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Floquet engineering of open quantum systems

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- I. Floquet theory for the Schrödinger equation
- II. Dissipation & transport
- III. Applications

<https://sigmundkohler.github.io/download/FloquetTutorial.pdf>



Laser physics since the 1970s

- strong ac fields acting on atoms
 - treatment beyond linear response
- Floquet theory for Schrödinger equation
 - discrete time translation and Floquet ansatz
 - properties of Floquet states
 - relation to Berry phase
 - analytical and numerical techniques
- Floquet-Bloch-Redfield theory
 - dissipation & decoherence
 - Floquet state population
 - efficient computation

~ 2000: qubits + microwaves

~ 2015: Floquet engineering, optical lattices, artificial gauge fields, ...

1 Floquet & Schrödinger equation

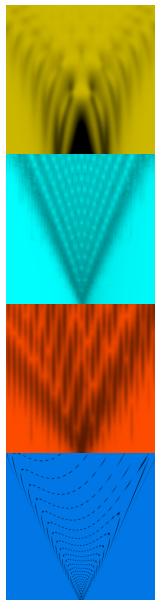
- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

3 Applications & miscellaneous topics

- LZSM Interference
- Floquet-Gibbs states
- Hidden symmetries
- Floquet degeneracies & topology
- Time-dependent Liouvillians
- Two-color Floquet theory
- Adiabatic Floquet theory
- Floquet scattering theory



- Time evolution of an eigenstate ($\hbar = 1$)

$$|\psi(t)\rangle = e^{-iE_n t} |\phi_n\rangle$$

Notation:

ψ : solution of Schrödinger equation

ϕ : other state vector, e.g., eigenstate

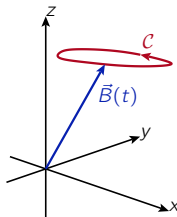
- energy $\leftrightarrow$ phase

for (periodically) time-dependent system ?

- Spin in magnetic field

$$\vec{B}(t) = \vec{B}(t + T):$$

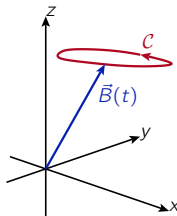
$$H(t) = \frac{1}{2} \vec{B}(t) \cdot \vec{\sigma}$$



- Spin in magnetic field

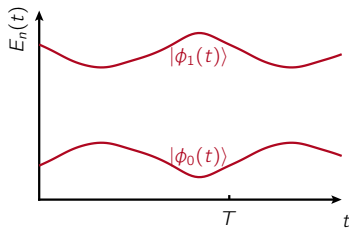
$$\vec{B}(t) = \vec{B}(t + T):$$

$$H(t) = \frac{1}{2} \vec{B}(t) \cdot \vec{\sigma}$$



- Quantum dynamics for $\dot{B} \ll B^2$:
state follows the eigenstate
adiabatically

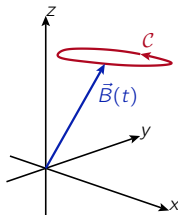
$$|\psi(t)\rangle \propto |\phi_n(t)\rangle$$



→ $|\psi(t)\rangle$ determined up to phase factor

After one period: $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$

$$\varphi = - \int_0^T dt E_n(t) + \gamma_C$$

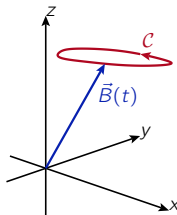


- dynamical phase $\leftrightarrow$ mean energy
- Berry phase γ_C
 - depends only on closed curve C in parameter space

M. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

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- Assumptions:
 - 1 $\vec{B}(t)$ changes adiabatically slowly
 - 2 initial state: eigenstate $|\phi_n(0)\rangle$

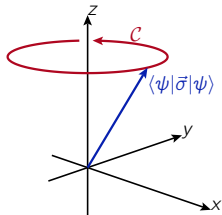
State vector

$$|\phi(t)\rangle = \cos \frac{\vartheta}{2} |\uparrow\rangle + e^{-i\varphi} \sin \frac{\vartheta}{2} |\downarrow\rangle$$

relative phase $\varphi = (2\pi/T)t$

[or any $\varphi(t)$ with $\varphi(T) - \varphi(0) = 2\pi$]

Motion of Bloch vector



$$\begin{aligned} \gamma &= \int_0^T dt \left(\langle \uparrow | \cos \frac{\vartheta}{2} + \langle \downarrow | e^{i\varphi} \sin \frac{\vartheta}{2} \right) \dot{\varphi} \sin \frac{\vartheta}{2} e^{-i\varphi} |\downarrow\rangle \\ &= \pi(1 - \cos \vartheta) = \frac{1}{2}(\text{solid angle}) \end{aligned}$$

Different perspective:

State vector undergoes **periodic** time-evolution

- $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$
- dynamics $|\psi(t)\rangle$ induced by some Hamiltonian $H(t)$

Remarks:

- no adiabatic condition
- $|\psi(t)\rangle$ need not be an eigenstate of $H(t)$
- $H(t)$ is not unique
- only condition: **cyclic time-evolution in Hilbert space**

$$|\psi(t)\rangle = e^{i\varphi(t)}|\phi(t)\rangle, \quad |\phi(t)\rangle = |\phi(t+T)\rangle$$

→ Phase acquired during cyclic evolution: $\varphi = f(T) - f(0)$

$$\frac{d\varphi}{dt} = -\langle \phi | H(t) | \phi \rangle + \langle \phi | i \frac{d}{dt} | \phi \rangle \implies \boxed{\varphi = \varphi_{\text{dyn}} + \gamma}$$

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Dynamical phase $\varphi_{\text{dyn}} = - \int_0^T dt \langle \phi(t) | H(t) | \phi(t) \rangle$

- depends on choice of $H(t)$
- reflects mean energy

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Aharonov-Anandan phase (“non-adiabatic Berry phase”)

$$\gamma = \int_0^T dt \langle\phi|i\frac{d}{dt}|\phi\rangle$$

- depends only on trajectory in Hilbert space — not in parameter space!
- adiabatic limit: $\gamma = \gamma_{\text{C}}$

Aharonov & Anandan, PRL 1987

Goal: propagator $U(t, t')$

- Time-independent system: diagonalize Hamiltonian $\rightarrow |\phi_n\rangle, E_n$

$$U(t, t') = U(t - t') = \sum_n e^{-iE_n(t-t')} |\phi_n\rangle \langle \phi_n|$$

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- Driven system:

$$i \frac{d}{dt} |\psi\rangle = H(t) |\psi\rangle \rightarrow \text{numerical integration}$$

problem 1: time-integration not efficient for long times

problem 2: no information about structure of U

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- Solution for $H(t) = H(t + T)$: “Bloch theory in time”

cf. $H(x)|\phi\rangle = \epsilon|\phi\rangle$ with $H(x) = H(x + a)$

$\rightarrow$ Bloch waves $\phi(x) = e^{iqx} \varphi(x)$, where $\varphi(x)$ is a -periodic

F. Bloch, Z. Phys. A **52**, 555 (1928)

Gaston Floquet (1883):

Ann. de l'Ecole Norm. Sup. **12**, 47 (1883)

Parametric oscillator (cf. Paul trap)

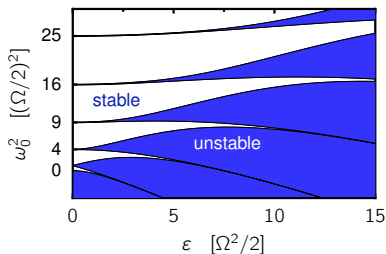
$$\ddot{x} + (\omega_0^2 + \varepsilon \cos \Omega t)x = 0$$

Floquet theorem:
solutions have the structure

$$x(t) = e^{\pm i\mu t} \xi(t)$$

where $\xi(t) = \xi(t + 2\pi/\Omega)$

(undriven limit: $\mu = \omega_0$, $\xi = \text{const}$)



μ real

→ oscillating solutions

μ imaginary

→ one solution unstable

- $H(t) = H(t + T)$
 - $t \rightarrow t + T$ is symmetry operation
 - solutions of Schrödinger equation with $|\psi(t+T)\rangle = e^{i\varphi}|\psi(t)\rangle$
- Floquet ansatz

$$|\psi(t)\rangle = e^{-iqt}|\phi(t)\rangle = e^{-iqt} \sum_k e^{-ik\Omega t} |\phi_k\rangle$$

- q quasienergy (cf. quasi momentum) → long-time dynamics
 - $|\phi(t)\rangle = |\phi(t+T)\rangle$, Floquet state → within driving period
- Floquet theorem: complete set of Floquet solutions of this form
- Schrödinger equation $i\partial_t|\psi\rangle = H(t)|\psi\rangle$ yields

$$\left(H(t) - i\frac{d}{dt}\right)|\phi(t)\rangle = q|\phi(t)\rangle$$

$|\phi(t)\rangle$ Floquet state with quasienergy q

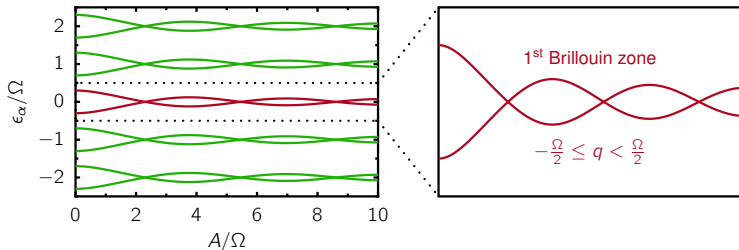
→ $e^{ik\Omega t}|\phi(t)\rangle$ Floquet state with $q + k\Omega$

→ Equivalent states: all correspond to the same $|\psi(t)\rangle$

$|\phi(t)\rangle$ Floquet state with quasienergy q

→ $e^{ik\Omega t}|\phi(t)\rangle$ Floquet state with $q + k\Omega$

→ Equivalent states: **all correspond to the same $|\psi(t)\rangle$**



■ all Brillouin zones equivalent, choice arbitrary

→ **quasienergies cannot serve for ordering!**

- Physical quantity / observable: mean energy

$$E = \frac{1}{T} \int_0^T dt \langle \psi(t) | H(t) | \psi(t) \rangle = \frac{1}{T} \int_0^T dt \langle \phi(t) | H(t) | \phi(t) \rangle$$

- All equivalent states have the same mean energy
[proof: insert $e^{ik\Omega t} |\phi(t)\rangle$]
- Floquet states can be ordered by their mean energy

■ Mean energy

$$E = \frac{1}{T} \int_0^T dt \langle \phi(t) | \{ H(t) - i\partial_t + i\partial_t \} | \phi(t) \rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = q|\phi(t)\rangle$

$$E = q + \frac{1}{T} \int_0^T dt \langle \phi(t) | i\partial_t | \phi(t) \rangle$$

■ Mean energy

$$E = \frac{1}{T} \int_0^T dt \langle \phi(t) | \{ H(t) - i\partial_t + i\partial_t \} | \phi(t) \rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = q|\phi(t)\rangle$

$$-q = -E + \frac{1}{T} \int_0^T dt \langle \phi(t) | i\partial_t | \phi(t) \rangle$$

■ Compare to

$$\varphi = \varphi_{\text{dyn}} + \gamma$$

■ Mean energy

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■ Compare to

$$\varphi = \varphi_{\text{dyn}} + \gamma$$

$(E - q)T$ is a geometric phase

Floquet equation as function of Ω and Ωt :

$$q(\Omega)\phi(\Omega t) = \left[H(\Omega t) - i\Omega \frac{\partial}{\partial \Omega t} \right] \phi(\Omega t)$$

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Compute derivative $(\partial/\partial \Omega)|_{\Omega t}$ and apply $\int \frac{dt}{T} \phi^+$ to obtain

$$\frac{\partial q}{\partial \Omega} = -\frac{\gamma}{2\pi} = \frac{q - E}{\Omega} \quad \rightarrow \quad \boxed{E = q - \Omega \frac{\partial q}{\partial \Omega}}$$

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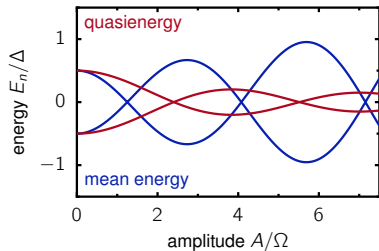
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E.g. for $\gamma = \gamma_{\text{ad}} + \mathcal{O}(\Omega)$

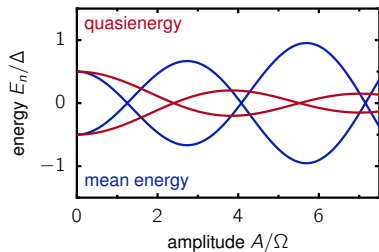
$$\rightarrow q = \text{const} - (\gamma_{\text{ad}}/2\pi)\Omega + \dots \rightarrow E = E_{\text{ad}} + \mathcal{O}(\Omega^2)$$

Fainshtein, Manakov, Rapoport, J. Phys. B (1978)



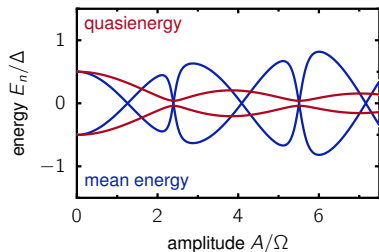
Driven **undetuned** two-level system

- exact crossings
(consequence of a symmetry)



Driven **undetuned** two-level system

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... with **small detuning**

- **quasi energies**
 - avoided crossings
- **mean energies**
 - exact crossings remain
 - additional crossings
 - do not follow from any eigenvalue equation

Goal: more formal treatment of $H(t) - i\partial_t$

- $|\phi(t)\rangle \in \mathcal{R} \otimes \mathcal{T}$ composite Hilbert space / Sambe space

Shirley, PR **138**, B979 (1965), Sambe, PRA **7**, 2203 (1973)

$\mathcal{T}$: Hilbert space of T -periodic functions with inner product

$$\langle f|g\rangle = \int_0^T f(t)^* g(t) \frac{dt}{T} = \sum_k f_k^* g_k$$

- extended Dirac notation:

- $|\phi(t)\rangle = \langle t|\phi\rangle$
- Fourier coefficient $|\phi_k\rangle = \langle k|\phi\rangle$

$$\text{e.g.: } |\phi(t)\rangle = \langle t|\phi\rangle = \sum_k \langle t|k\rangle \langle k|\phi\rangle = \sum_k e^{-ik\Omega t} |\phi_k\rangle$$

- $H - i\partial_t$ is hermitian (in Sambe space!)
- Floquet states $|\phi_\alpha\rangle$ orthonormal and complete in $\mathcal{R} \otimes \mathcal{T}$

$$\langle\langle\phi_\alpha^{(k)}|\phi_\beta^{(k')}\rangle\rangle = \delta_{\alpha\beta}\delta_{kk'}$$

? but in $\mathcal{R}$?

- $H - i\partial_t$ is hermitian (in Sambe space!)

→ Floquet states $|\phi_\alpha\rangle$ orthonormal and complete in $\mathcal{R} \otimes \mathcal{T}$

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? but in $\mathcal{R}$?

- Consider $\langle\phi_\alpha(t)|\phi_\beta(t)\rangle = \sum_k \lambda_k e^{-ik\Omega t}$ since T -periodic with the Fourier coefficient

$$\lambda_k = \frac{1}{T} \int_0^T dt e^{ik\Omega t} \langle\phi_\alpha(t)|\phi_\beta(t)\rangle = \langle\langle\phi_\alpha|\phi_\beta^{(k)}\rangle\rangle = \delta_{\alpha\beta}\delta_{k,0}$$

→ Floquet states orthogonal at equal times

- propagator in terms of Floquet states

$$U(t, t') = \sum_{\alpha} |\psi_{\alpha}(t)\rangle \langle \psi_{\alpha}(t')| = \sum_{\alpha} e^{-iq_{\alpha}(t-t')} |\phi_{\alpha}(t)\rangle \langle \phi_{\alpha}(t')|$$

- long-time dynamics (depends on $t - t'$)
- dynamics within driving period (“micro motion”, depends on t and t')

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- long-time dynamics (depends on $t - t'$)
- dynamics within driving period (“micro motion”, depends on t and t')

- one-period propagator for kicked systems

$$H(t) = H_0 + K \sum_n \delta(t - nT)$$

$$\rightarrow U(T) = e^{-iH_0 T} e^{-iK}$$

- ✓ easy to compute
- ✓ provides quasienergies
- ✗ only long-time dynamics (stroboscopic)

Solve eigenvalue problem

$$\{H(t) - i\partial_t\}|\phi\rangle = q|\phi\rangle$$

Solve eigenvalue problem

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Straightforward in Fourier representation (“Floquet matrix”)

$$H_0 + H_1 \cos(\Omega t) - i \frac{d}{dt} \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & H_0 + 2\Omega & \frac{1}{2}H_1 & 0 & 0 & 0 & \cdots \\ \cdots & \frac{1}{2}H_1 & H_0 + \Omega & \frac{1}{2}H_1 & 0 & 0 & \cdots \\ \cdots & 0 & \frac{1}{2}H_1 & H_0 & \frac{1}{2}H_1 & 0 & \cdots \\ \cdots & 0 & 0 & \frac{1}{2}H_1 & H_0 - \Omega & \frac{1}{2}H_1 & \cdots \\ \cdots & 0 & 0 & 0 & \frac{1}{2}H_1 & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- 1 direct diagonalization of $H(t) - i\partial_t$ in Sambe space
 - conceptually simple $\rightarrow$ first choice
 - increasingly difficult with smaller frequency
- 2 analytical tool: **perturbation theory**
strong driving: $H_1 \cos(\Omega t) - i\partial_t$ as zeroth order

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strong driving: $H_1 \cos(\Omega t) - i\partial_t$ as zeroth order
- 3 numerical propagation: $U(T, 0) \rightarrow e^{-iqT}$ and $|\phi(0)\rangle \rightarrow |\phi(t)\rangle$
- 4 (t, t') formalism (very efficient propagation scheme)
- 5 matrix-continued fractions (convenient for time-dep. Liouvillians)

Starting point: **Rabi-like Hamiltonian** with $1 \ll \langle a^\dagger a \rangle \approx n_0$

$$H = \Omega a^\dagger a + g(a^\dagger + a)X + \dots$$

Classical field / coherent state

$$a \longrightarrow \langle a \rangle = \sqrt{n_0} e^{-i\Omega t}$$

→ driving: $2g\sqrt{n_0} \cos(\Omega t)X$

✓ intuitive

✗ assumes separability

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New basis: “phase states”

$$|\theta\rangle = \sum_{k=-n_0}^{\infty} e^{-ik\theta} |n_0 - k\rangle$$

→ $H = \Omega(n_0 - i\partial_\theta) + 2g\sqrt{n_0} \cos(\theta)X$

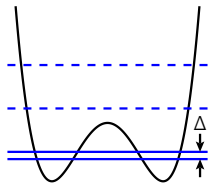
cf. $\mathcal{H} = H(t) - i\partial_t$

✓ entanglement survives

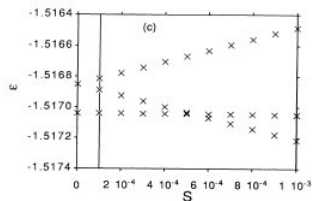
✓ Anandan phase $\leftrightarrow n_0 - \langle a^\dagger a \rangle$

- 1 role of quasienergy crossings
- 2 perturbation theory (two-level approximation)
- 3 basic example of Floquet engineering

Driven double-well potential $H(t) = H_{\text{DW}} + Sx \cos(\Omega t)$

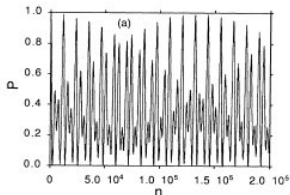


quasienergies $q_\alpha(S)$



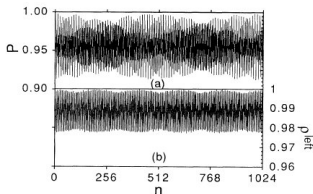
- ? tunnel oscillations influenced by driving
- ? dynamics at quasienergy crossing

Occupation $P_{\text{left}}(nT)$



far from crossing:

■ tunnel oscillations



at crossing:

■ particle stays in left well

→ “coherent destruction of tunneling” by ac field

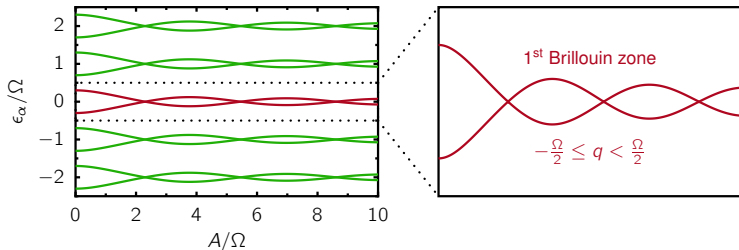
Grossmann *et al.*, PRL 1991

Analytical understanding → two-level approximation

Driven two-level system

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

quasienergy spectrum



Analytical approach for $\Delta \ll \Omega$: high-frequency limit

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

- zeroth order propagator

$$U_{\text{dr}}(t, 0) = \exp\left(-\frac{iA}{2\Omega}\sin(\Omega t)\sigma_z\right)$$

- transformation and rotating-wave approximation (i.e. time-average)

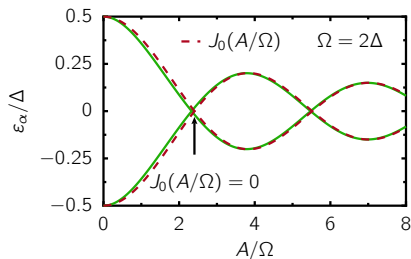
$$H(t) \longrightarrow -\frac{\Delta}{2}U_{\text{dr}}^\dagger(t, 0)\sigma_x U_{\text{dr}}(t, 0) \longrightarrow -\frac{\Delta}{2}J_0(A/\Omega)\sigma_x$$

→ tunnel-matrix element renormalized by Bessel function J_0

$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

■ quasienergies

$$q_{\pm} = \pm \frac{\Delta}{2} J_0(A/\Omega)$$



$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

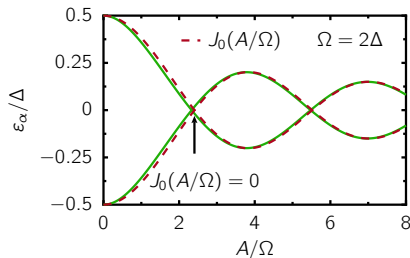
- quasienergies

$$q_{\pm} = \pm \frac{\Delta}{2} J_0(A/\Omega)$$

- mean energies $E = q - \Omega(\partial q / \partial \Omega)$

$$E_{\pm} = \pm \frac{\Delta}{2} \left[J_0(A/\Omega) - \frac{A}{\Omega} J_1(A/\Omega) \right]$$

no need to compute Floquet states!



Bessel function $J_1(x) = \frac{d}{dx} J_0(x)$

$$H(t) = -\frac{\Delta}{2}\sigma_z + \frac{A}{2}\sigma_x \cos(\Omega t)$$

- close to resonance: $\delta = \Delta - \Omega \ll \Delta$, small amplitude: $A \ll \Delta$

$$H(t) = -\frac{\Delta}{2}\sigma_z + \frac{A}{2}\sigma_x \cos(\Omega t)$$

- close to resonance: $\delta = \Delta - \Omega \ll \Delta$, small amplitude: $A \ll \Delta$

$$\mathcal{H}_0 = \frac{\Omega}{2}\sigma_z - i\frac{\partial}{\partial t} \quad \mathcal{H}_1 = \frac{\delta}{2}\sigma_z + \frac{A}{2}\cos(\Omega t)\sigma_x$$

- 2nd order perturbation theory

$$\mathcal{H} \approx \frac{1}{2} \begin{pmatrix} \delta + A^2/16\Omega & A/2 \\ A/2 & -\delta - A^2/16\Omega \end{pmatrix} \quad \begin{array}{l} \text{Rabi Hamiltonian} \\ \text{beyond RWA} \end{array}$$

- absorption maximum at avoided crossing: $\Delta - \Omega_{\text{res}} + A^2/16\Omega_{\text{res}} = 0$:

$$\Omega_{\text{res}} \approx \Delta + \frac{A^2}{16\Delta} \quad \text{Bloch-Siegert shift}$$

Some standard references

■ Classic work:

- Shirley, Phys. Rev. 138, B979 (1965)
- Sambe, Phys. Rev. A 7, 2203 (1973)

■ Reviews:

- Grifoni, Hänggi, Phys. Rep. 304, 229 (1998)
- Hänggi, Chap.5 of “Quantum transport and dissipation” (1998)
<http://www.physik.uni-augsburg.de/theo1/hanggi/Papers/Chapter5.pdf>
- Eckardt, Rev. Mod. Phys. 89, 011004 (2017)

- 1 Proof the Floquet theorem: A T -periodic Hamiltonian has a fundamental matrix of the form $P(t)e^{-iCt}$ with $P(t) = P(t + T)$.
- 2 Derive the Rapoport relation $E = q - \Omega(\partial q / \partial \Omega)$.
- 3 Perform the perturbation theory for the TLS at high-frequency, $\Delta \ll \Omega$.
- 4 Compute the Bloch-Siegert shift to order $\mathcal{O}(A^2)$.
- 5 Discretize the loop integral for the Berry phase.
- 6 Given $H(t)$ and an adiabatic eigenstate $|u(t)\rangle$ with energy $E(t)$.
Write the ansatz for adiabatic following in Floquet form.
Which gauge transformations are still allowed?
- 7 Compute numerically the quasienergies of the driven TLS.
- 8 Compute the stability borders of the Mathieu equation.
Hint: assume integer μ and compute the corresponding ω_0^2

1 Floquet & Schrödinger equation

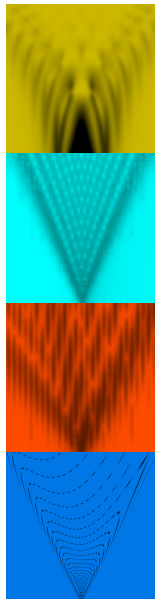
- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

3 Applications & miscellaneous topics

- LZSM Interference
- Floquet-Gibbs states
- Hidden symmetries
- Floquet degeneracies & topology
- Time-dependent Liouvillians
- Two-color Floquet theory
- Adiabatic Floquet theory
- Floquet scattering theory



Heuristic approach

coupling of qubit to electromagnetic environment $\rightarrow$ sponaneous decay

$$|\psi\rangle \longrightarrow \begin{cases} \sigma_- |\psi\rangle & \text{decay with probability } \alpha \ll 1 \\ |\psi\rangle + |\delta\psi\rangle & \text{no decay, probability } 1 - \alpha \end{cases}$$

- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2} \sigma_+ \sigma_- |\psi\rangle$

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- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$
- corresponding density operator

$$\rho \longrightarrow \rho + \frac{\alpha}{2} \left(2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_- \right)$$

- continuous limit with coherent time-evolution $\rightarrow$ master equation

$$\frac{d}{dt}\rho = -i[H, \rho] + \frac{\gamma}{2} \left(2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_- \right)$$

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

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Fulfilled by a **Markovian** master equation iff of “**Lindblad form**”

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G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

- Interpretation: incoherent transitions $|\psi\rangle \rightarrow Q_n|\psi\rangle$

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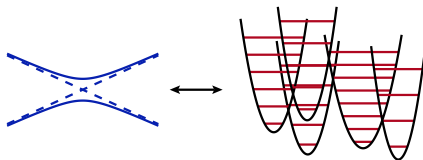
- Interpretation: incoherent transitions $|\psi\rangle \rightarrow Q_n|\psi\rangle$

✗ Critique

- request for Markovian evolution unphysical
- axiomatic, not based on physical model
- high-temperature limit typically wrong
i.e. not the Klein-Kramers or the Smoluchowski equation

Caldeira-Leggett model

Magalinskii 1959; Caldeira, Leggett 1981

Coupling of a **system** to **bath of harmonic oscillators**

$$H = H_{\text{system}}(t) + X \sum_{\nu} \gamma_{\nu} (b_{\nu}^{\dagger} + b_{\nu}) + \sum_{\nu} \omega_{\nu} b_{\nu}^{\dagger} b_{\nu}$$

- eliminate bath
- equation of motion for reduced density operator
 - interpretation: bath “measures” system operator X

Total density operator $R \approx \rho \otimes \rho_{\text{bath,eq}}$

$$\dot{R} = -i[H_{\text{total}}, R]$$

2nd order perturbation theory in system-bath coupling

$$\begin{aligned} \frac{d}{dt}\rho = & -i[H_{\text{sys}}, \rho] - i \int_0^{(t-t_0) \rightarrow \infty} d\tau \mathcal{A}(\tau) [X, [\tilde{X}(-\tau), \rho(t-\tau)]_+] \\ & - \int_0^{(t-t_0) \rightarrow \infty} d\tau \mathcal{S}(\tau) [X, [\tilde{X}(-\tau), \rho(t-\tau)]] \end{aligned}$$

- Heisenberg operator $\tilde{X}(-\tau) = U(\tau) X U^\dagger(\tau)$
- bath correlation functions $\mathcal{A}, \mathcal{S}$
- non-Markovian
- short system-bath correlation time: Markov approximation

- anti-symmetric correlation function

$$\mathcal{A}(\tau) = -i\langle[\xi(\tau), \xi(0)]\rangle$$

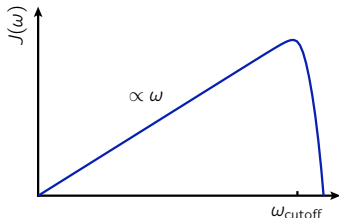
- Fourier transformed: spectral density $\longrightarrow$ continuum limit

$$\mathcal{A}(\omega) = \pi \sum_{\nu} |\gamma_{\nu}|^2 \delta(\omega - \omega_{\nu}) \longrightarrow J(\omega)$$

- here: Ohmic with cutoff

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_{\text{cutoff}}}$$

- dimensionless dissipation strength α

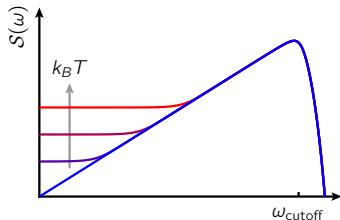


■ symmetric bath correlation function

$$S(\tau) = \frac{1}{2} \langle [\xi(\tau), \xi(0)]_+ \rangle$$

$$S(\omega) = J(\omega) \coth \left(\frac{\omega}{2k_B T} \right)$$

$$= \begin{cases} 4\pi\alpha k_B T & \text{high } k_B T \\ 2\pi\alpha\omega & \text{low } k_B T \end{cases}$$

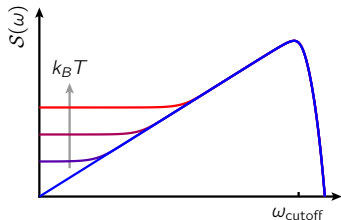


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$$= \begin{cases} 4\pi\alpha k_B T & \text{high } k_B T \\ 2\pi\alpha\omega & \text{low } k_B T \end{cases}$$



- $S(\omega)$ evaluated at transition frequencies

→ dissipation strength depends on spectrum / coherent dynamics

- Ohmic, short memory times (e.g. for $\gamma < k_B T$)

→ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_S, \rho] + i\gamma[X, \{[H_S, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics dissipation decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau \mathcal{S}(\tau) \tilde{X}(-\tau)$

- Ohmic, short memory times (e.g. for $\gamma < k_B T$)

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coherent dynamics dissipation decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau \mathcal{S}(\tau) \tilde{X}(-\tau)$

- not of Lindblad form
 - ✗ positivity might be violated
 - ✓ happens only on unphysically small time scales
- high-temperature limit: Fokker-Planck equation

- Decomposition into energy basis and rotating-wave approximation
- rate equation for the populations (Pauli master equation)

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} \left[w_{\alpha \leftarrow \alpha'} \rho_{\alpha'\alpha'} - w_{\alpha' \leftarrow \alpha} \rho_{\alpha\alpha} \right]$$

with the golden-rule rates

$$w_{\alpha \leftarrow \alpha'} = J(E_{\alpha} - E_{\alpha'}) \left| \langle \phi_{\alpha} | X | \phi_{\alpha'} \rangle \right|^2 n_{\text{th}}(E_{\alpha} - E_{\alpha'})$$

- notice: $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega) + 1$

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– notice: $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega) + 1$

✓ detailed balance: $\frac{w_{\alpha \leftarrow \alpha'}}{w_{\alpha' \leftarrow \alpha}} = e^{-(E_{\alpha} - E_{\alpha'})/k_B T}$

✓ Lindblad form

✗ high-temperature limit typically wrong

full Bloch-Redfield: golden rule for non-diagonal $\rho_{\alpha\beta}$

Driven system $\rightarrow$ noise term becomes time-dependent

$$\dot{\rho} = \dots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau S(\tau) \tilde{X}(t - \tau, t)$$

Driven system $\rightarrow$ noise term becomes time-dependent

$$\dot{\rho} = \dots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau S(\tau) \tilde{X}(t - \tau, t)$$

Central idea:

- 1 adapted basis: Floquet states $|\phi_\alpha(t)\rangle \rightarrow$ captures coherent dynamics
- 2 master equation in Floquet basis

$$\frac{d}{dt} \rho_{\alpha\beta} = -i(q_\alpha - q_\beta) \rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta, \alpha'\beta'}(t) \rho_{\alpha'\beta'}$$

where $\mathcal{L}(t) = \mathcal{L}(t + T)$

- 3 rotating-wave approximation

$$\frac{d}{dt}\rho_{\alpha\beta} = -i(q_{\alpha} - q_{\beta})\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}(t) \rho_{\alpha'\beta'}$$

$$i(q_{\alpha} - q_{\beta} - k\Omega)\rho_{\alpha\beta}^{(k)} = \sum_{\alpha',\beta',k'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}^{(k-k')} \rho_{\alpha'\beta'}^{(k')} \quad (\text{if } \rho \text{ is } T\text{-periodic})$$

	Liouvillian	Density operator	Remark
Convolution	full $\mathcal{L}$	time periodic	best Markovian long-time solution
Moderate RWA	$\mathcal{L}_{\alpha\beta,\alpha'\beta'}^{(0)}$	transients	coherent dynamics, depends on BZ (!)
Full RWA	$\mathcal{L}_{\alpha\alpha,\alpha'\alpha'}^{(0)}$	populations only	efficient, Pauli type, analytic calculations

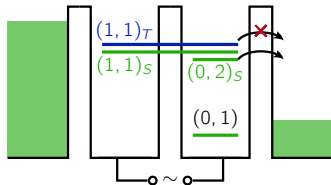
→ full RWA (Pauli master equation)

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} w_{\alpha \leftarrow \alpha'} \rho_{\alpha'\alpha'} - \sum_{\alpha} w_{\alpha' \leftarrow \alpha} \rho_{\alpha\alpha}$$

with the golden-rule rates

$$w_{\alpha \leftarrow \alpha'} = \sum_k J(q_\alpha - q_{\alpha'} + k\Omega) \left| \sum_{k'} \langle \phi_{\alpha, k+k'} | X | \phi_{\alpha', k} \rangle \right|^2 n_{\text{th}}(q_\alpha - q_{\alpha'} + k\Omega)$$

- sidebands contribute to $w_{\alpha \leftarrow \alpha'}$
... but NOT as independent states!
- no detailed balance between forward / backward rates



- (many-body) Floquet states
 - evaluate rates $w_{\alpha \leftarrow \alpha'}$
- dc current, counting statistics

	Dissipation	Transport
Environment	harmonic oscillators	electron source / drain
Coupling of mode ν	$X(a_\nu^\dagger + a_\nu)$	$c^\dagger c_\nu + c_\nu^\dagger c$
Absorption / tunnel in	$n_{\text{th}}(\omega)$	$f(\epsilon - \mu)$
Emission / tunnel out	$1 + n_{\text{th}}(\omega)$	$1 - f(\epsilon - \mu)$
“Ohmic”	$J(\omega) \propto \omega$	$\Gamma(\omega) = \text{const}$

1 Floquet & Schrödinger equation

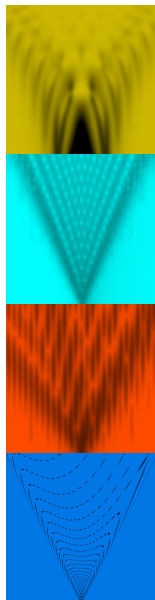
- Geometric phases
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2 Quantum dissipation and transport

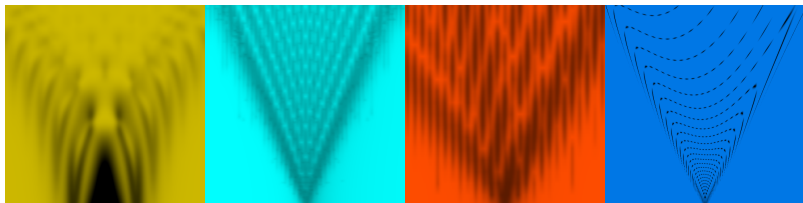
- System-bath model
- Floquet-Bloch-Redfield formalism

3 Applications & miscellaneous topics

- LZSM Interference
- Floquet-Gibbs states
- Hidden symmetries
- Floquet degeneracies & topology
- Time-dependent Liouvillians
- Two-color Floquet theory
- Adiabatic Floquet theory
- Floquet scattering theory

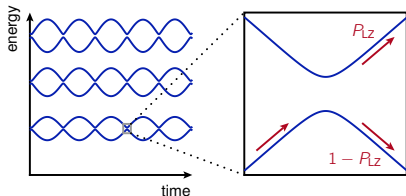


Landau-Zener-Stückelberg-Majorana Interference



Experiments: Stefan Ludwig (PDI Berlin)
 Jason Petta (UCLA)
 Gang Cao & Guo-Ping Guo (Hefei)

Quantum system in AC-field, $H(t)$

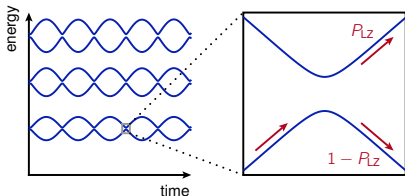


- non-adiabatic transition probability

$$P_{LZ} = e^{-\pi \Delta^2 / 2 \hbar v}$$

Landau, Zener, Stückelberg,
Majorana, 1932

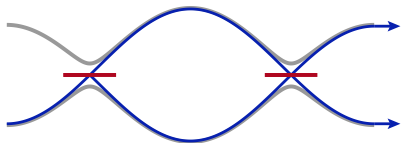
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- non-adiabatic transition probability

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Landau, Zener, Stückelberg,
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- beam splitter, interference
- Landau-Zener-(Stückelberg-Majorana) interferometry

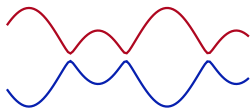
$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \quad g(t) = \epsilon + A\cos(\Omega t)$$

1 (avoided) crossing requires $A > |\epsilon|$

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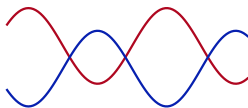
- 1 (avoided) crossing requires $A > |\epsilon|$
- 2 relative phase between dominant paths

adiabatic: $P_{LZ} \ll 1$



$$\varphi(T) = \int_0^T dt |g(t)|$$

diabatic: $1 - P_{LZ} \ll 1$



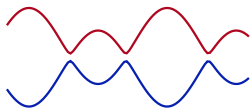
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→ fringes for $\varphi(T) = 2\pi k$

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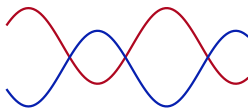
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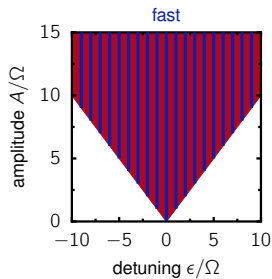
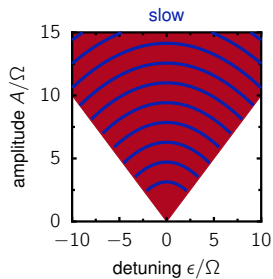
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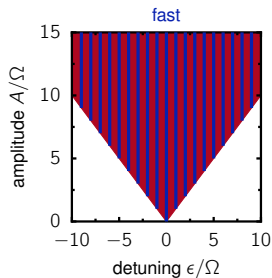
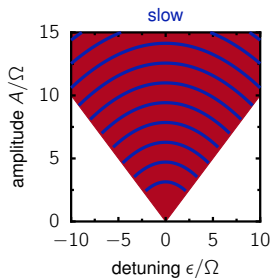
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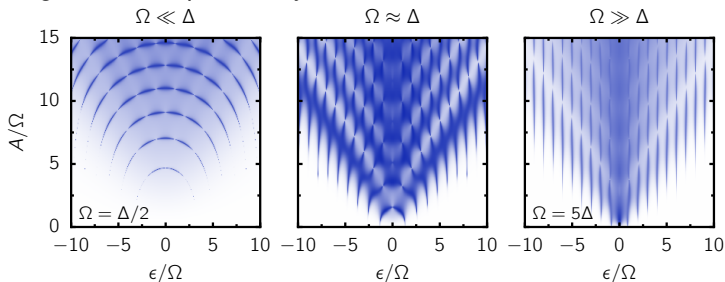
$$\varphi(T) = \int_0^T dt g(t) = \epsilon T$$

$\epsilon = k\Omega$ “ k -photon resonance”



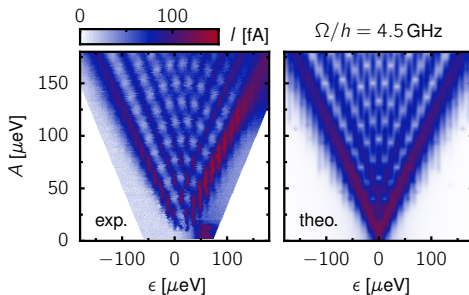
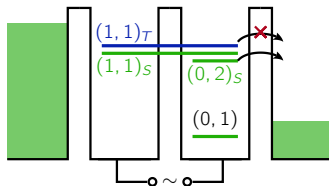


E.g. excitation probability of a TLS:

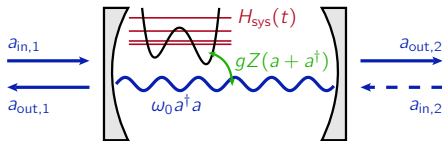


DC current:

$$I(t) = e_0 \Gamma_R \langle n_R(t) \rangle \rightarrow \overline{\langle n_R(t) \rangle}^T$$

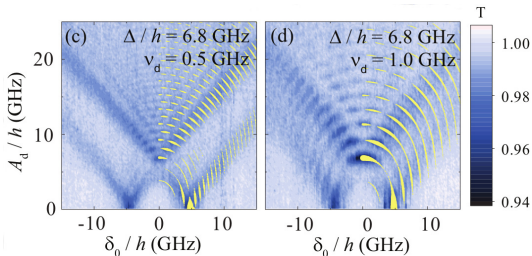


Forster *et al.*, PRL 2014



Dispersive frame:
effective cavity frequency

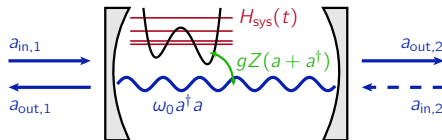
$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{qb} - \omega_0} \sigma_z$$



Theory for readout of
driven system?

Koski *et al.*, PRL 2018

Qubit-cavity Hamiltonian



$$H = H_{\text{sys}}(t) + gZ(a^\dagger + a) + \omega_0 a^\dagger a$$

- Backaction: cavity $\rightarrow$ qubit $\rightarrow$ cavity
- Cavity equation (input/output formalism)

$$\frac{d}{dt}a = -i\omega_0 a - \frac{\kappa}{2}a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\text{in},\nu} - igZ$$

- (non equilibrium) Kubo formula $Z(t) = g \int dt' \chi(t - t') a(t')$
with the response function (may depend on the initial state!)

$$\chi(t) = -i \langle [Z(t), Z] \rangle \theta(t - t')$$

$$\rightarrow -i\omega a = -i(\omega_0 + g^2 \chi(\omega)) a - \frac{\kappa}{2} a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\text{in},\nu}$$

- measured quantity: (non-equilibrium) susceptibility

System Hamiltonian: $H_{\text{qb}} = \frac{\epsilon}{2}\sigma_z$ coupled via $Z = \sigma_x$

Heisenberg operator

$$Z(t) = \sigma_x(t) = \sigma_x \cos(\epsilon t) + \sigma_y \sin(\epsilon t)$$

susceptibility: $\chi(t) = -i\langle [Z(t), Z(0)] \rangle \theta(t)$

$$g^2\chi(\omega_0) = \frac{g^2}{\epsilon - \omega_0} \langle \sigma_z \rangle + \frac{g^2}{\epsilon + \omega_0} \langle \sigma_z \rangle$$

RWA

non-RWA

(cf. Zueco, Reuther, SK, Hänggi, PRA 2009)

Response of periodically driven system

$$\chi(t, t') = -i \langle [Z(t), Z(t')] \rangle_{\text{non-eq}} = \chi(t+T, t'+T)$$

such that

$$\chi(t, t - \tau) = \sum_k e^{-ik\Omega t} \int d\omega e^{-i\omega\tau} \chi^{(k)}(\omega)$$

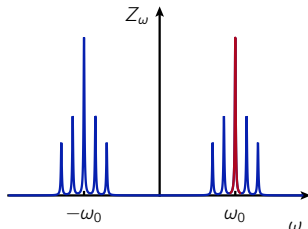
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- resonant cavity driving, $\omega = \omega_0$
- response $Z(t)$ acquires sidebands
- good cavity limit, $\kappa \ll \omega_0, \Omega$



Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta, \alpha, k} \frac{(p_\alpha - p_\beta) |Z_{\beta\alpha, k}|^2}{q_\alpha - q_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory $\rightarrow$ quasi-energies q_α
- Floquet-Bloch-Redfield $\rightarrow$ populations p_α

Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta, \alpha, k} \frac{(p_\alpha - p_\beta) |Z_{\beta\alpha, k}|^2}{q_\alpha - q_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory $\rightarrow$ quasi-energies q_α
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Resonance conditions

cavity response:

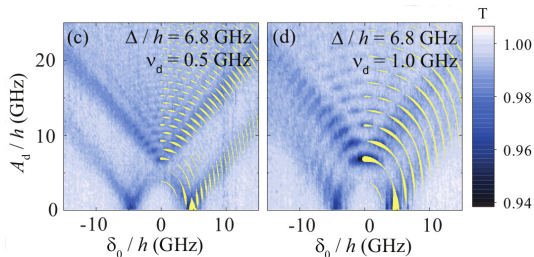
$$\Delta q = \omega_0 + k\Omega$$

cf. population:

$$\Delta q = k\Omega$$

e.g. Ivakhnenko *et al.*, Phys.Rep. 2023

$\rightarrow$ Agree only for low-frequency oscillator!

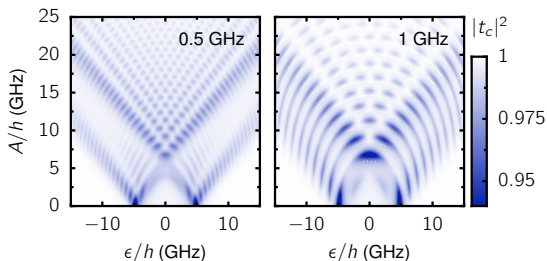


Double QD in GaAs

→ two-level system

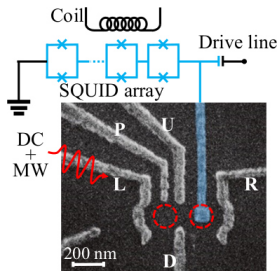
Koski *et al.*, PRL 2018

(yellow: heuristic theory)

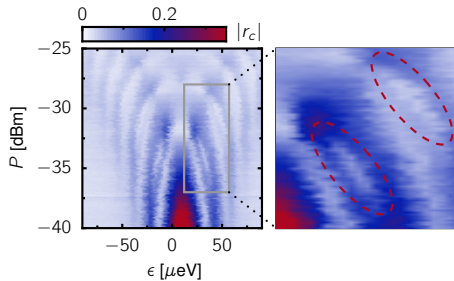


Present theory

SK, PRA 2018

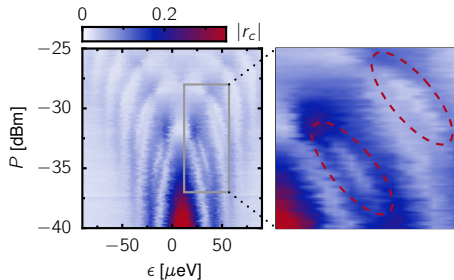


Chen *et al.*, Phys. Rev. B **103**, 205428 (2021)



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD $\rightarrow$ two-level sys.



Experiment (Cao & Guo, Hefei)

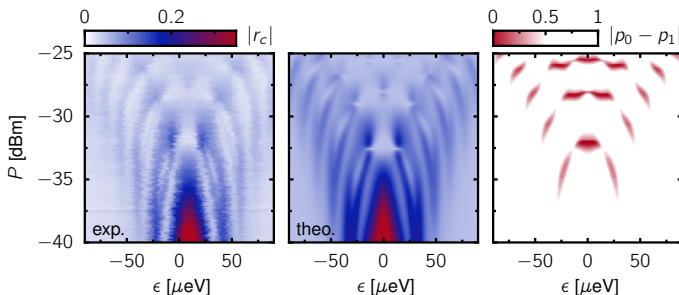
- holes in LZSM pattern
- GaAs DQD → two-level sys.

Recap: susceptibility (two-level system)

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{q_1 - q_0 + \omega_0 + k\Omega + i\gamma/2}$$

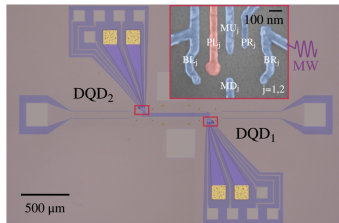
response determined by

- resonance condition for cavity signal
- Floquet state population

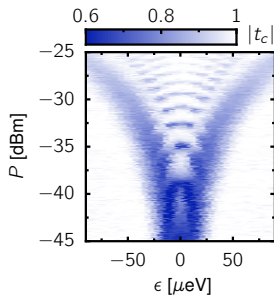


- competing resonance conditions
 - holes in fringes when $p_0 \approx p_1 \approx 1/2$
- cavity response provides information about Floquet state population

Readout for ultra-strong coupling

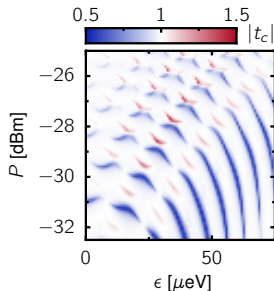
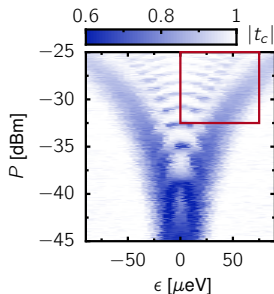


Gu *et al.*, Phys. Rev. Lett. **120**, 2333602 (2023)



Experiment (Cao & Guo, Hefei)

- two DQDs interacting via cavity
- DQD-cavity coupling large



Experiment (Cao & Guo, Hefei)

- two DQDs interacting via cavity
- DQD-cavity coupling large

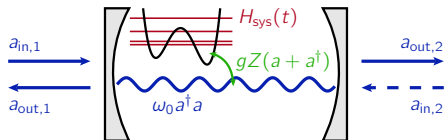
Theory

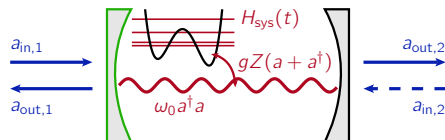
- (too) simple generalization:

$$\chi = \sum_{\text{all DQDs}} \chi_{\text{DQD}}$$

- transmission > 1
- lasing? No!

→ beyond linear response regime





Perturbation: **mirror**

$$H_1 = -i \sum_{\ell=L,R} \sqrt{\kappa_\ell} a_{\text{in},\ell}^\dagger a + \text{H.c.}$$

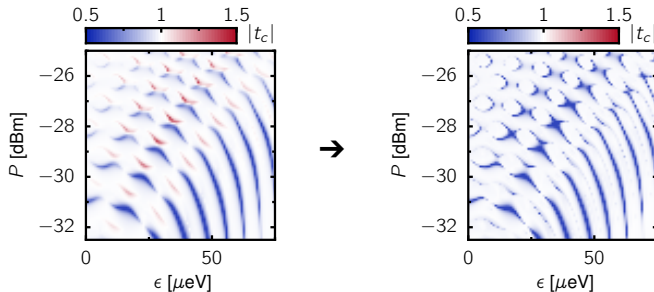
Corresponding susceptibility

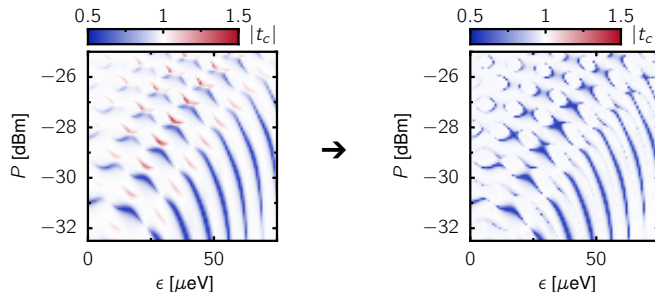
$$\chi(t, t') = -i \langle [a(t), a^\dagger(t')] \rangle \theta(t - t')$$

Transmission amplitude (Fourier component $k = 0$, evaluated at ω_0)

$$S_{12}(\omega) = -i \sqrt{\kappa_1 \kappa_2} \chi^{(0)}(\omega)$$

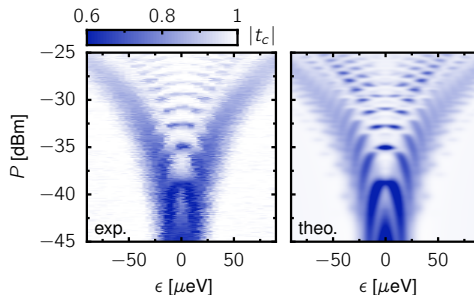
Gu *et al.*, PRL 2023



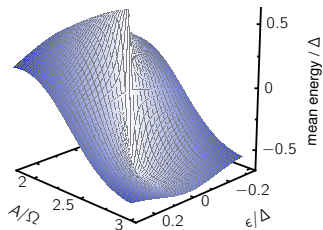
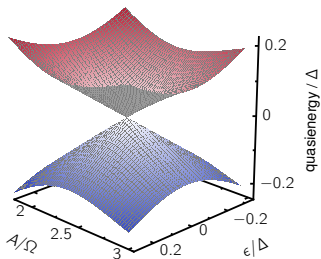


Experiment vs. theory

- inhomogeneous broadening
- includes DQD-DQD interaction via cavity



Dissipation at conical intersections



SK, Phys. Rev. A **110**, 052218 (2024)

SK, J. Chem. Phys. **162**, 154101 (2025)

Stationary state of driven dissipative quantum system

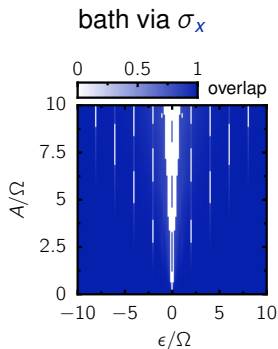
mean energy: $p_{\alpha} \propto e^{-E_{\alpha}/kT}$

quasi energy: $p_{\alpha} \propto e^{-q_{\alpha}/kT}$

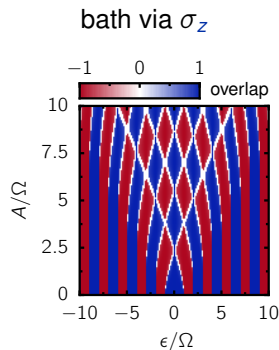
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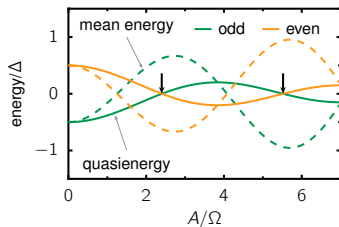


mean-energy state



(anti-) Floquet-Gibbs state

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t)$$

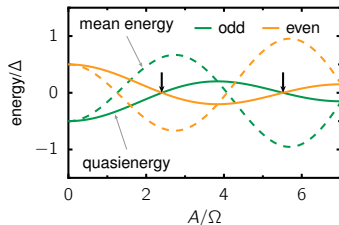


■ spatio-temporal symmetry G :

$$\sigma_x \text{ \& } (t \rightarrow t + T/2)$$

→ even / odd states

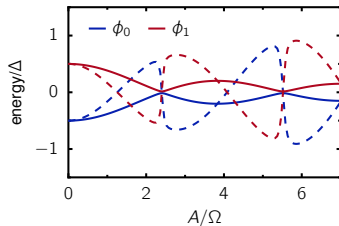
$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$



■ spatio-temporal symmetry G :

$$\sigma_x \text{ \& \; } (t \rightarrow t + T/2)$$

→ even / odd states

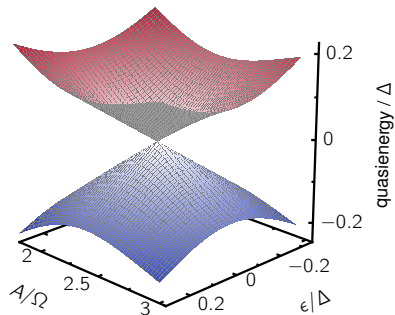


with detuning

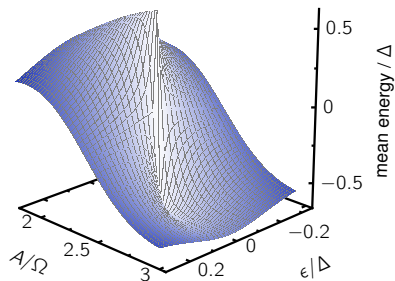
■ quasienergies: **anti-crossing**

■ mean energies: **exact crossing**

quasienergy



mean energy



On circle around tip:

- constant quasienergy
- maximal variation of mean energy

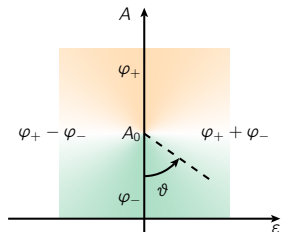
Two-state approximation for ϕ_0

■ basis: φ_- / φ_+ at tip

■ $\vartheta = 0 \longrightarrow \vartheta = \pi$:

$$\phi_0 \longrightarrow \phi_1$$

$$p_0 \longrightarrow p_1 = 1 - p_0$$



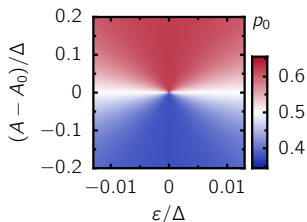
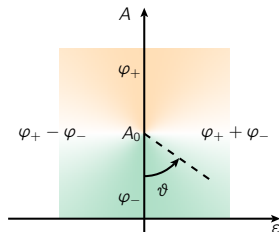
Two-state approximation for ϕ_0

■ basis: φ_- / φ_+ at tip

■ $\vartheta = 0 \longrightarrow \vartheta = \pi$:

$$\phi_0 \longrightarrow \phi_1$$

$$p_0 \longrightarrow p_1 = 1 - p_0$$



Population for σ_x -coupling

- angles with $p_0 = p_1 = 1/2$
 - full mixture, max. entropy
- behaves like mean energy of ϕ_0

cf. discontinuity along A -axis

Engelhardt *et al.*, PRL 2019

Golden-rule rate with $S_{01}(t) = \langle \phi_0(t) | S | \phi_1(t) \rangle \rightarrow$ sidebands S_k

Generic

- Ohmic $J(\omega) = \frac{\pi}{2} \alpha \omega$
- $k = 0$ suppressed

Golden-rule rate with $S_{01}(t) = \langle \phi_0(t) | S | \phi_1(t) \rangle \rightarrow$ sidebands S_k

Generic

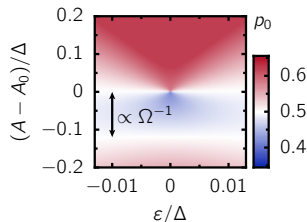
- Ohmic $J(\omega) = \frac{\pi}{2} \alpha \omega$

$\rightarrow k = 0$ suppressed

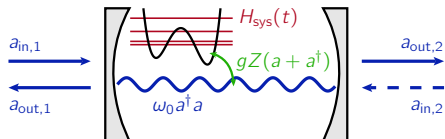
Exception: $S = \sigma_z$

- time-independent for large Ω

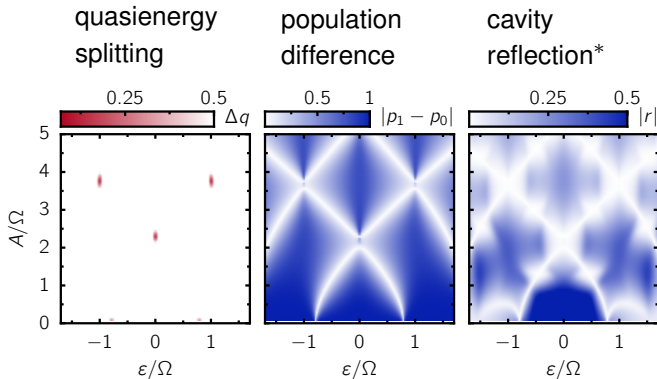
$\rightarrow k \neq 0$ suppressed



- Crossover to Floquet-Gibbs:
two lines with $p_0 = 1/2$ merge
with increasing Ω
- $\rightarrow$ Measurable signature of σ_z -coupling



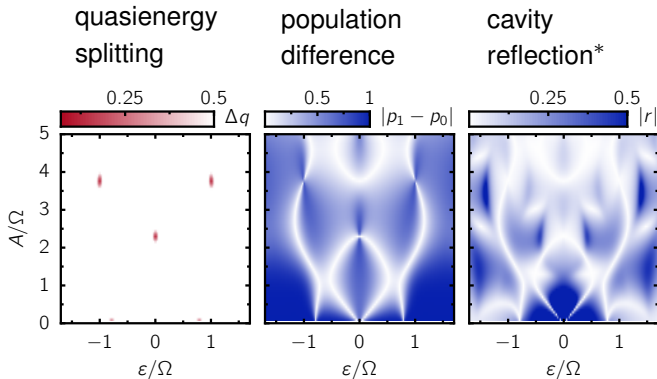
Bath coupling via σ_z



→ experimental signature of bath coupling operator

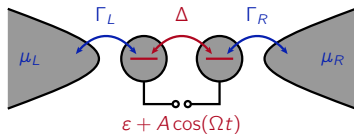
* for DQD and cavity of Chen *et al.*, PRB 2021, $\Omega/2\pi = 10$ GHz

Bath coupling via σ_x



→ experimental signature of bath coupling operator

* for DQD and cavity of Chen *et al.*, PRB 2021, $\Omega/2\pi = 10$ GHz



- zero bias
 - no interaction
 - no spin
- master eq. for populations P_i

$$\frac{d}{dt} \begin{pmatrix} P_L \\ P_R \end{pmatrix} = \begin{pmatrix} -\Gamma_L & \Gamma_R \\ \Gamma_L & -\Gamma_R \end{pmatrix} \begin{pmatrix} P_L \\ P_R \end{pmatrix}$$

(numerics: beyond RWA)

Special case $\Gamma_L = \Gamma_R$:

$$P = \sum_k f(q + k\Omega - \mu) \langle \phi_k | \phi_k \rangle$$

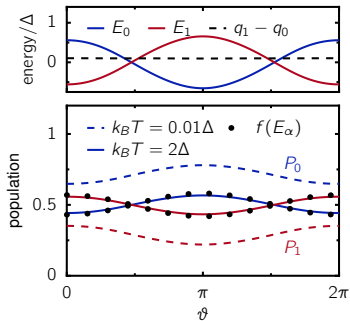
Special case $\Gamma_L = \Gamma_R$:

$$P = \sum_k f(q + k\Omega - \mu) \langle \phi_k | \phi_k \rangle$$

use $f(x) \approx \frac{1}{2} + \frac{x}{2k_B T}$

$$P \approx f(E - \mu)$$

→ mean-energy state



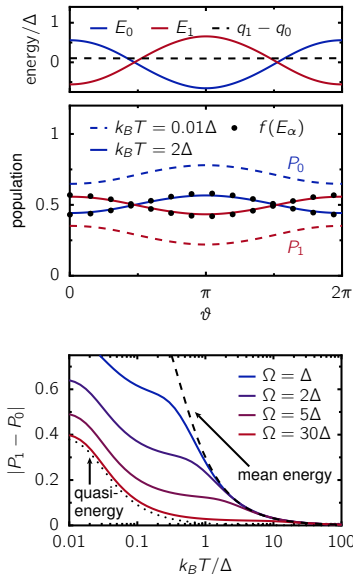
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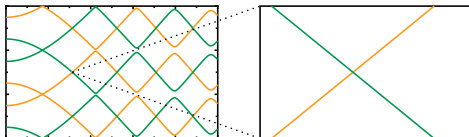
Mean energy state

- dissipation:
(almost) any bath coupling
- intermediate / high
temperature $kT \gtrsim \hbar\Omega$
- open TLS: $\Gamma_L \approx \Gamma_R$

Floquet-Gibbs state

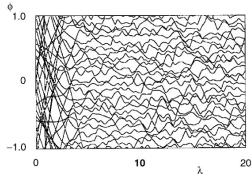
- huge driving frequency,
 $\hbar\Omega \gtrsim 25\Delta$
- coupling to environment
via σ_z or $\Gamma_L \approx \Gamma_R$

Hidden Symmetries in Floquet Systems



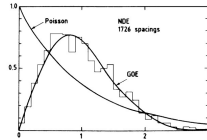
SK & Casado-Pascual, Quantum **10**, 2112 (2026)

■ Spectra of classically chaotic systems



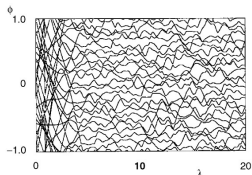
Haake, book
on Q. Chaos

→ level repulsion → anti crossings

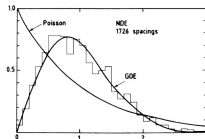


Bohigas 1983

■ Spectra of classically chaotic systems



Haake, book
on Q. Chaos



Bohigas 1983

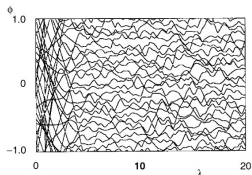
→ level repulsion → anti crossings

■ symmetry: Hamiltonian block diagonal (here: even/odd)

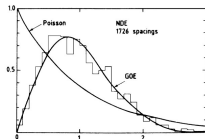
$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

→ allows exact level crossings

■ Spectra of classically chaotic systems



Haake, book
on Q. Chaos



Bohigas 1983

→ level repulsion → anti crossings

■ symmetry: Hamiltonian block diagonal (here: even/odd)

$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

→ allows exact level crossings

→ Exact level crossings indicate symmetries / integrals of motion

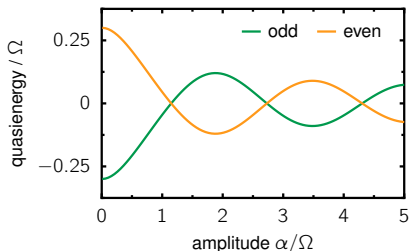
TLS Hamiltonian

$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t)$$

generalized parity $G = \sigma_x e^{(T/2)\partial_t}$

→ even / odd states

→ exact crossings Peres, PRL 1991



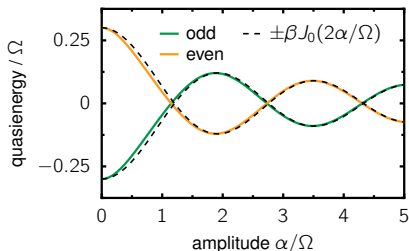
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High-frequency approximation

$$q_{\pm} = \pm\beta J_0(2\alpha/\Omega)$$

TLS Hamiltonian

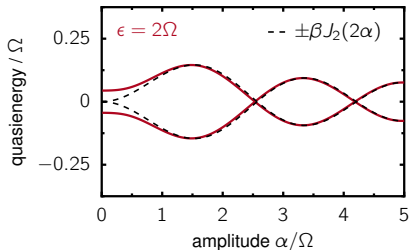
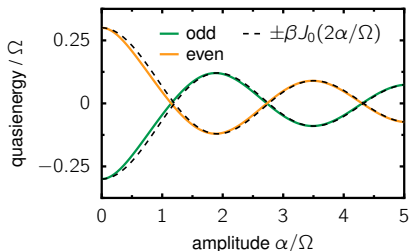
$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{n\Omega}{2}\sigma_z$$

generalized parity $G = \sigma_x e^{(T/2)\partial_t}$

High-frequency approximation

$$q_{\pm} = \pm\beta J_0(2\alpha/\Omega) \longrightarrow \pm\beta J_n(2\alpha/\Omega)$$

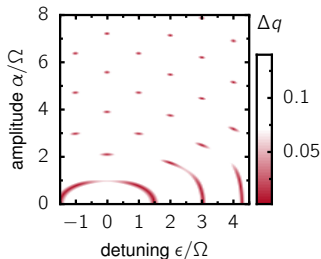
■ suggests that crossings are exact



$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

Quasienergy splittings tiny at
isolated points

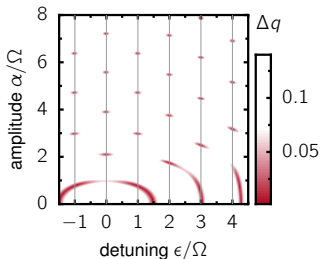
(intermediate frequency,
beyond Bessel function approx.)



$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

Quasienergy splittings tiny at isolated points

(intermediate frequency, beyond Bessel function approx.)



- crossings exact for integer detuning ?
- underlying symmetry / integral of motion ?

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

- ansatz for time non-local LR invariant: $J(t) = Q(t)e^{(T/2)\partial_t}$
[motivation: for $\epsilon = 0 \rightarrow Q = \sigma_x$]

$$i\partial_t Q = \left[\frac{\epsilon}{2}\sigma_z + \beta\sigma_x, Q \right] + \{ \alpha\sigma_z, Q \} \cos(\Omega t)$$

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

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$$i\partial_t Q = \left[\frac{\epsilon}{2} \sigma_z + \beta \sigma_x, Q \right] + \{ \alpha \sigma_z, Q \} \cos(\Omega t)$$

- trivial (and useless) solution: projector on Floquet state

$$J = |\phi_\ell(t)\rangle\langle\phi_\ell(t)| \rightarrow Q = |\phi_\ell(t)\rangle\langle\phi_\ell(t + T/2)|$$

depends on state $\ell \rightarrow$ cannot be used to classify state

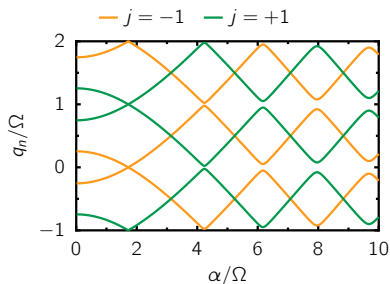
- 1 Fourier ansatz $Q(t) = \sum_k e^{-ikt} Q_k$
 - 2 matrix recurrence relation
 - 3 scalar recurrence
 - 4 $\epsilon/\Omega = n = \text{integer}$: recurrence terminates at $k_{\max} = n$
 - 5 $J^2 = 1$, i.e. $\mathbb{Z}_2$ symmetry
- eigenvalues $j = \pm 1$, “even / odd” Floquet states

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- eigenvalues $j = \pm 1$, “even / odd” Floquet states

Such $J = Qe^{(T/2)\partial_t}$ exists for any integer value of ϵ/Ω

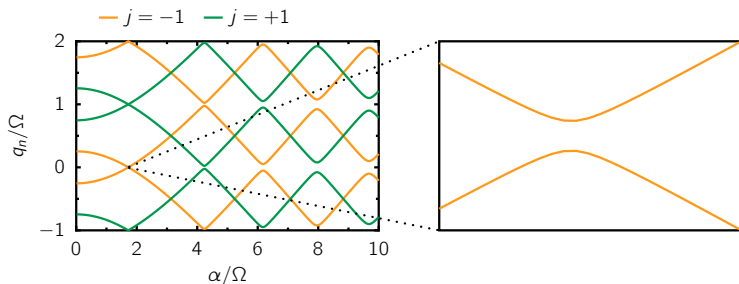
For $\epsilon = \Omega$

$$Q(t) = \frac{1}{\sqrt{\alpha^2 + \beta^2}} \begin{pmatrix} \beta & \alpha e^{-i\Omega t} \\ -\alpha e^{i\Omega t} & \beta \end{pmatrix} \quad \text{with} \quad J^2 = QQ^\dagger = 1$$



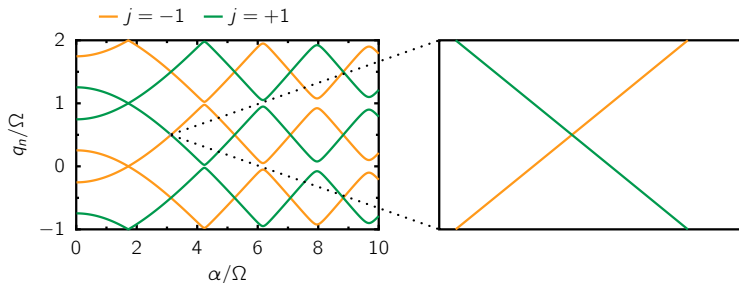
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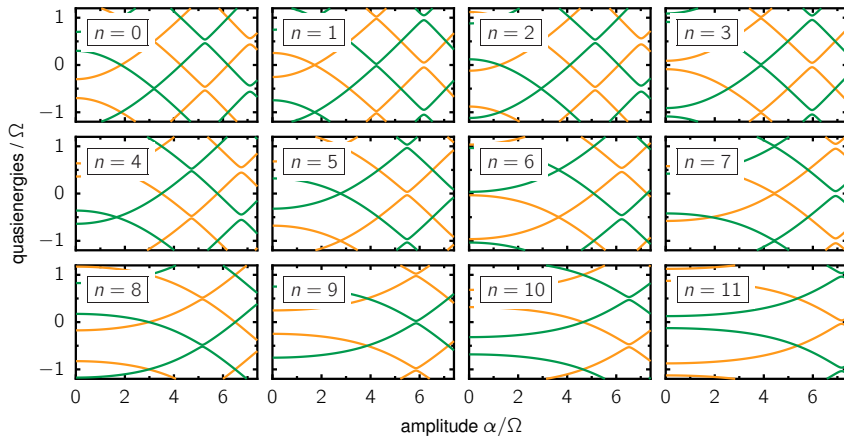
For $\epsilon = \Omega$

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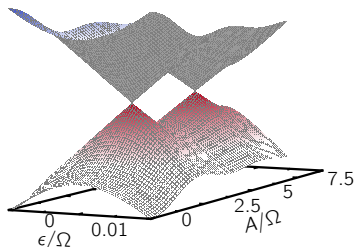


→ accidental degeneracy due to hidden symmetry

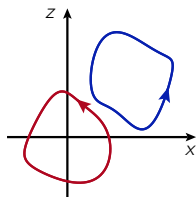
- numerical iteration: J with eigenvalues $j = +1$ and $j = -1$



Topological charges at Floquet degeneracies



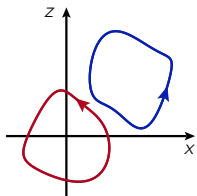
SK & Guéry-Odelin, arXiv:0926.00975



Varying B -field in xz plane

$$H = x(t)\sigma_x + z(t)\sigma_z, \quad E_{\pm} = \pm\sqrt{x^2 + z^2} = \pm|\mathbf{r}|$$

Berry phase = half solid angle: $\pm\pi$ or 0



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Berry curvature and potential

$$\gamma = \oint_C d\mathbf{r} \cdot \langle u | i\partial_{\mathbf{r}} | u \rangle = \int_F d^2\mathbf{f} \cdot \boldsymbol{\Omega}, \quad \Omega_{\kappa} = i\epsilon_{\kappa\mu\nu} \langle \partial_{\mu} u | \partial_{\nu} u \rangle$$

■ For real Hamiltonian: $\boldsymbol{\Omega} = \mathbf{0}$ — unless singular cf. $\Delta \frac{1}{r} = -4\pi\delta(\mathbf{r})$

→ $\boldsymbol{\Omega} = \pi\delta(\mathbf{r})$

→ Dirac cone has topological charge (formally a magnetic monopole)

T -periodic Hamiltonian with
slowly varying parameter

$$H(\mathbf{x}(t), t)$$

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slowly varying parameter

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- Floquet adiabatic theorem

$$|\psi(t)\rangle = e^{i\varphi(t)} |\phi(\mathbf{x}(t), t)\rangle$$

Floquet state $|\phi(\mathbf{x}, t)\rangle$ for fixed $\mathbf{x}$

- Equation of motion for phase

$$\dot{\varphi} = -q + \dot{\mathbf{x}} \cdot \langle \phi | i \partial_{\mathbf{x}} | \phi \rangle$$

P. Weinberg *et al.*, Phys. Rep. 2017

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- Dynamical phase $\leftrightarrow$ quasienergy

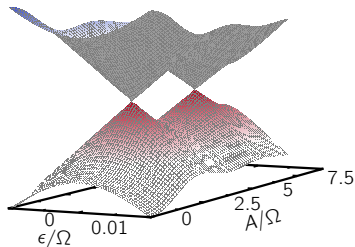
$$\varphi_{\text{dyn}} = -q(t_f - t_0)$$

- Floquet-Berry phase

$$\gamma_{\text{FB}} = \oint d\mathbf{x} \cdot \langle \langle \phi(\mathbf{x}) | i\partial_{\mathbf{x}} | \phi(\mathbf{x}) \rangle \rangle$$

formally like Berry,
but now in Sambe space

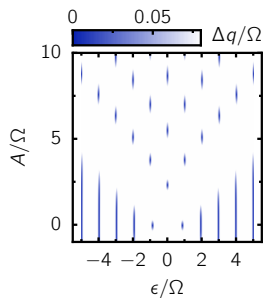
- only difference:
 T_{loop}/T must be integer



- 1 Quasienergies form Dirac cones
- 2 Floquet-Berry phase

→ **Topological charges ?**

Regions with tiny splitting



→ isolated degeneracy points

Two types of degeneracies:

- 1 integer detuning
- 2 m -photon resonance

$$E_e - E_g = m\Omega$$

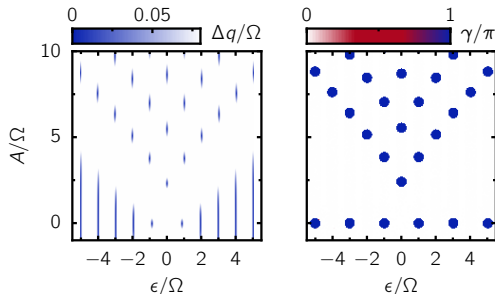
Floquet states for $A = 0$:

$$|g\rangle \text{ and } e^{im\Omega t}|e\rangle$$

both with quasienergy E_g

- Topological charge?
- How to detect it?

- FB loop integral for circle around (ϵ, A)
- detects topologically non-trivial points



Integer detuning

- perturbation theory like above
- Dirac cones
- topological charge

m -photon resonance

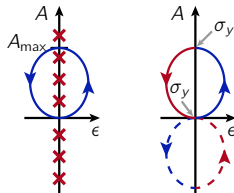
- lowest order perturbation $\propto A^m$

$$\mathbf{B}_{\text{eff}} = (\epsilon - \epsilon_0, 0, \beta_m A^m)$$

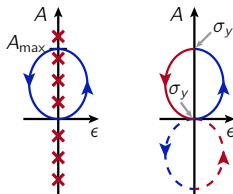
winds around degeneracy for odd m only

- topological charge iff m odd

- start at $\epsilon = A = 0$
- loop with $A \rightarrow A_{\max}$
encloses n degeneracies
- ✓ $\gamma_{\text{FB}} = n\pi \pmod{2\pi}$
- ✗ dynamical phase dominates



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Chirality of $\mathcal{H}(t) = \frac{\Delta}{2}\sigma_x + \frac{1}{2}[\epsilon + A\cos(\Omega t)]\sigma_z - i\partial_t$

Unitary mapping in Sambe space

$$S = \sigma_y \otimes (t \rightarrow -t)$$

$$\rightarrow S\mathcal{H}(t)S^{-1} = -\mathcal{H}(-t)$$

links Floquet states

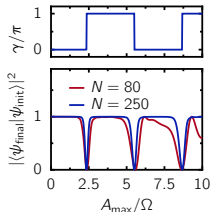
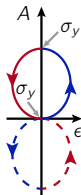
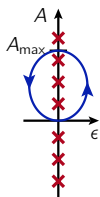
$$\sigma_y|\phi_{-}(t)\rangle = |\phi_{+}(-t)\rangle$$

$$\sigma_y|\phi_{-}(kT)\rangle = |\phi_{+}(+kT)\rangle$$

$$\blacksquare q_{-} = -q_{+}$$

→ opposite dynamical phase

$$|\psi_{\text{initial}}\rangle = |g\rangle \longrightarrow |\psi_{\text{final}}\rangle = e^{i\gamma} |\psi_{\text{initial}}\rangle$$

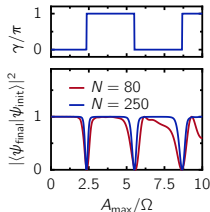
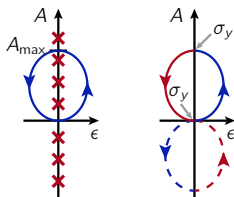


$$\rightarrow \gamma_{\text{FB}} = n\pi \pmod{2\pi}$$

■ sufficient fidelity:

$$T_{\text{loop}} \gtrsim 100T$$

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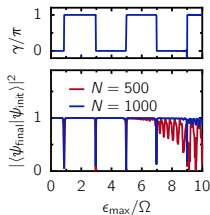
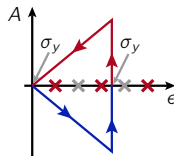
$$\rightarrow \gamma_{\text{FB}} = n\pi \pmod{2\pi}$$

■ sufficient fidelity:

$$T_{\text{loop}} \gtrsim 100T$$

■ jumps at odd- m resonances only

■ requires larger T_{loop}



Possible measurement: Ramsey interference

Periodically time-dependent Liouvillians

$$\frac{d}{dt}P = L(t)P$$

Master equation of type

$$\frac{d}{dt}P = L(t)P$$

- Floquet-Bloch-Redfield beyond moderate RWA
- time-dependent system with Lindblad dissipator

$$\dot{\rho} = -i[H(t), \rho] + \gamma(2a^\dagger \rho a - a^\dagger a \rho - \rho a^\dagger a)$$

- very weak dissipation
- transport problem with large bias

- long-time solution T -periodic
- Floquet ansatz with “quasienergy” zero

$$P(t) = \sum_k e^{-ik\Omega t} p_k$$

$$\frac{d}{dt}P = L(t)P \text{ with}$$

$$L(t) = L_0 + 2L_1 \cos(\Omega t)$$

→ kernel of tridiagonal Floquet matrix

$$L_0 + 2L_1 \cos(\Omega t) - \partial_t \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & L_0 + 2i\Omega & L_1 & 0 & 0 & 0 & \cdots \\ \cdots & L_1 & L_0 + i\Omega & L_1 & 0 & 0 & \cdots \\ \cdots & 0 & L_1 & L_0 & L_1 & 0 & \cdots \\ \cdots & 0 & 0 & L_1 & L_0 - i\Omega & L_1 & \cdots \\ \cdots & 0 & 0 & 0 & L_1 & L_0 - 2i\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- ansatz $P(t) = \sum_k e^{-ik\Omega t} p_k$ yields

$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate $p_{k-1} = -L_1^{-1} \{ (L_0 - ik\Omega) p_k + L_1 p_{k+1} \}$

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- idea: truncate and iterate $p_{k-1} = -L_1^{-1} \{ (L_0 - ik\Omega) p_k + L_1 p_{k+1} \}$
 $\times$ fails, L_1 generally singular
- solution: ansatz $p_k = S_k L_1 p_{k\mp 1}$ ($k \gtrless 0$) leads to

$$S_k = - (L_0 + ik\Omega + L_1 S_{k\pm 1} L_1)^{-1} \longrightarrow S_{\pm 1} \quad (1)$$

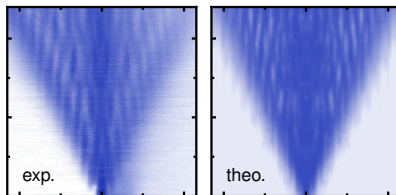
$$0 = (L_1 S_{-1} L_1 + L_0 + L_1 S_1 L_1) p_0 \quad (2)$$

- truncate at $\pm k_0$, iterate (1), and solve (2)
- time-averaged $P(t) = p_0 \rightarrow$ time-averaged expectation values

Risken, "The Fokker-Planck Equation"

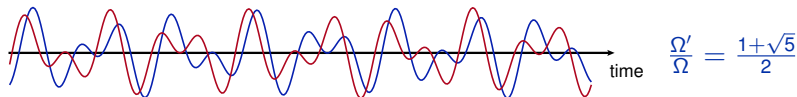
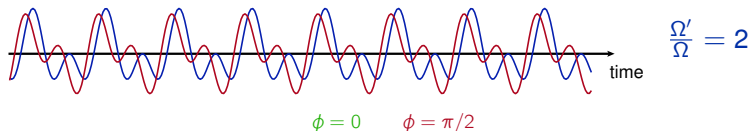
Appendix of Forster *et al.*, PRB 2015

LZSM Interference for Bichromatic Driving

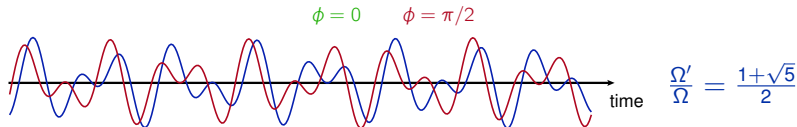
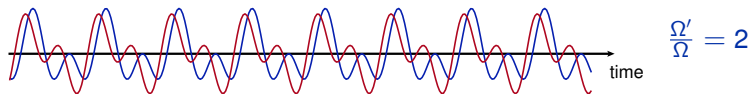


Forster, Mühlbacher, Blattmann, Schuh, Wegscheider, Ludwig & SK
PRB **92**, 245422 (2015)

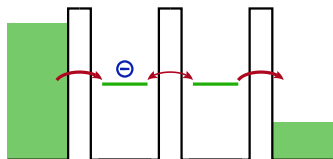
- $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$
 Ω', Ω commensurable vs. incommensurable
 $\rightarrow g(t)$ periodic vs. quasi-periodic



- $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$
 Ω', Ω commensurable vs. incommensurable
→ $g(t)$ periodic vs. quasi-periodic



- quasi-periodic:
 - dynamical phases (quasi) random
 - ? regular LZSM pattern, symmetries



E.g. Lindblad approach:

- source-to-dot tunneling

$$|\psi\rangle \longrightarrow c_1^\dagger |\psi\rangle$$

$$\rightarrow \dot{\rho} = \dots + \Gamma \left(c_1^\dagger \rho c_1 - \frac{1}{2} c_1 c_1^\dagger \rho - \frac{1}{2} \rho c_1 c_1^\dagger \right)$$

$$\frac{d}{dt} P = L(t)P \quad \text{where} \quad L(t) = -i[H_{\text{DQD}}(t), \cdot] + L_{\text{in}} + L_{\text{out}}$$

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L'_1 \cos(n'\Omega t)$$

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L'_1 \cos(n'\Omega t)$$

- long-time solution periodic, “Floquet solution with eigenvalue 0”

$$\rho(t) = \rho(t + 2\pi/\Omega) = \sum_k e^{-ik\Omega t} \rho_k$$

- homogeneous set of equations for ρ_k
- ρ_0 , time-averaged expectation values, e.g. dc current

- $\frac{d}{dt}\rho = L(t)\rho$ with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\omega t)$$

- auxiliary angular coordinate $\omega t \longrightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t, \theta)\mathcal{P}$$

$$\mathcal{L}(t, \theta) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

- $2\pi/\Omega$ -periodic time-dependence
- solve by usual Floquet tools — here: matrix-continued fractions

- $\frac{d}{dt}\rho = L(t)\rho$ with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\omega t)$$

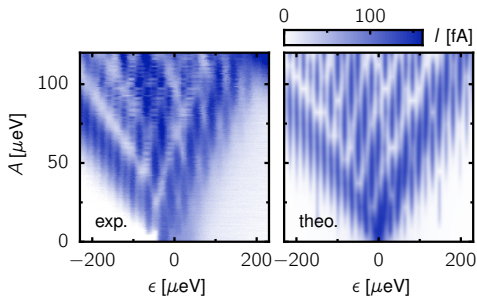
- auxiliary angular coordinate $\omega t \longrightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t, \theta)\mathcal{P}$$

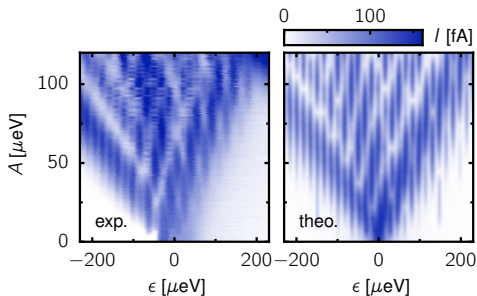
$$\mathcal{L}(t, \theta) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

- $2\pi/\Omega$ -periodic time-dependence
- solve by usual Floquet tools — here: **matrix-continued fractions**
 - connection $\rho(t) = \mathcal{P}(t, \theta)|_{\theta=\omega t}$

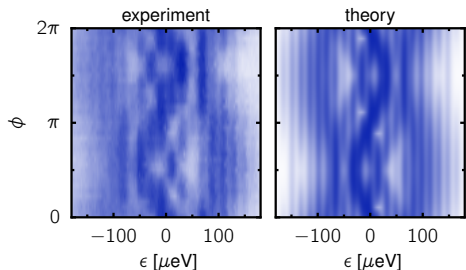
cf. t - t' formalism, see Peskin & Moiseyev, J.Chem.Phys. 1993



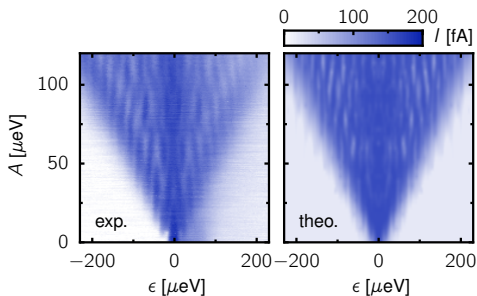
- $g(t) = \sin(\Omega t) + \eta \sin(2\Omega t + \phi)$
[4.8 GHz, $\eta = 1.25$]
- pattern asymmetric ...



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[4.8 GHz, $\eta = 1.25$]
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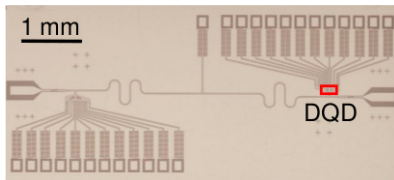
- ... and ϕ -dependent
- phase calibration



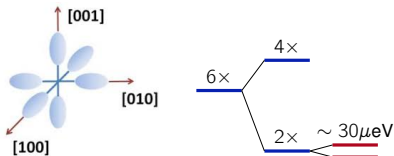
- golden ratio $\frac{\Omega_2}{\Omega_1} = \frac{1 + \sqrt{5}}{2}$
- pattern symmetric
and ϕ -independent

Forster *et al.*, PRB 2015

Dispersive readout of LZSM patterns in Si/SiGe DQDs (under low-frequency driving)

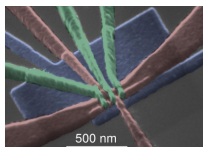


Mi, SK, Petta, Phys. Rev. B **98**, 161404(R) (2018)

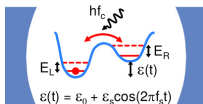


Level structure

- strain, confinement
- valley splitting
- valley qubit



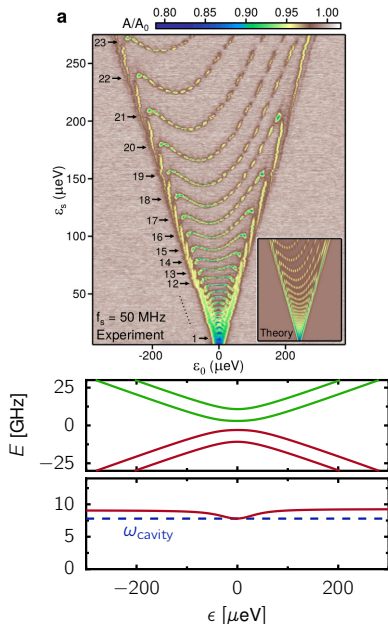
P2 (To Cavity)



J. Petta, U Princeton

Si double quantum dots

- position, spin, valley
- coupling to cavity
- “dispersive readout”



Experiment (J.R. Petta, Princeton)

- very slow driving: 50–100 MHz
→ non-eq. population tiny

- heuristic resonance condition:
 $\bar{E}_1 - \bar{E}_0 = \omega_{\text{cavity}} + k\Omega$
(mean energies!)

Computational problem:

- amplitude: up to 300 GHz
→ Floquet theory requires
 $\gtrsim 10^4$ sidebands

■ adiabatic eigenstates $H(t)|u(t)\rangle = (\bar{E} + \delta E(t))|u(t)\rangle$

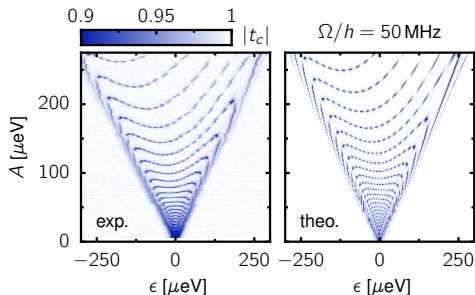
$$\rightarrow |\psi(t)\rangle = e^{-i\bar{E}t} e^{i\varphi(t)} |u(t)\rangle$$

→ adiabatic Floquet states

$$|\phi(t)\rangle = e^{i\varphi(t)} |u(t)\rangle \quad \text{with} \quad \varphi(t) = - \int_0^t dt' \delta E(t') \quad T \text{ periodic!}$$

$$\text{with quasienergies } q = \bar{E} = \frac{1}{T} \int_0^T dt E(t)$$

[here: time-reversal symmetry $\rightarrow \langle u | \partial_t | u \rangle = 0$, i.e., no Berry phase]



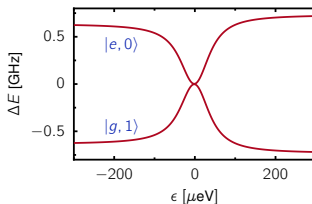
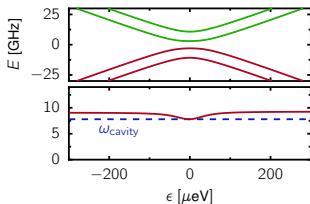
Dressed-state interpretation

■ LZSM between states

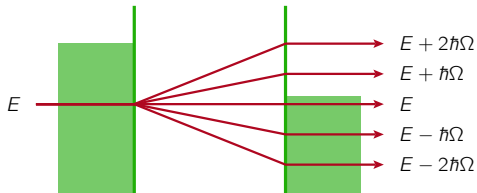
$$|g, 1\rangle, |e, 0\rangle$$

→ cavity-assisted LZSM

Mi, SK, Petta, PRB 2018

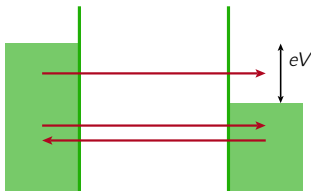


Floquet scattering theory

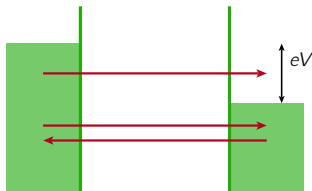


SK, Lehmann, Hänggi, Phys. Rep. **406**, 379 (2005)

- Landauer (1957): “conductance is transmission”



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- current

$$I = \frac{e}{h} \int dE \, T(E) [f(E + eV) - f(E)]$$

- transmission of an electron with energy E

$$T(E) = \Gamma_L \Gamma_R |\langle 1 | G(E) | N \rangle|^2$$

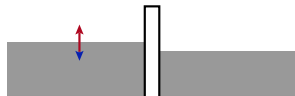
Fischer, Lee, PRB 1981; Meir, Wingreen, PRL 1992

- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{\text{ac}} \cos(\Omega t)$$

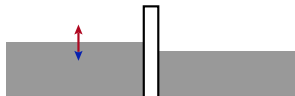
- time-dependent energy shift by $eV_{\text{ac}} \cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp\left(-iEt - i\frac{eV_{\text{ac}}}{\Omega} \sin(\Omega t)\right)$$



- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{ac} \cos(\Omega t)$$



- time-dependent energy shift by $eV_{ac} \cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp\left(-iEt - i\frac{eV_{ac}}{\Omega} \sin(\Omega t)\right) = \sum_k J_k(eV_{ac}/\Omega) e^{-i(E+k\Omega)t}$$

- sidebands occupied with probability $J_k^2(\dots)$
- energy $k\Omega$ corresponds to additional DC bias voltage $k\Omega/e$

$$I(V_0, V_{ac}) = \sum_k J_k^2\left(\frac{eV_{ac}}{\Omega}\right) I_0(V_0 + k\Omega/e)$$

DC conductivity determines the current !

- Approach rather heuristic
- Rigorous derivation?
- When is Tien-Gordon theory applicable?

Transport and driving:

Green's function and Landauer formula for time-dependent situation

Transport and driving:

Green's function and Landauer formula for time-dependent situation

■ Floquet equation

with self-energy $\Sigma = |1\rangle \frac{i\Gamma_L}{2} \langle 1| + |N\rangle \frac{i\Gamma_R}{2} \langle N|$

$$\left(H(t) + \Sigma - i \frac{d}{dt} \right) |\varphi_\alpha(t)\rangle = (\epsilon_\alpha - i\gamma_\alpha) |\varphi_\alpha(t)\rangle$$

■ propagator in the presence of the contacts

$$G(t, t - \tau) = \sum_k e^{ik\Omega t} \int dE e^{-iE\tau} \underbrace{\sum_{\alpha, k'} \frac{|\varphi_{\alpha, k+k'}\rangle \langle \varphi_{\alpha, k'}|}{\epsilon - (\epsilon_\alpha + k'\Omega - i\gamma_\alpha)}}_{G^{(k)}(E)}$$

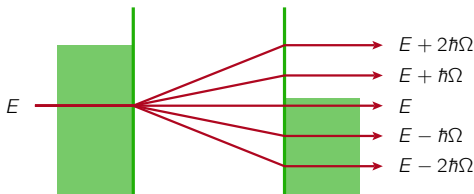
propagation under absorption/emission of $|k|$ photons

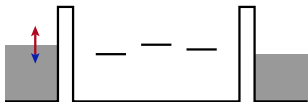
■ dc current

$$I = \frac{e}{h} \sum_k \int dE \left\{ T_{LR}^{(k)}(E) f(E - \mu_L) - T_{RL}^{(k)}(E) f(E - \mu_R) \right\}$$

■ transmission under absorption of k photons

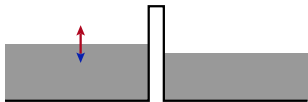
$$T_{LR}^{(k)}(E) = \Gamma_L \Gamma_R |\langle 1 | G^{(k)}(E) | N \rangle|^2 \neq T_{RL}^{(\pm k)}(E \pm k\Omega)$$



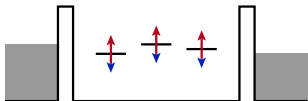


applicable for

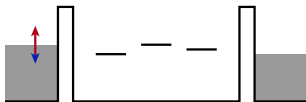
- AC bias voltage



- tunnel barriers
(studied by Tien & Gordon)

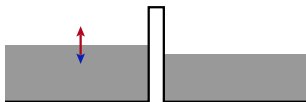


- uniform AC gate voltage

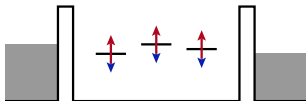


applicable for

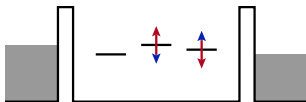
- AC bias voltage



- tunnel barriers
(studied by Tien & Gordon)



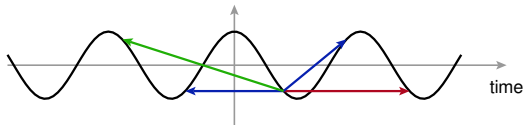
- uniform AC gate voltage



but not for

- non-uniform gating
- dipole force

$$H_{\text{dipole}} \propto x \cos(\Omega t)$$



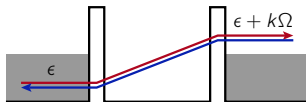
- 1 time periodicity $t \longrightarrow t + T$ → Floquet theory applicable
- 2 time reversal $t \longrightarrow -t$ → Floquet states real
- 3 generalized parity $(x, t) \longrightarrow (-x, t + T/2)$
e.g. symmetric potential with dipole driving → Floquet states even/odd
- 4 time-reversal parity $(x, t - T/4) \longrightarrow (-x, T/4 - t)$
– combination of the other three

Consequences for scattering probabilities ?

Symmetry-related processes have the same probability

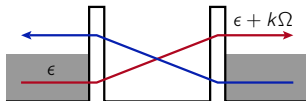
time-reversal

$$t \rightarrow -t$$



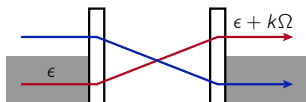
generalized parity

$$(x, t \rightarrow -x, t + \frac{T}{2})$$



time-reversal parity

$$(x, t \rightarrow -x, -t)$$



1 Floquet & Schrödinger equation

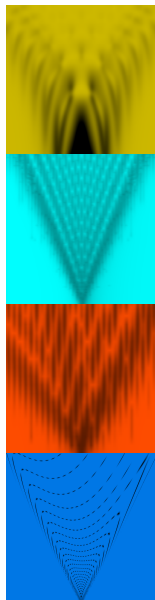
- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

3 Applications & miscellaneous topics

- LZSM Interference
- Floquet-Gibbs states
- Hidden symmetries
- Floquet degeneracies & topology
- Time-dependent Liouvillians
- Two-color Floquet theory
- Adiabatic Floquet theory
- Floquet scattering theory



Experimental collaborations

- Stefan Ludwig (PDI Berlin)
- Jason R. Petta (UCLA)
- Guo-Ping Guo & Gang Cao (Hefei)

