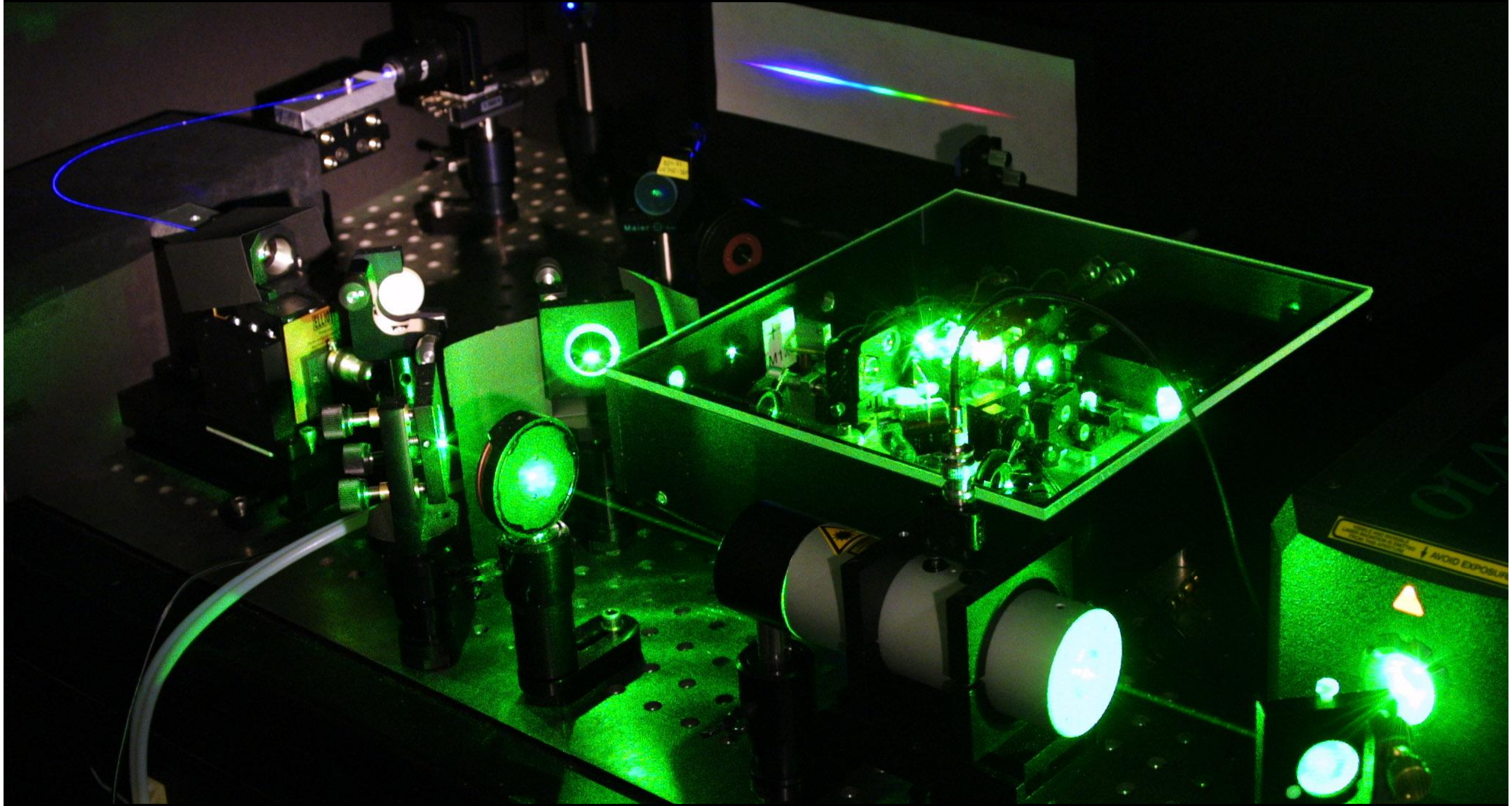
The Frequency Comb (R)evolution

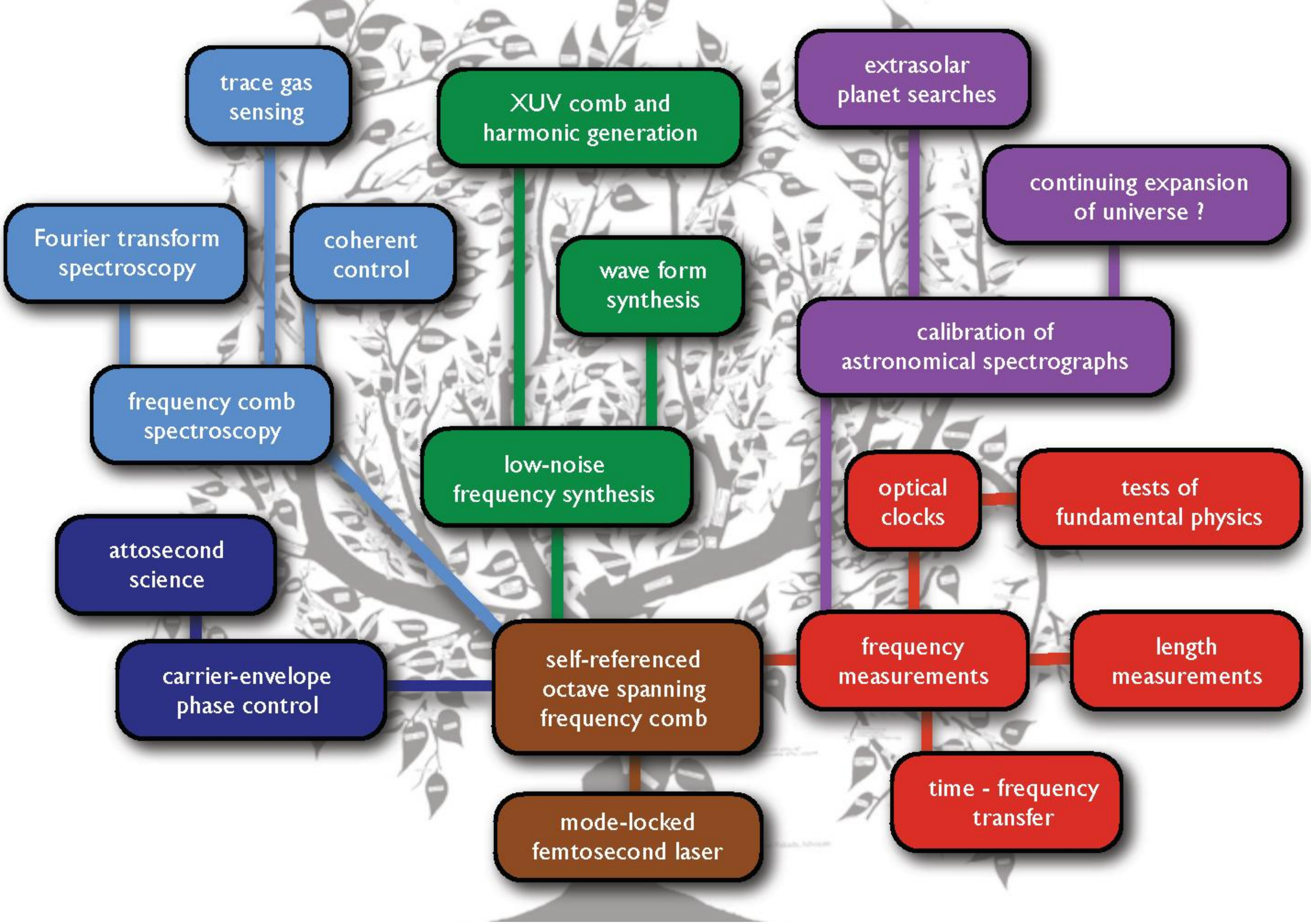


Thomas Udem

Max-Planck Institut für Quantenoptik Garching/Germany

- The History of the Comb
- Derivation of the Comb
- Self-Referencing

Frequency combs - evolutionary tree



trace gas sensing

XUV comb and harmonic generation

extrasolar planet searches

Fourier transform spectroscopy

coherent control

continuing expansion of universe ?

wave form synthesis

calibration of astronomical spectrographs

frequency comb spectroscopy

low-noise frequency synthesis

optical clocks

tests of fundamental physics

attosecond science

carrier-envelope phase control

self-referenced octave spanning frequency comb

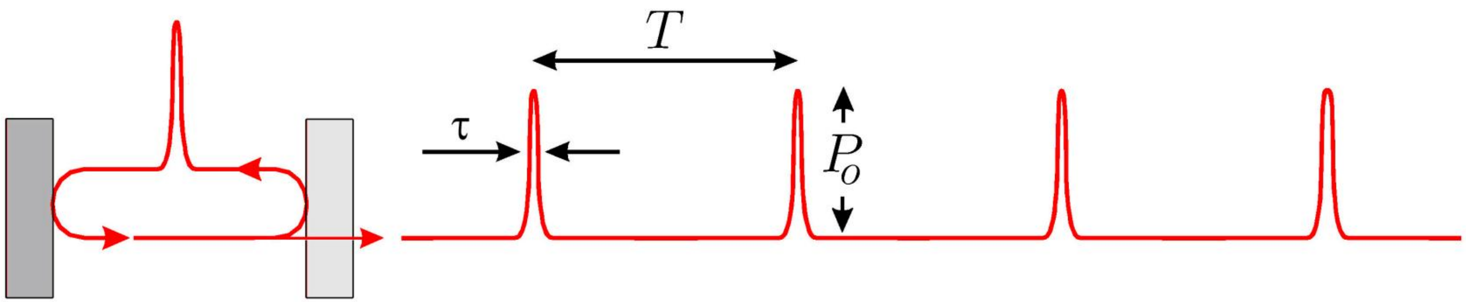
frequency measurements

length measurements

mode-locked femtosecond laser

time - frequency transfer

Mode Locked Laser as a Comb Generator

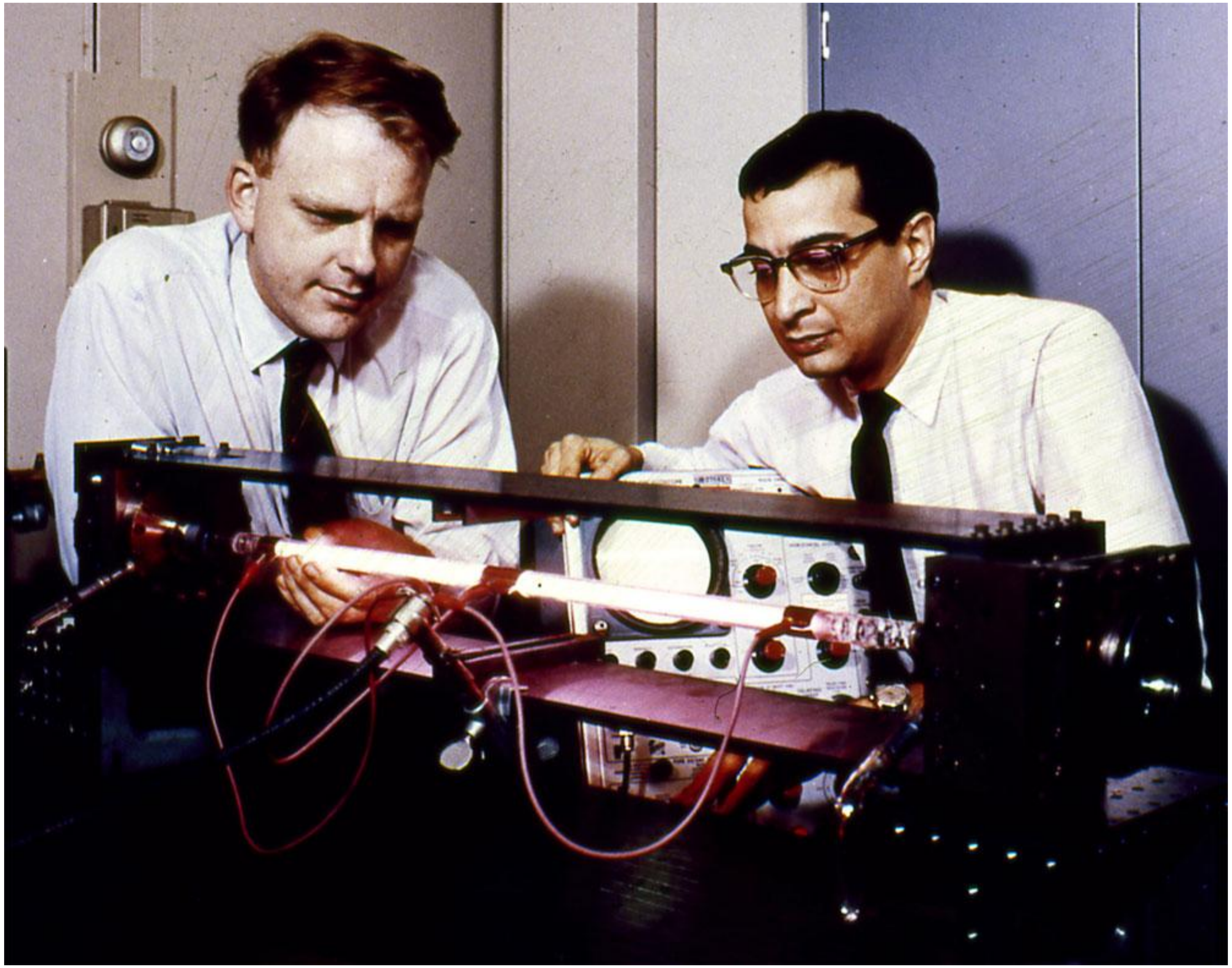


Typical Ti:sapphire Kerr-lens mode locked laser:

- pulse repetition rate T^{-1} = 100 MHz … 1 GHz
- pulse duration τ = 10 fs
- spectral width = $1/\tau$ = 100 THz
- $N = 10^5 \ldots 10^6$ modes phase synchronized

The History of the Comb

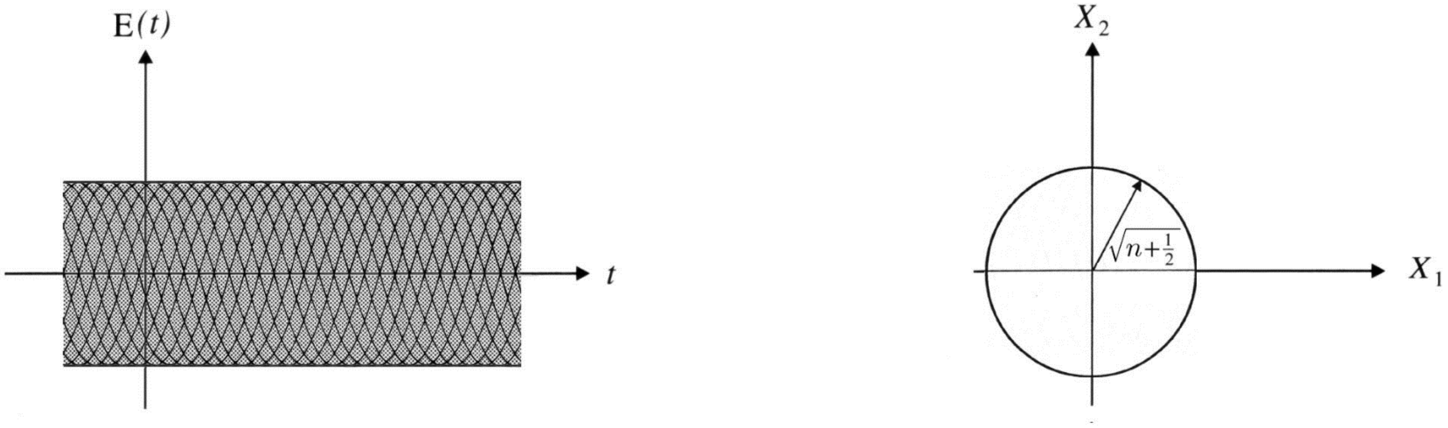
The First Continuous Wave Laser



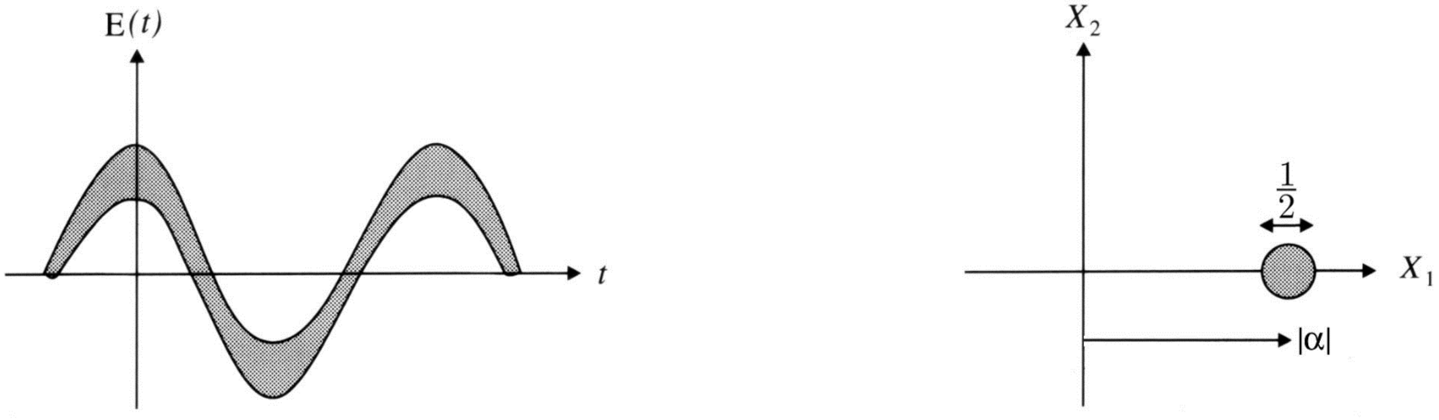
William Bennett and Ali Javan with the first continuous wave laser 1961

Coherent Waves with Frequencies?

Fock state

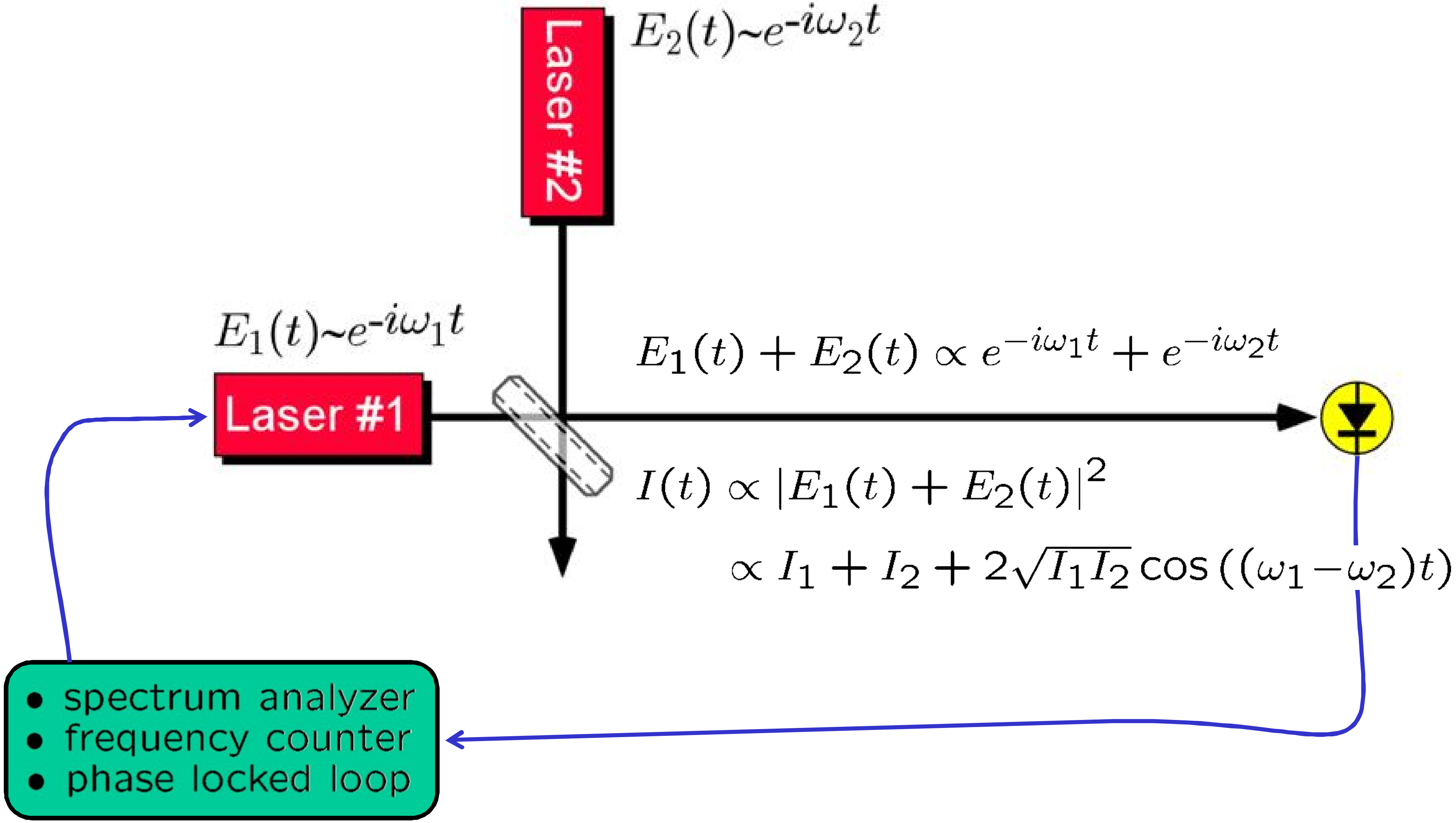


coherent state



Scully, Zubairy - Quantum Optics

Laser Beat Notes



Laser #2

$E_2(t) \sim e^{-i\omega_2 t}$

$E_1(t) \sim e^{-i\omega_1 t}$

Laser #1

$$E_1(t) + E_2(t) \propto e^{-i\omega_1 t} + e^{-i\omega_2 t}$$

$$I(t) \propto |E_1(t) + E_2(t)|^2$$

$$\propto I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left((\omega_1 - \omega_2)t\right)$$

- spectrum analyzer
- frequency counter
- phase locked loop

Laser Beat Notes

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 52, NUMBER 1 JANUARY, 1962

Letters to the Editor

Frequency Characteristics of a Continuous-Wave He–Ne Optical Maser

A. Javan,* E. A. Ballik, and W. L. Bond
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received August 31, 1961)

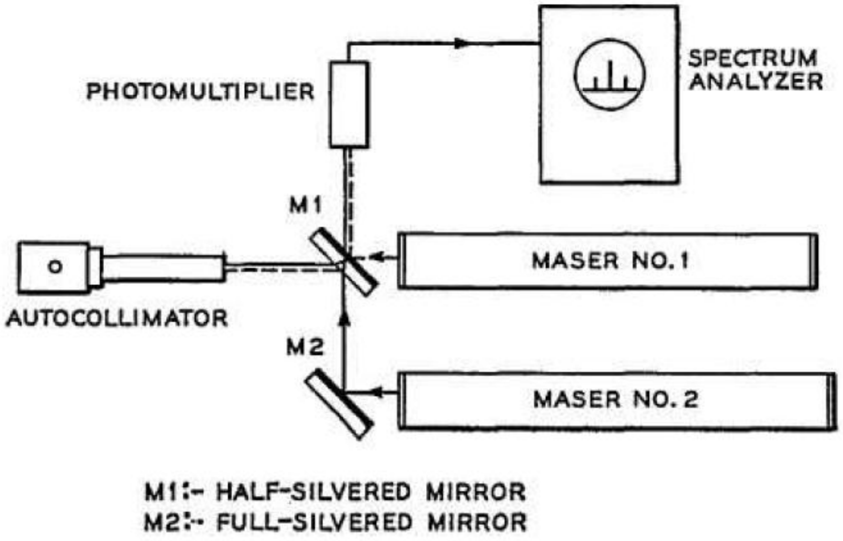


Fig. 1. Method of observing beat signals between two optical masers. The mirrors are adjusted so that both wave fronts are parallel and coincident.

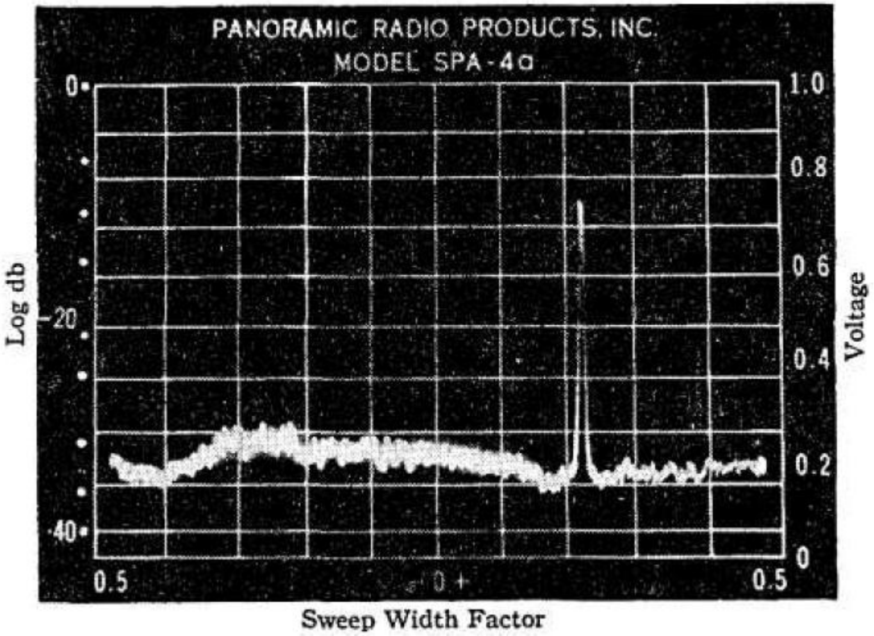
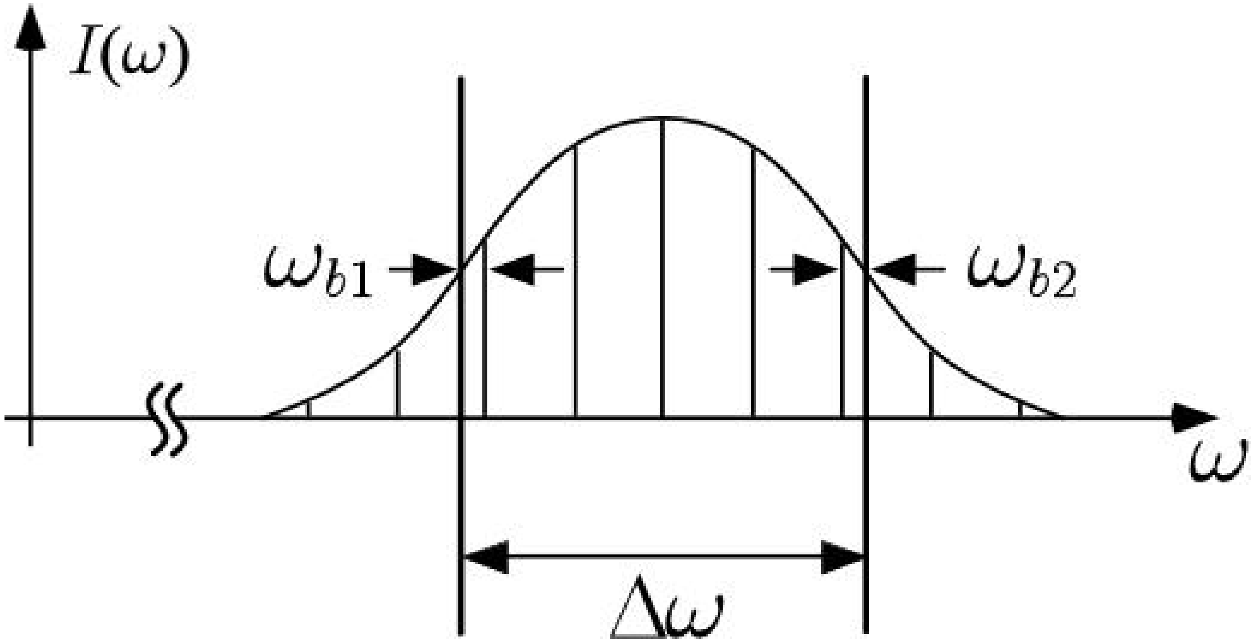


Fig. 2. Beat signal for single-mode operation of each maser. The frequency of the beat is about 5 Mc/sec.

Increasing the Measureable Frequency Difference

direct measurement of optical beat frequencies is limited by the detector bandwidth to a few 100 GHz with some tricks to a few THz.



High-Resolution Two-Photon Spectroscopy with Picosecond Light Pulses

J. N. Eckstein, A. I. Ferguson, and T. W. Hänsch
Department of Physics, Stanford University, Stanford, California 94305

(Received 11 January 1978)

We have demonstrated the feasibility of Doppler-free two-photon spectroscopy with a train of picosecond standing-wave light pulses from a synchronously pumped mode-locked cw dye laser. The actively controlled mode spectrum provides a means for accurate measurements of large frequency intervals. From a multipulse spectrum of the sodium 3*s*-4*d* transition we have determined a new value of the 4*d* fine-structure splitting, 1028 ± 0.4 MHz.

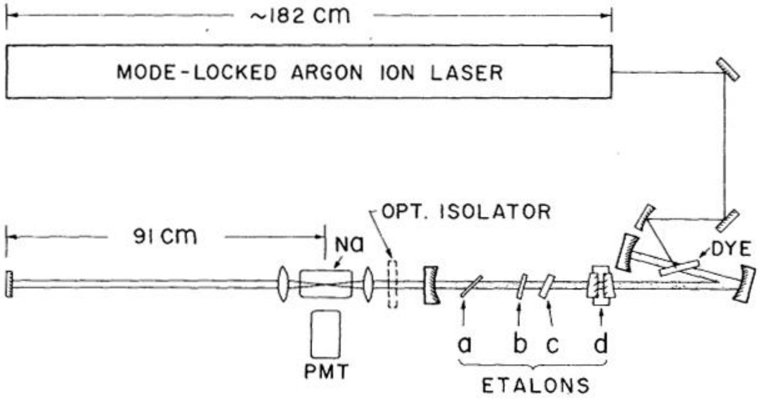
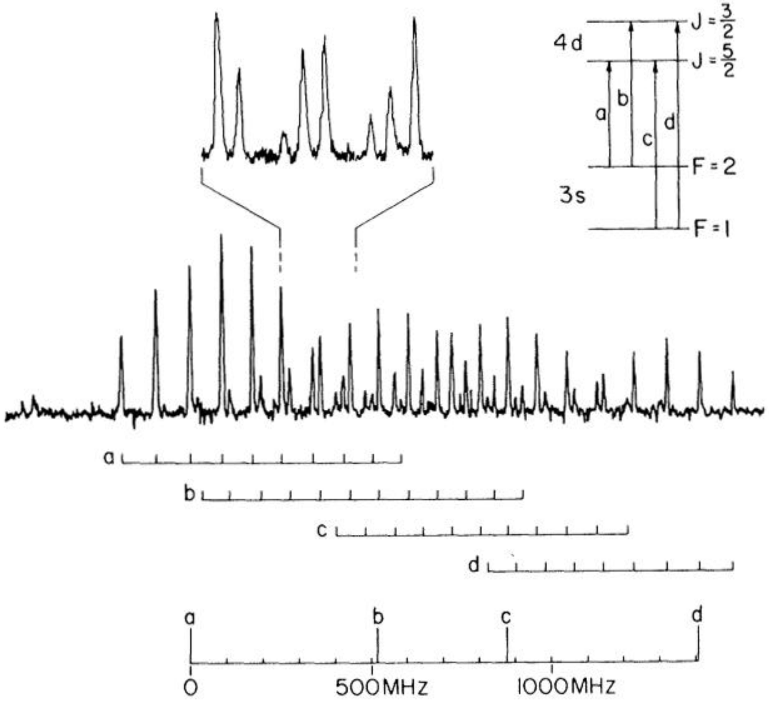
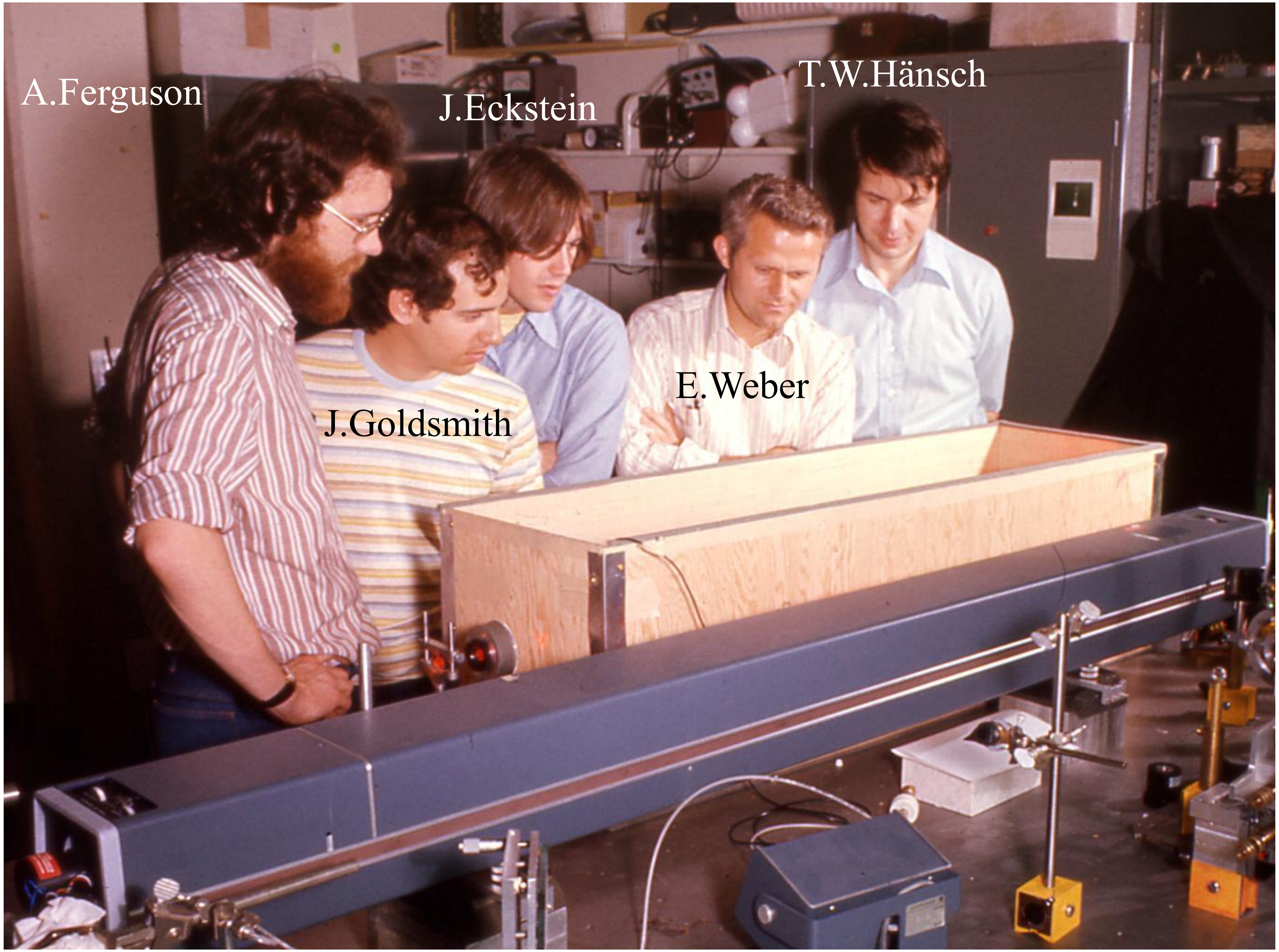


FIG. 1. Scheme of apparatus.

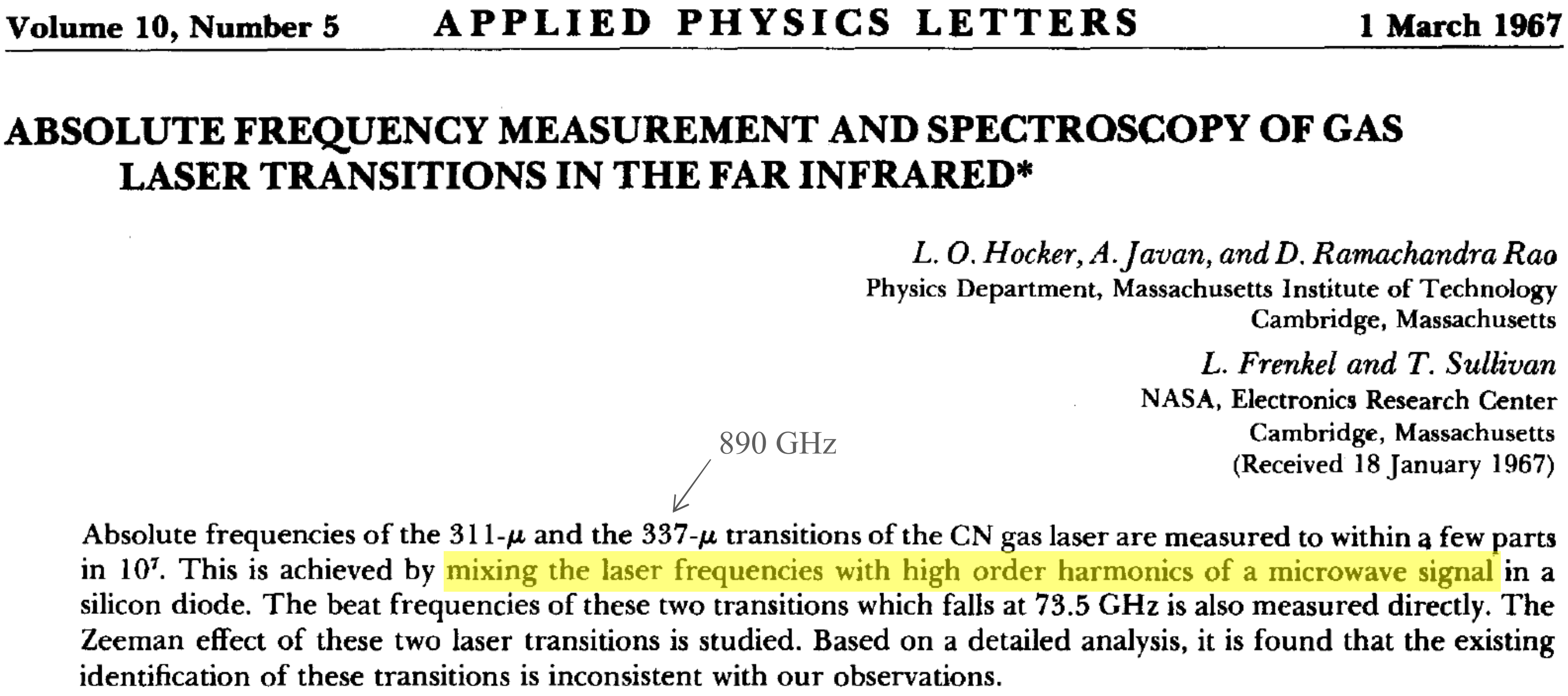


Stanford Mode Locked Laser 1978



Carrier envelope offset phase and frequency described in detail in Jim Eckstein‘s Thesis Stanford 1978

Frequency Differences and „Absolute“ Frequencies



Volume 10, Number 5 **APPLIED PHYSICS LETTERS** **1 March 1967**

ABSOLUTE FREQUENCY MEASUREMENT AND SPECTROSCOPY OF GAS LASER TRANSITIONS IN THE FAR INFRARED*

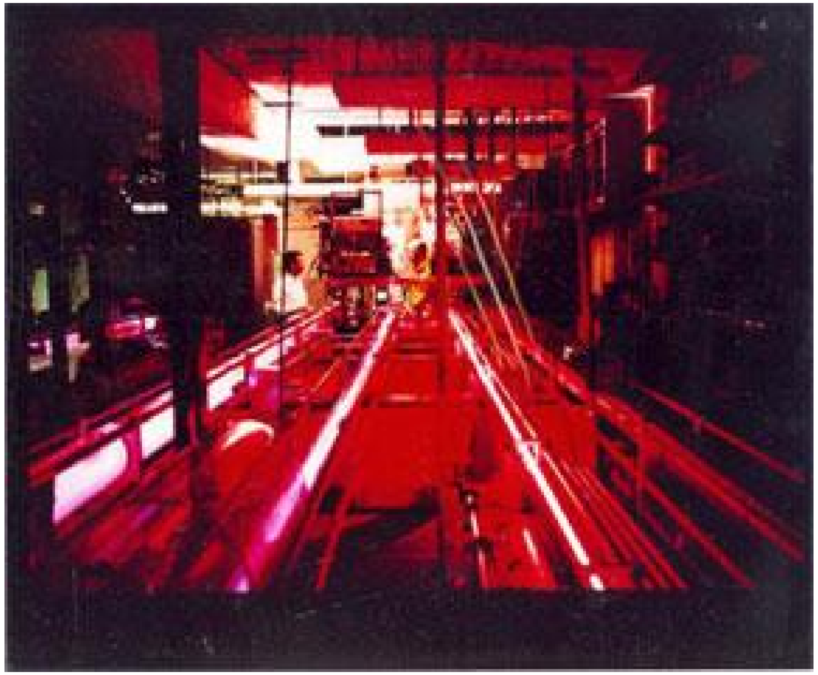
L. O. Hocker, A. Javan, and D. Ramachandra Rao
Physics Department, Massachusetts Institute of Technology
Cambridge, Massachusetts

L. Frenkel and T. Sullivan
NASA, Electronics Research Center
Cambridge, Massachusetts
(Received 18 January 1967)

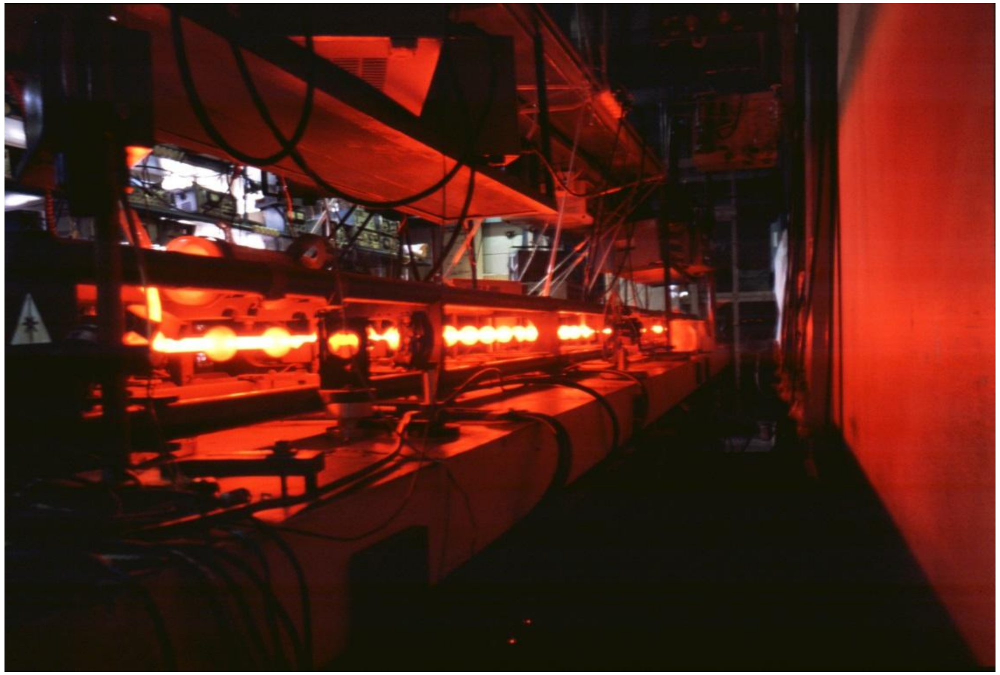
890 GHz

Absolute frequencies of the 311-μ and the 337-μ transitions of the CN gas laser are measured to within a few parts in 10^7. This is achieved by mixing the laser frequencies with high order harmonics of a microwave signal in a silicon diode. The beat frequencies of these two transitions which falls at 73.5 GHz is also measured directly. The Zeeman effect of these two laser transitions is studied. Based on a detailed analysis, it is found that the existing identification of these transitions is inconsistent with our observations.

Harmonic Frequency Chains

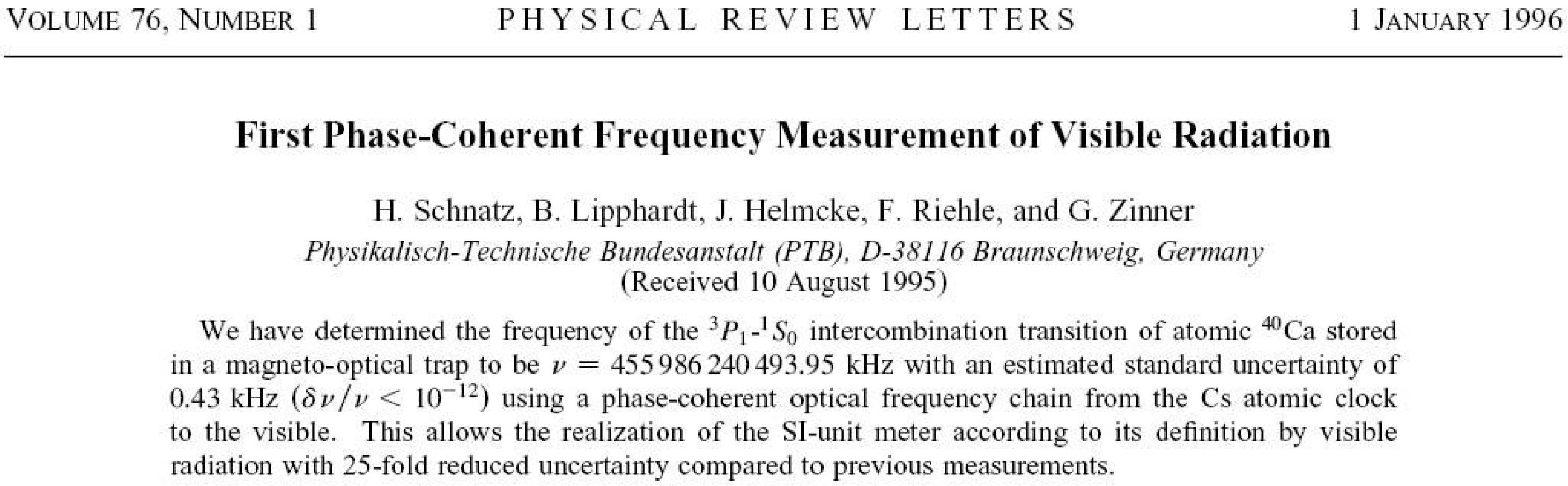


Boulder



Novosibirsk

Harmonic Frequency Chains



VOLUME 76, NUMBER 1 PHYSICAL REVIEW LETTERS 1 JANUARY 1996

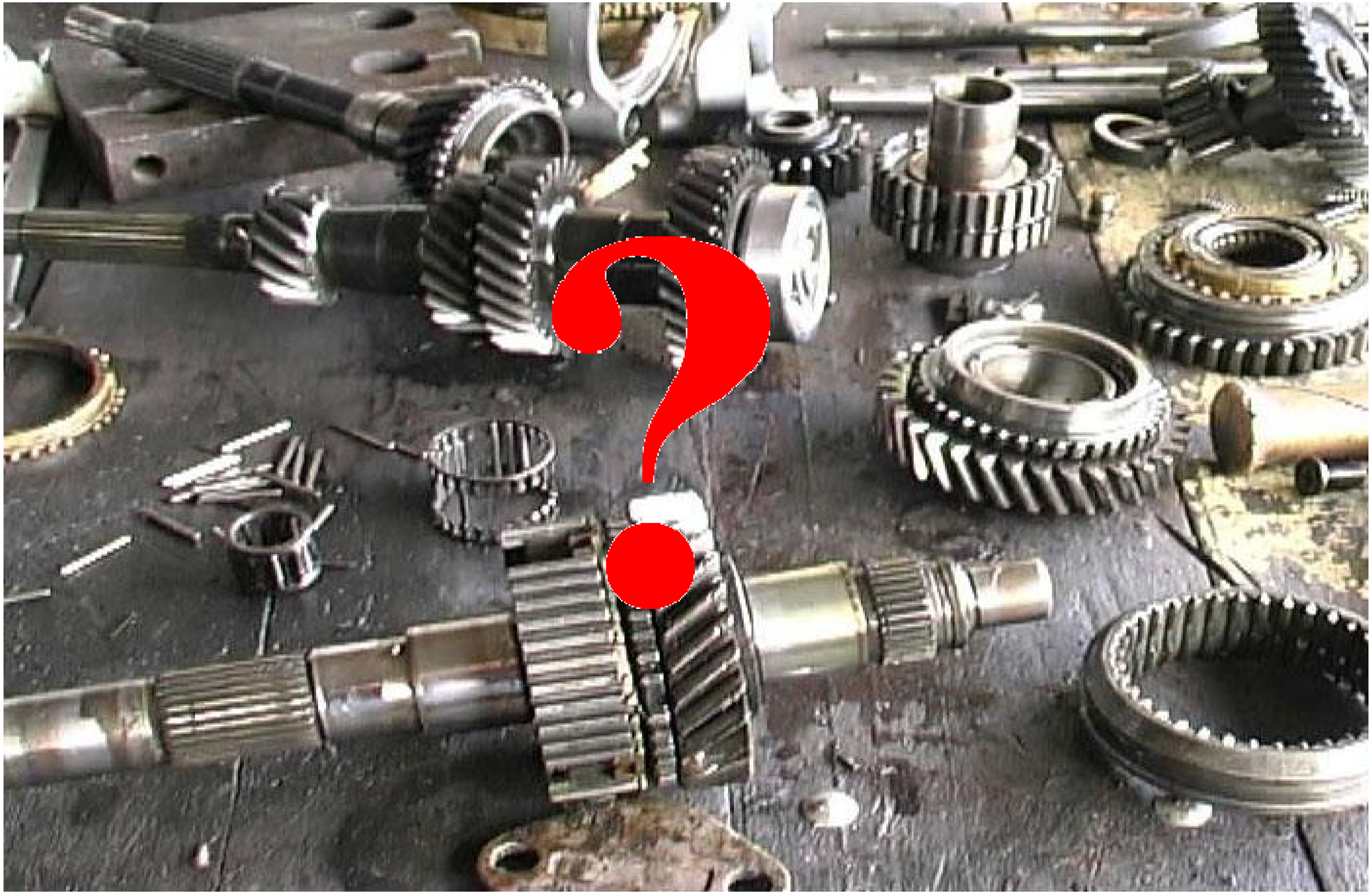
First Phase-Coherent Frequency Measurement of Visible Radiation

H. Schnatz, B. Lipphardt, J. Helmcke, F. Riehle, and G. Zinner
Physikalisch-Technische Bundesanstalt (PTB), D-38116 Braunschweig, Germany

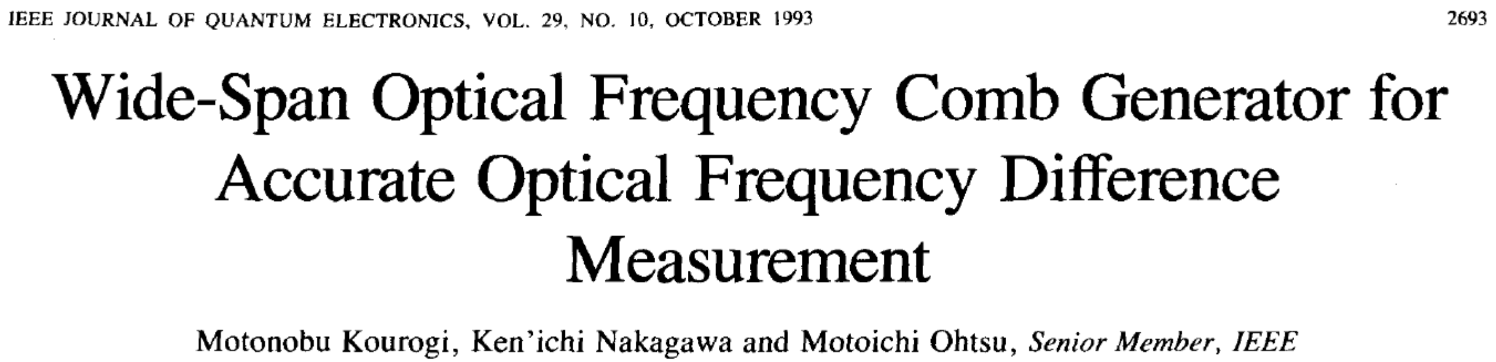
(Received 10 August 1995)

We have determined the frequency of the $^3P_1$-$^1S_0$ intercombination transition of atomic ^{40}Ca stored in a magneto-optical trap to be $\nu = 455\,986\,240\,493.95$ kHz with an estimated standard uncertainty of 0.43 kHz ($\delta\nu/\nu < 10^{-12}$) using a phase-coherent optical frequency chain from the Cs atomic clock to the visible. This allows the realization of the SI-unit meter according to its definition by visible radiation with 25-fold reduced uncertainty compared to previous measurements.

How to Improve the Optical Counter



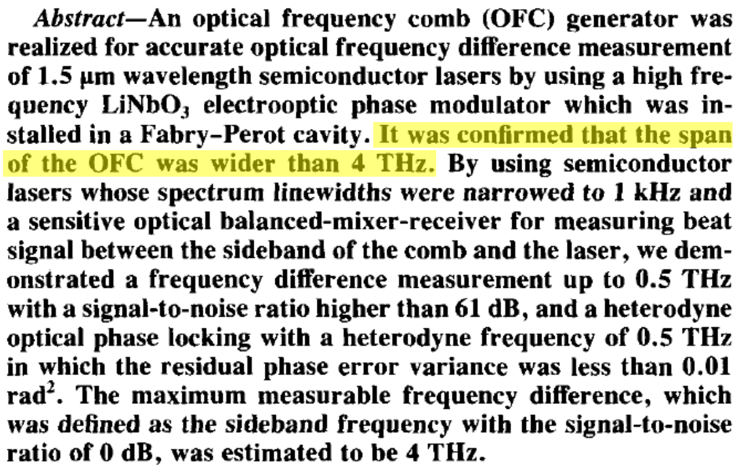
Kourogi Type Frequency Comb Generator



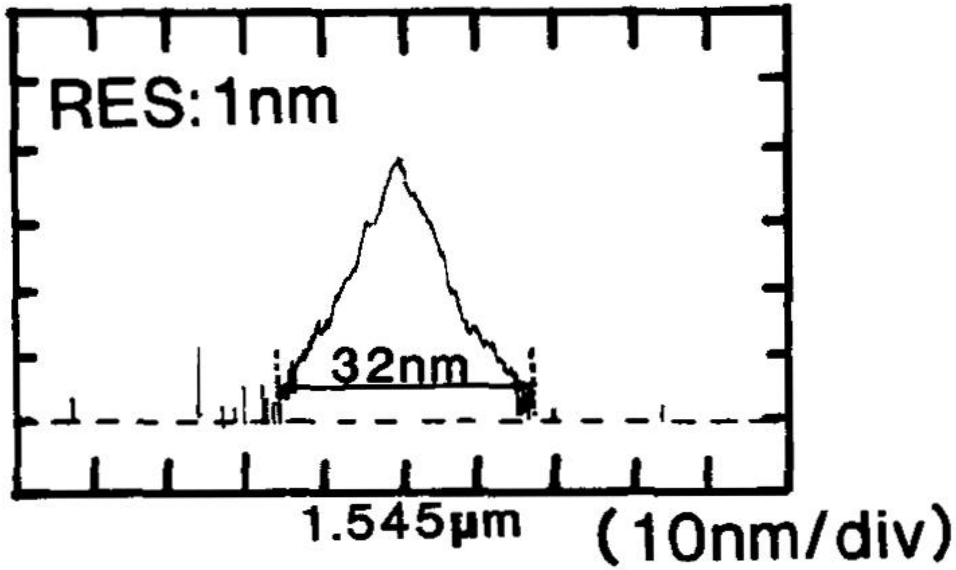
IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 29, NO. 10, OCTOBER 1993 2693

Wide-Span Optical Frequency Comb Generator for Accurate Optical Frequency Difference Measurement

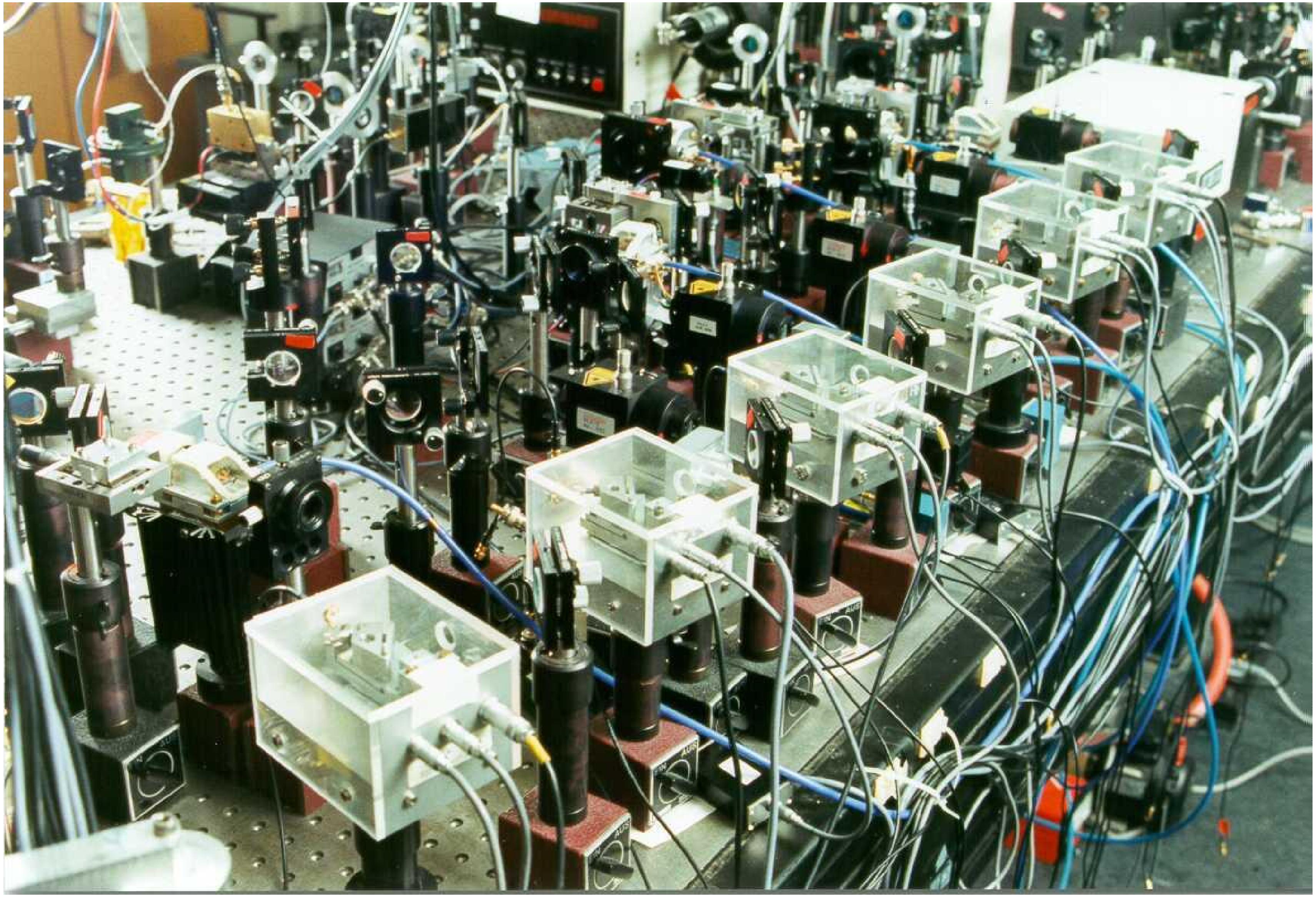
Motonobu Kourogi, Ken'ichi Nakagawa and Motoichi Ohtsu, *Senior Member, IEEE*



***Abstract*—An optical frequency comb (OFC) generator was realized for accurate optical frequency difference measurement of 1.5 μm wavelength semiconductor lasers by using a high frequency $LiNbO_3$ electrooptic phase modulator which was installed in a Fabry–Perot cavity. It was confirmed that the span of the OFC was wider than 4 THz. By using semiconductor lasers whose spectrum linewidths were narrowed to 1 kHz and a sensitive optical balanced-mixer-receiver for measuring beat signal between the sideband of the comb and the laser, we demonstrated a frequency difference measurement up to 0.5 THz with a signal-to-noise ratio higher than 61 dB, and a heterodyne optical phase locking with a heterodyne frequency of 0.5 THz in which the residual phase error variance was less than 0.01 rad^2. The maximum measurable frequency difference, which was defined as the sideband frequency with the signal-to-noise ratio of 0 dB, was estimated to be 4 THz.**

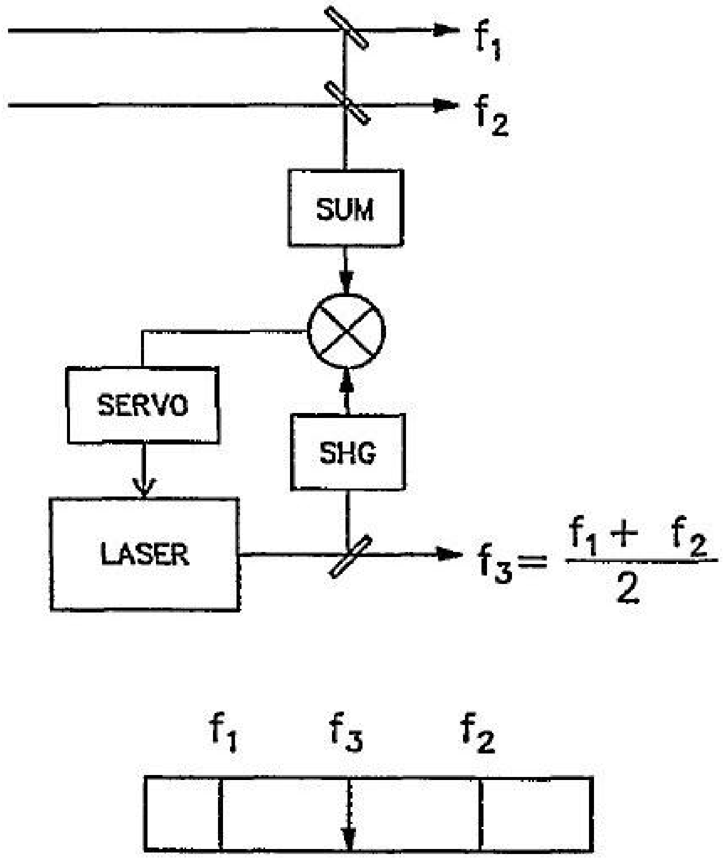


Kourogi's Comb Generator in our Lab

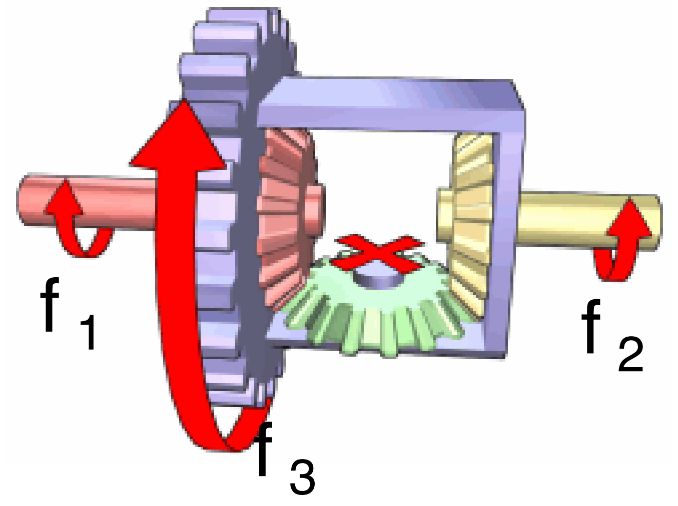


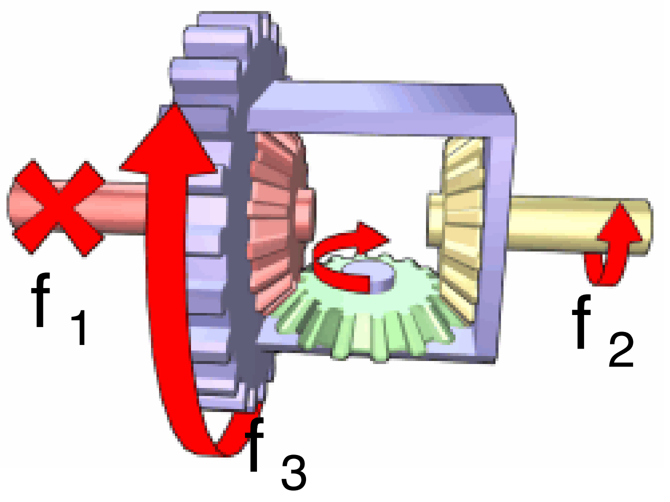
Opt. Lett. 23, 1387 (1998)

Optical Interval Dividers



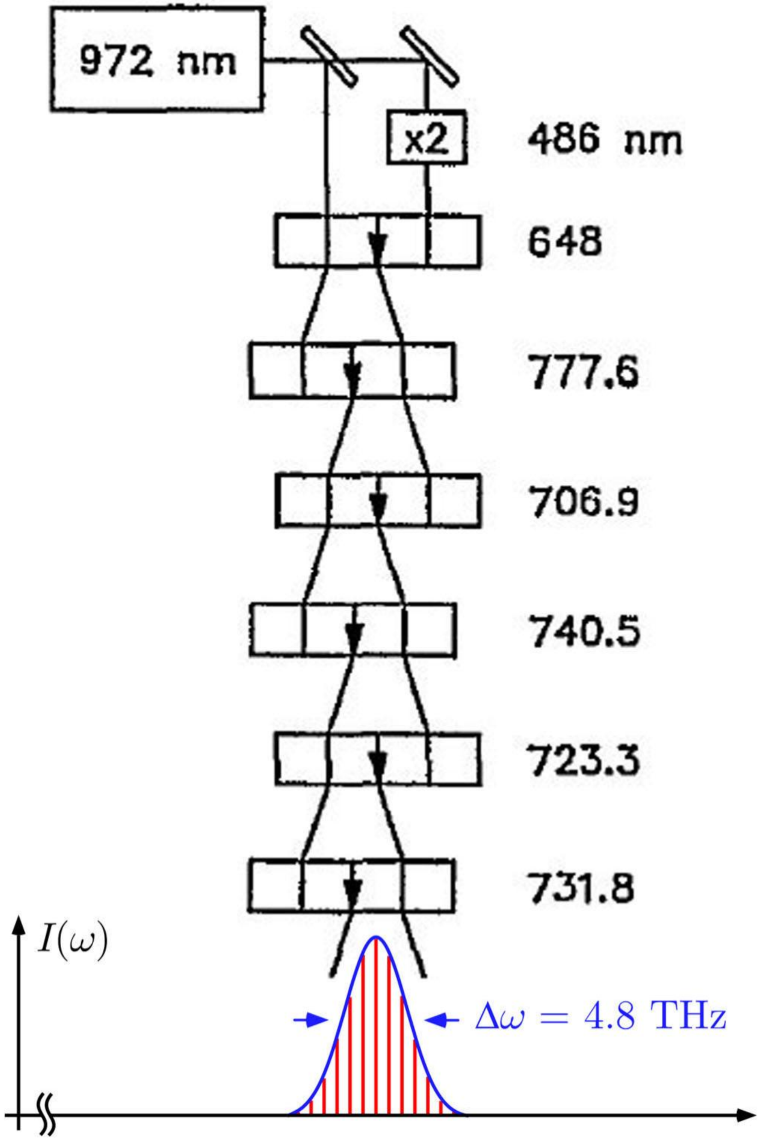
Optical Interval Divider (Differential Gear)



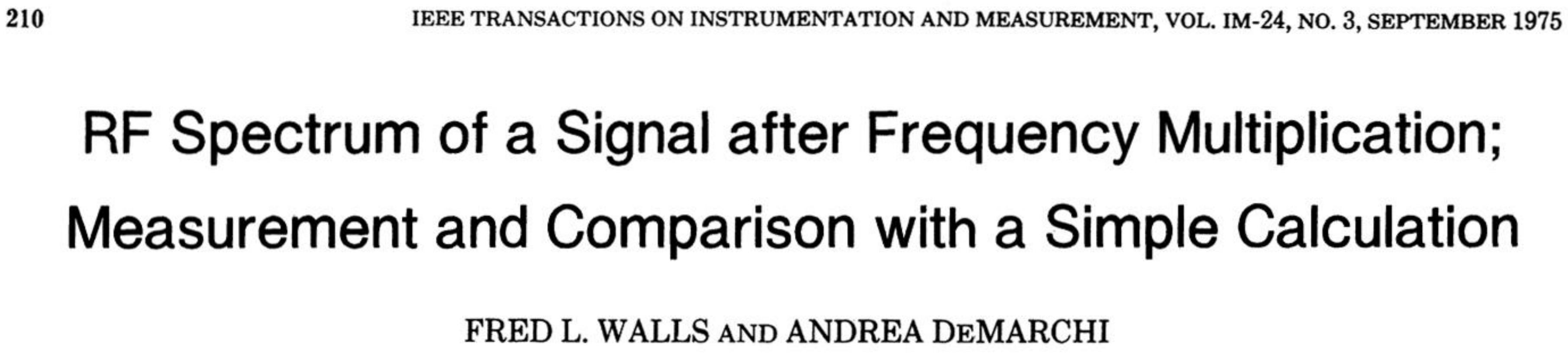


T.W.Hänsch in “The Hydrogen Atom” G.F.Bassani, M.Inguscio, T.W.Hänsch eds, Springer 1989

Optical Counter



Phase Noise



210 IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. IM-24, NO. 3, SEPTEMBER 1975

RF Spectrum of a Signal after Frequency Multiplication; Measurement and Comparison with a Simple Calculation

FRED L. WALLS AND ANDREA DEMARCHI

$$\sin(\omega t + \varphi(t)) \xrightarrow{\text{frequency } \times N} \sin(N\omega t + N\varphi(t))$$

- power in $\varphi(t)$ increases by N^2 !
- noise increases by 120dB for $N = 10^6$!

Phase Noise: Why the Comb shouldn‘t work

Frequency Control of Semiconductor Lasers, Wiley & Sons 1996

Absolute Measurement of Optical Frequencies

H. R. TELLE

Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

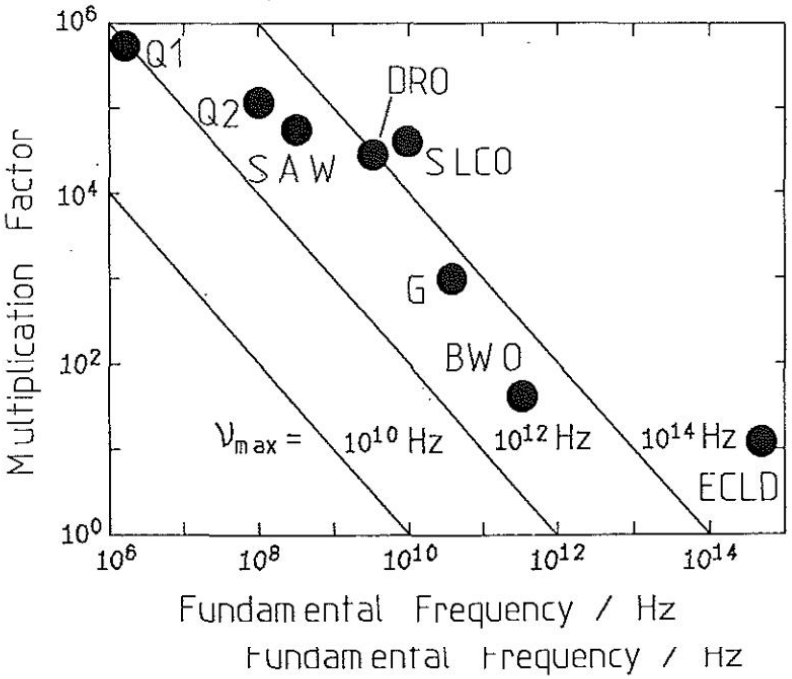
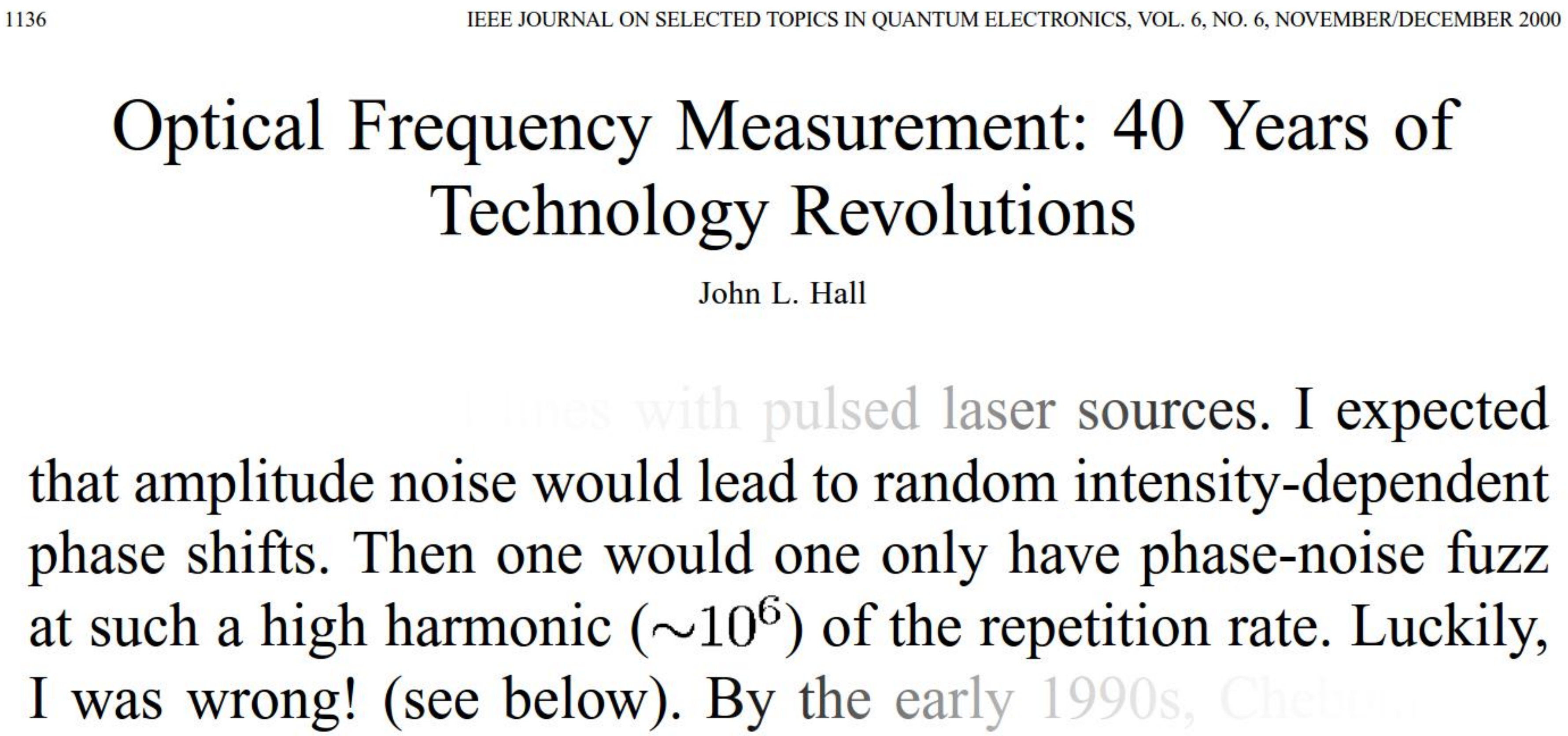


FIGURE 5.11 Carrier collapse due to multiplied phase-noise pedestal—maximum useful multiplication factors of some typical transfer oscillators [6, 58–60]. Q1, Q2, quartz oscillators; SAW, surface acoustic wave oscillator; DRO, dielectric resonator oscillator; SLCO, sapphire loaded-cavity oscillator; G, Gunn oscillator, BWO: backward wave oscillator; ECLD, external cavity laser diode.

Phase Noise: Why the Comb shouldn‘t work



1136 IEEE JOURNAL ON SELECTED TOPICS IN QUANTUM ELECTRONICS, VOL. 6, NO. 6, NOVEMBER/DECEMBER 2000

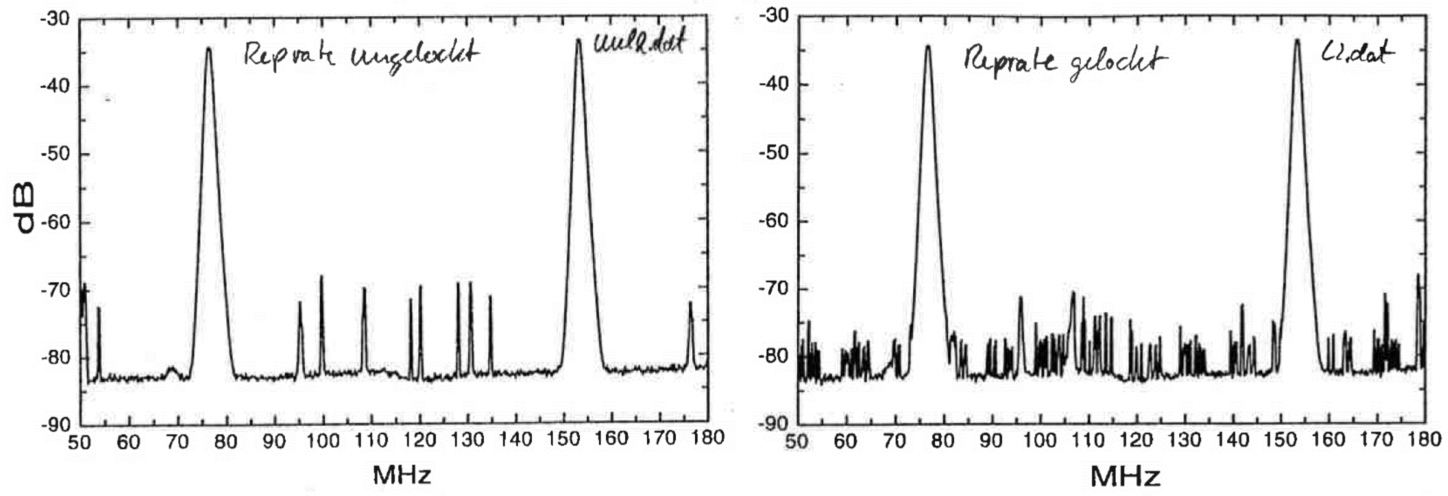
Optical Frequency Measurement: 40 Years of Technology Revolutions

John L. Hall

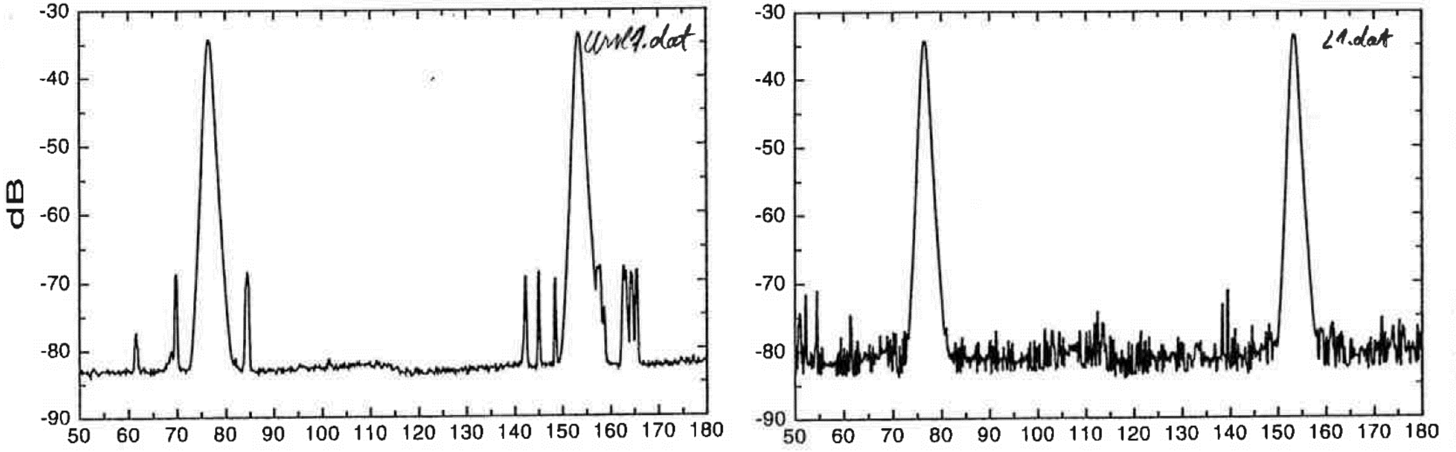
lines with pulsed laser sources. I expected that amplitude noise would lead to random intensity-dependent phase shifts. Then one would one only have phase-noise fuzz at such a high harmonic ($\sim 10^6$) of the repetition rate. Luckily, I was wrong! (see below). By the early 1990s, Chebo

Phase Noise

RBW: 1MHz Loopfilter (Seite 21) $R_2 = 1\,M\Omega$

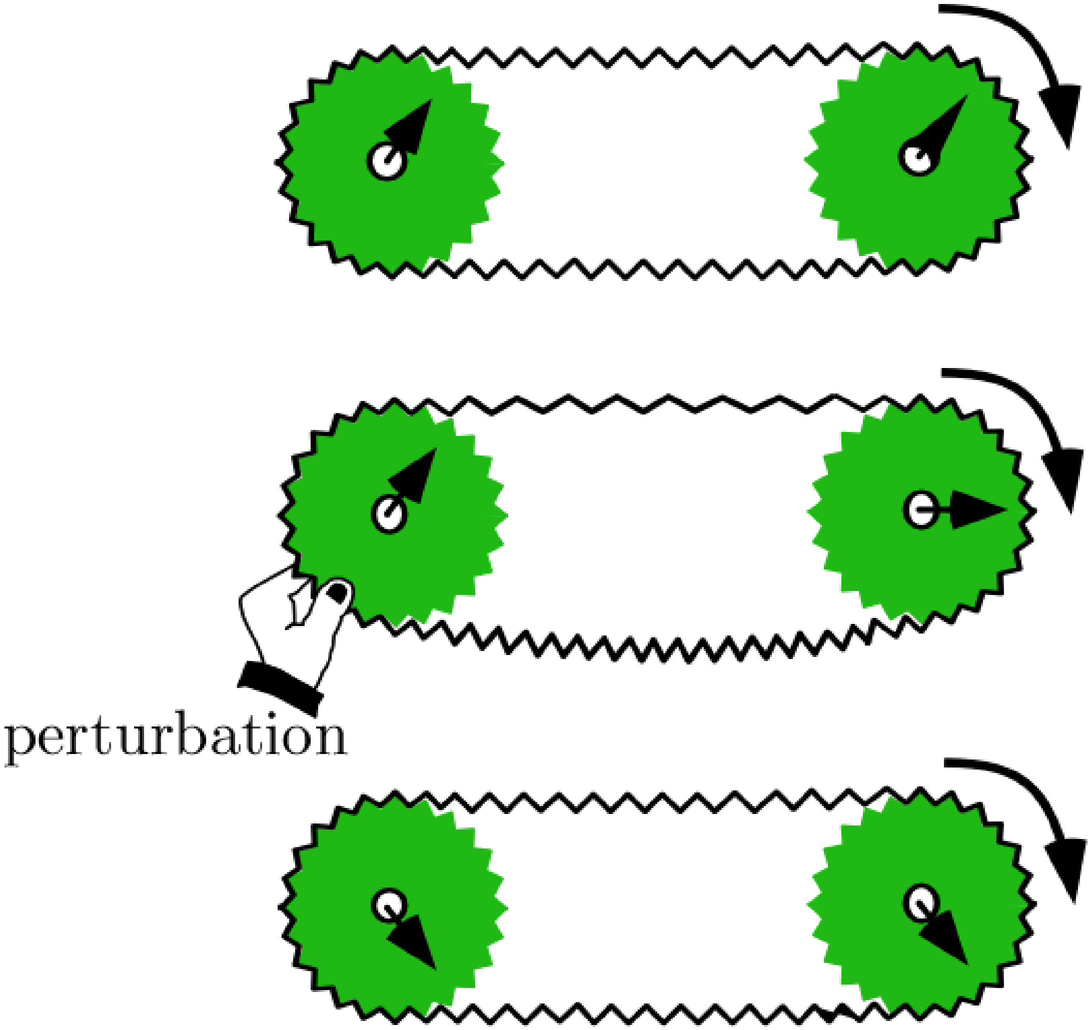


wie oben nur mit $R_2 = 0\,\Omega$ (keine Dämpfung!)

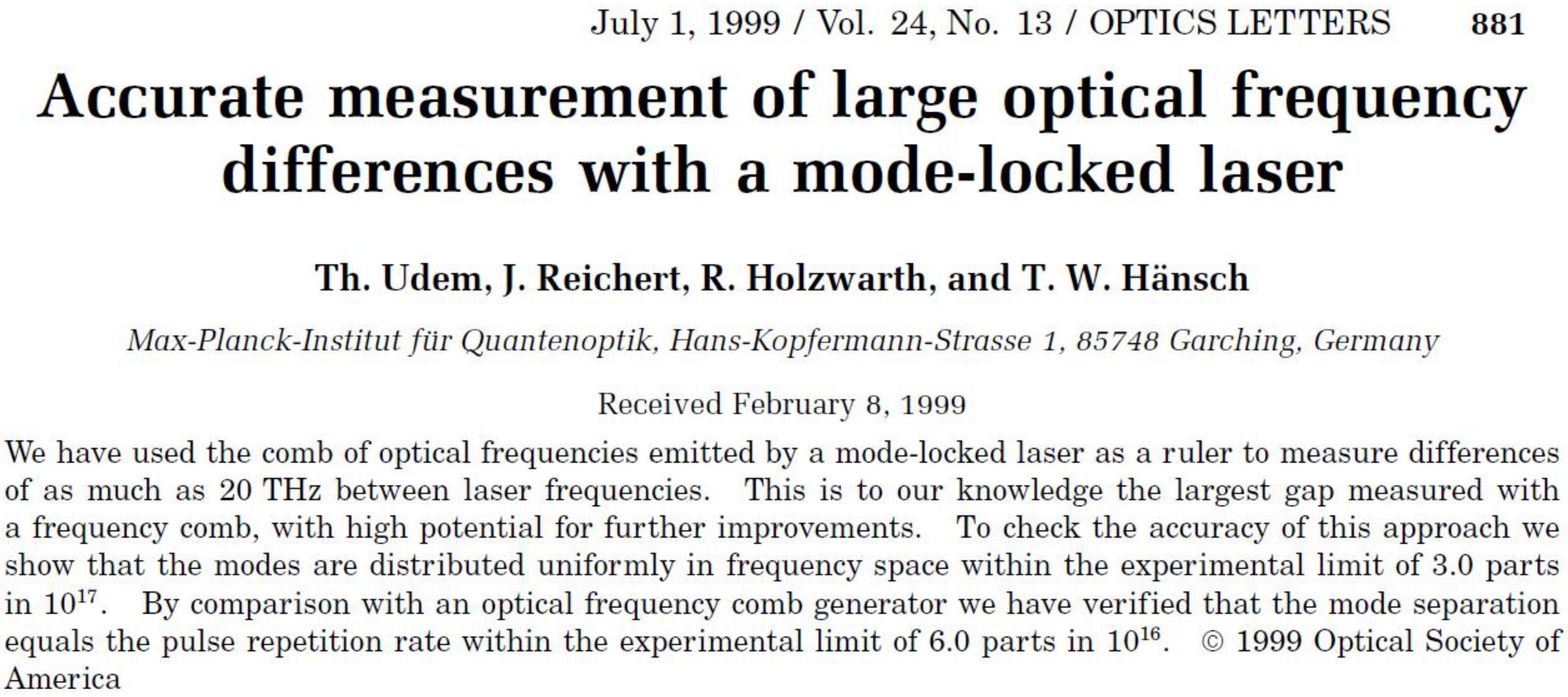


Th. Udem, lab book 1994

Phase Locked Loop



Testing the Mode Spacing Constancy



July 1, 1999 / Vol. 24, No. 13 / OPTICS LETTERS **881**

Accurate measurement of large optical frequency differences with a mode-locked laser

Th. Udem, J. Reichert, R. Holzwarth, and T. W. Hänsch

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany

Received February 8, 1999

We have used the comb of optical frequencies emitted by a mode-locked laser as a ruler to measure differences of as much as 20 THz between laser frequencies. This is to our knowledge the largest gap measured with a frequency comb, with high potential for further improvements. To check the accuracy of this approach we show that the modes are distributed uniformly in frequency space within the experimental limit of 3.0 parts in 10^{17}. By comparison with an optical frequency comb generator we have verified that the mode separation equals the pulse repetition rate within the experimental limit of 6.0 parts in 10^{16}.

Testing the Mode Spacing Constancy

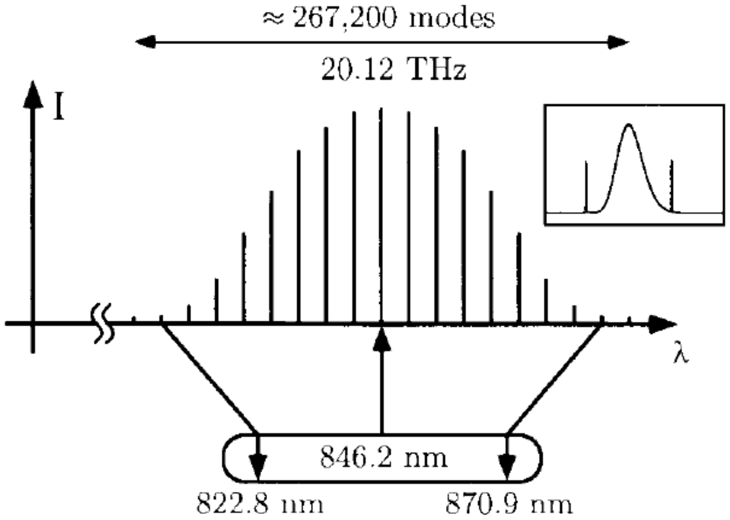


Fig. 2. The uniform distribution of the modes of a mode-locked laser is verified by comparison with an optical frequency interval divider. Inset, measured spectrum of the frequency comb together with the 822.8- and the 870.9-nm laser diodes drawn into it.

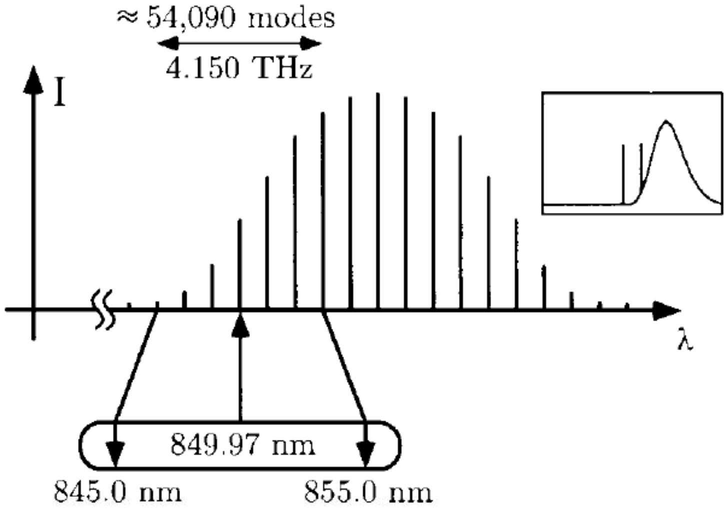


Fig. 3. Verifying the uniform distribution of the modes of a mode-locked laser on one side of its spectrum. Inset, measured spectrum of the frequency comb together with the 845- and 855-nm laser diodes drawn into it.

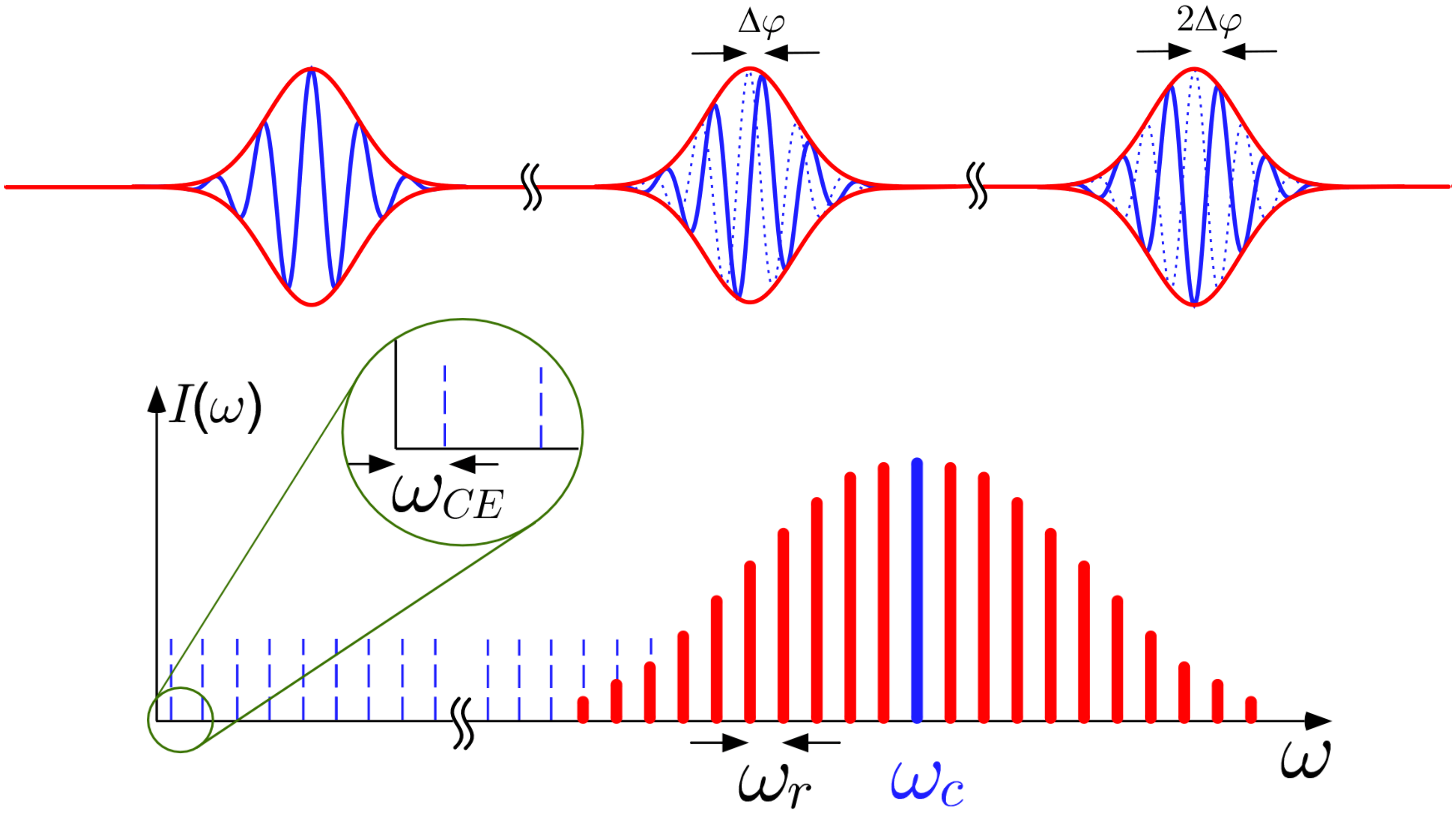
Table 1. Results from the Setup of Fig. 2 with Statistical Uncertainties Derived from the Data[a]

Gate Time (s)	Mean Deviation (mHz) from 40 MHz	Relative Deviation	Approved Reading	Cycle-Slip Threshold	Number of Cycle Slips
1	-0.6 ± 2.4	1.2×10^{-16}	8442	0.5 Hz	202
10	-1.93 ± 0.73	9.5×10^{-17}	2936	50 mHz	257
100	0.54 ± 0.67	3.4×10^{-17}	338	5 mHz	179

Derivation of the Comb

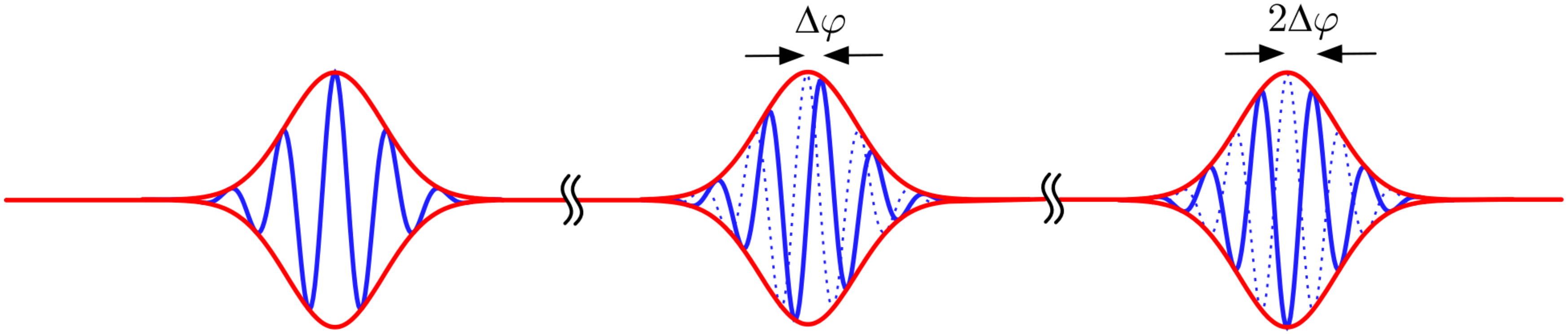
(from the pulse train)

Mode Locked Laser

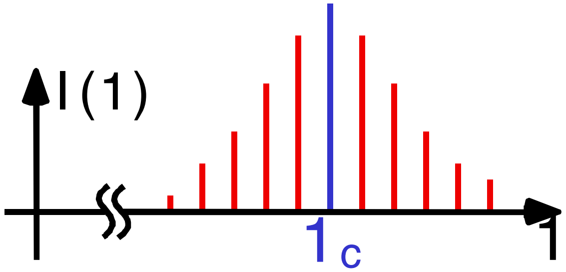


$$\boxed{1_n = n1_r + 1_{CE} \text{ with } 1_{CE} < 1_r}$$

Mode Locked Laser



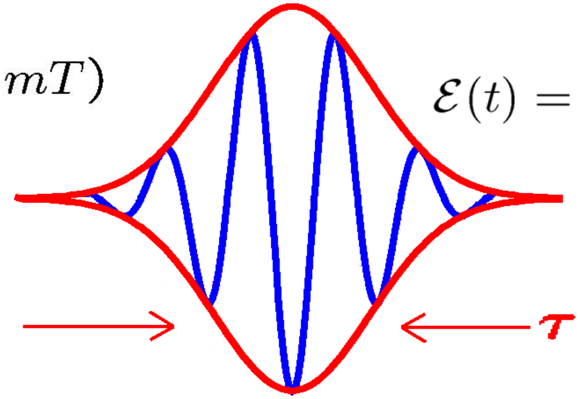
$$E(t) = A(t)\, e^{-i\omega_c t} = \sum_{m=-\infty}^{+\infty} A_m\, e^{-im\omega_r t - i\omega_c t}$$



$$\boxed{\omega_m = m\omega_r + \omega_C}$$

Spectrum of N+1 Pulses

$$E(t) = \frac{E_o}{\sqrt{N}} \sum_{m=0}^{N} e^{im\Delta\varphi} \mathcal{E}(t - mT)$$



$$\mathcal{E}(t) = E_o e^{-t^2/2\tau^2} \cos(\omega_c t + \varphi)$$

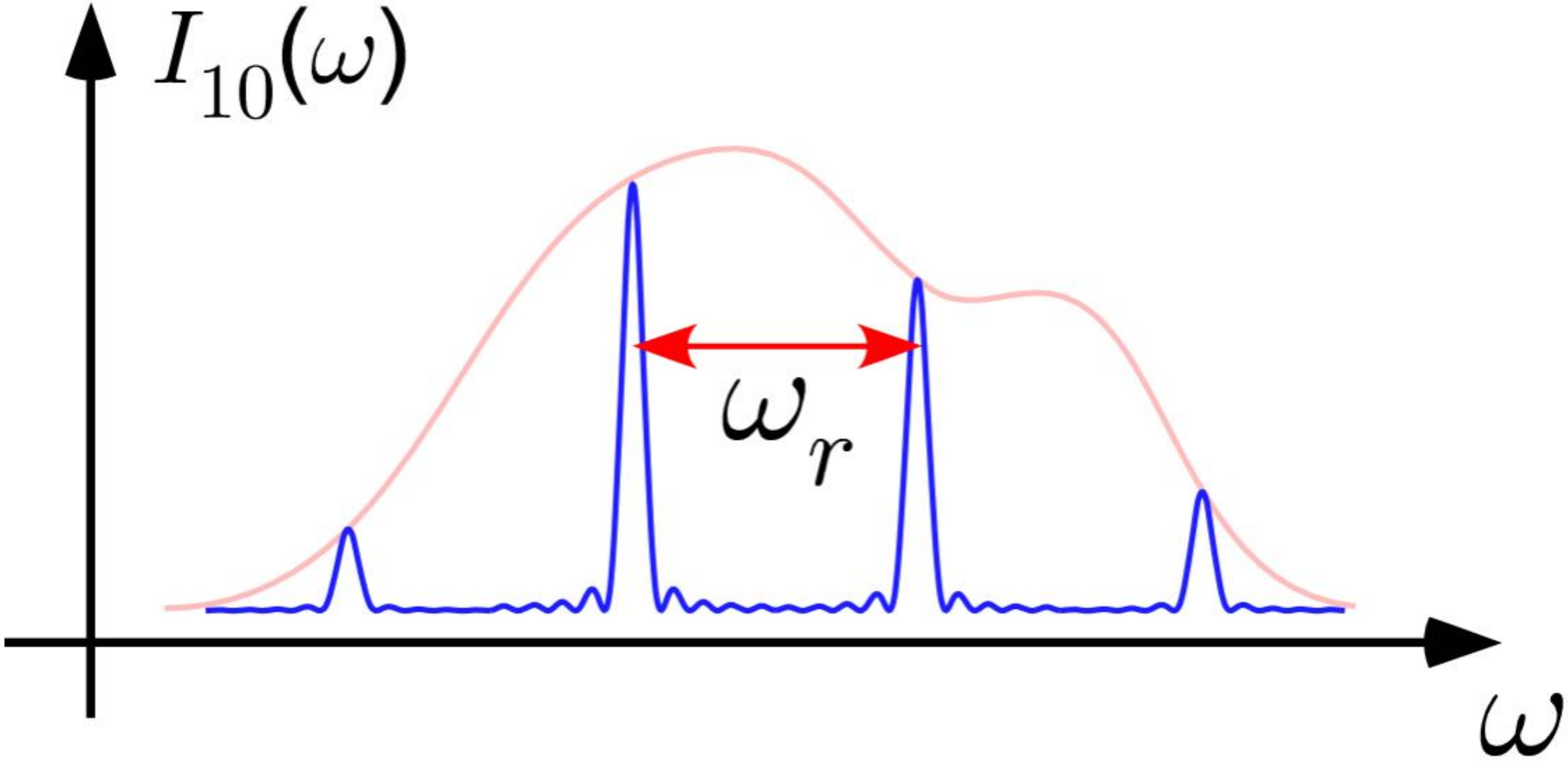
$$\mathcal{FT}\{\mathcal{E}(t - \Delta t)\} = e^{-i\omega\Delta t} \mathcal{FT}\{\mathcal{E}(t)\}$$ (shift theorem)

$$\tilde{E}(\omega) = \frac{E_o \tilde{\mathcal{E}}(\omega)}{\sqrt{N}} \sum_{m=0}^{N} e^{-im(\omega T - \Delta\varphi)} = \frac{E_o \tilde{\mathcal{E}}(\omega)}{\sqrt{N}} \frac{1 - e^{-iN(\omega T - \Delta\varphi)}}{1 - e^{-i(\omega T - \Delta\varphi)}}$$

$$I(\omega) \propto |\tilde{\mathcal{E}}(\omega)|^2 \Rightarrow I_N(\omega) = \frac{1 - \cos(N(\omega T - \Delta\varphi))}{N(1 - \cos(\omega T - \Delta\varphi))} \; I(\omega)$$

Spectrum of N+1 Pulses

$$I(\omega) \propto |\tilde{\mathcal{E}}(\omega)|^2 \Rightarrow I_N(\omega) = \frac{1-\cos(N(\omega T - \Delta\varphi))}{N(1-\cos(\omega T - \Delta\varphi))}\ I(\omega)$$



Fourier limited Line Width of the Modes

$$I(\omega) \propto |\tilde{\mathcal{E}}(\omega)|^2 \Rightarrow I_N(\omega) = \frac{1-\cos(N(\omega T - \Delta\varphi))}{N(1-\cos(\omega T - \Delta\varphi))} \; I(\omega)$$

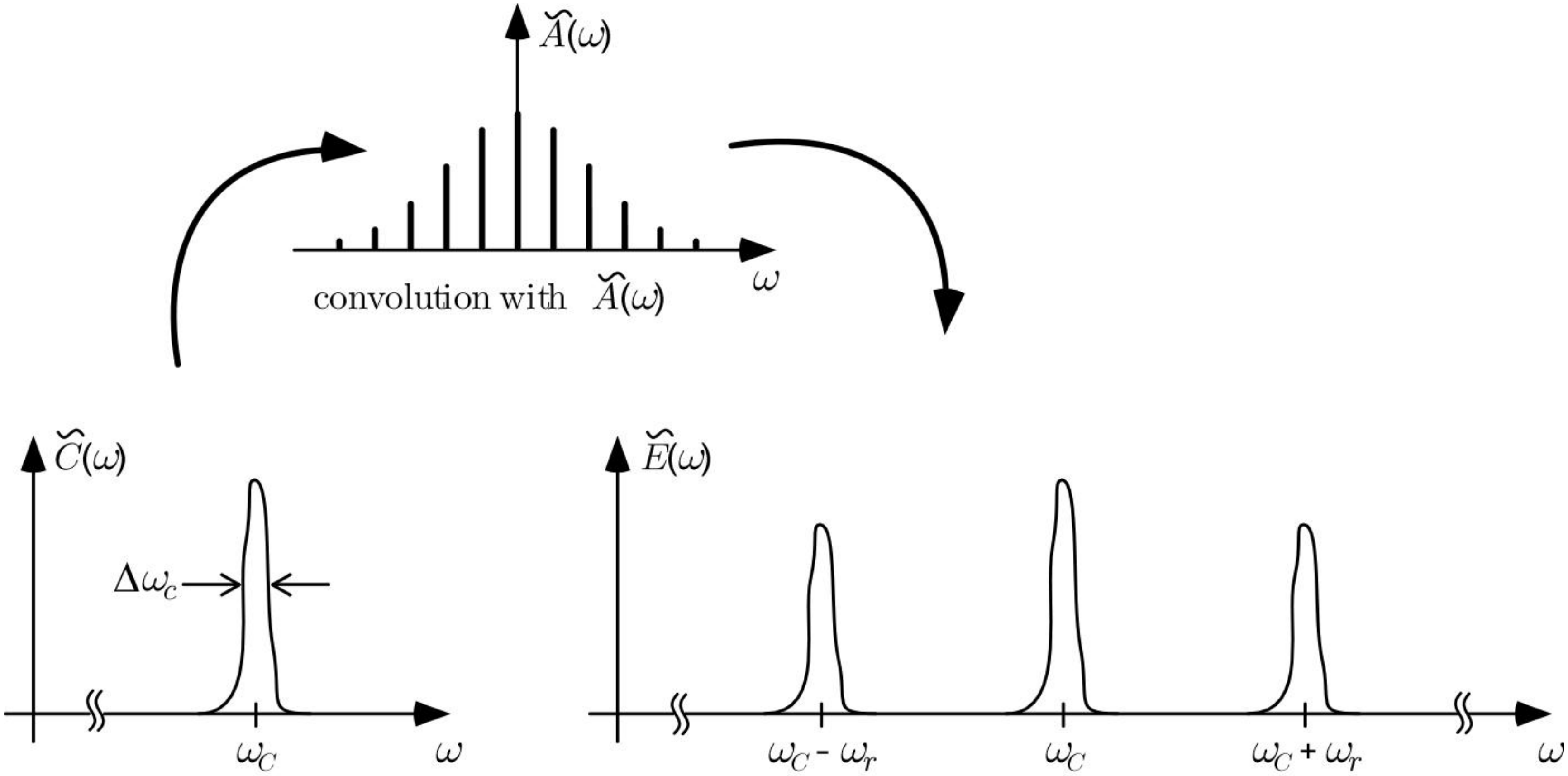
$$\frac{1}{2\pi} \lim_{N\to\infty} \frac{1-\cos(Nx)}{N(1-\cos(x))} \approx \frac{1}{\pi} \lim_{N\to\infty} \frac{1-\cos(Nx)}{Nx^2} = \delta(x)$$

$$I_N(\omega) \to I(\omega) \sum_n \delta(\omega T - \Delta\varphi - 2\pi n) \to \omega_n = \Delta\varphi/T + 2\pi n/T$$

$$\text{line width} \approx \frac{1}{2\pi}\sqrt{\frac{24}{T^2(N^2-1)}} \to 1/t_{obs} \quad \text{with} \quad N = t_{obs}/T$$

alternatively: line width $\approx \omega_r/N$

Line Width of real Lasers

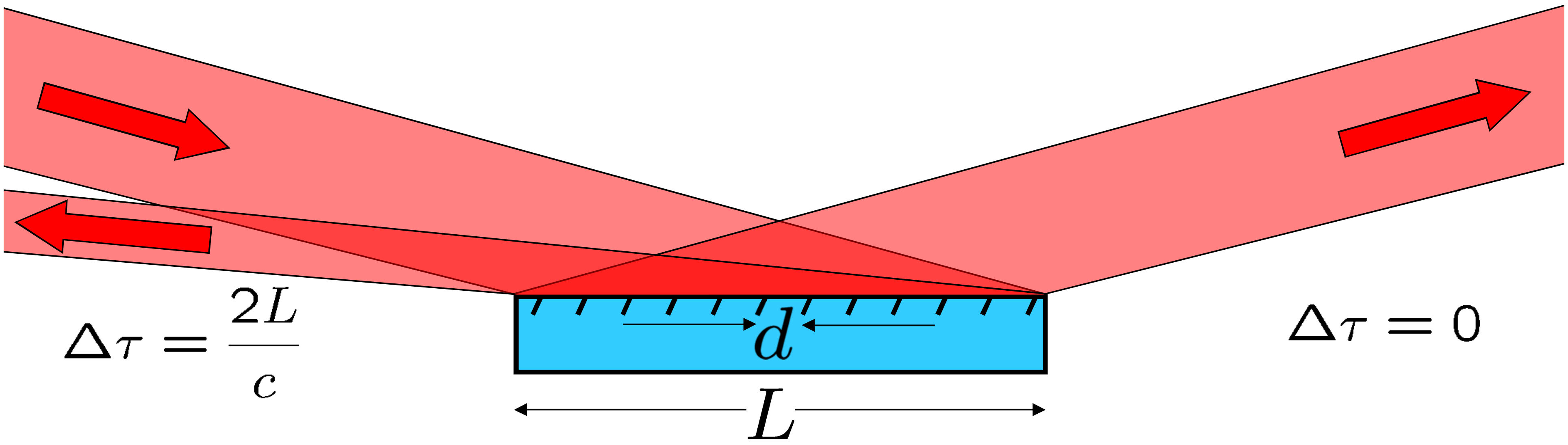


Resolving the Modes of the Comb

$$\frac{\lambda}{\Delta\lambda} = \frac{n\omega_r + \omega_{CE}}{\omega_r} = \frac{\omega_n}{\omega_r}$$

$$d = \frac{\lambda}{2} \Longrightarrow L = \text{length of laser cavity}$$

$$\Delta\nu = \frac{\nu}{N} = \frac{\nu d}{L} = \frac{\nu\lambda}{2L} \Longrightarrow \Delta\nu\Delta\tau = 1$$



$$\Delta\tau = \frac{2L}{c}$$

$$\Delta\tau = 0$$

Self-Referencing

How to Measure the Comb Offset

Measure any frequency difference between different harmonics of the same laser (or comb).

$$N(n\omega_r + \omega_{CE}) - M(n'\omega_r + \omega_{CE}) \underset{\substack{\Big\uparrow \\ Nn - Mn' = 0}}{=} (M - N)\omega_{CE}$$

bandwidth requirement: $\Delta n = \frac{M-N}{M} < n >$

15 December 1999

Optics Communications 172 (1999) 59–68

OPTICS
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Measuring the frequency of light with mode-locked lasers [1]

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Received 1 July 1999; received in revised form 20 August 1999; accepted 1 September 1999

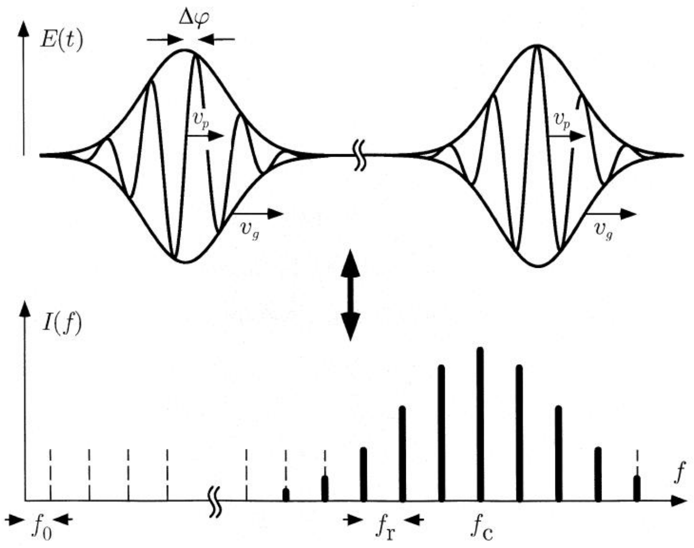
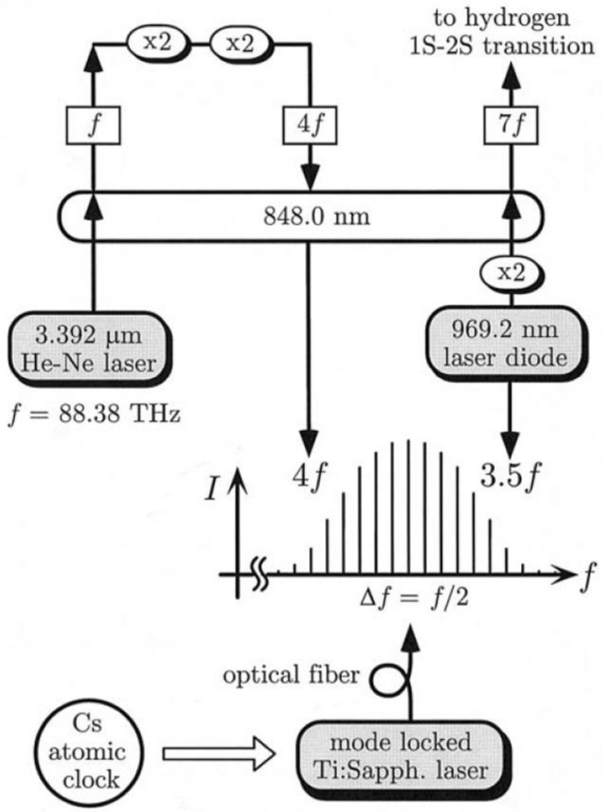
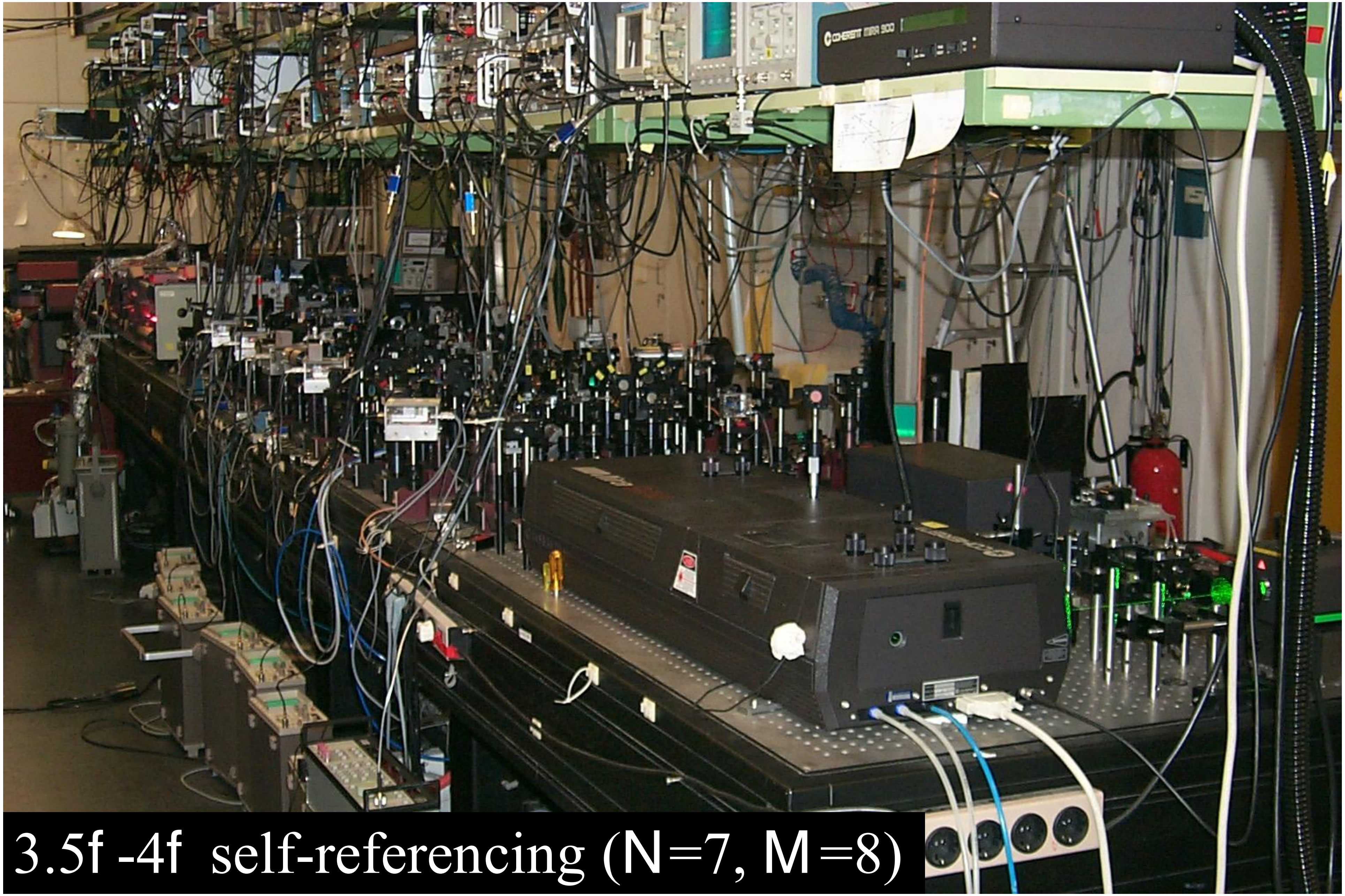


Fig. 1. Two consecutive pulses of the pulse train emitted by a mode-locked laser and intensity spectrum of the train. Within the cavity, the envelope is traveling with the group velocity v_g which, in general, differs from the phase velocity of the carrier v_p. The carrier phase relative to the envelope changes from pulse to pulse by $\Delta\varphi$. The modes are offset from being integer multiples of the pulse repetition rate f_r by $f_0 = (\Delta\varphi/2\pi) f_r$.

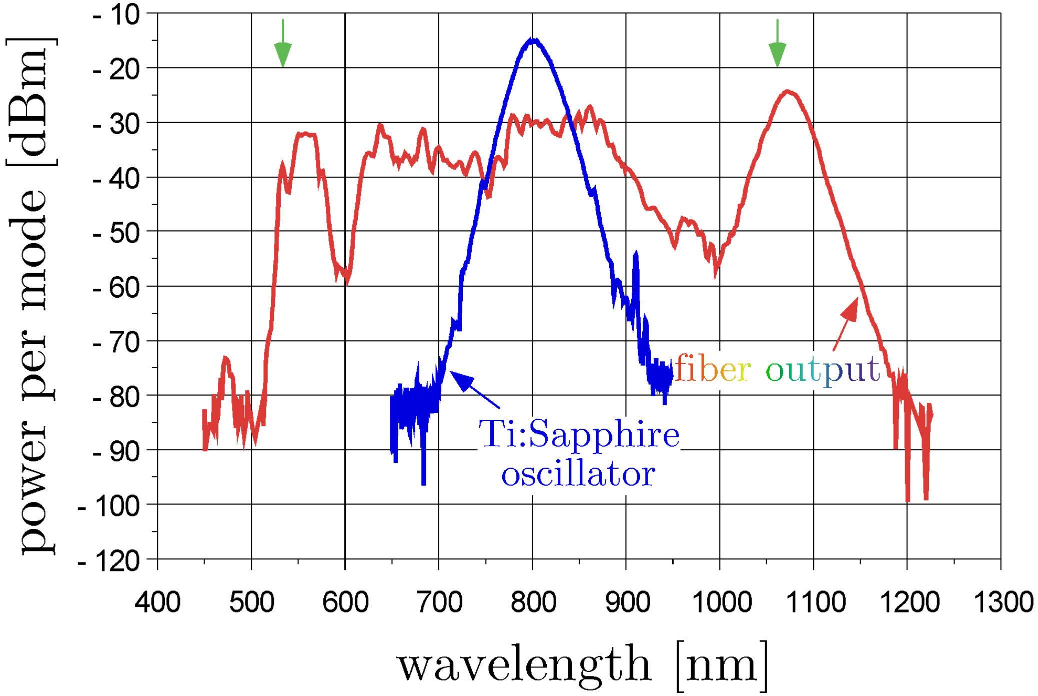


The first self-referenced Frequency Comb



3.5f -4f self-referencing (N=7, M=8)

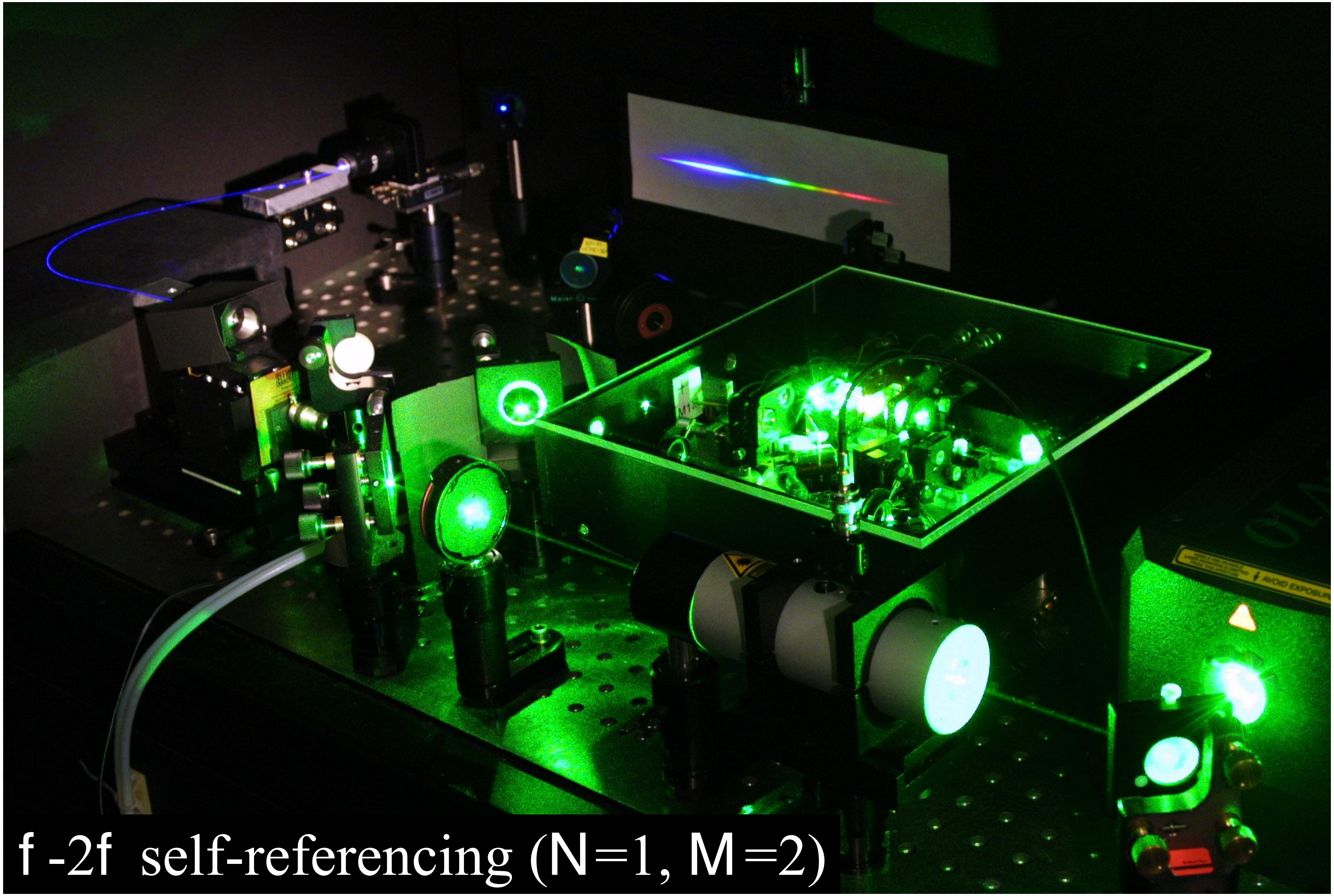
Generating an Octave Spanning Comb



Photonic crystal fiber:
William Wadsworth
Jonathan Knight
Tim Birks
Phillip Russell
U. of Bath England

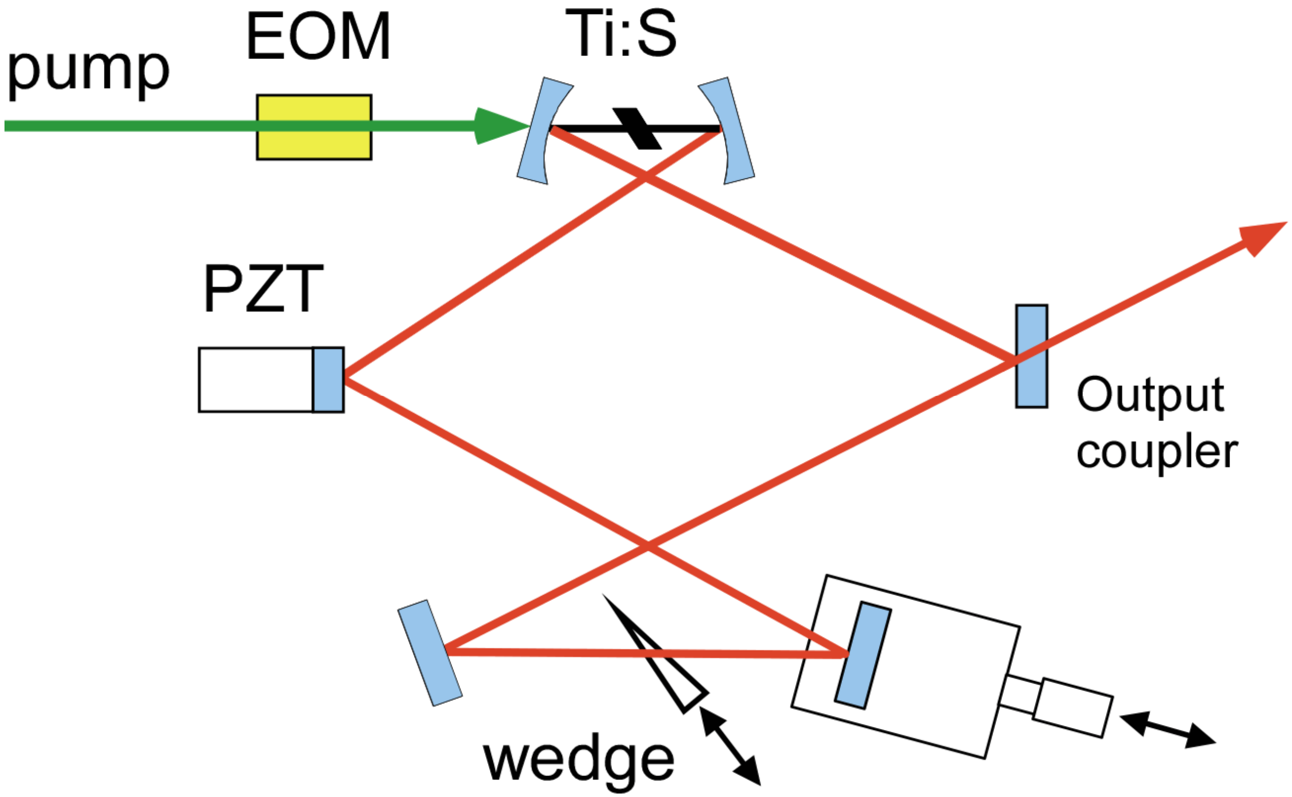
note: if the action of the fiber is the same for all the pulses the field stays stricly periodic. This property is the only one necessary to derive $\omega_n = n\omega_r + \omega_{CE}$

A much more compact Device



f -2f self-referencing (N=1, M=2)

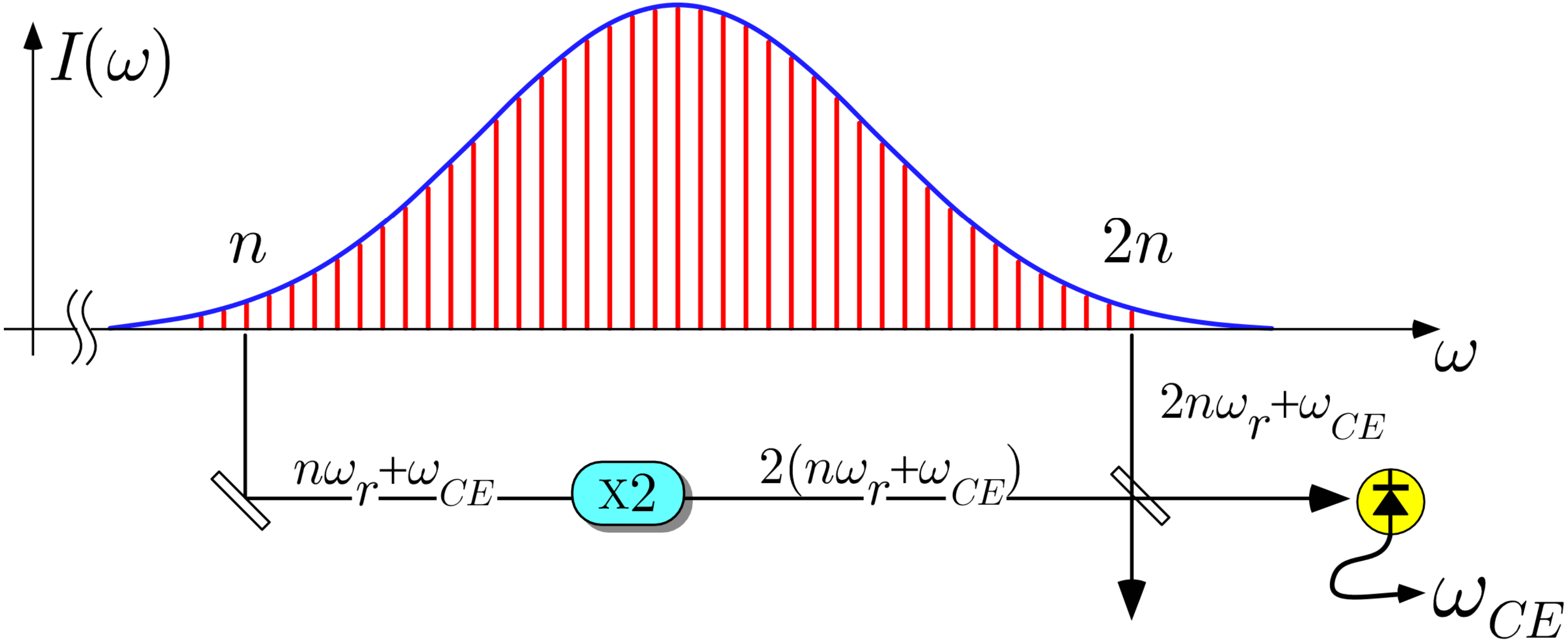
Compact Ringlaser



- wedge and EOM for slow and fast 1_{CE} control.
- translation stage and PZT for slow and fast 1_r control.

Simplest way of Self Referencing: M=2 N=1

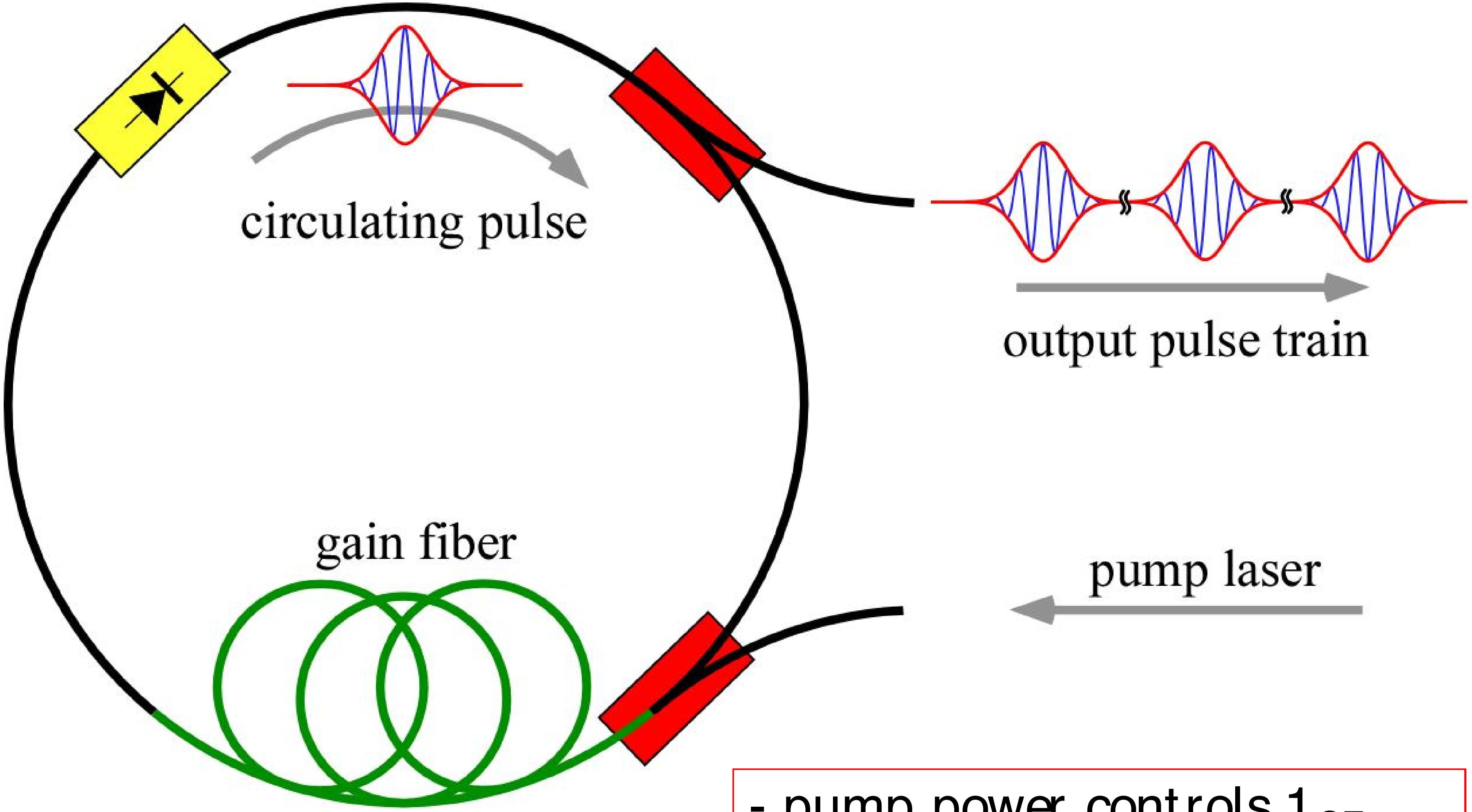
It is simple to detect 1_{CE} of an octave wide frequency comb:



$$1_{CE} = 2(n1_r + 1_{CE}) - (2n1_r + 1_{CE})$$

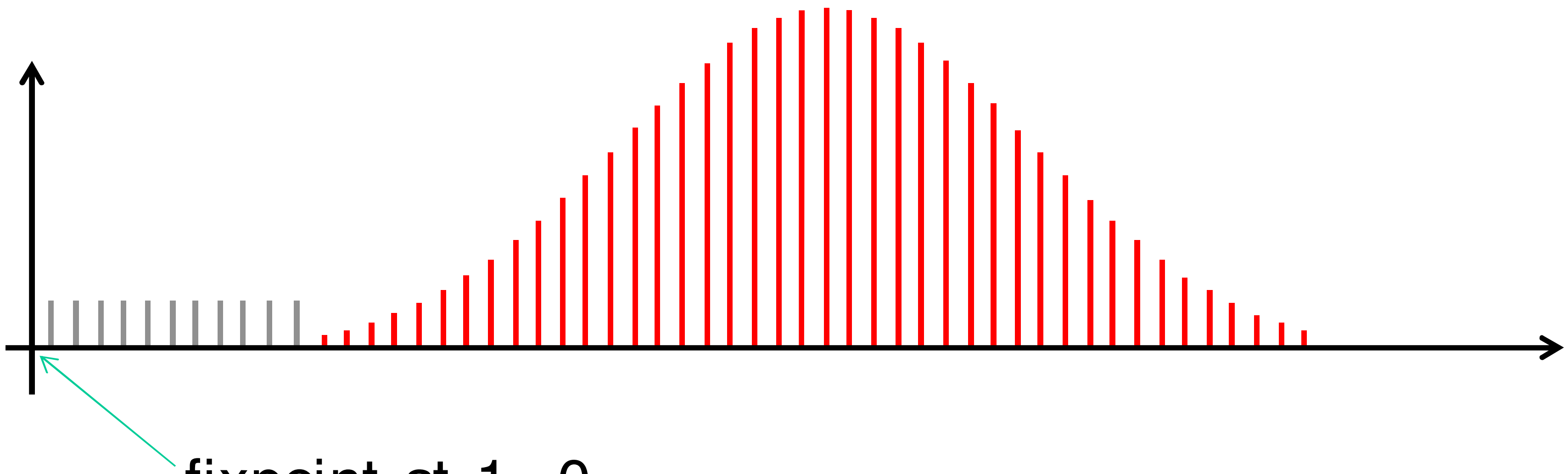
J. Reichert *et al.*, PRL 84, 3232 (2000)

Fiberlaser



- pump power controls 1_{CE}
- fiber stretcher for 1_r control

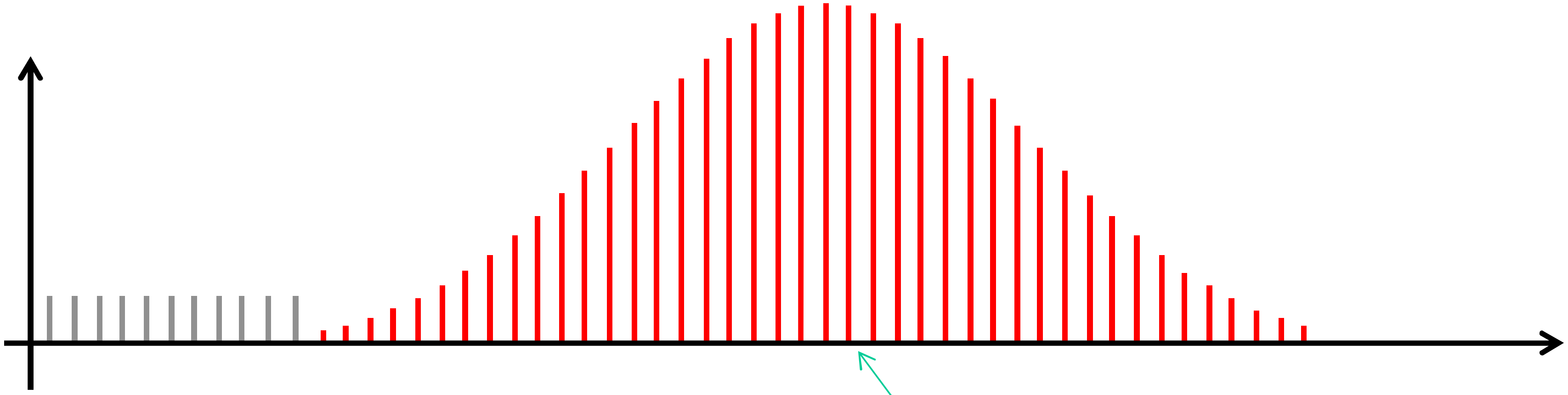
Fixpoint Concept I



fixpoint at 1=0

- Cavity length L „elastic tape“ : $(n1_r + 1_{CE})\,(1 + \Delta L/L)$

Fixpoint Concept II



fixpoint at $m = 0$ ($1 = 1_C$)

- Pump power „accordion“ : $m1_r(1+\alpha) + 1_C$

➡ Fixpoint important for locking and noise compensation!

➡ Better to enumerate mode number m from fixpoint.

Controlling the Frequency Comb

depends on the cavity length

$$1_n = n1_r + 1_{CE}$$

depends on the pump power

we can measure and control

$1_r = 2\pi/T$ and $1_{CE} = \Delta 7/T$

Optical Frequency Counter

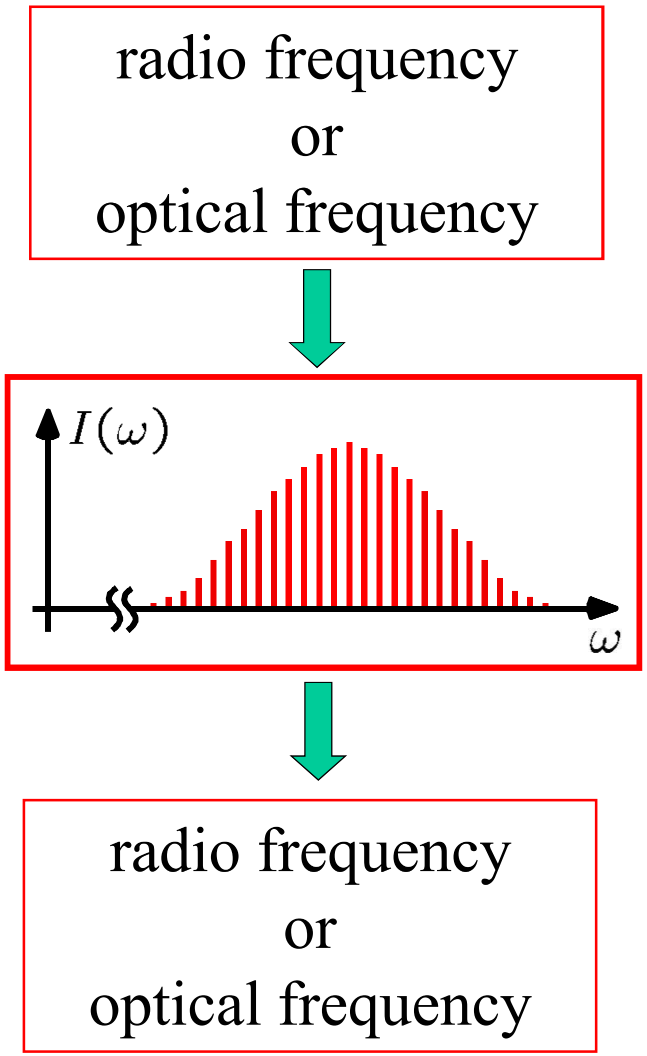
locked to a Cs atomic clock

$$1_n = n1_r + 1_{CE}$$

every mode can be used for
optical frequency measurement

a million stabilized lasers in a single beam!

Frequency Conversions with the Comb



radio frequency
or
optical frequency

radio frequency
or
optical frequency

radio frequency ➡ optical frequency

locked to a Cs clock

$$\omega_n = n\omega_r + \omega_{CE}$$

optical frequencies

optical frequency ➡ radio frequency

locked to a stable laser

locked to ω_r

$$\omega_n = n\omega_r + \omega_{CE}$$

countable clock output

optical frequency ➡ optical frequency

locked to a stable laser

locked to ω_r

$$\omega_n = n\omega_r + \omega_{CE}$$

measure another laser

Science 293, 825 (2001)

Thank you for your Attention