

ASYMPTOTIC DIRICHLET PROBLEM FOR THE MINIMAL GRAPH EQUATION ON CARTAN-HADAMARD MANIFOLDS

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I will review the recent research on solvability/nonsolvability of the asymptotic Dirichlet problem for the minimal graph equation

$$\operatorname{div} \frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} = 0$$

on Cartan-Hadamard manifolds M . In joint works with Jaime Ripoll and Jean-Baptiste Casteras we have studied the asymptotic Dirichlet problem under sectional curvature bounds

$$-(b \circ \rho)^2(x) \leq \operatorname{Sect}_x \leq -(a \circ \rho)^2(x) < 0,$$

where $a, b: [0, \infty) \rightarrow (0, \infty)$ are smooth functions subject to certain growth conditions and $\rho(x) = d(x, o)$ is the distance of x to a fixed point $o \in M$.

On the other hand, in another paper with Ripoll we showed, by modifying Borbély's example, that there are 3-dimensional Cartan-Hadamard manifolds M , with sectional curvatures ≤ -1 , where the asymptotic Dirichlet problem is not solvable.