

Old Tools for New Puzzles: From Toponium Physics to the Exotic Nature of $X(3872)$

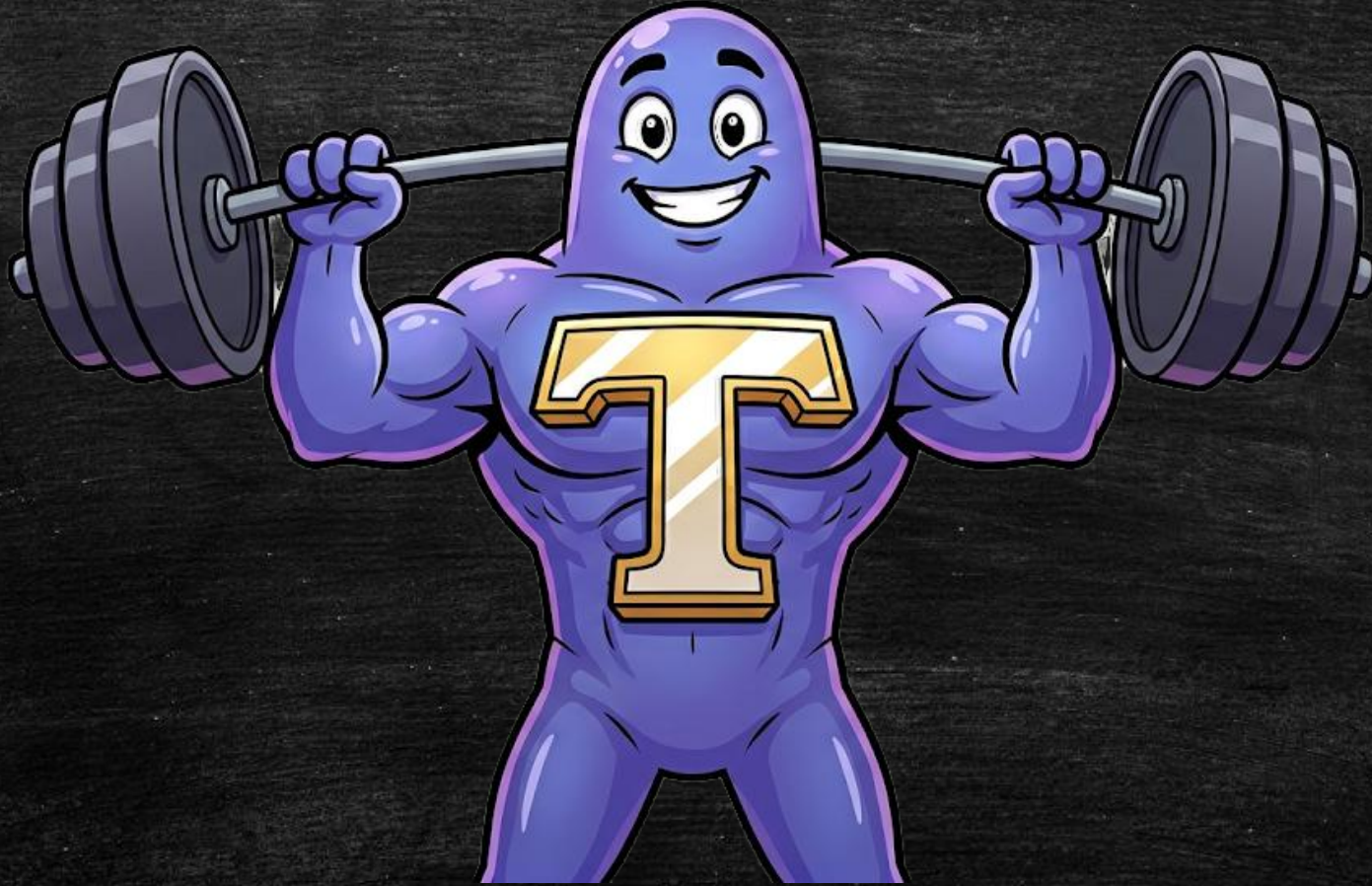
Based on:

P. Nason, E. Re and L. Rottoli, JHEP 10 (2025) 149, arXiv:2505.00096 [hep-ph]; P. Nason, $t\bar{t}$ production near threshold at the LHC, QWG2025
A. Carducci, G. Cianti, P. D'Annibali, D. G. and A.D. Polosa, arXiv:2605.12274 [hep-ph]

Top Quark

$$m_t = 173 \text{ GeV}$$

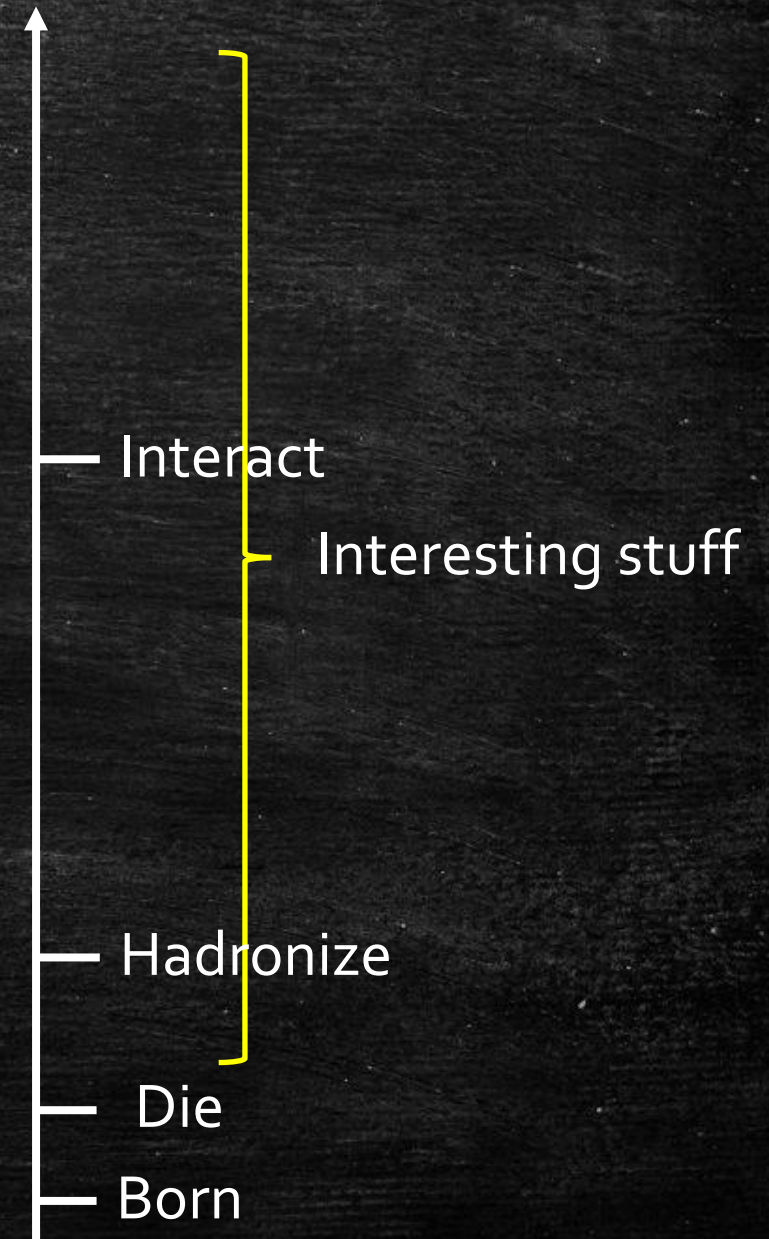
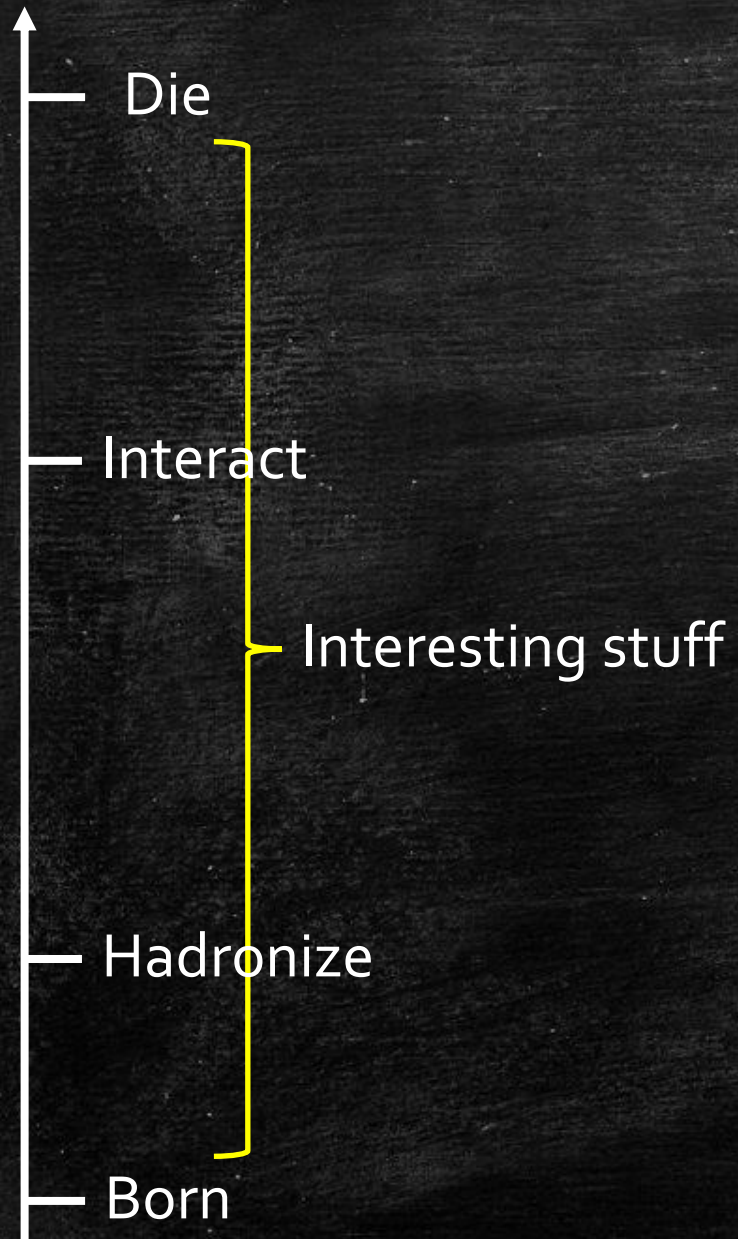
$$\Gamma_t = 1.4 \text{ GeV}$$



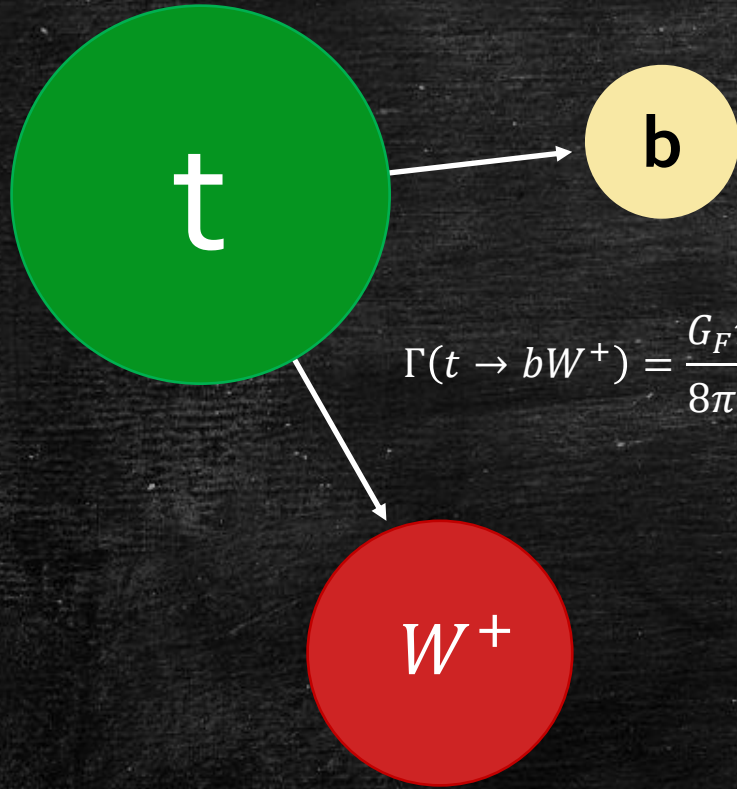
$$m_b = 4.2 \text{ GeV}$$

$$m_W = 80 \text{ GeV}$$

Why is t interesting?

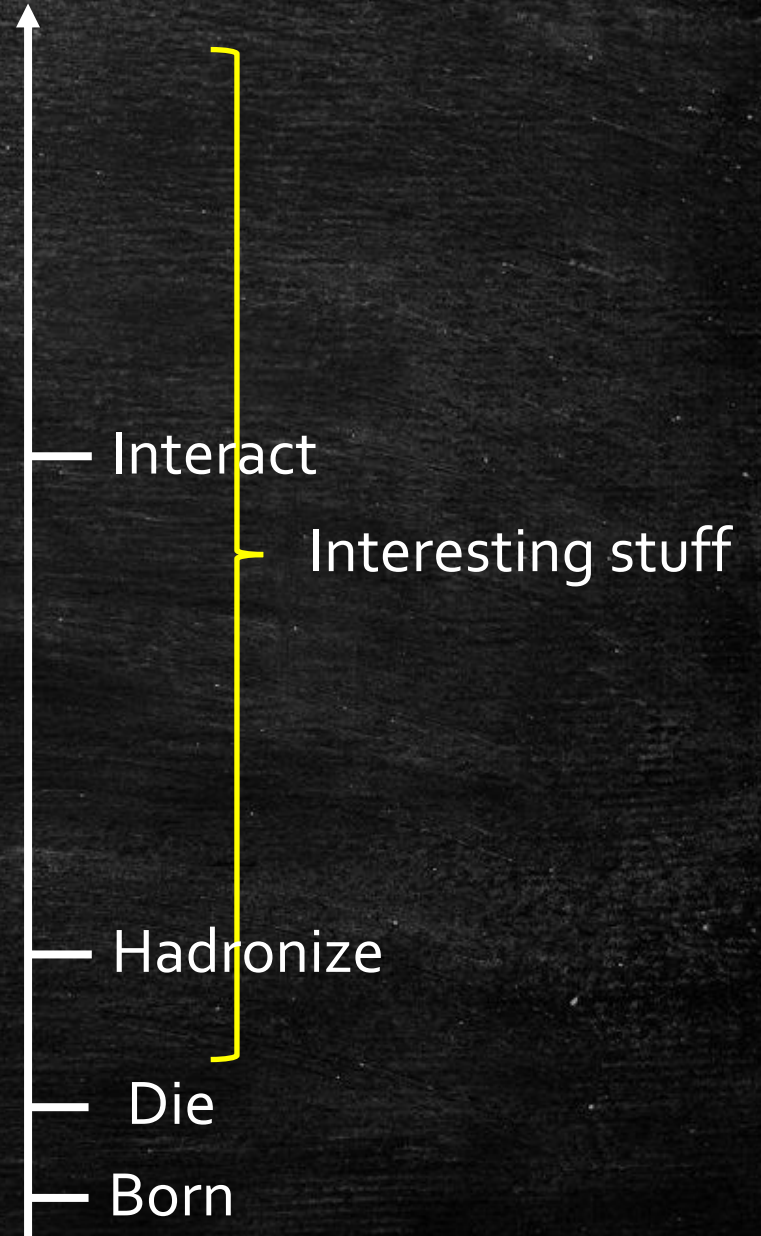


Hadronization problem

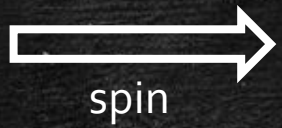


$$\Gamma(t \rightarrow bW^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + \frac{2m_W^2}{m_t^2}\right) \sim \frac{m_t^3}{m_W^2}$$

$$\tau_{\text{dec}} \sim 1 \text{ fm}/c \left(\frac{m_W}{m_t}\right)^3 < \tau_{\text{had}} \sim \frac{1}{\Lambda_{\text{QCD}}} \sim 1 \text{ fm}/c$$

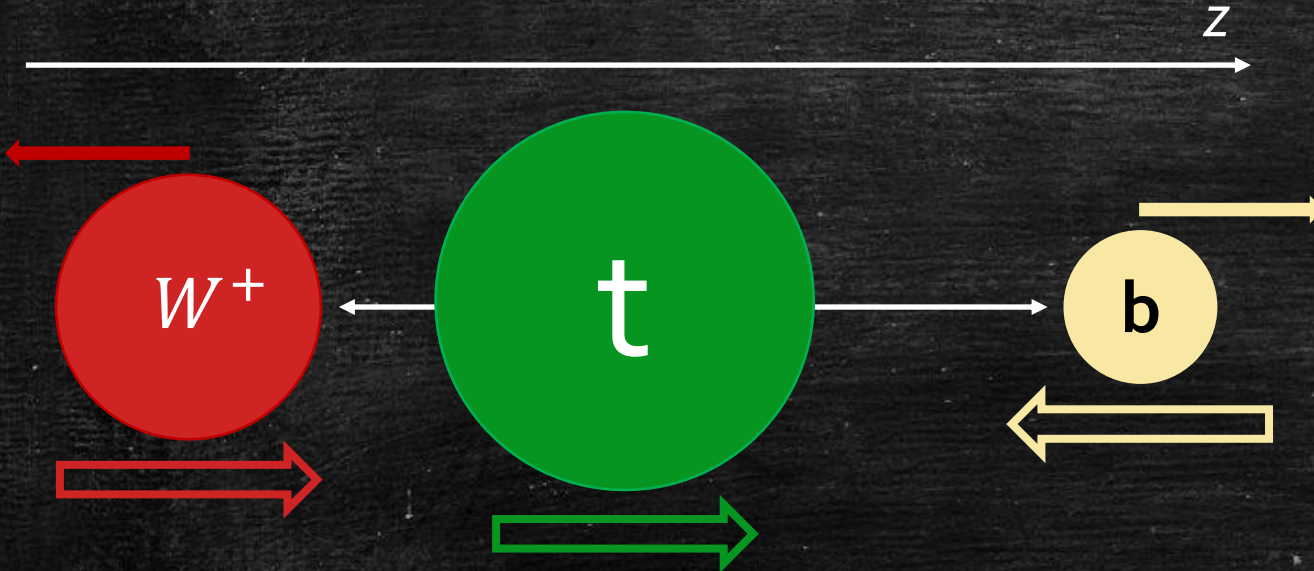


Top decay: spin heredity



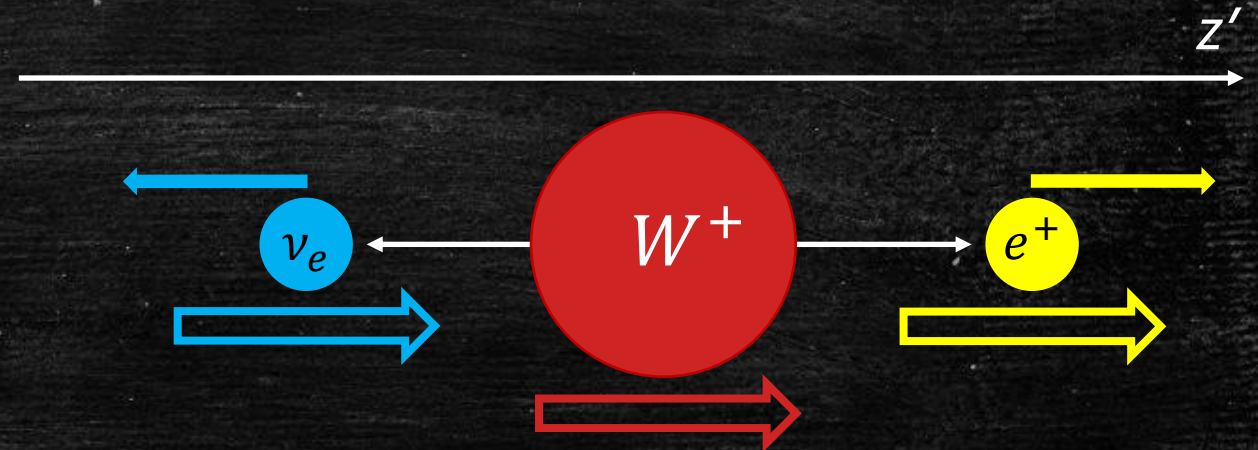
Chirality=Elicity

$$\tau_{\text{dec}} \sim 1 \text{ fm}/c \left(\frac{m_W}{m_t} \right)^3 < \tau_{\text{int}} \sim m_t$$

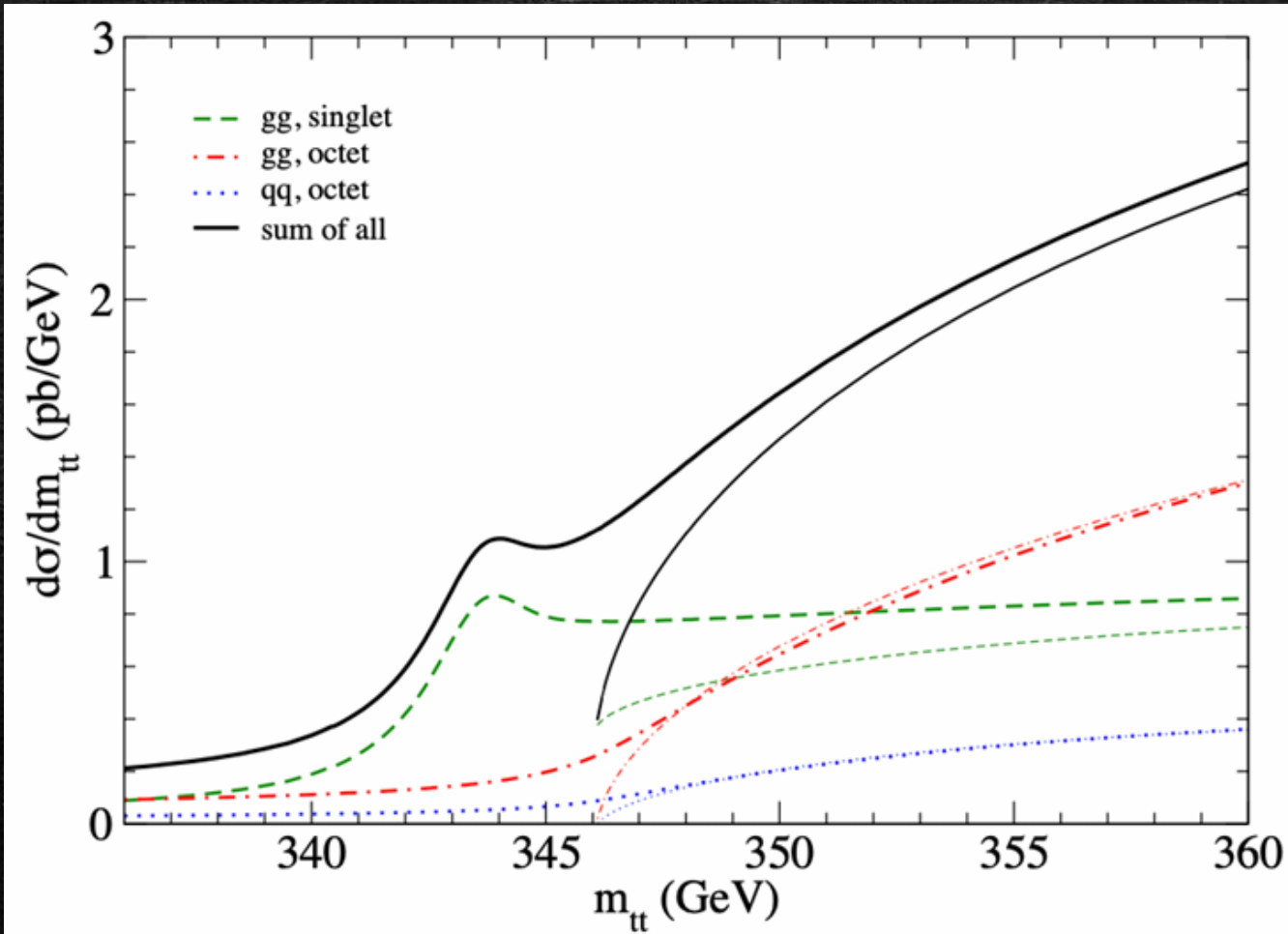


The antilepton direction coincides with the spin of the top

$$\frac{d\Gamma}{d\cos\theta_e} \sim 1 + \cos\theta_e$$

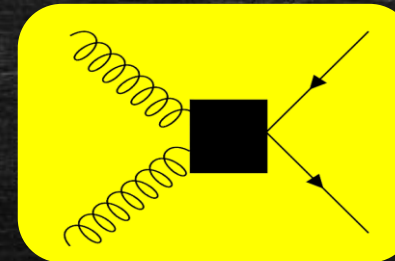


$t\bar{t}$ threshold production

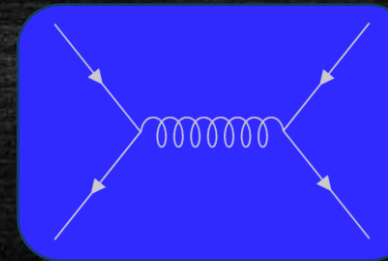


F. Maltoni, (Basic) Introduction to toponium physics, LHCTopWG Meeting — 11-13 Nov 2024 — CERN

Production of maximally entangled state



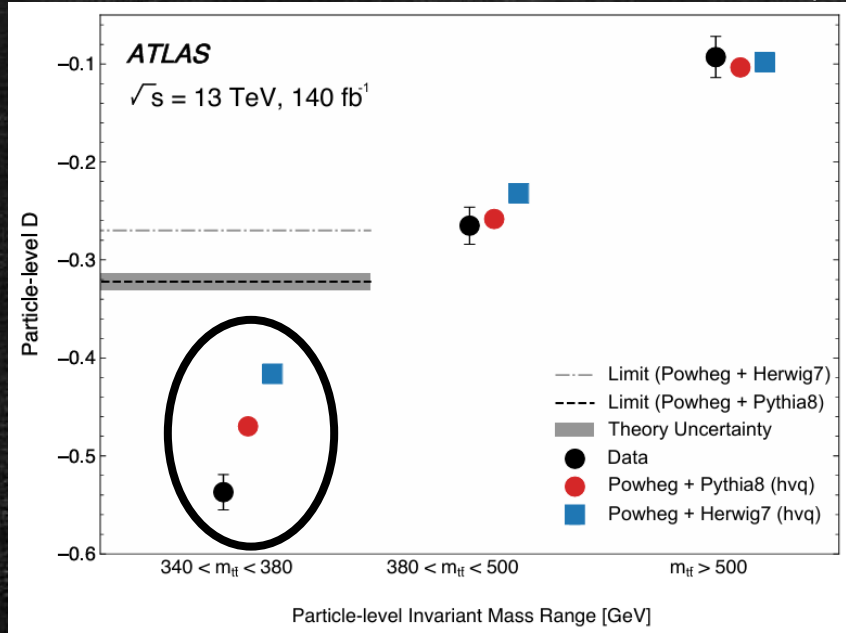
$q\bar{q}$ has $S_{q\bar{q}} = 0$
(Landau-Yang theorem)



subleading

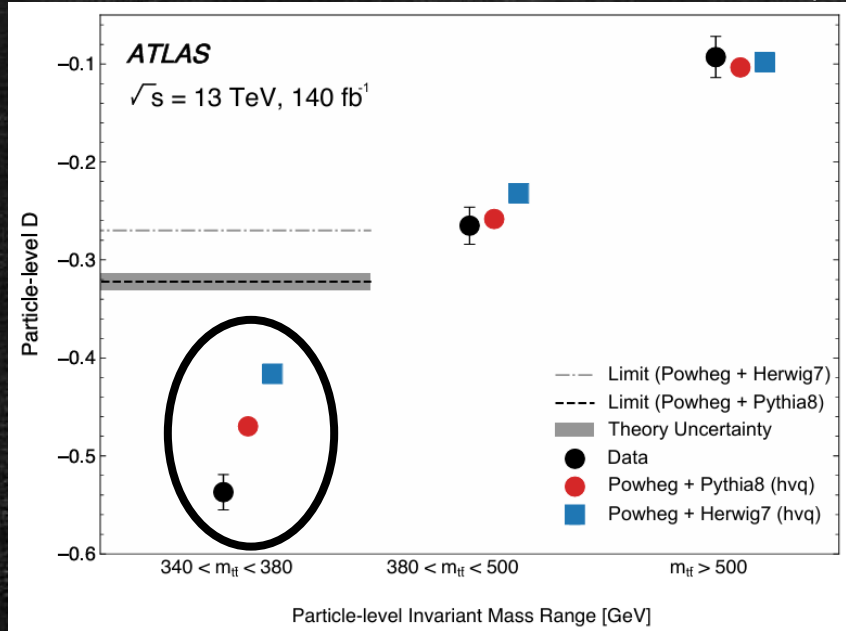
Entanglement and Invariant mass

Atlas, Nature 633 (2024) 8030, 542-547, arXiv:2311.07288 [hep-ex]



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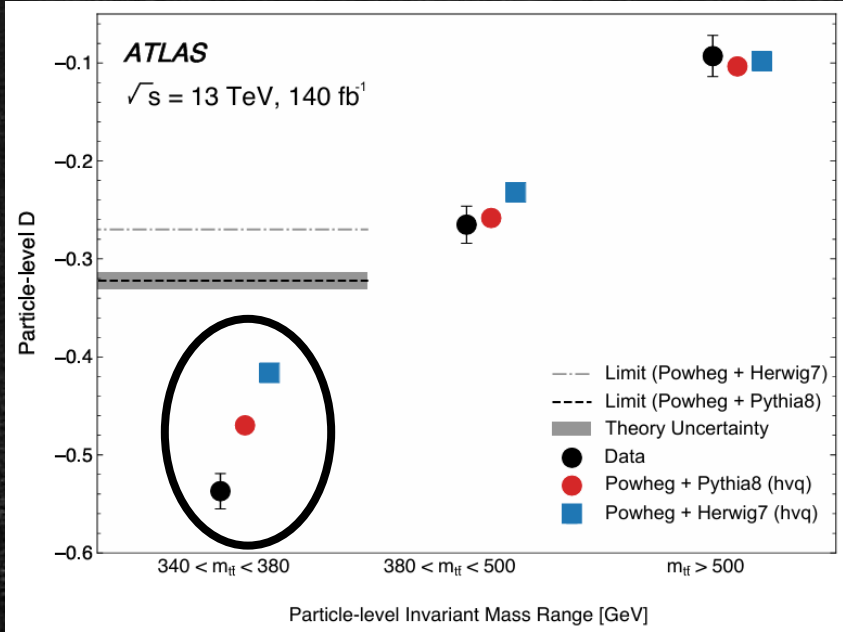
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The discrepancy could be due to the effect of a $\eta_{t\bar{t}}$

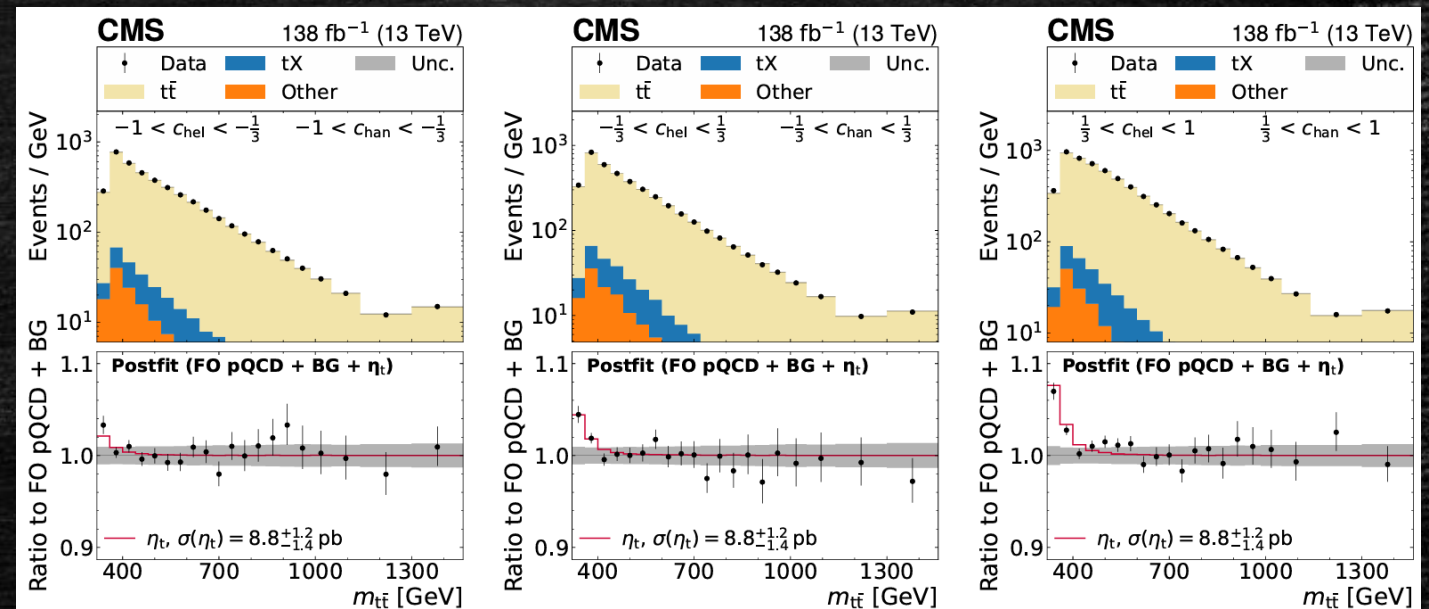
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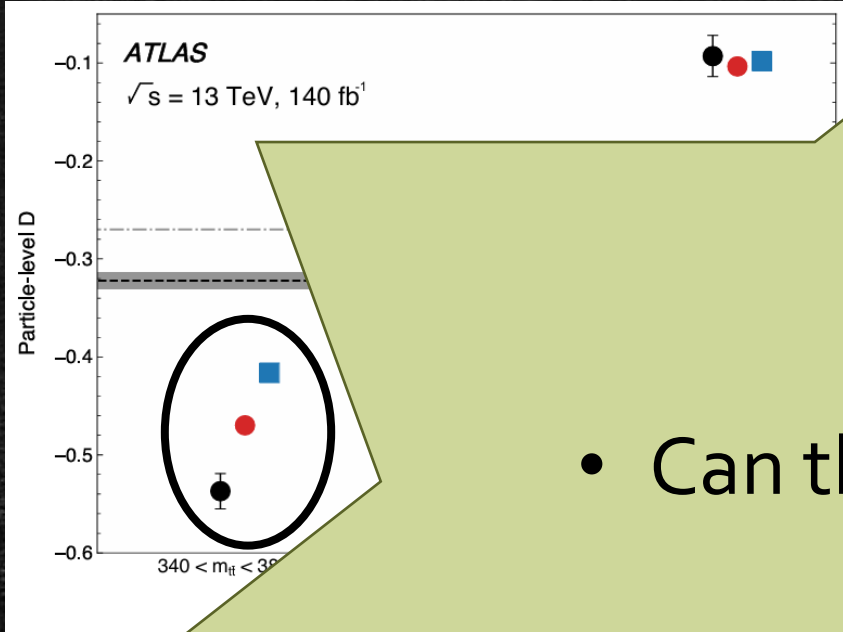
Near threshold excess



CMS, Rept.Prog.Phys. 88 (2025) 8, 087801, arXiv:2503.22382 [hep-ex]

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Atlas, Nature 633 (2024) 8030, 542-547, arXiv:2311.07288 [hep-ex]

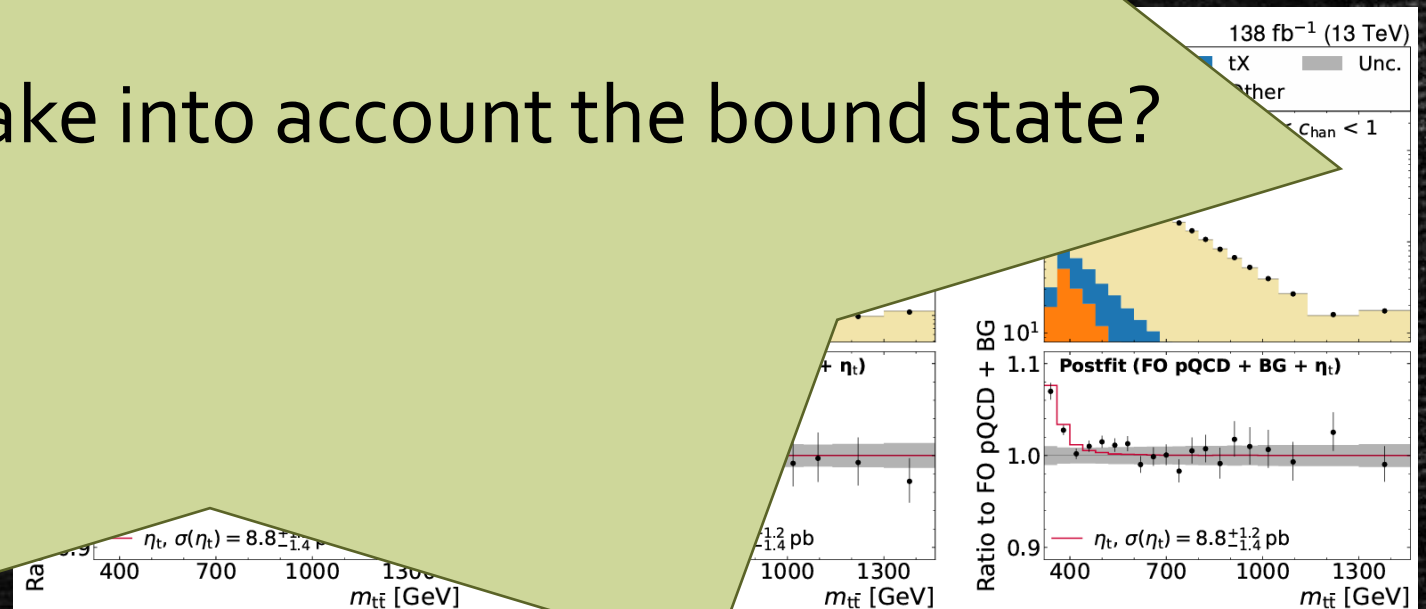


- Can the top quark hadronize?

- If so, how do we take into account the bound state?

Near three

ct of a $\eta_{t\bar{t}}$



CMS, Rept.Prog.Phys. 88 (2023) 8, 087801, arXiv:2503.22382 [hep-ex]

Toponium formation

Top-Antitop potential

$$V(r) = -C_F \frac{\alpha_S}{r} + \cancel{kr}$$

- $\alpha_S = 0.1 - 0.2$
- $k = 0.176 \text{ GeV}^2$



$$\tau_{\text{orb}} \sim \frac{r}{v} \sim \frac{1}{m_t \alpha_S^2}$$

$$E_1 = -m_e \frac{\alpha^2}{2}$$

$$p \sim m_e \alpha$$

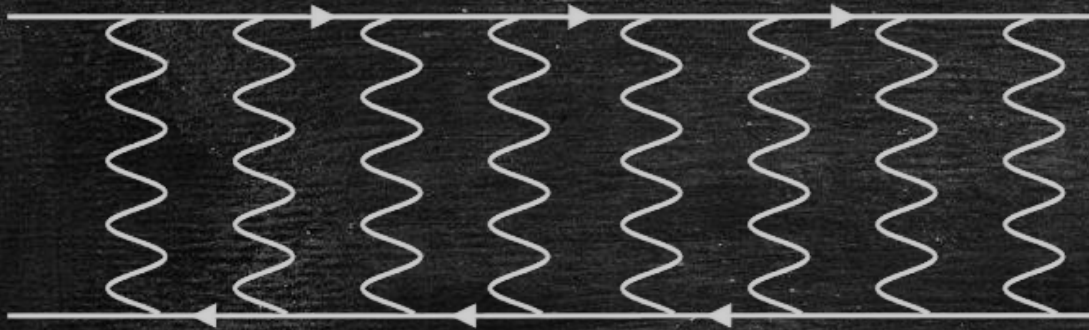
$$r \sim 1/m_e \alpha$$



$$\tau_{\text{dec}} \sim 1 \text{ fm}/c \left(\frac{m_W}{m_t} \right)^3 \sim \tau_{\text{orb}} \sim \frac{1}{m_t \alpha_S^2} \sim 1 \text{ fm}/c \frac{1}{10}$$



How to take into account a Bound state?



A bound state is a non perturbative phenomenon



Is it included in perturbative calculation?

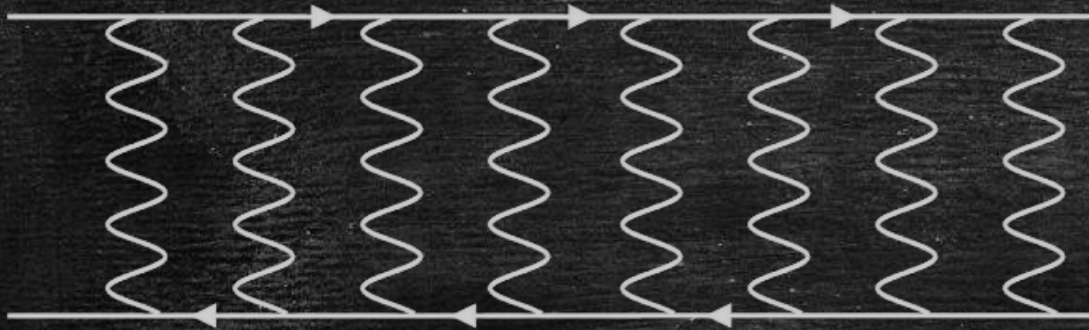
$$\sigma \sim \underbrace{|\psi(0)|^2}_{\alpha^5} \alpha^2 \delta(E - E_B)$$

This contribution exists only if $\alpha > 0$



$$\sigma \sim \alpha^5 + |\alpha|^5$$

How to take into account a Bound state?



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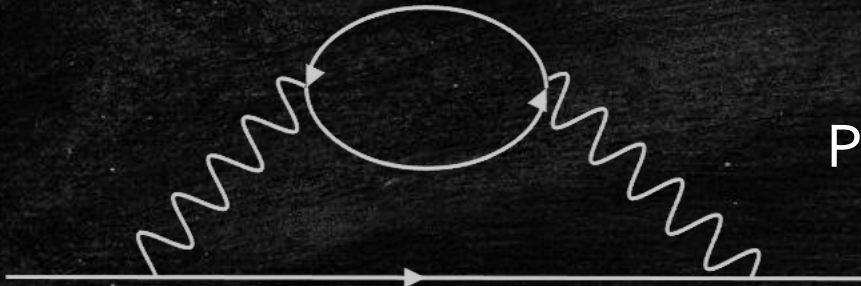
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Positronium contribution to g-2

K. Melnikov et al., Phys.Rev.D 90 (2014) 1, 017301; M. I. Eides, Phys.Rev.D 90 (2014) 5, 057301; M. A. Braun, Zh. Eksp. Teor. Fiz. 54, 1220 (1967) [Sov. Phys. JETP 27, 652 (1968)]

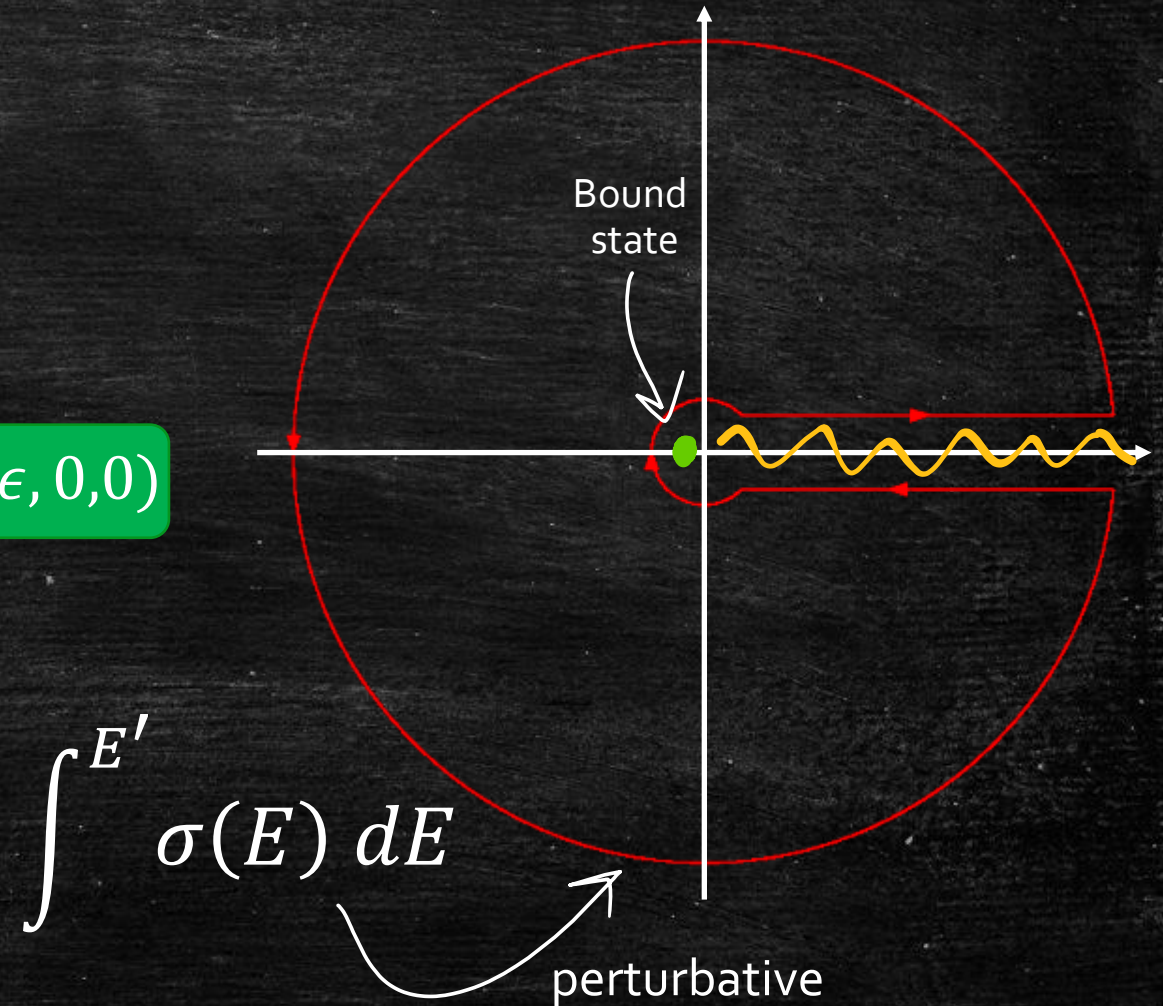
Resolvent and Optical Theorem

$$\sigma(E) \sim \rho(E) \sim \text{Im}[R(E + i\epsilon, 0, 0)]$$

$$R(E, 0, 0) = \sum_n \frac{|\psi_n(0)|^2}{E - E_n}$$

$$\text{Im}[R(E + i\epsilon, 0, 0)] \sim R(E + i\epsilon, 0, 0) - R(E - i\epsilon, 0, 0)$$

The information of the bound state must be present in the perturbative expansion



Delta potential

$$\left(-\frac{1}{2m} \frac{d^2}{dx^2} - \lambda \delta(x) \right) \psi(x) = E \psi(x) \quad \longrightarrow \quad E_B = -\frac{m\lambda^2}{2} = -\frac{m^2 \lambda^2}{2m}$$

Perturbation Theory

$$R(E, 0, 0) = \frac{1}{H_0 - E} - \frac{1}{H_0 - E} V \frac{1}{H_0 - E}$$

$$\text{Im}[R(E + i\epsilon, 0, 0)] = \theta(E) \sqrt{\frac{m}{2E}} + \pi \frac{\lambda m}{2} \delta(E)$$



$$\rho(E) = \theta(E) \frac{m}{\pi k} + \frac{\lambda m}{2} \delta(E)$$

Exact Result

$$R(E, 0, 0) = \theta(\lambda) \frac{m\lambda}{E - E_B} + \frac{1}{\pi} \int_0^\infty dk \frac{k^2}{k^2 + m^2 \lambda^2} \frac{1}{E - E_k}$$

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$$\rho(E) = \theta(E) \frac{m}{\pi k} + \theta(\lambda) \lambda m \delta(E) - \frac{\lambda^2 m}{\pi k} \left(\frac{m}{k} \right)^2$$

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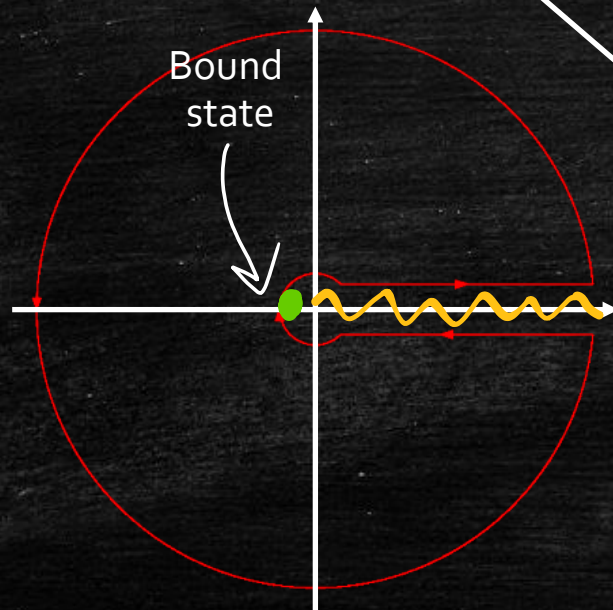
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Perturbation Theory

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Exact Result

$$\begin{aligned} \rho(E) &= \\ &= \theta(E) \frac{m}{\pi k} \frac{k^2}{k^2 + m^2 \lambda^2} + \theta(\lambda) \lambda m \delta(E - E_B) \end{aligned}$$

$$\int_{-\infty}^{E'} dE \rho(E) = \frac{1}{\pi} k' \left(-\frac{\lambda m}{\pi} \arctan\left(\frac{k'}{m\lambda}\right) + \lambda m \theta(\lambda) \right)$$

$$\lambda m \left[\theta(\lambda) - \frac{1}{2} \frac{|\lambda|}{\lambda} \right]$$

Analytic

$$= \frac{1}{\pi} k' + \frac{\lambda m}{2}$$

Toponium: conclusions

P. Nason, E. Re and L. Rottoli, JHEP 10 (2025) 149, arXiv:2505.00096 [hep-ph]

Top-Antitop potential

$$V(r) = -C_F \frac{\alpha_S}{r}$$

$$\begin{aligned}\rho_l(\mathcal{E}) &= \frac{1}{2\pi r_l^3} \zeta(3) \delta(\mathcal{E}) + \frac{m^{3/2}}{4\pi^2} \sqrt{\mathcal{E}} \left(1 + \frac{b_l}{2v} + \frac{b_l^2}{12v^2} \right) + \mathcal{O}(\alpha_s^4) \\ &= \frac{m^2}{4\pi^2} \left(v + \frac{b_l}{2} + \frac{b_l^2}{12v} + \frac{\zeta(3)}{4\pi^2} b_l^3 m \delta(\mathcal{E}) + \mathcal{O}(\alpha_s^4) \right).\end{aligned}$$

$$b_l \propto \alpha_S$$

Toponium: conclusions

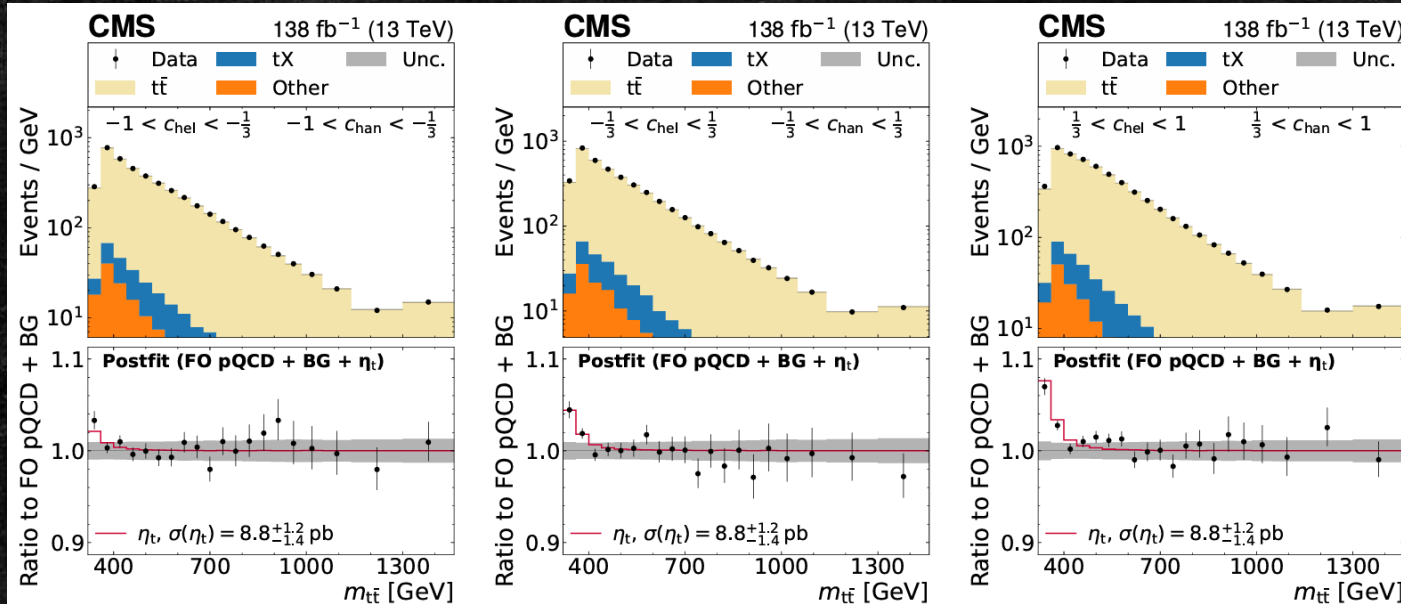
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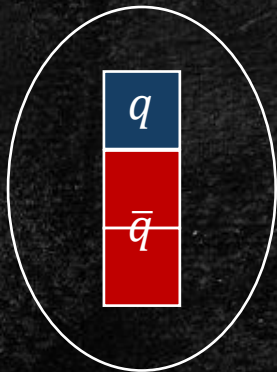
Bound states contribution doesn't have to be added as a nonperturbative effect. It arises in perturbation theory.

Exotic Hadrons

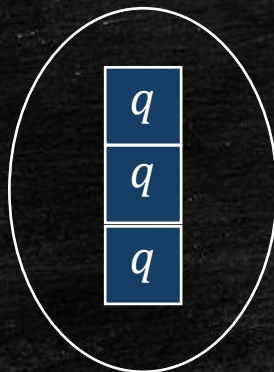
Exotic hadrons are subatomic particles composed of quarks and gluons, but which consist of more than three valence quarks or have an explicit valence gluon content.

R. Jaffe, Phys. Rev. D 15, 267 (1977) DOI: 10.1103/PhysRevD.15.267; H. Fritzsch and M. Gell-Mann, Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia (1972).

- Pieces: **quarks**, **anti-quarks** and **gluons** 
- **$SU(3)_C$ rule:** every combination is allowed as long as it respects **confinement i.e. boxes forms columns of three elements**



Meson



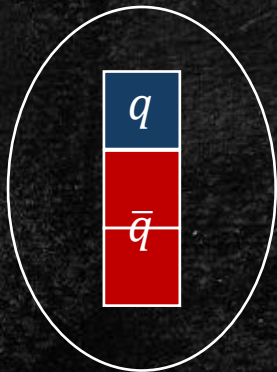
Baryon

Exotic Hadrons

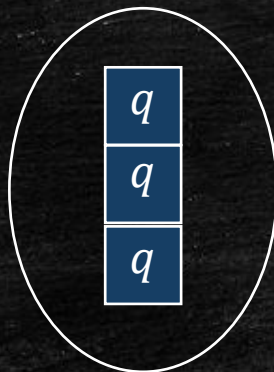
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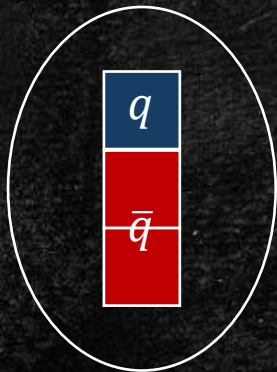
Glueball

Exotic Hadrons

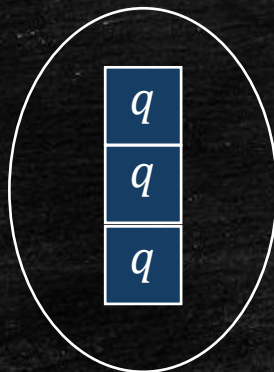
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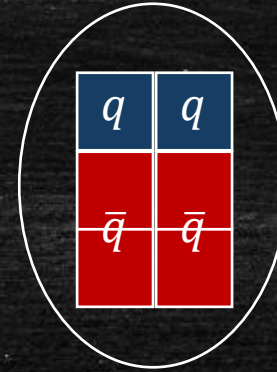
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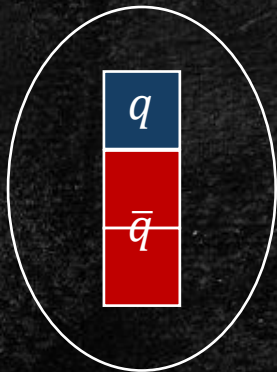
Tetraquark

Exotic Hadrons

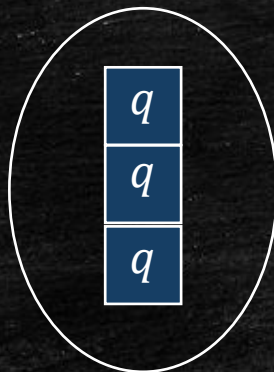
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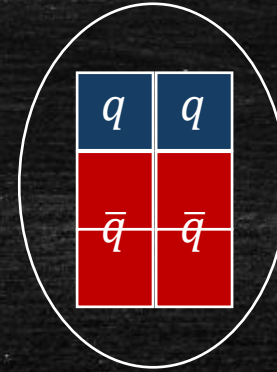
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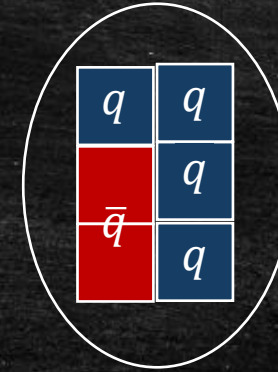
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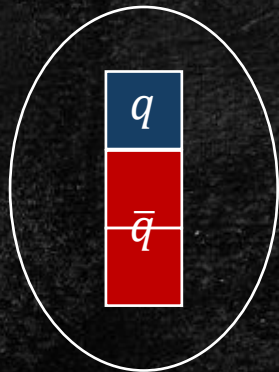
Pentaquark

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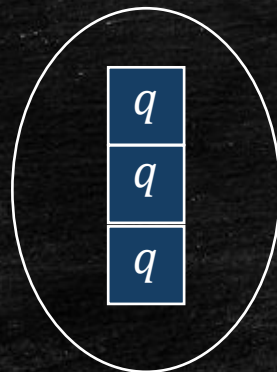
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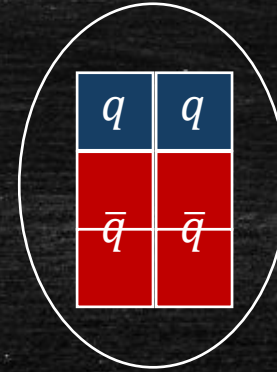
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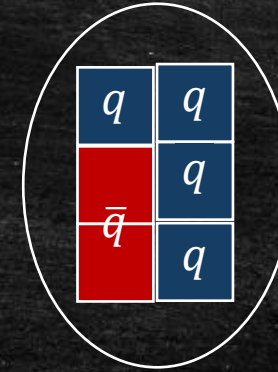
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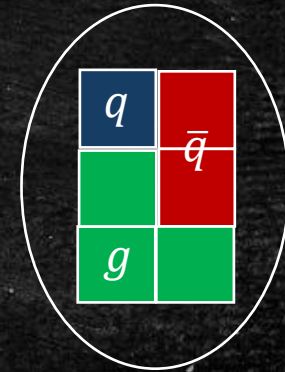
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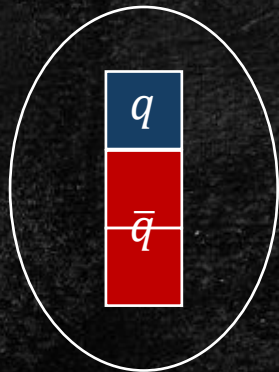
Hybrid

Exotic Hadrons

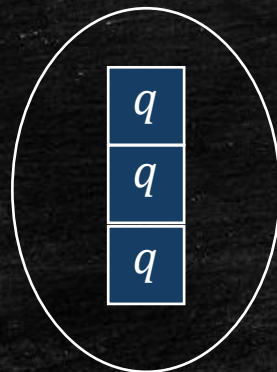
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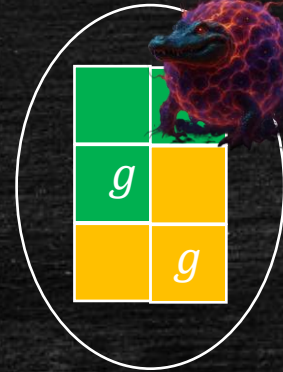
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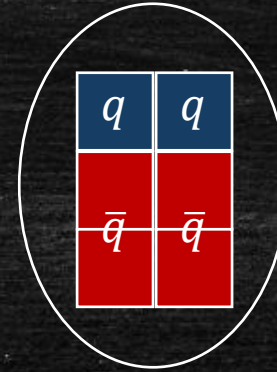
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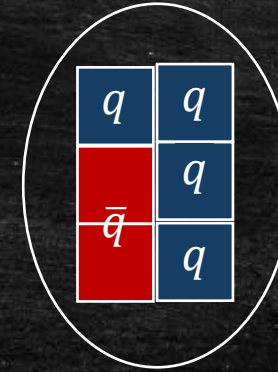
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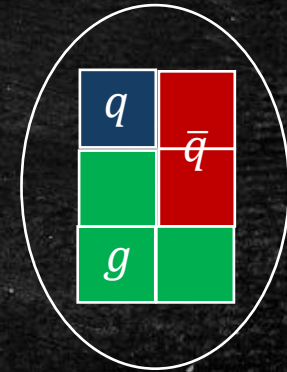
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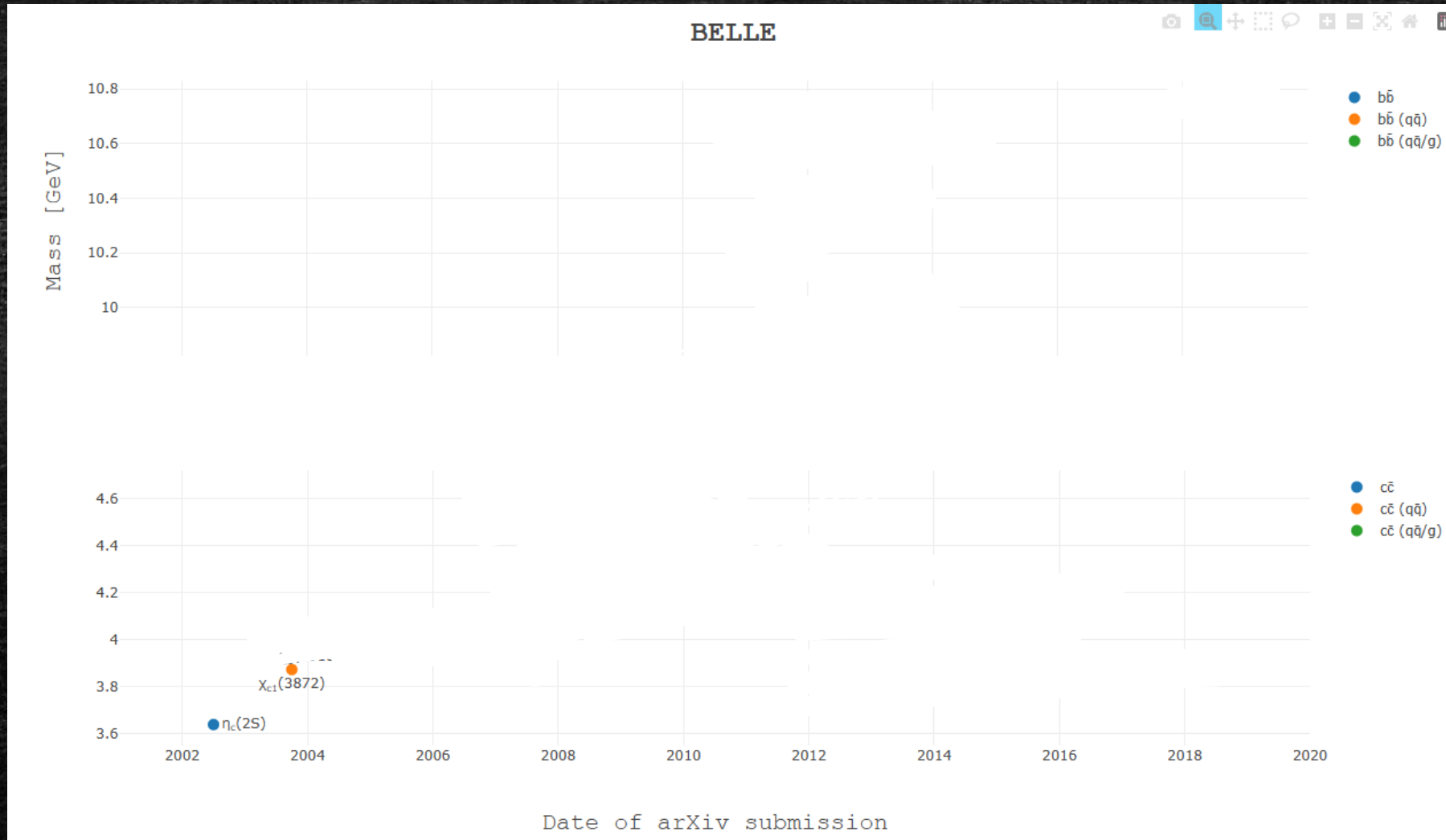


Hybrid

EXOTIC HADRONS

Exotic Hadrons

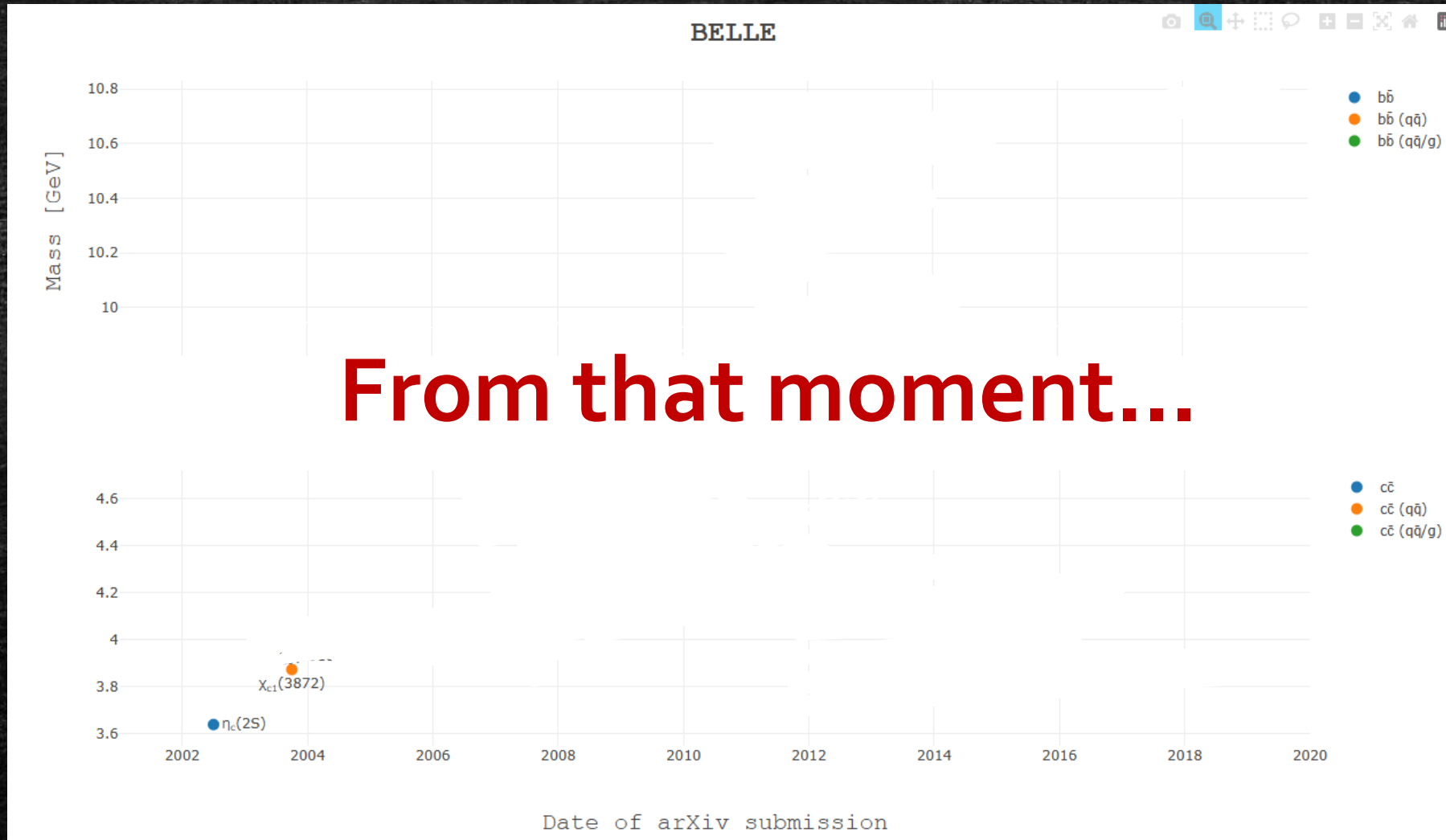
Although the existence of exotic states had already been theorized in the 1970s, the first exotic particle was discovered in 2003: the $X(3872)$.



QWG Exotics hub: <https://qwg.ph.nat.tum.de/exoticshub/>

Exotic Hadrons

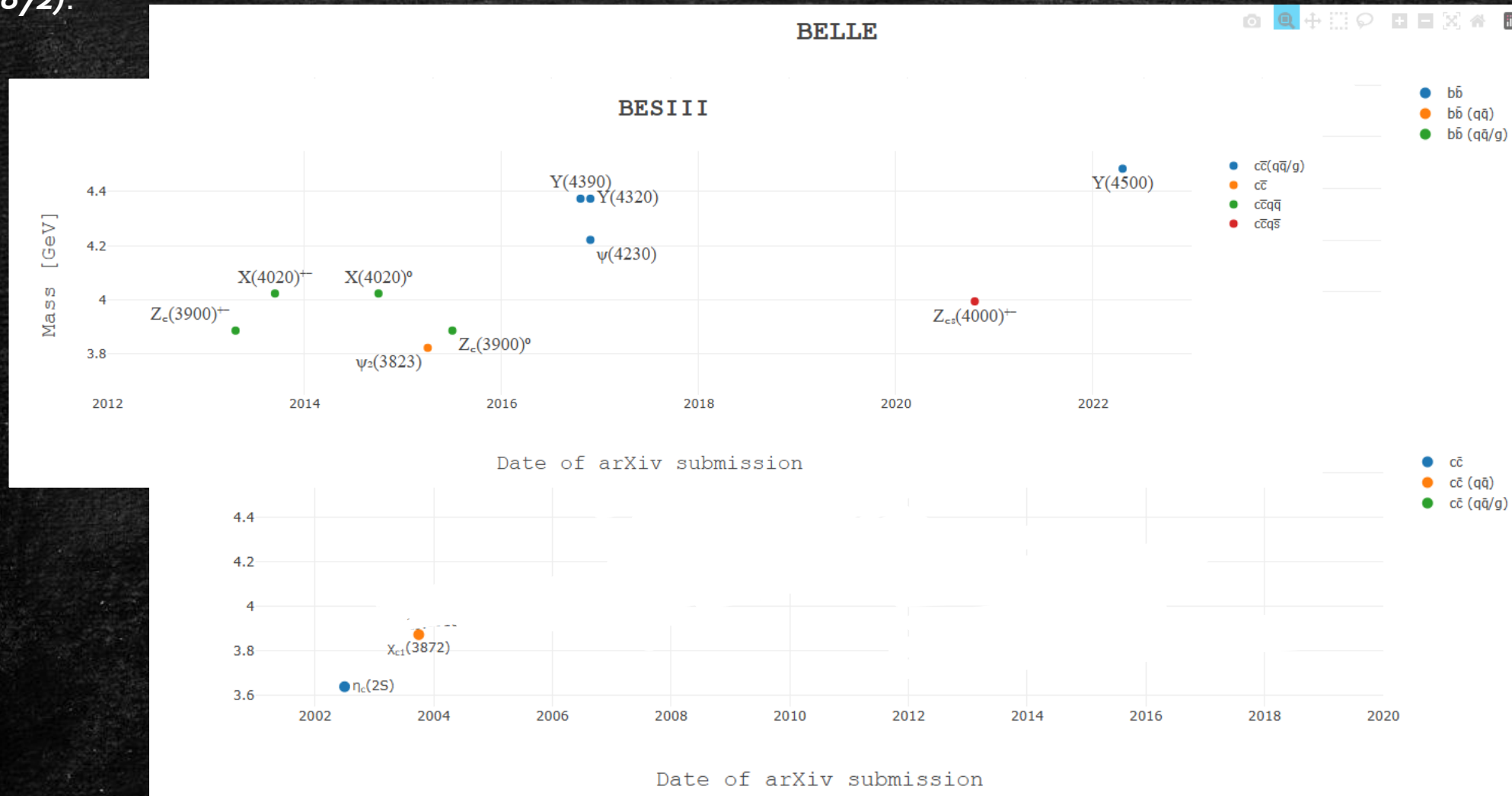
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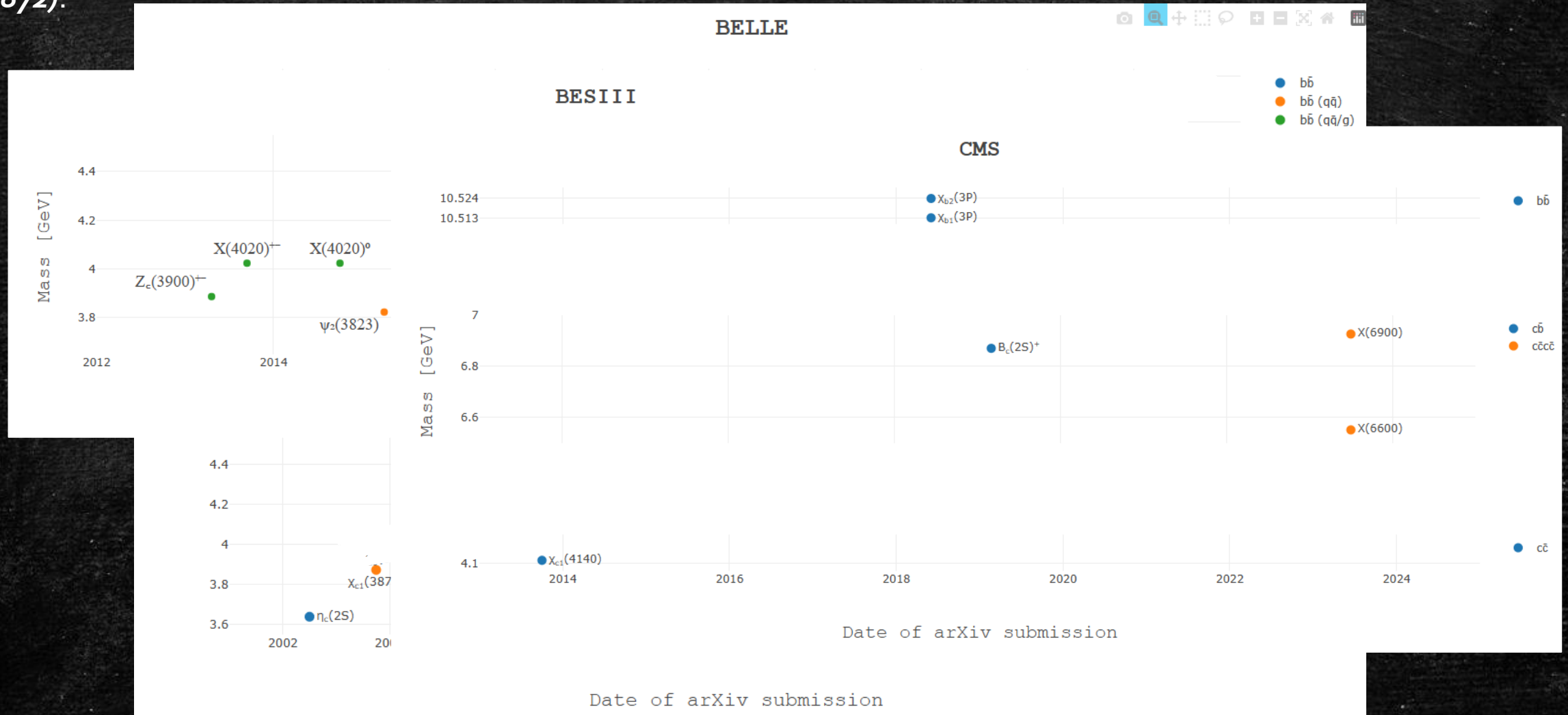
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OWG Exotics hub: <https://qwg.ph.nat.tum.de/exoticshub/>

Exotic Hadrons

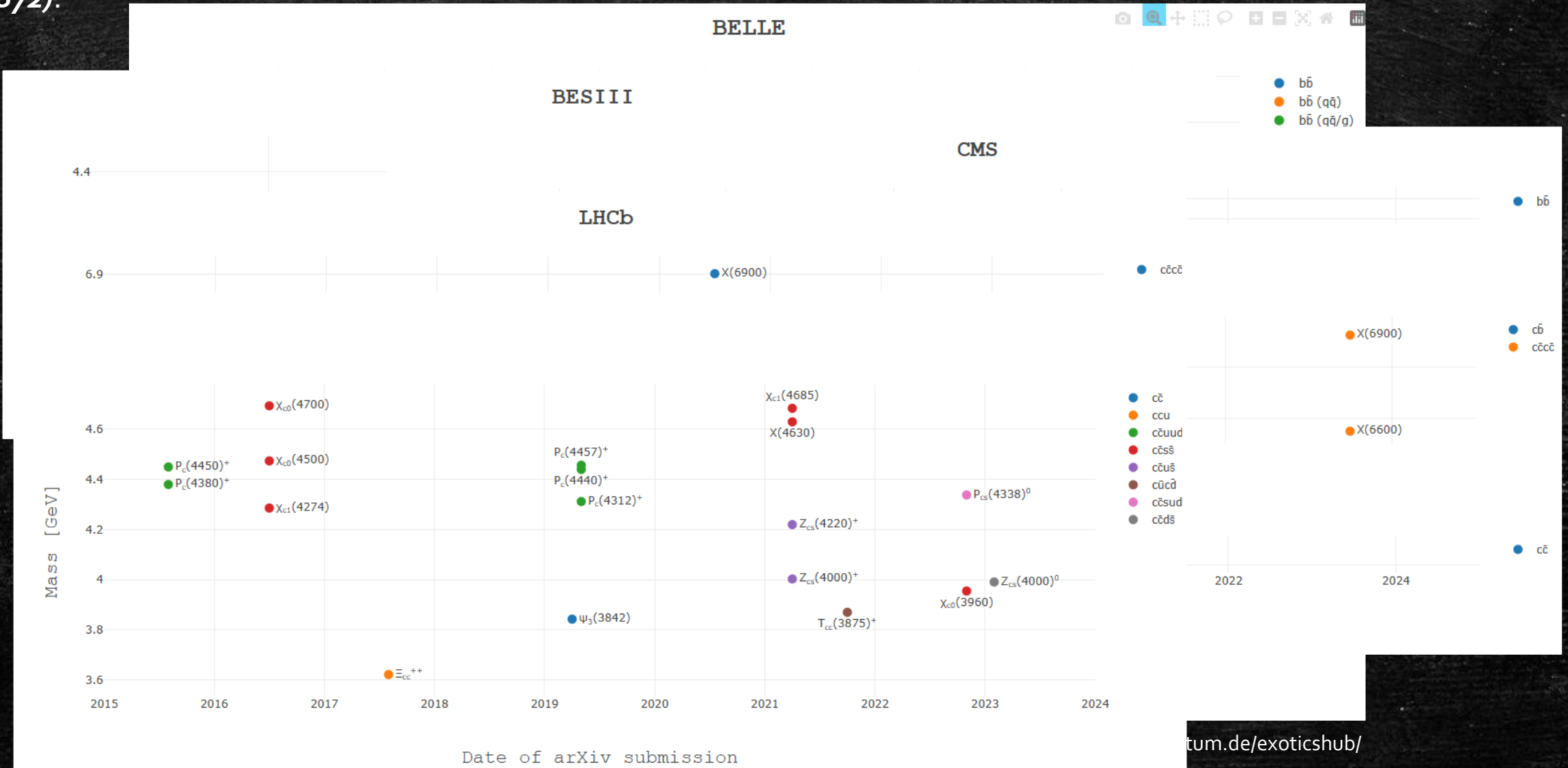
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[tum.de/exoticshub/](https://exoticshub.tum.de/exoticshub/)

Exotic Hadrons

- 2025: first Glueball!

Discovery of a Glueball-like particle $X(2370)$ at BESIII

Yanping Huang^a

Shan Jin^b

Peng Zhang^a

^a*Institution of high energy physics, Beijing 100049, People's Republic of China*

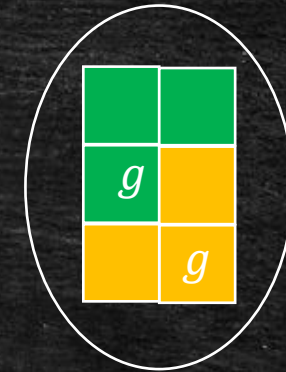
huangyp@ihep.ac.cn

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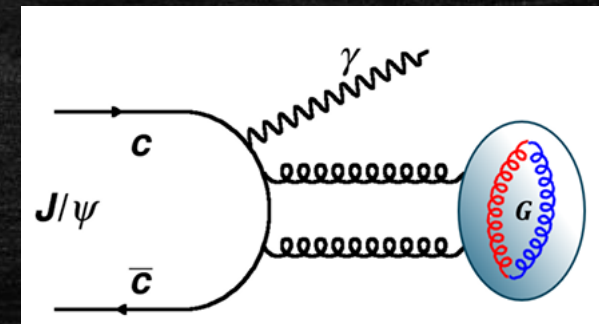
^b*Nanjing University, Nanjing 210093, People's Republic of China*

jins@ihep.ac.cn

Radiative decays of the J/ψ particle are of gluon-rich environment, providing an ideal place for hunting glueballs. The $X(2370)$ particle was first discovered in $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$ process in 2011 with the BESIII experiment at BEPCII Collider, and later it was confirmed in $J/\psi \rightarrow \gamma K \bar{K} \eta'$ decays. In 2024, with a sample of 10 billion J/ψ events collected at the BESIII detector, the spin-parity of the $X(2370)$ was determined to be 0^{-+} for the first time in the partial wave analysis of $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$ process. Recently, new decay modes of $X(2370) \rightarrow K_S^0 K_S^0 \pi^0$, $\pi^0 \pi^0 \eta$ and $a^0 \pi^0$ were observed. The mass, spin-parity quantum numbers, production and decay properties of the $X(2370)$ particle are consistent with the features of the lightest pseudoscalar glueball.



Glueball



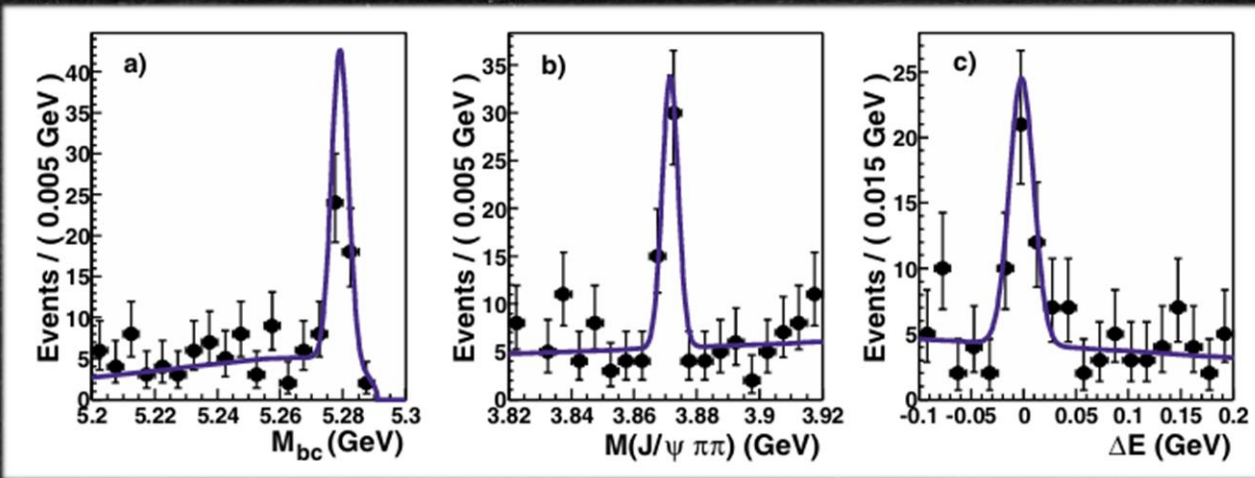
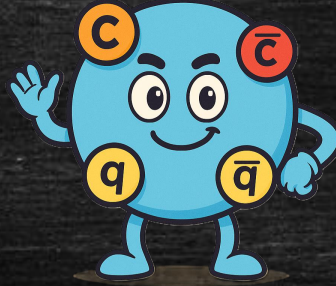
$X(3872)$: Introduction

Quantum numbers: $[c\bar{c}q\bar{q}] \ I(J^{PC}) = 0(1^{++})$

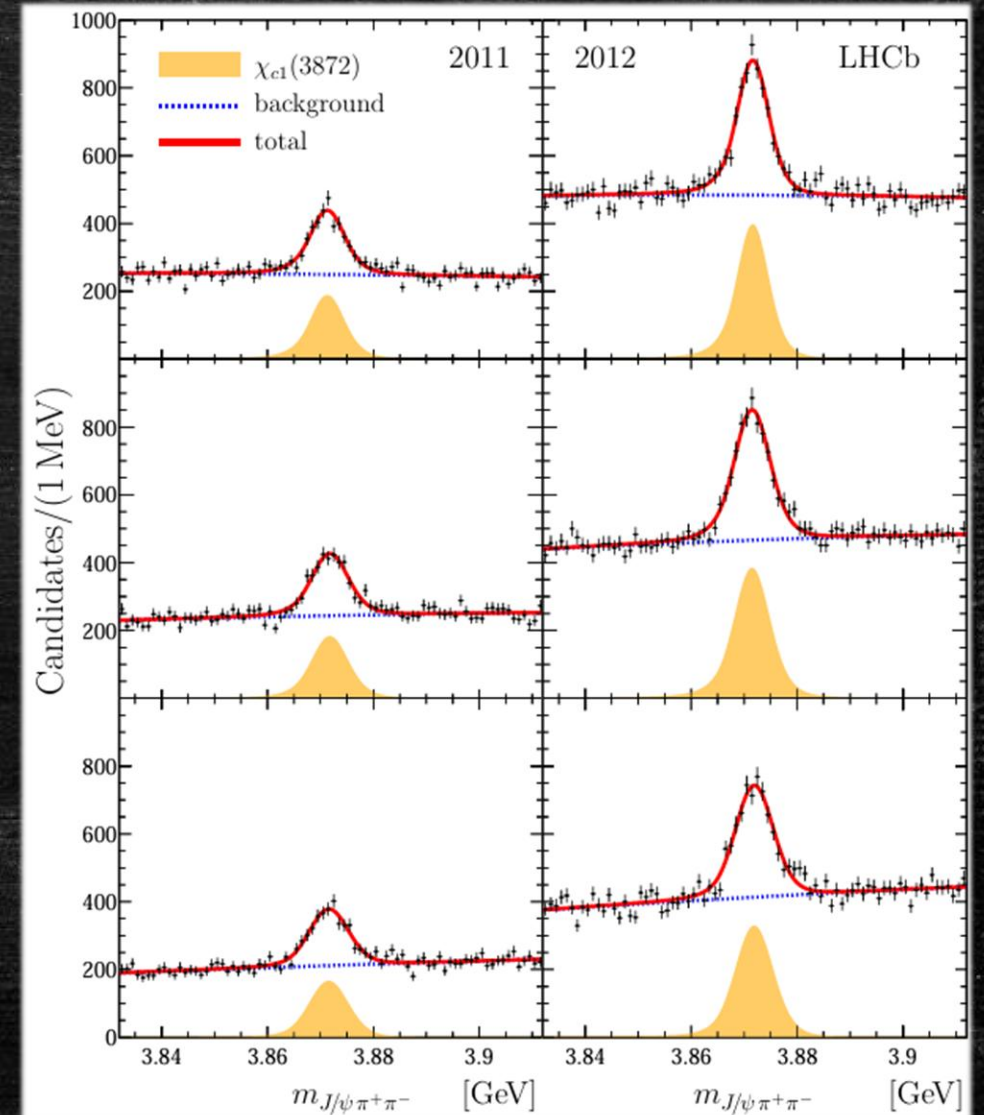
Mass: $3871.64 \pm 0.06 \text{ MeV}$ ($m_{th} = 3871.69$)

Main decay channels:

- $X(3872) \rightarrow D^0 \bar{D}^{*0} \rightarrow D^0 \bar{D}^0 \pi^0$
- $X(3872) \rightarrow J/\psi \pi^+ \pi^-$



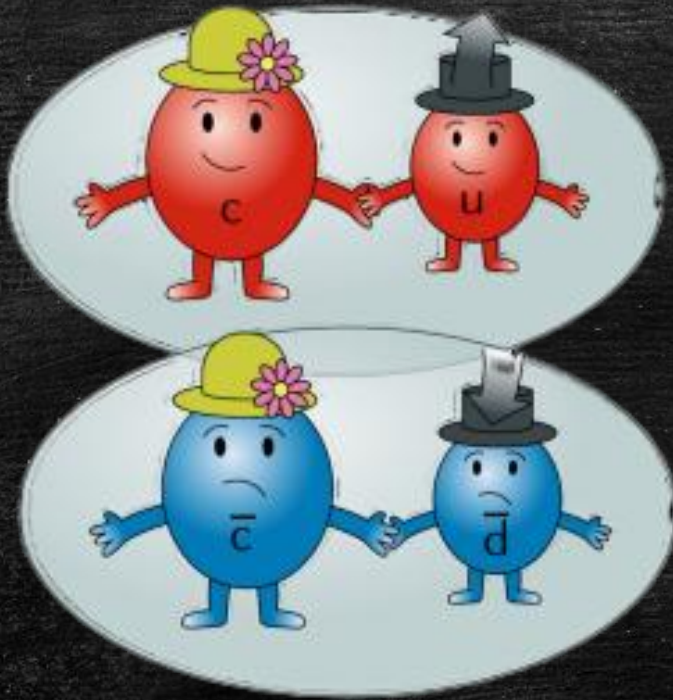
Belle collaboration, Phys.Rev.Lett.91:262001,2003 [arXiv:hep-ex/0309032]



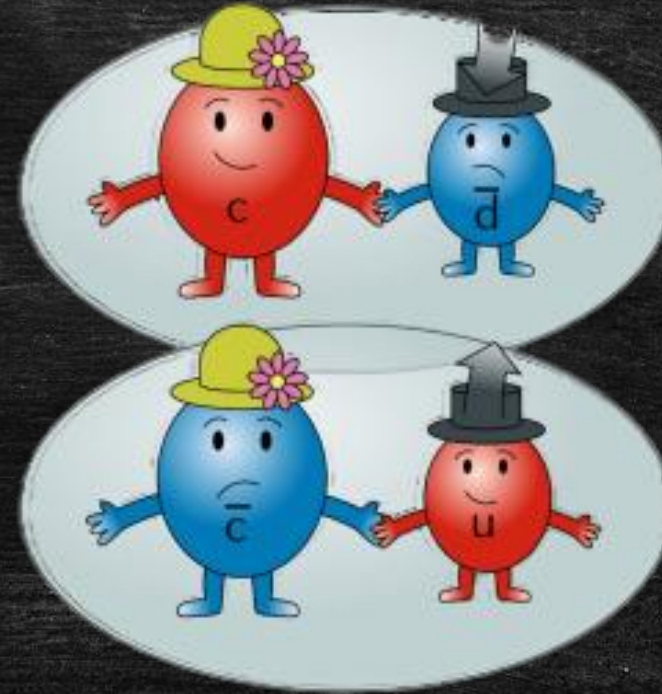
LHCb collaboration, Phys. Rev. D 102, 092005 (2020) [arXiv:2005.13419 [hep-ex]]

Compact objects vs Molecules

The $X(3872)$'s closeness to the $D^0\bar{D}^{*0}$ threshold makes it challenging to clearly determine whether the state is molecular or compact.



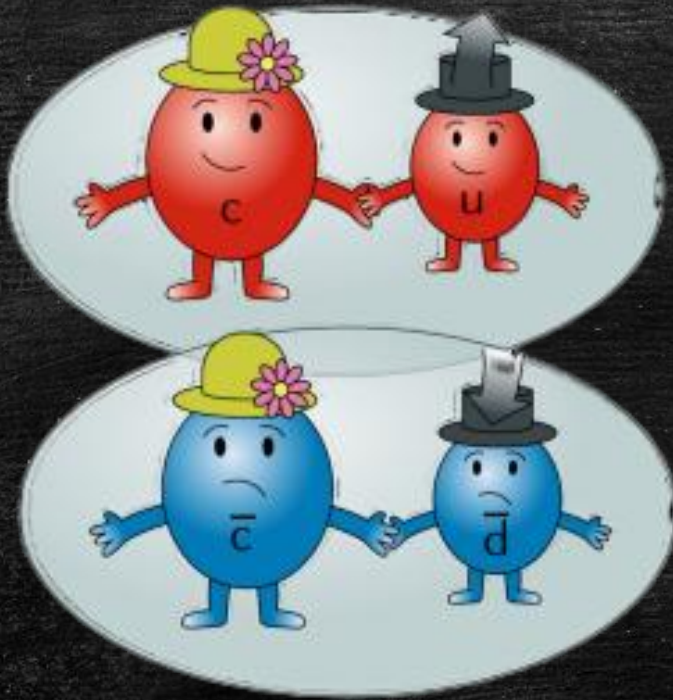
Tetraquark



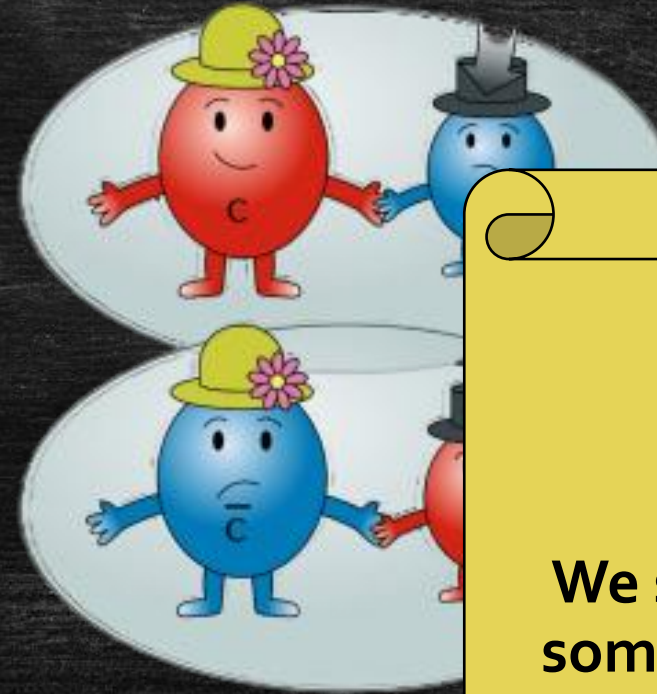
Molecule

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Tetraquark



Molecule



IDEA

We start with
something we
know...
the DEUTERON

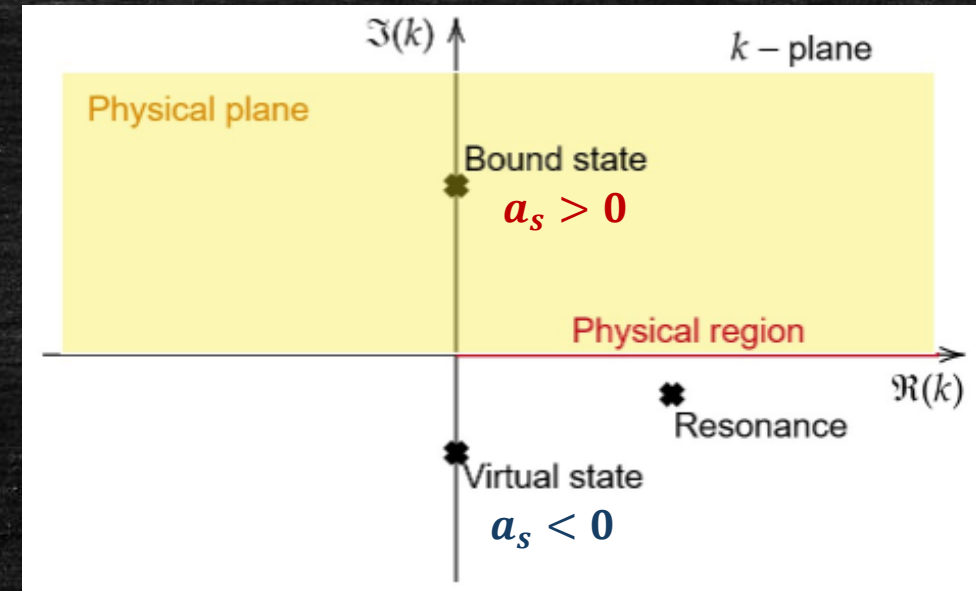
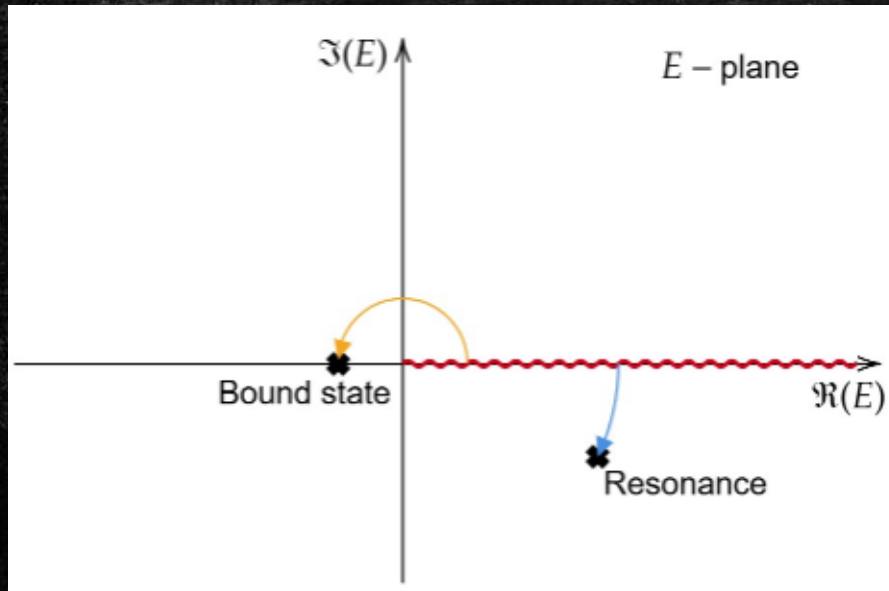
Non-relativistic EFT: Nature of Poles

At low energies, the scattering amplitude depends on two parameters.

Limit $k \rightarrow 0$: Effective Range Expansion

$$f(\theta) \simeq f_0 = \frac{1}{k \cot \delta_0 - ik}$$

$$k \cot \delta_0 = -\frac{1}{a_s} + \frac{1}{2}r_0k^2 + \mathcal{O}(k^4) \quad (\text{ERE})$$



Non-relativistic EFT: Nature of Poles

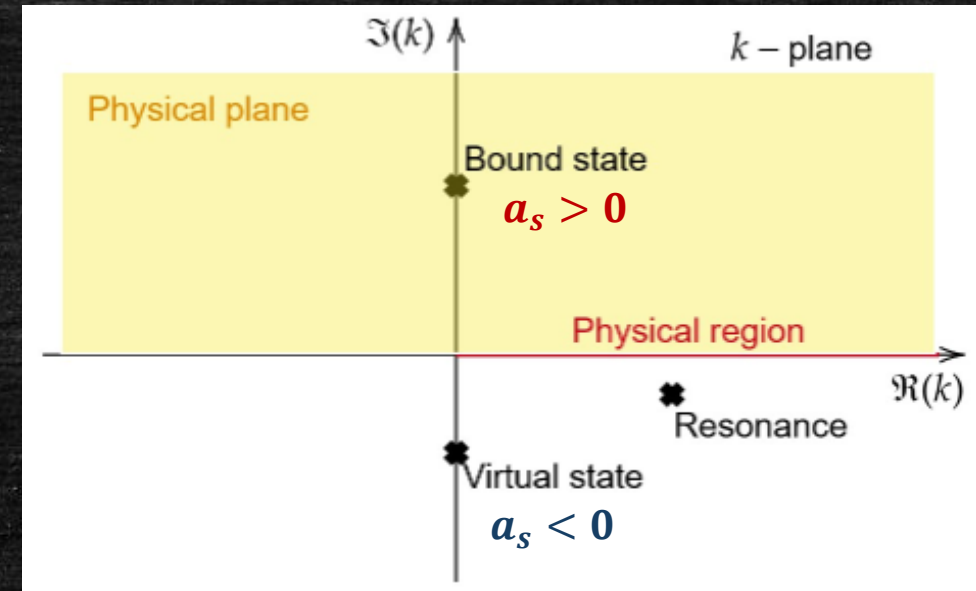
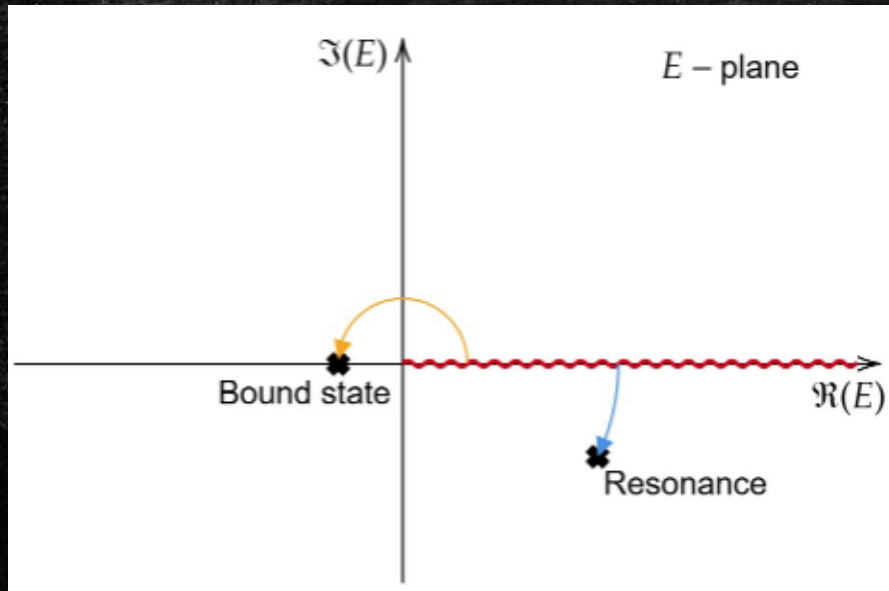
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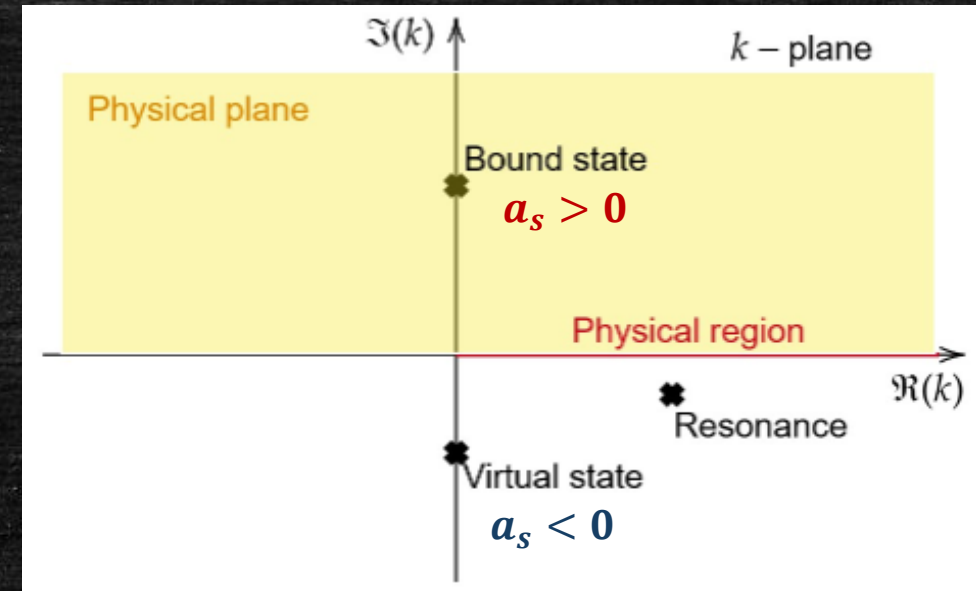
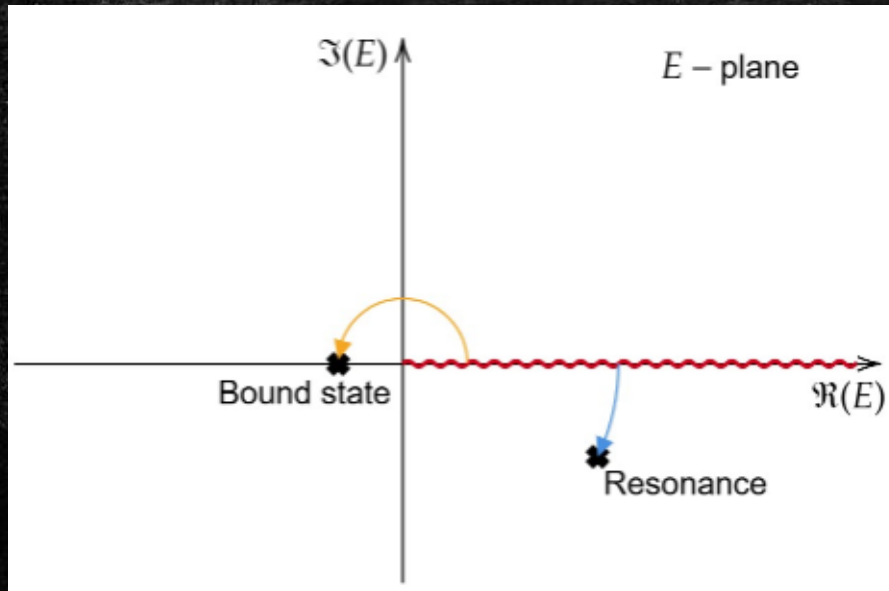
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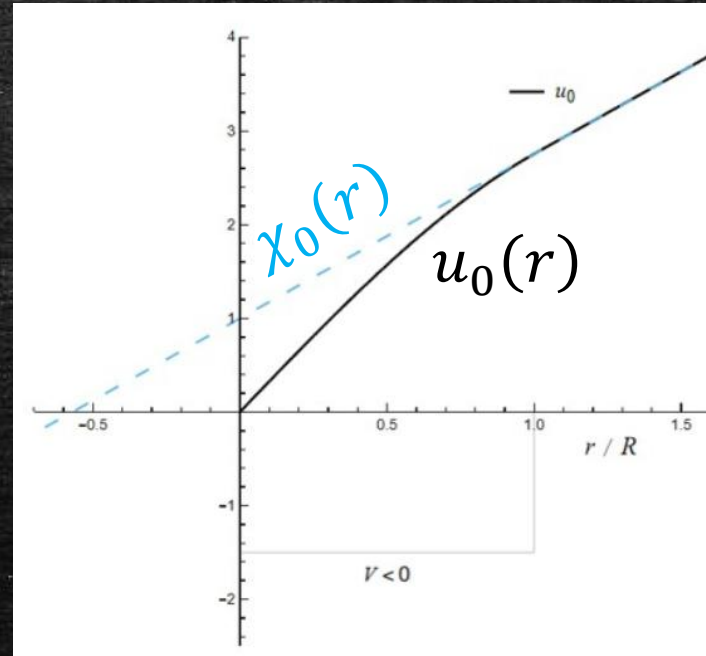
Effective range



Non-relativistic EFT: Effective range

The sign of r_0 is related to whether the particle is a molecule or a compact state (Weinberg 1965).

$$r_0 = 2 \int_0^R \chi_0^2(r) - u_0^2(r) dr$$



A compact state produce a negative r_0

Weinberg Phys. Rev. 137, B672 (1965)

In the presence of a shallow bound state and if the potential is purely attractive (or **has a repulsive core**), then $r_0 > 0$

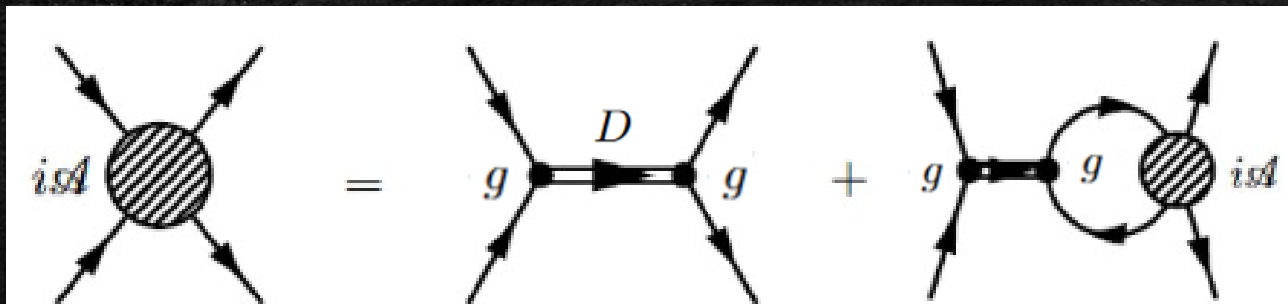
D.G. [arXiv:2605.18207 [hep-ph]]

Proton-Neutron: the dibarion field

$$\mathcal{L} = N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N + \boxed{\sigma} D^\dagger \left(i\partial_t + \frac{\nabla^2}{4M} - \mu \right) D - \boxed{g D^\dagger (NN)} + h.c.$$

D. B. Kaplan, Nucl. Phys. B 494(1997), 471–484, nucl-th/9610052.

The dibarion's kinetic term is defined up to a sign $\sigma = \pm 1$



$$f = -\frac{M}{4\pi} \frac{\sigma g^2}{E - \mu + i \sigma g^2 \frac{M}{4\pi} k}$$

D. B. Kaplan, Nucl. Phys. B 494(1997), 471–484, nucl-th/9610052.

$$f \propto \left(-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik \right)^{-1}$$

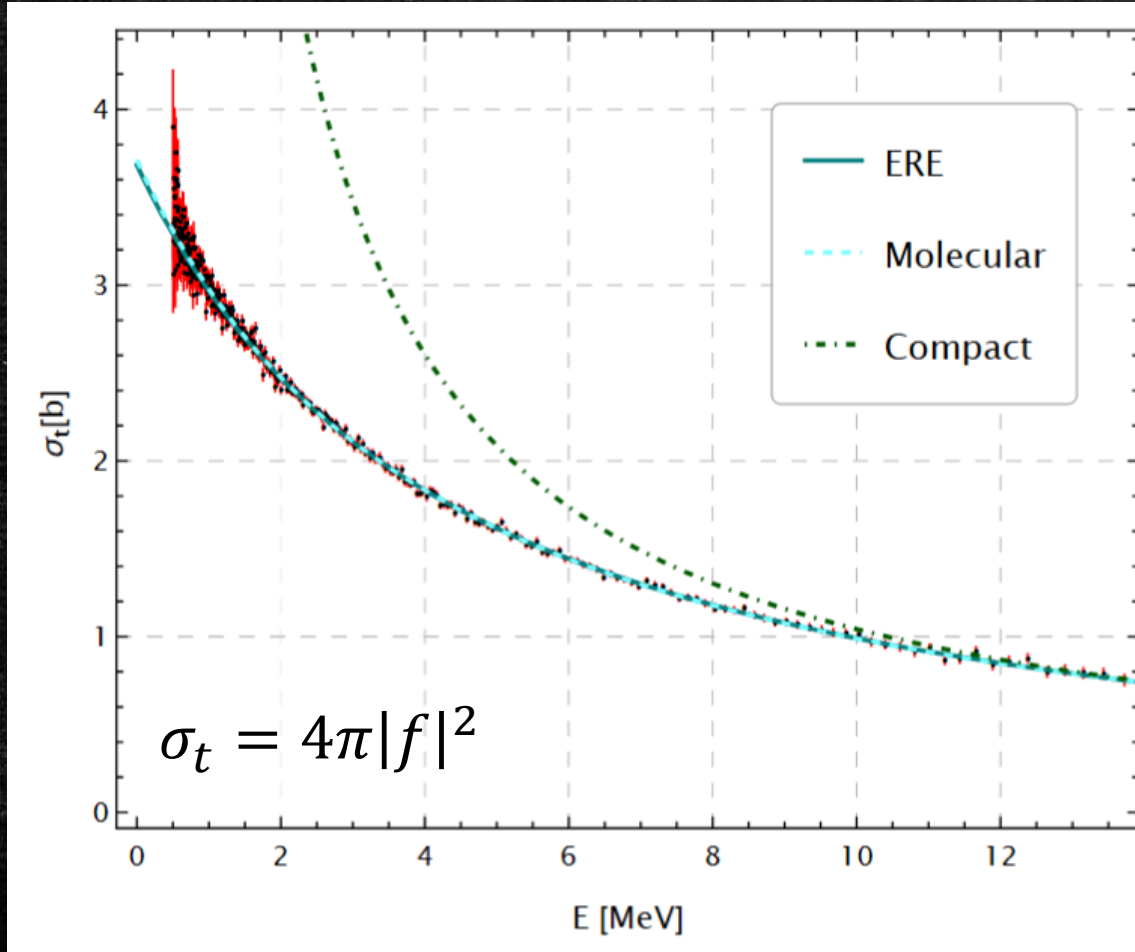
Proton-Neutron scattering NRQFT

$$\sigma D^\dagger \left(i\partial_t + \frac{\nabla^2}{4M} - \mu \right) D$$

Kinetic Term

Compact: $f(k) = -\frac{\tilde{g}^2}{E - \mu + i\tilde{g}^2 k}$
 $\sigma = +1$

Molecular: $f(k) = \frac{\tilde{g}^2}{E - \mu - i\tilde{g}^2 k}$
 $\sigma = -1$



A. Carducci, G. Cianti, P. D'Annibali, D. G. and A.D. Polosa, arXiv:2605.12274 [hep-ph]

What about the $X(3872)$?

$$D = \begin{pmatrix} D^+ \\ D^0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{kin} = & D^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_D} - \begin{pmatrix} \Delta_1 & 0 \\ 0 & 0 \end{pmatrix} \right) D + \bar{D}^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_D} - \begin{pmatrix} 0 & 0 \\ 0 & \Delta_1 \end{pmatrix} \right) \bar{D} + \\ & + D^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_{D^*}} - \begin{pmatrix} \Delta_2 & 0 \\ 0 & 0 \end{pmatrix} \right) D + \bar{D}^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_{D^*}} - \begin{pmatrix} 0 & 0 \\ 0 & \Delta_2 \end{pmatrix} \right) \bar{D} + \\ & \pm X^\dagger \left(i\partial_t + \frac{\nabla^2}{2(m_D + m_{D^*})} - m_F \right) X. \end{aligned}$$

A lot of fields...

and interactions

$$\begin{aligned} \mathcal{L}_{int} \supset & -\frac{\lambda_S}{2} (\bar{D}D)_+^\dagger \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} (\bar{D}D)_+ - \frac{\lambda_T}{2} (\bar{D}D)_+^\dagger \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} (\bar{D}D)_+ \\ & - \frac{g}{\sqrt{2}} X^\dagger (\bar{D}^0 D^0)_+ + \frac{g}{\sqrt{2}} X^\dagger (D^- D^+)_+. \end{aligned}$$

$X(3872)$ compositeness from its Lineshape

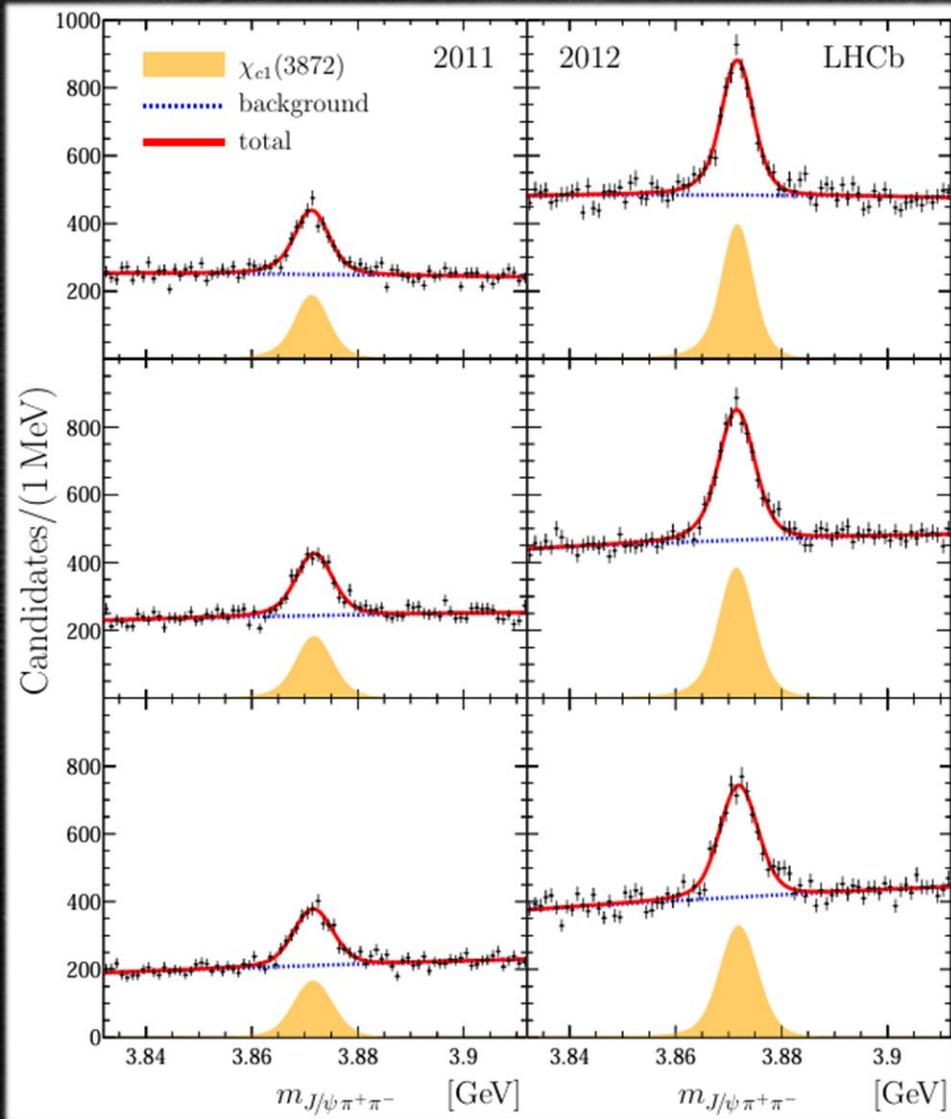
$$f_{\text{comp}}(E) = - \frac{\tilde{g}^2}{E - m_F + i\tilde{g}^2\sqrt{2mE} - \tilde{g}^2\sqrt{2m(\Delta - E)} + i\frac{\Gamma_r}{2}}$$

Flatté parametrization $\longrightarrow$ used by the LHCb collaboration

$$f_{\text{mol}}(E) = - \frac{\tilde{g}^2}{-E + m_F + i\tilde{g}^2\sqrt{2mE} - \tilde{g}^2\sqrt{2m(\Delta - E)} + i\frac{\Gamma_r}{2}}$$

Inverted sign (Kaplan) parametrization

$X(3872)$ compositeness from its Lineshape



$$\frac{\tilde{g}^2}{\bar{E} - \tilde{g}^2 \sqrt{2m(\Delta - E)} + i \frac{\Gamma_r}{2}}$$

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Optical Theorem

$$f_{\text{mol}}(E) = - \frac{\tilde{g}^2}{-E + m_F + i\tilde{g}^2\sqrt{2mE} - \tilde{g}^2\sqrt{2m(\Delta - E)} + i\frac{\Gamma_r}{2}}$$

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$$\frac{1}{a} = \pm \frac{m_F}{\tilde{g}^2} - \sqrt{2m\Delta} + i\frac{\Gamma_r}{2}$$

$$\text{Im}[f(\theta = 0)] = \frac{k}{2\pi} \sigma_t$$

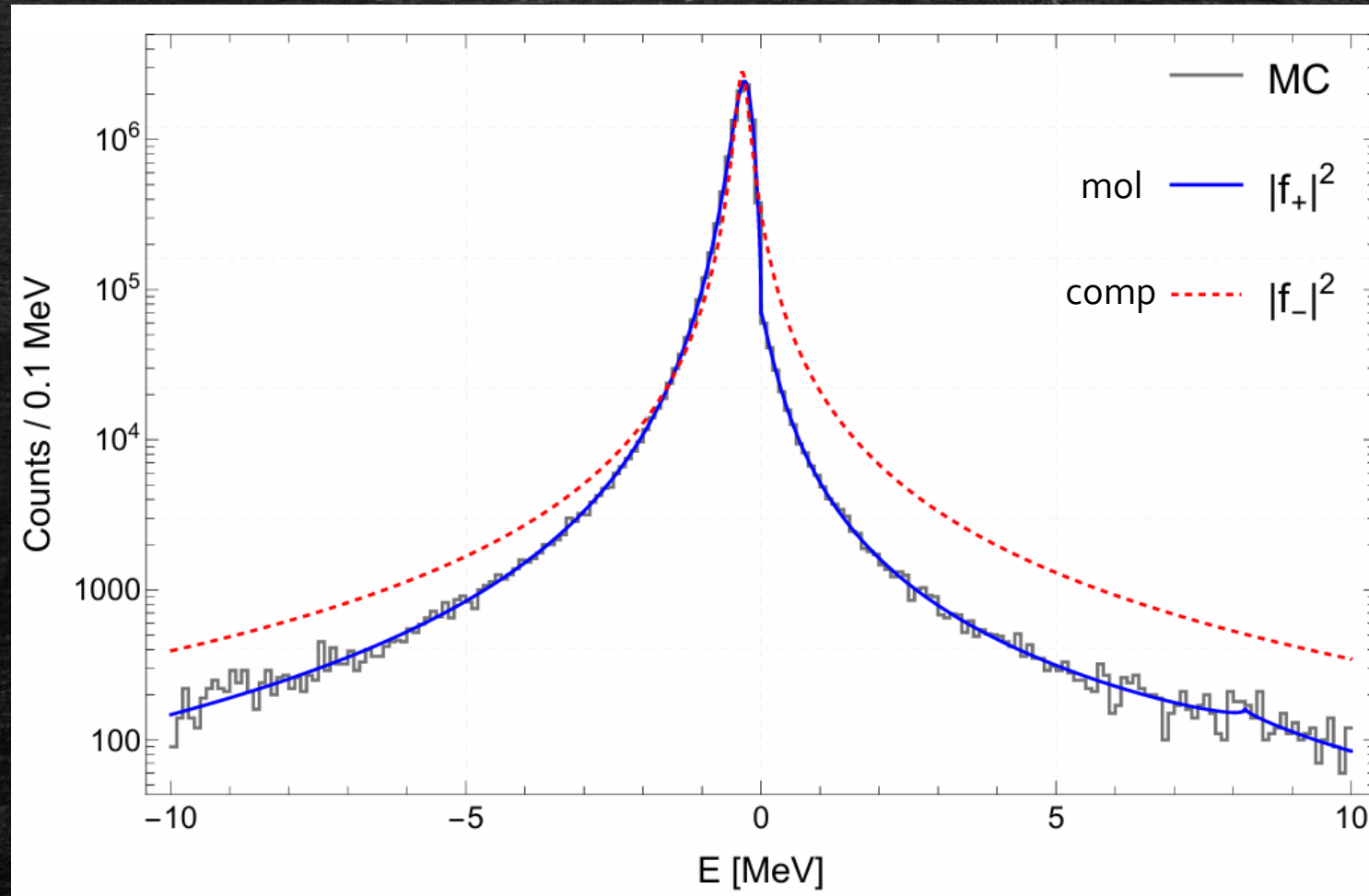


$$\sigma_t(k=0) = \cancel{\sigma_{\text{el}}(0)} + \sigma_{\text{inel}}(0) \sim \frac{1}{k}$$

Bethe 1/v law

$$\text{Im}[f(k=0)] = -\text{Im}(a) \propto \text{Im}\left(\frac{1}{a}\right)$$

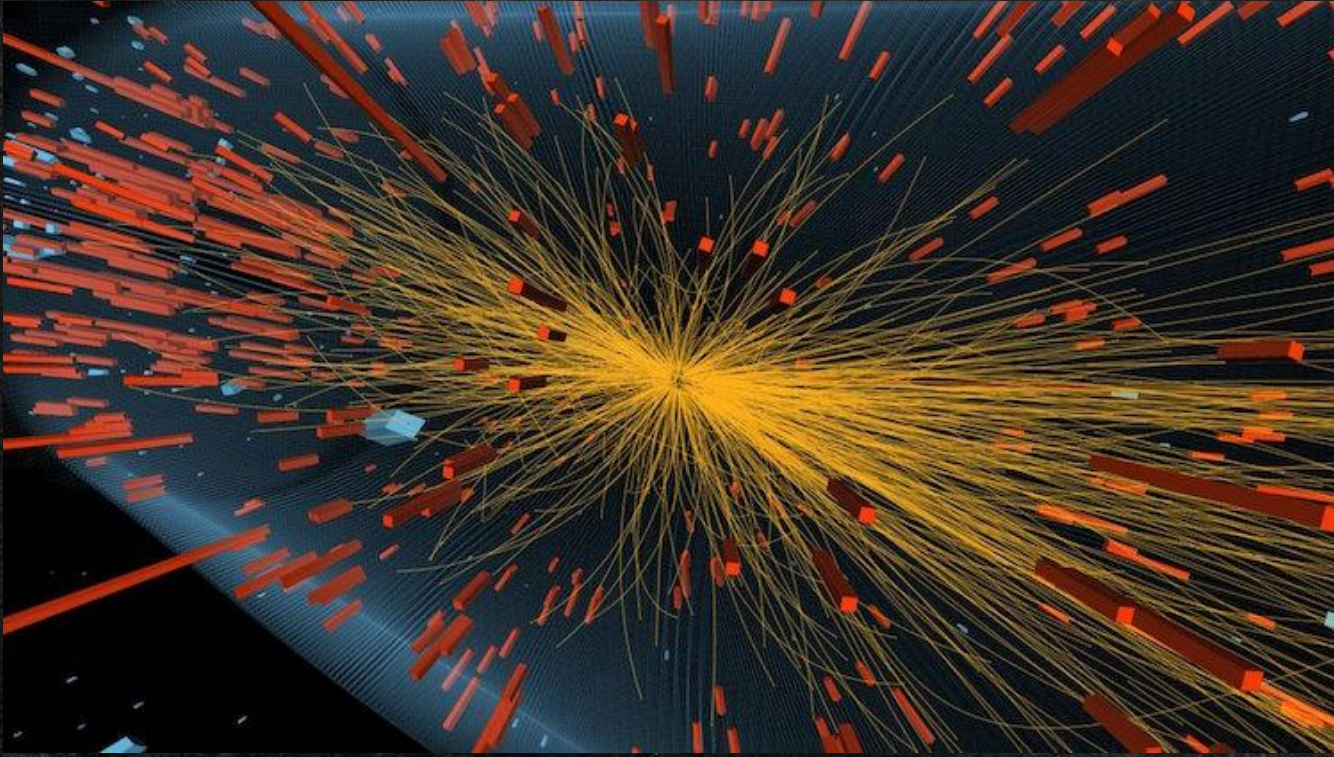
Molecular vs Compact Lineshape



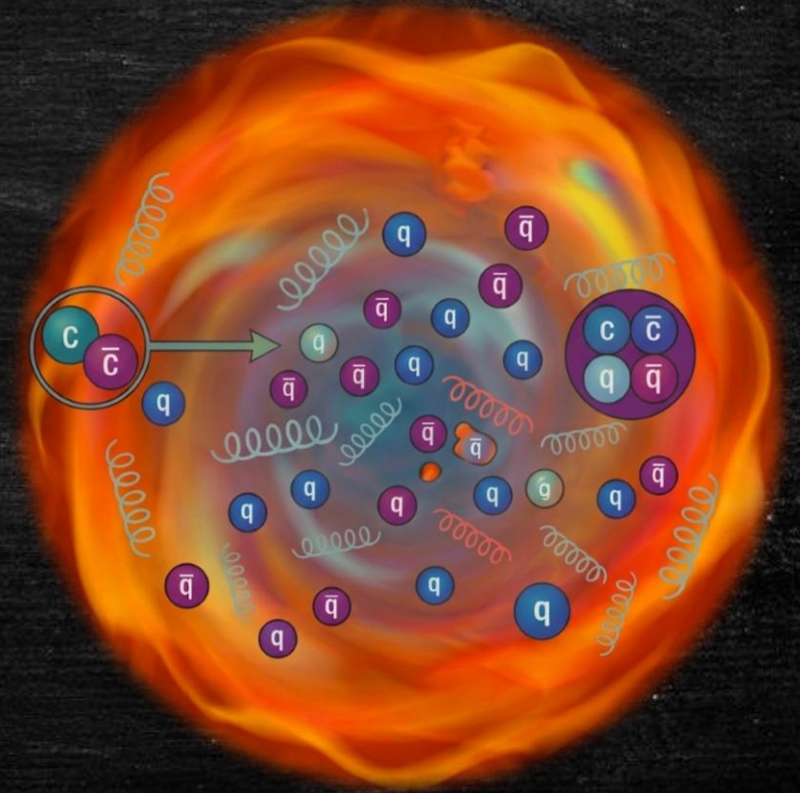
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What's next?

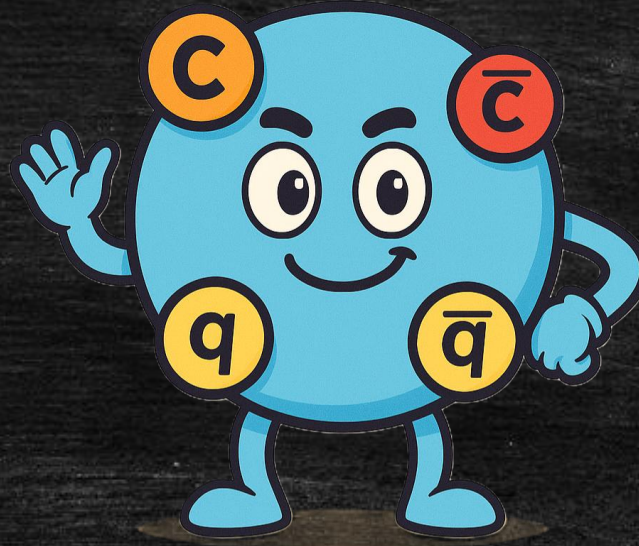
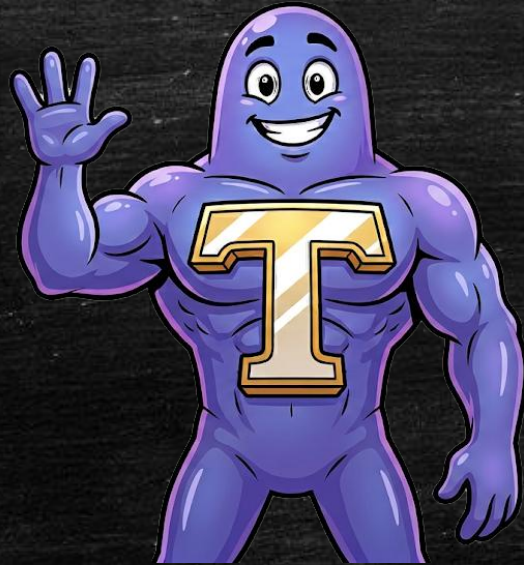


Formation of exotic states in QGP



Takeaway

It is remarkable how theoretical tools originally developed to address different problems are now being successfully repurposed to tackle new frontier questions in hadron spectroscopy.



- Hadronic spectroscopy is very cool!