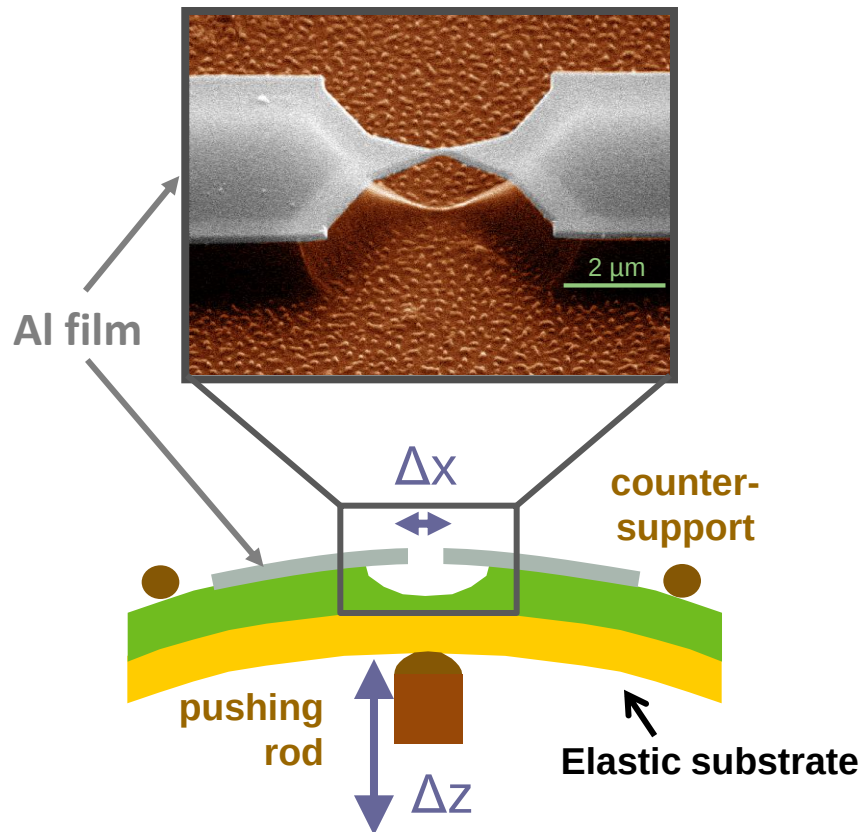
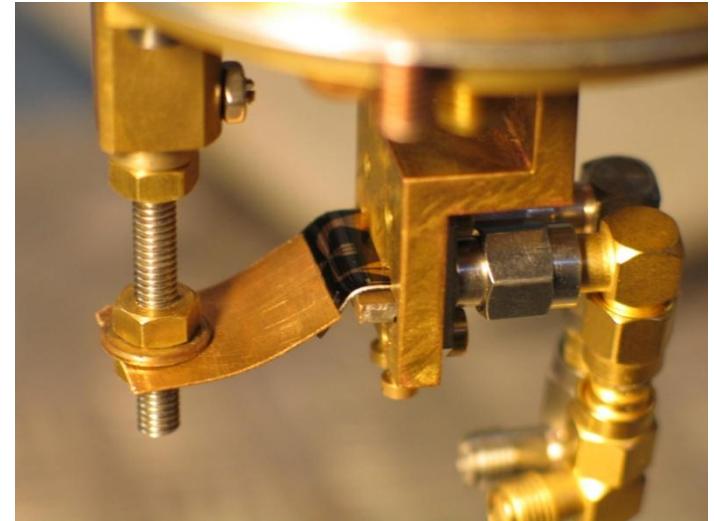
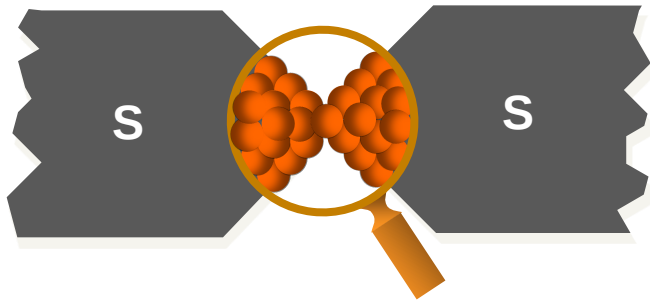
SQUID – various applications



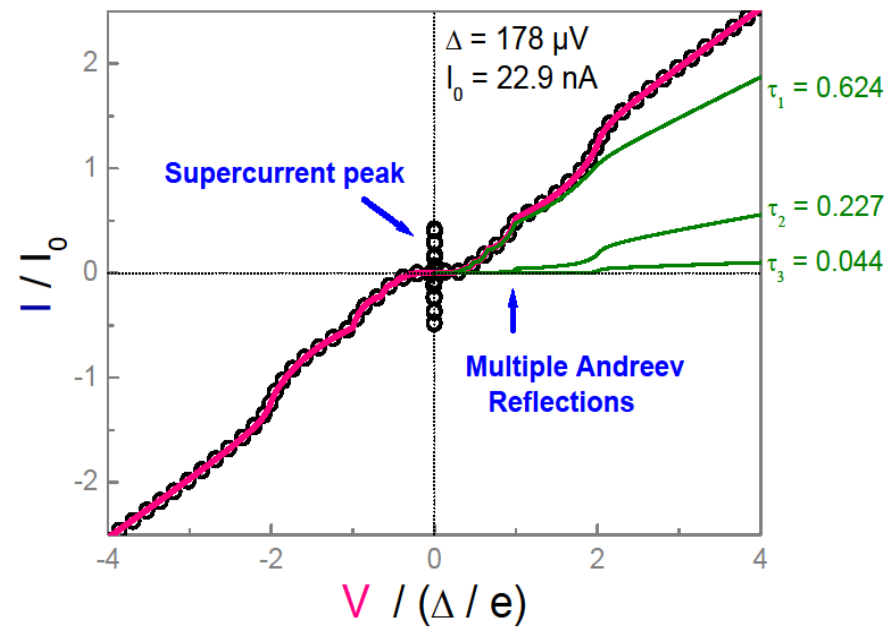
URZĄDZENIA WYKORZYSTUJĄCE SQUID zwykle wymagają dodatkowych elementów. Magnetometr zawiera „transformator strumienia”, który składa się z pętli pomiarowej połączonej z cewką sprzężoną z interferometrem. Pole magnetyczne wzbudza prąd w pętli, powodując jego przepływ przez cewkę i powstanie strumienia pola magnetycznego przenikającego interferometr. W gradiometrze dwie oddalone od siebie pętle pomiarowe nawinięte w przeciwnych kierunkach są poddawane jednoczesnemu działaniu pola magnetycznego. Strumień pola magnetycznego przenika interferometr tylko wtedy, gdy natężenie tego pola w obu pętlach jest różne. W woltomierzu mierzone napięcie wywołuje przepływ prądu o natężeniu równym ilorazowi wartości tego napięcia i wielkości oporu połączonego z cewką sprzężoną z nadprzewodnikowym interferometrem kwantowym.

Atomic contact

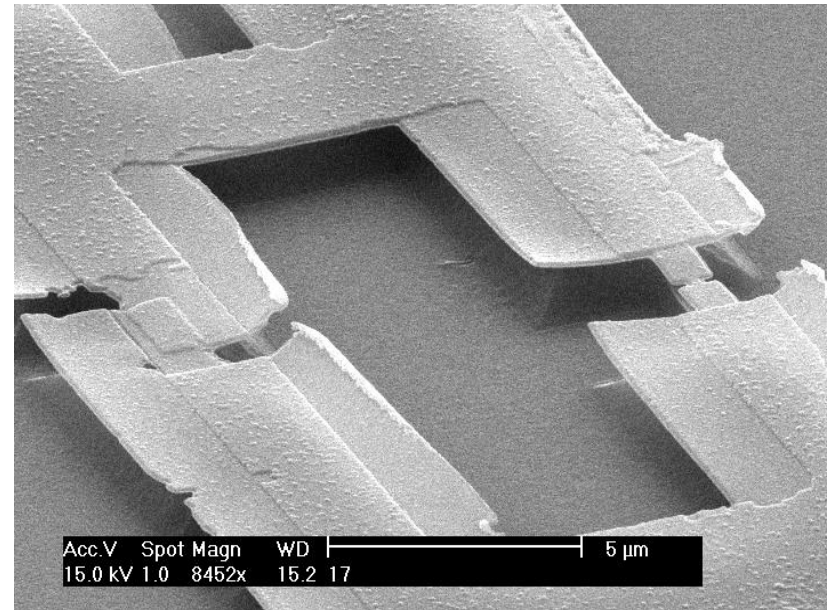
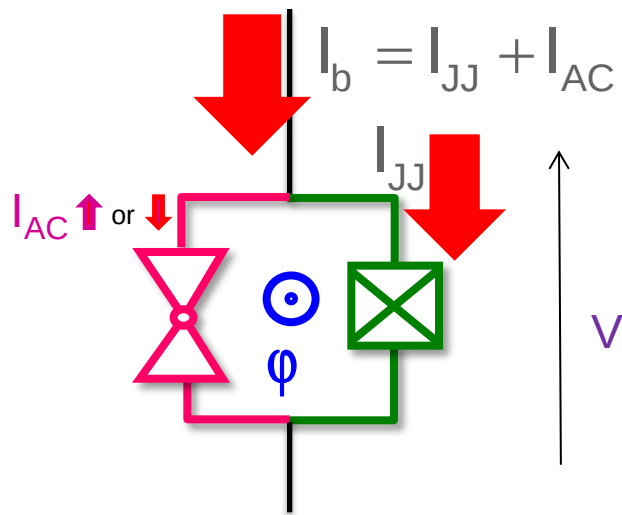
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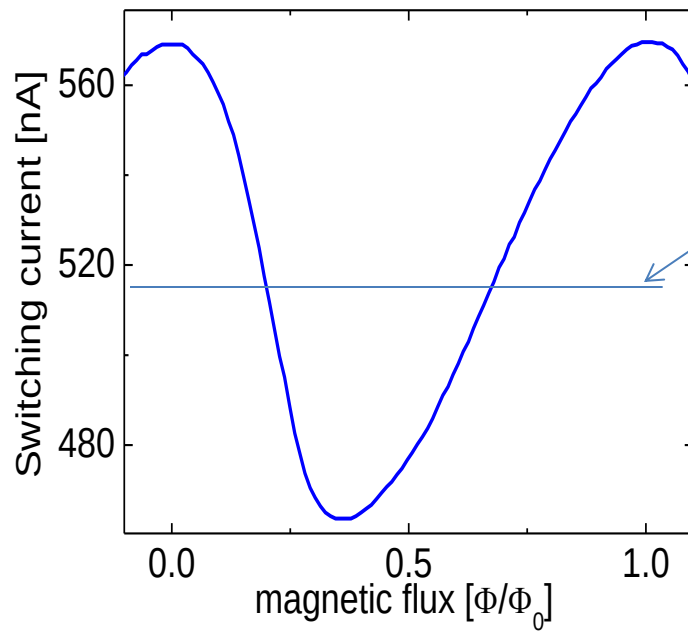
few channels, $\{\tau_i\}$ tunable



Atomic Squid...



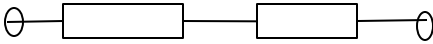
Interference pattern for ATOMIC SQUID



I₀-switching current
of junction alone

When SQUID switches, phase across
JJ is the same independently of applied
magnetic flux => interference pattern
is current-phase relation of atomic
contact

Voltage vs. phase

Normal metal	Superconductor
Voltage drop forces current	Phase drop imposes current
R $r \ll R$  All voltage drops on R	The biggest phase drop in the loop on the weakest weak link