



Detection of Gravitational Waves with laser interferometers: focus on Virgo Laser and optics

Winter College on Optics, Trieste, February 24th, 2016

Eric Genin European Gravitational Observatory on behalf of the LIGO Scientific and VIRGO collaborations







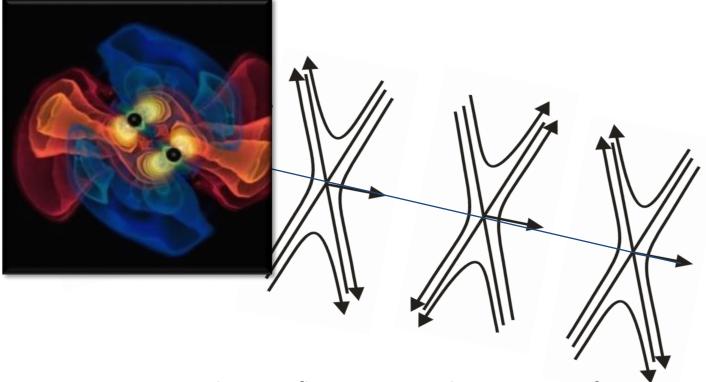


- GW detection/GW150914
- □ A Laser interferometer to detect Gravitational waves
- □ Advanced Virgo/Ligo: Laser and optics



Introduction to Gravitational Waves

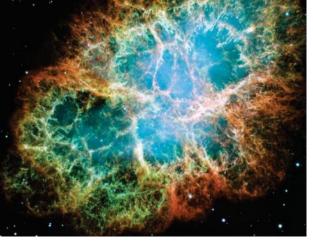




Gravitational waves are propagating dynamic fluctuations in the curvature of spacetime ('ripples' in space-time)

Predicted by Einstein 100 years ago; confirmation by Hulse/Taylor/Weisberg

Emitted from accelerating mass distributions Sourced by the time-dependence of the quadrupole mass moment Practically, need massive objects at speeds approaching the speed of light GWs carry *direct* information about the relativistic motion of bulk matter *Winter college on Optics, Trieste, February 24, 2016* E. Genin



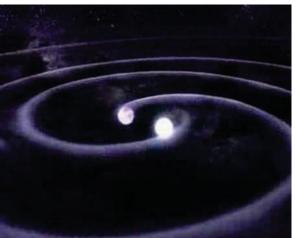
Supernovae



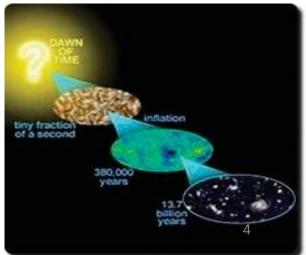
Rotating neutron stars

Hunting the GW signals

Coalescent Binary Sytem



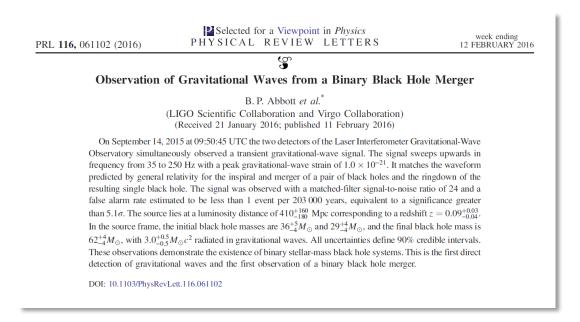
GW stochastic background



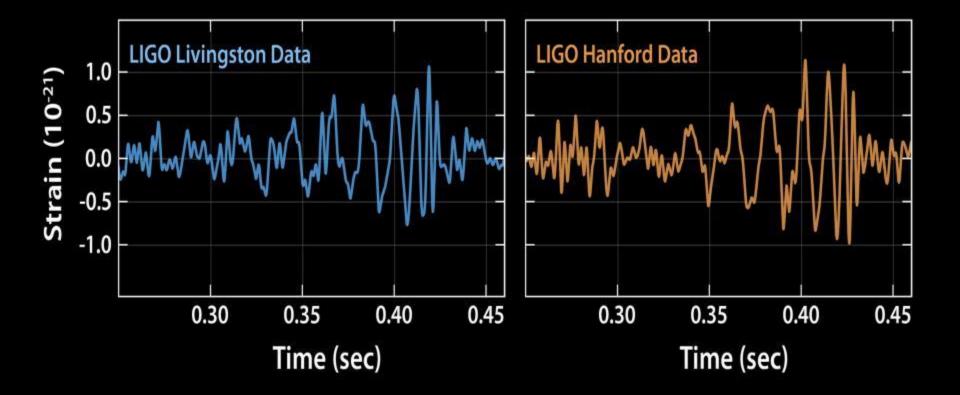


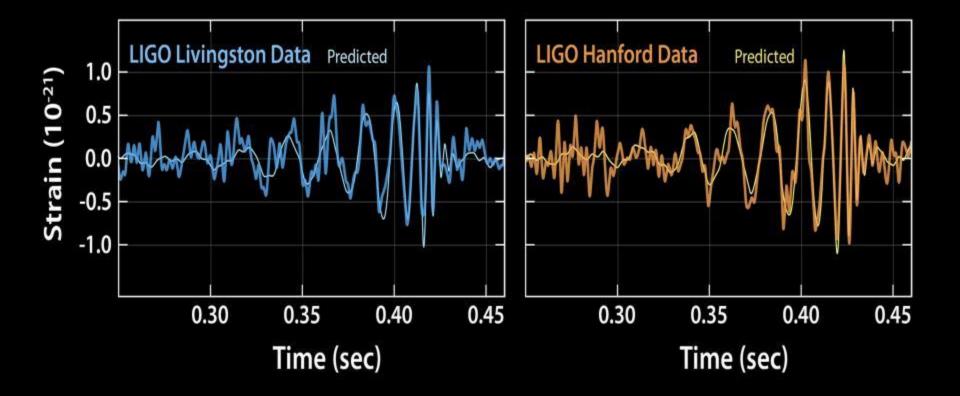
The first event ((()))VIRG

- On September 14, 2015 at 09:50:45 UTC the LIGO Hanford, WA, and Livingston, LA, observatories detected a coincident signal.
- □ The event was flagged as GW150914
- Exhaustive investigations of instrumental and environmental disturbances were performed, giving *no evidence* that GW150914 is an instrumental artifact
- \rightarrow Opens new perspectives, a new way to look at the Universe.



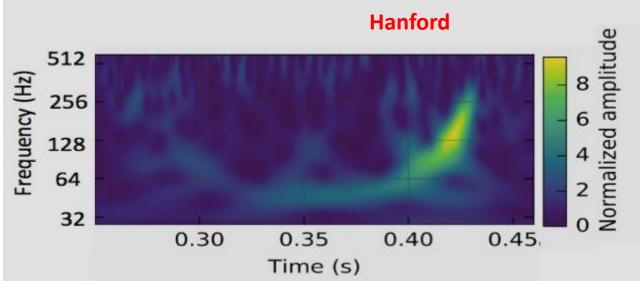
http://journals.aps.org/prl/pdf/10.1103/PhysRevLett.116.061102

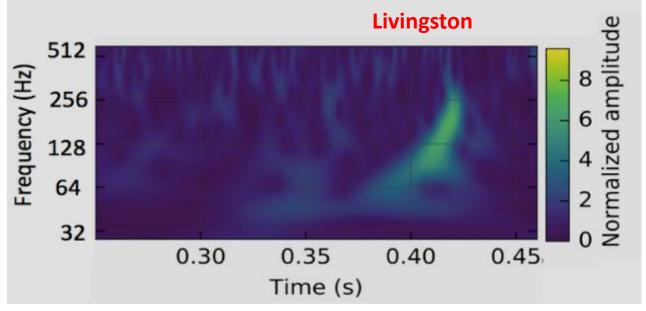






((O))/VIRGD





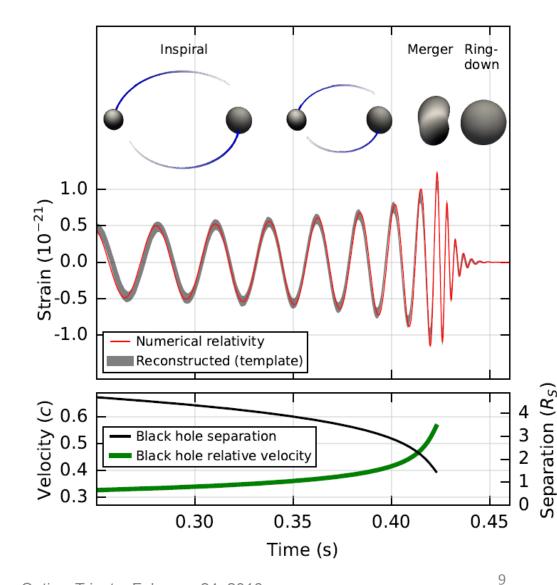
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Estimated GW Strain Amplitude ((O)) VIRG for GW150914

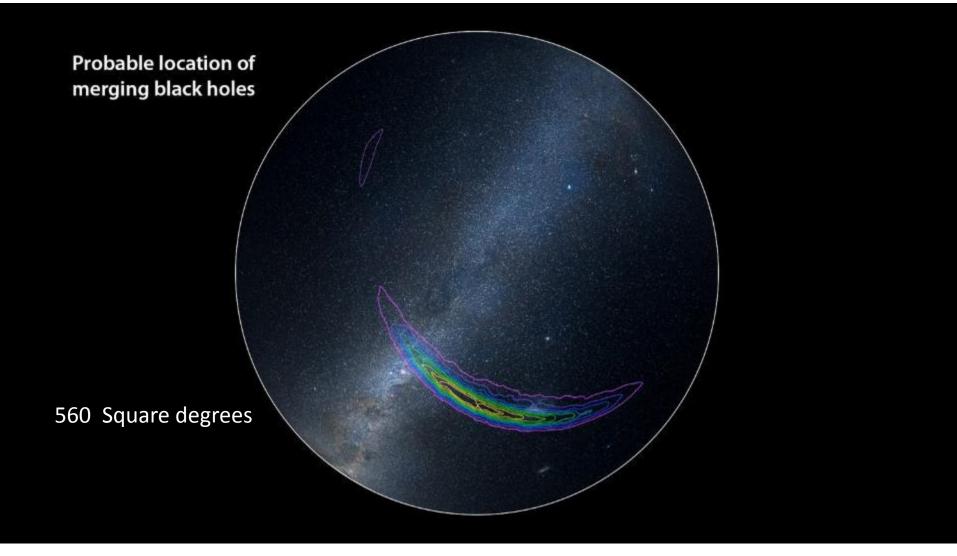
Full bandwidth waveforms without filtering. Numerical relativity models of black hole horizons during coalescence

Effective black hole separation in units of Schwarzschild radius (R_s =2GM/c²); and effective relative velocities given by post-Newtonian parameter v/c = (GMpf/c³)^{1/3}











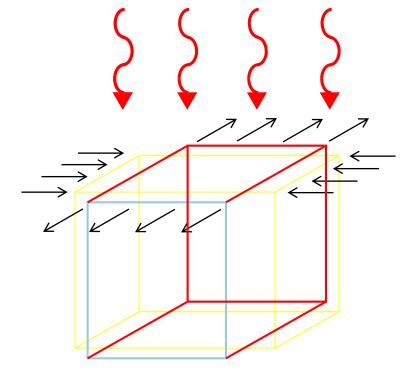


A Laser interferometer to detect GW

The physical Effect

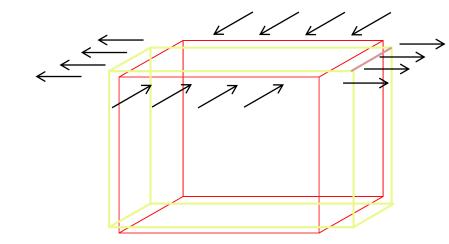
GW squeeze and stretch the space in perpendicular directions

- \rightarrow Deformation of elastic bodies
- \rightarrow Displacement of free masses



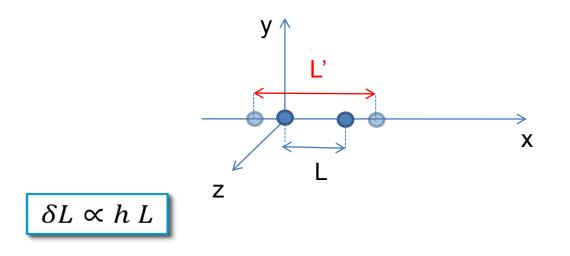
To detect GW:

monitor distances between free masses





The effect of Gravitational (((O))) VIRGO Waves on free falling masses



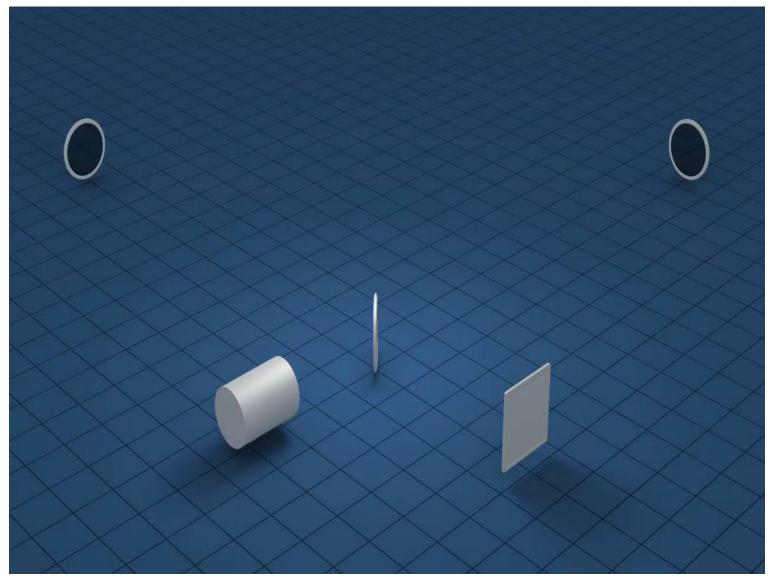
Very weak amplitude: $h \approx 10^{-21}$

The distance between two masses separated by \sim Km will change by $\delta L \approx 10^{-18}$ m

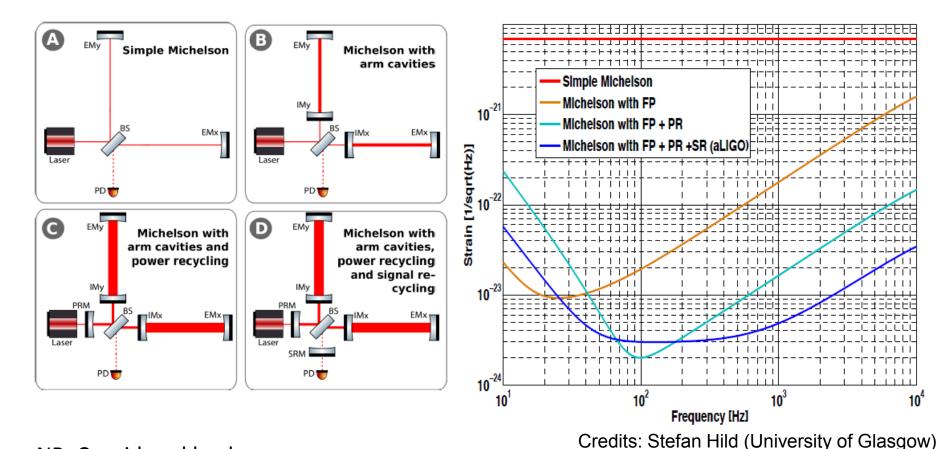
"That is comparable to a hair's-width change in the distance from the Sun to Alpha Centauri, its nearest star".



The solution: use a Michelson Interferometer



How to improve the Michelson interferometer sensitivity

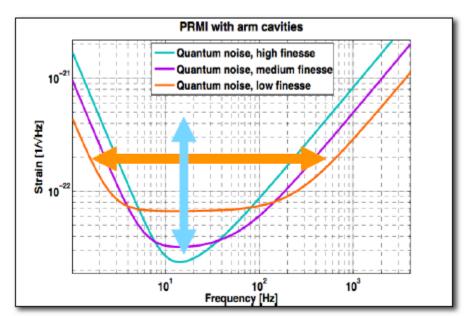


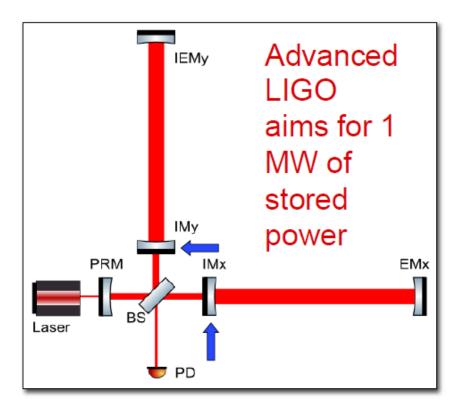
NB: Considered km long arms.

Arm cavity property

Increasing the storage time in the arms by using arm cavities.

Finesse of the arm cavities determines bandwidth of GW detector.





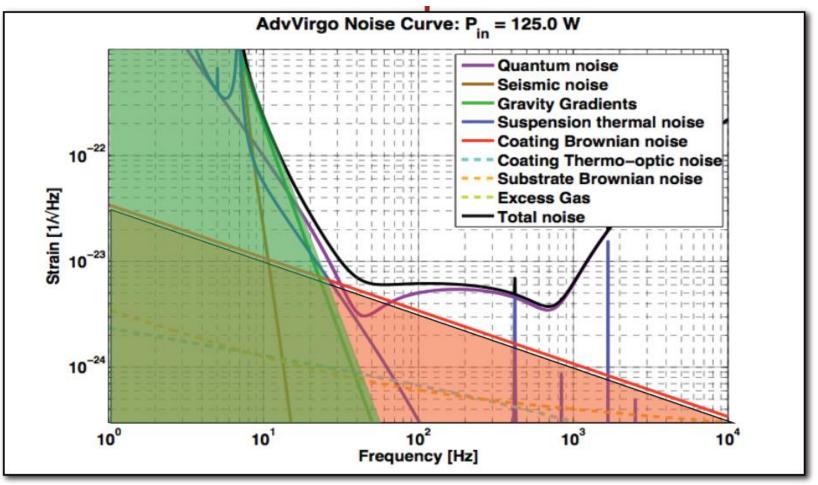
Credits: Stefan Hild (University of Glasgow)

NB: Considered km long arms.



Typical sensitivity curve for Advanced Virgo





Credits: Stefan Hild (University of Glasgow)

Mostly limited by quantum noise over the whole bandwidth.

But also by gravity gradient noise at low frequency

and coating thermal noise in mid frequency range



Location in the sky (IO)/VIRGO

GW laser interferometers are not pointing telescopes,

- Sky location can be reconstructed through the time of arrival of GW radiation at the different detector sites, as well as the relative amplitude and phase of the GWs in different detectors.
- → It is mandatory to have a network of interferometer to better localize the source in the sky



H1- Hanford – Washington state





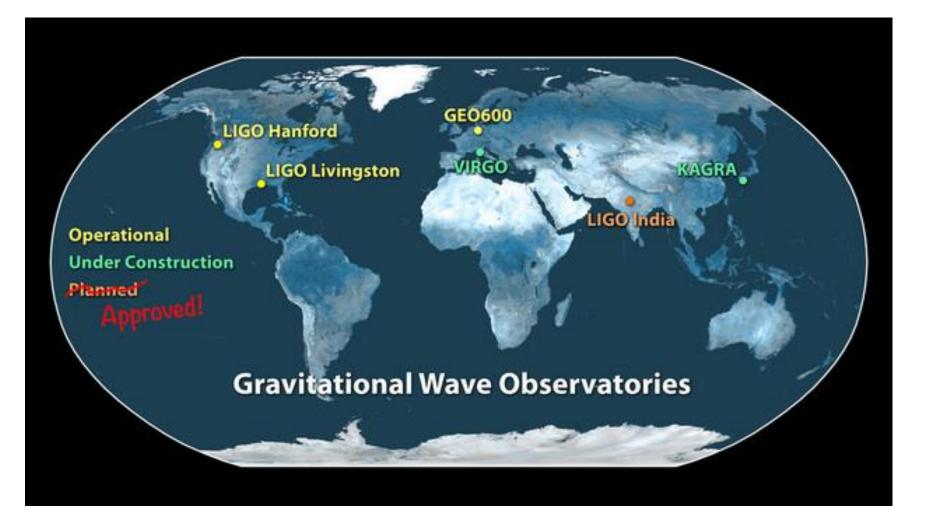
GEO600 – Hannover - Germany



L1- Livingston – Louisiana state



Expected network in coming years







((O))VIRGD

LIGO upgrade concluded

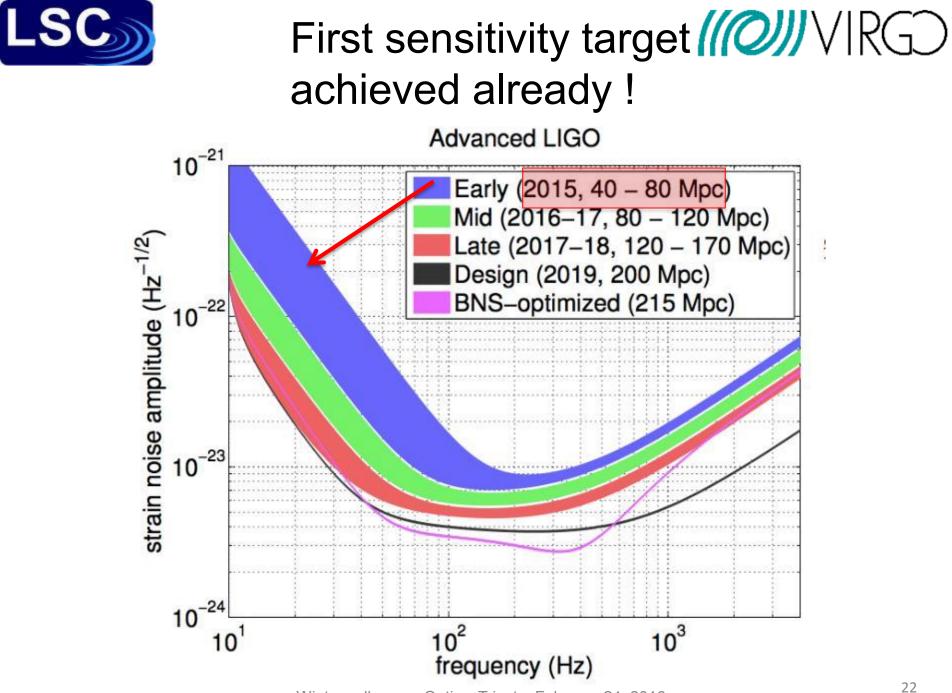
First data taking period from September 2015 to January 2016

H1- Hanford – Washington state

VIRGO will end the upgrade in 2016

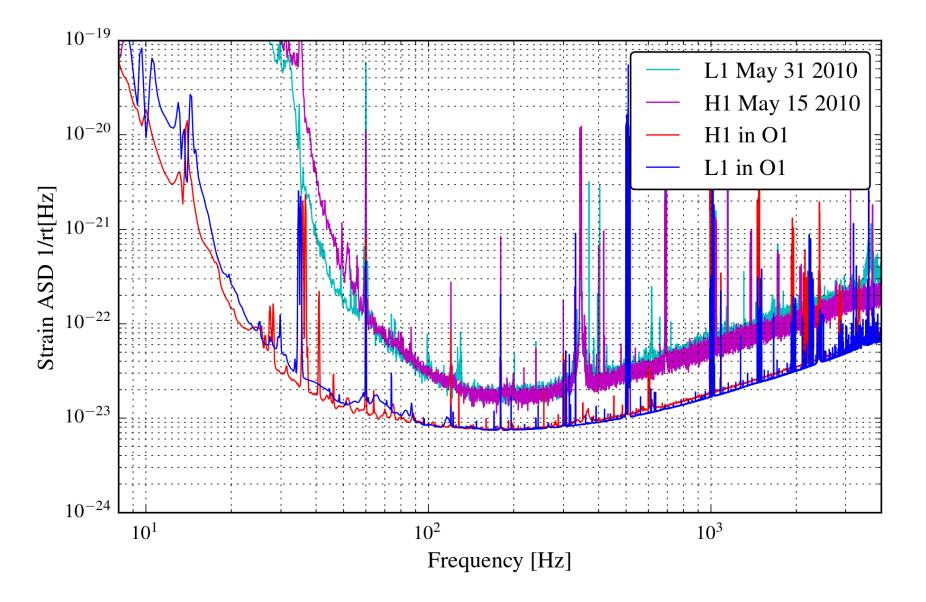


L1- Livingston – Louisiana state



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From LIGO to aLIGO: Sensitivity improvements



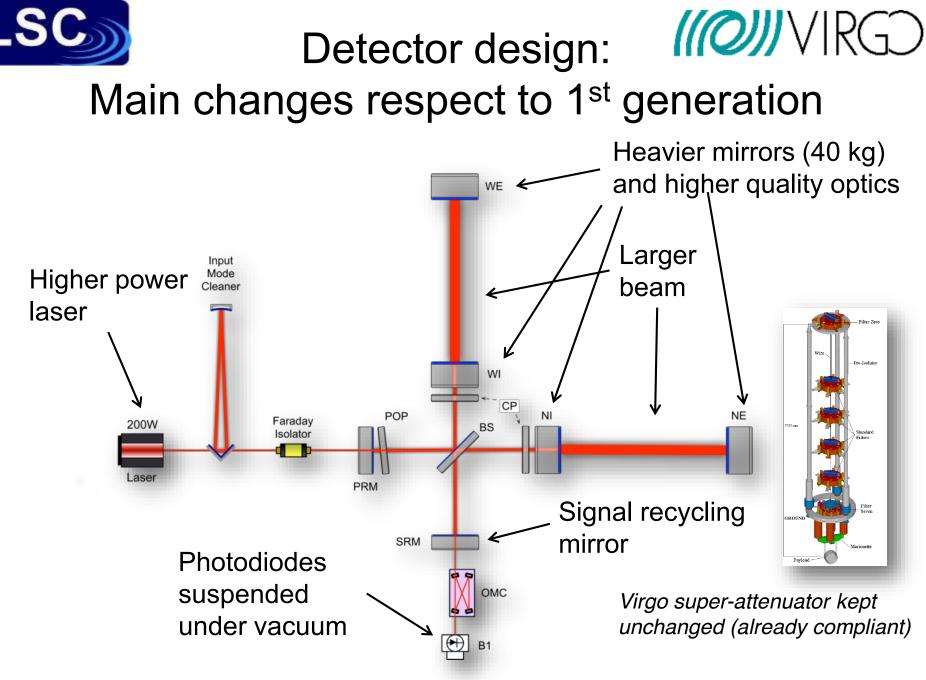


What is ADVANCED VIRGO?

- Advanced Virgo (AdV): upgrade of the Virgo interferometric detector of gravitational waves
- Participated by scientists from Italy and France (former founders of Virgo), The Netherlands, Poland and Hungary
- □ Funding approved in Dec. 2009
- Construction in progress. End of installation: Spring 2016
- □ First science data in 2016

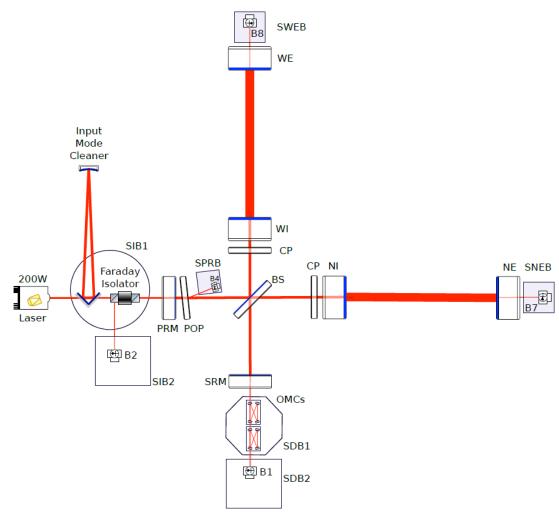
5 European countries 19 labs, ~200 authors

APC Paris ARTEMIS Nice EGO Cascina INFN Firenze-Urbino INFN Genova INFN Napoli INFN Perugia INFN Perugia INFN Pisa INFN Roma La Sapienza INFN Roma Tor Vergata INFN Roma Tor Vergata INFN Roma Tor Vergata INFN Trento-Padova LAL Orsay – ESPCI Paris LAPP Annecy LKB Paris LMA Lyon NIKHEF Amsterdam POLGRAW(Poland) RADBOUD Uni. Nijmegen RMKI Budapest





Virgo Overview ((O))/VIRGD



Advanced Virgo project baseline design

NB: 3km arm cavities linewidth=100Hz

AdV Overview, Part I Subsystem and Parameters AdV design (TDR) Initial Virgo Sensitivity Binary Neutron Star Inspiral Range 12 Mpc 134 Mpc $3.5 \cdot 10^{-24} / \sqrt{\text{Hz}}$ $4 \cdot 10^{-23} / \sqrt{\text{Hz}}$ Anticipated Max Strain Sensitivity Instrument Topology Michelson Interferometer Michelson Power Enhancement Arm cavities and Arm cavities and Power Recycling Power Recycling Signal Enhancement Signal Recycling n.a. Laser and Optical Powers Laser Wavelength $1064\,\mathrm{nm}$ $1064\,\mathrm{nm}$ >175 TEM₀₀ W Optical Power at Laser Output $20 \,\mathrm{W}$ Optical Power at Interferometer Input $125 \,\mathrm{W}$ 8 W Optical Power at Test Masses 6 kW 650 kW Optical Power on Beam Splitter 0.3 kW 4.9 kW Test Masses Mirror Material Fused Silica Fused Silica Main Test Mass Diameter $35\,\mathrm{cm}$ $35\,\mathrm{cm}$ Main Test Mass Weight $42 \, \mathrm{kg}$ 21 kg Beam Splitter Diameter $55\,\mathrm{cm}$ $23\,\mathrm{cm}$ Test Mass Surfaces and Coatings Coating Material Ti doped Ta₂O₅ Ta₂O₅ Roughness* < 0.05 nm< 0.1 nm $0.5 \,\mathrm{nm} \,\mathrm{RMS}$ Flatness $< 8 \,\mathrm{nm} \,\mathrm{RMS}$ Losses per Surface 37.5 ppm 250 ppm (measured) Test Mass RoC Input Mirror: 1420 m Input Mirror: flat End Mirror: 1683 m End Mirror: 3600 m Beam Radius at Input Mirror $48.7\,\mathrm{mm}$ $21\,\mathrm{mm}$ Beam Radius at End Mirror $58\,\mathrm{mm}$ $52.5\,\mathrm{mm}$ Finesse 443 50 Thermal Compensation Thermal Actuators CO₂ Lasers and CO₂ Lasers Ring Heater Actuation points Compensation plates Directly on mirrors and directly on mirrors Hartmann sensors Sensors n.a. and phase cameras

AdV figures vs Virgo (Extract of AdV technical design report)



The laser system ((O)) VIRGO

- The scope is to deliver a stable laser beam @ 1064 nm with the requested power, frequency stability and with small power fluctuations. So that the interferometer sensitivity can be achieved.
- We are relying on continuous technologic developments which allow us to start with a 20 W injection locked laser. This laser system has been further improved to deliver 50 Watts.
- A new more powerful (able to deliver 200 W CW at 1064 nm) is being developed: based on optical fiber laser technology.

 \rightarrow Challenging but seems to be able deliver the required power with the requested stability.

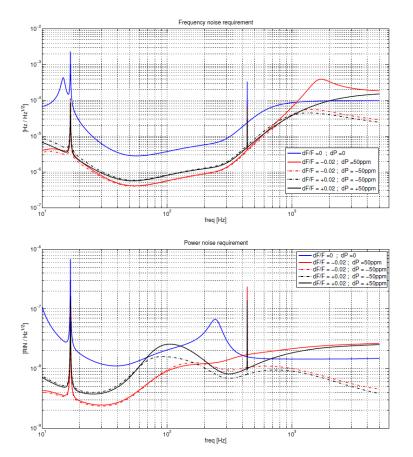


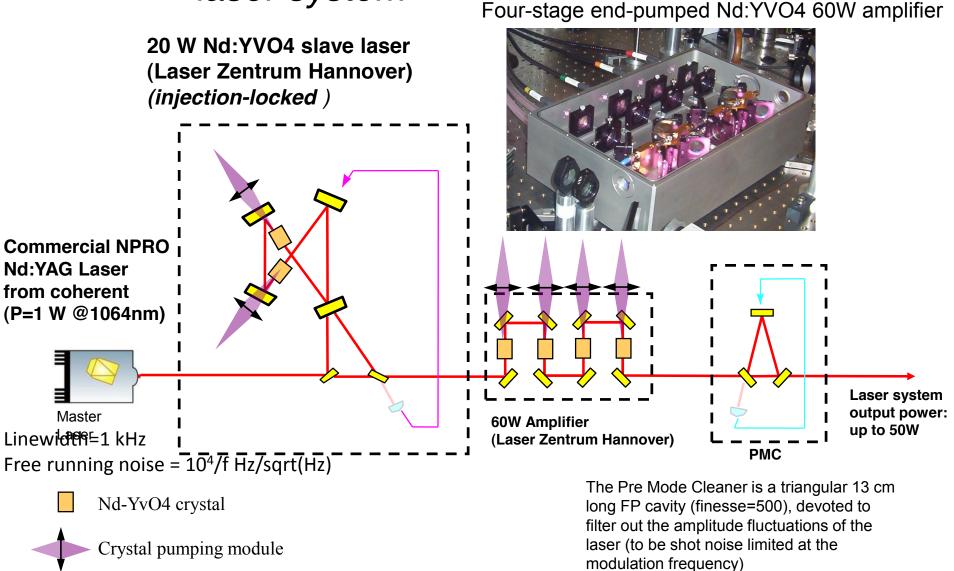
Figure 8.7: Dual recycled full power configuration, frequency and power noise requirements with a safety factor of ten used to draw the requirements from nominal sensitivity. Different values of finesse and loss asymmetries are used. Top: laser frequency noise at interferometer input. Bottom: laser intensity noise at the interferometer input. Blue curve: no defects. Red curves: dF/F = -2%; black curves: dF/F = +2%. Solid curves: dP = +50 ppm, dashed curves: dP = -50 ppm.

Requirements in term of frequency and power noise Over the whole detector bandwidth

Laser frequency stability required for arm cavity locking: 1 Hz rms over 1 s.



The Virgo/AdV (first phase) ((O))VIRGO laser system

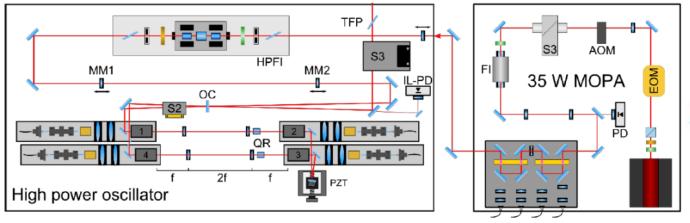




The aLigo laser system



200 W Nd:YVO4 slave laser (Laser Zentrum Hannover) (injection-locked)





Credits: O. Puncken (LZH)

Laser Amplifier (Laser Zentrum Hannover) (Description Commercial NPRO Nd:YAG Laser from coherent (P=2 W @1064nm) Linewidth=1 kHz

Free running noise = 10⁴/f Hz/sqrt(Hz)

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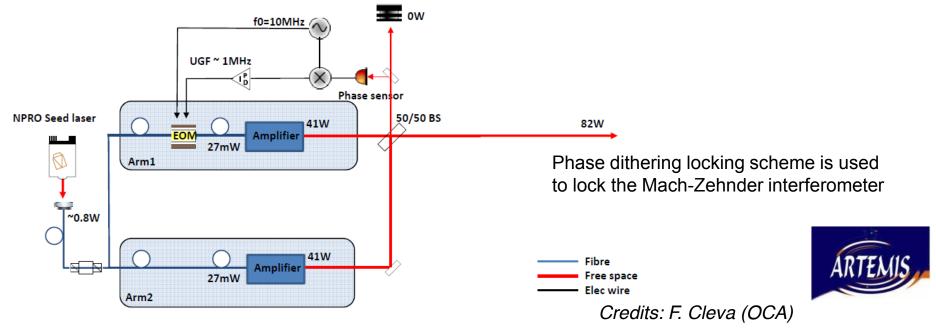
29



Development of a new laser source

- This laser using fiber amplifiers is currently being developed at Observatoire de la Cote d'Azur (Nice, France).
- Based on commercial 50 W fiber amplifier module produced by ALS (Bordeaux, France). Other applications: Yb-doped crystal and glass lasers pumping, Parallel pumping : Er fiber and amplifiers, Atoms traping and laser cooling, Non-linear frequency generation in the visible

□ Principle: sum coherently several laser amplifier modules up to get the required laser output power (200 W).





C

The input optics system



Input Mode Cleaner SIB1 Faraday Isolator	 and the Interferometer. Main components: Electro optic modulation at to control the optical cavi Input Mode Cleaner cavit and beam jitter noise Faraday isolator: isolates the interferometer. Mode matching optics: Ad it on the interferometer to from the Laser bench to the interferometer to from the Laser bench to the interferometer. 	blates the Laser from the back-reflected light of cs: Adjust the beam dimension to properly match eter to reduce as much as possible the light lost ch to the ITF aser frequency pre-stabilization and in data-taking	
图 B2	Parameter	Requirement	
SIB2	Transmission to the ITF	$> 70\% \ TEM_{00}$	
	Non-TEM ₀₀ power	< 5%	
	Intensity noise	$2 \times 10^{-9} / \sqrt{(Hz)}$ at 10 Hz	
	Beam Jitter	$< 10^{-10} \text{ rad} / \sqrt{(Hz)} (f > 10 \text{ Hz})$	

Requirements from the Technical report

Frequency noise (for lock acquisition)

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<1 Hz r.m.s



Electro optic modulator



□ Function: Phase modulate the laser beam at RF modulation frequencies needed for the control of the interferometer. We use the heterodyne detection technique which is commonly used to detect and analyze signals (radars, astronomy, telecommunications).

□ Requirements:

□ Withstand 200W CW laser power @1064nm.

Limited thermal lensing effect.

Maximum modulation depth = 0.2 rad.

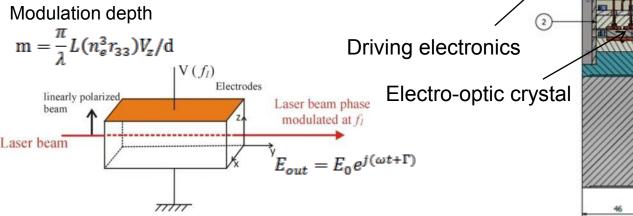
□ Provide 5 RF modulation frequencies (6.27, 8.36, 22, 304, 56.43, 131.67 MHz).

Low Residual Amplitude modulation (RAM) noise.

□ Principle:

Phase shift induced by the electric field

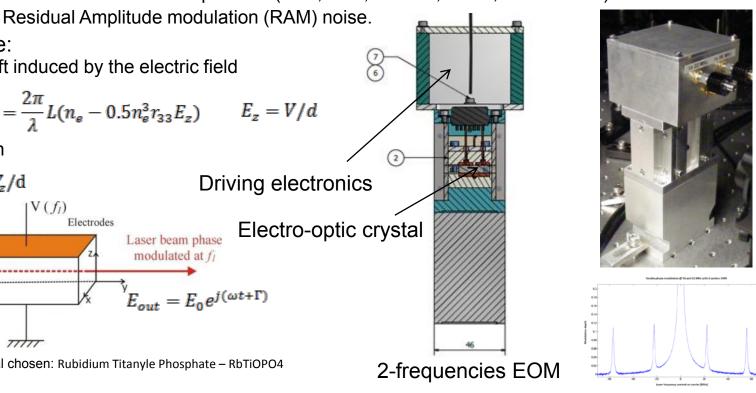
$$\Gamma = \frac{2\pi}{\lambda} n_z L = \frac{2\pi}{\lambda} L(n_e - 0.5 n_e^3 r_{33} E_z) \qquad E_z = V/d$$



Electro optic material chosen: Rubidium Titanyle Phosphate – RbTiOPO4

Applications:

- Optical cavities locking (heterodyne detection)
- Frequency- modulation spectroscopy (low RAM required)
- Telecommunications?





Input Mode Cleaner cavity

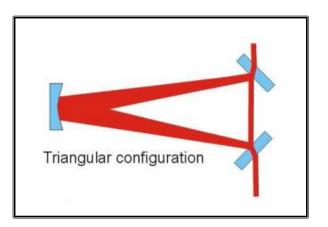
- □ Function: Beam spatial filtering, filter out beam jitter (1/*F*), to be used in Laser frequency stabilization loop, filter out frequency and power noise above its pole
- □ Main characteristics:
 - □ 144 m long suspended triangular resonant cavity (FSR=1.045 MHz)
 - □ *F* = 1000
- → Cut-off frequency (cavity pole)≈ 500 Hz.

Parameter	Measured value	
FSR	1045137 Hz +/- 0.5 Hz	
IMC Length	143.4225 m	
Pole	520Hz +/- 2 Hz	
Finesse	1005 +/- 4	
Round-Trip losses	222 ppm+/-24 ppm	
IMC cavity throughput	92.9 % +/- 0.5%	
Transmission (mirror #1)	T1=3015 +/-15 ppm	
Transmission (mirror #2)	T2=3015 +/-15 ppm	
Absorption flat mirrors	<1ppm / mirror	
Absorption End mirror	3ppm	
Mwphalfrom phile	Cenity pole frequency [, = 4.4950817283+c2] +/- 9.0053592583+c2]	
1 Experimental data	1,000	

Applications:

- Laser Frequency stabilization
- Laser beam cleaning (M² close to 1)

IOIIVIRG



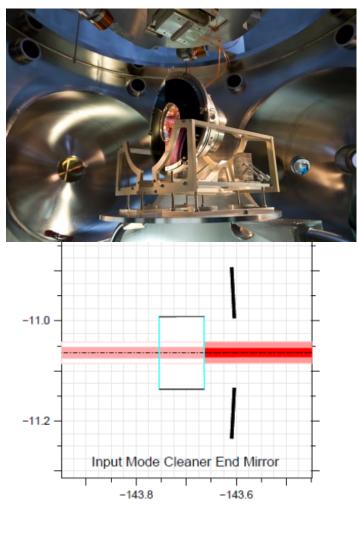
Example of IMC cavity pole measurement (injecting power noise before the cavity) Winter college on Optics, Trieste, February 24, 2016



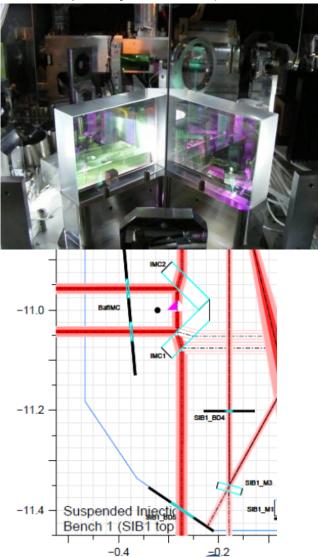
Input Mode Cleaner cavity: A few pictures

((O))VIRGD

MC end mirror in MC tower



IMC dihedron (input and output flat mirrors optically contacted) on SIB1



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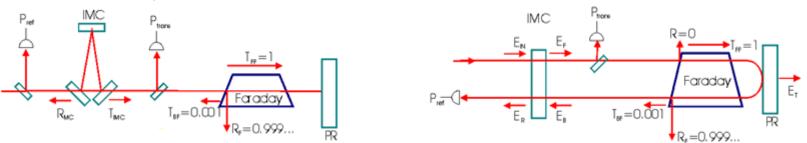


Faraday isolator



☐ Function:

- □ avoid to create a spurious cavity Input Mode Cleaner/ Interferometer.
 - Due to the fact that IMC cavity is long (144m), we have a small angle of incidence on 1 mirror of the cavity and the back-scattered light from this optics can easily be recoupled in the IMC cavity



- have an easy way to get the interferometer reflection (to be used for the interferometer control).
- □ avoid to re-inject light in the laser system and damage it.
- In order to reduce these effects, we have to install a Faraday isolator between the IMC and the interferometer.



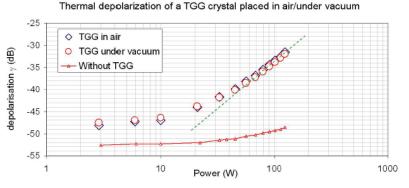
Faraday isolator



- Due to the high power of the laser inside the Faraday isolator which is installed under vacuum, we have to cope with several spurious effects:
 - □ Thermal lensing inside the magneto optic crystal [1]
 - □ Verdet constant change with temperature [2]
 - Thermally induced depolarization [3] Birefringence induced by laser beam heating

Verdet constant

Laser power



 ${d heta \over dT} \!=\! {dV \over dT} n L B =$

Material absorption

Reference:

The Virgo Collaboration, "In-vacuum optical isolation changes by heating in a Faraday isolator," Appl. Opt. 47, 5853-5861 (2008)
 The Virgo Collaboration, "In-vacuum Faraday isolation remote tuning," Appl. Opt. 49, 4780-4790 (2010)
 Mosca, S. and Canuel, B. and Karimi, E. and Piccirillo, B. and Marrucci, L. and De Rosa, R. and Genin, E. and Milano, L. and Santamato, E., Photon self-induced spin-to-orbital conversion in a terbium-gallium-garnet crystal at high laser power, Phys. Rev. A, vol. 82, issue 4 (2010).

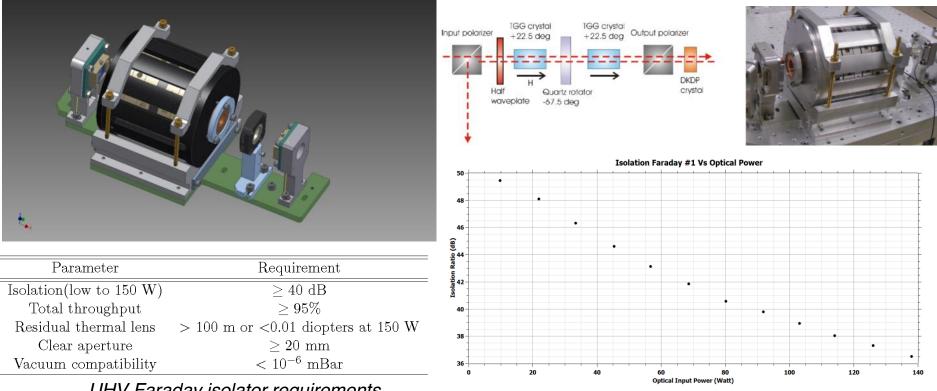
Mean rotation angle



Faraday isolator



A vacuum compatible Faraday isolator has been developed in collaboration with the Institute of Applied Physics and the University of Florida (LIGO group)



UHV Faraday isolator requirements

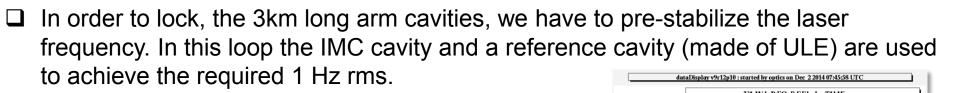
Isolation ratio vs laser input power

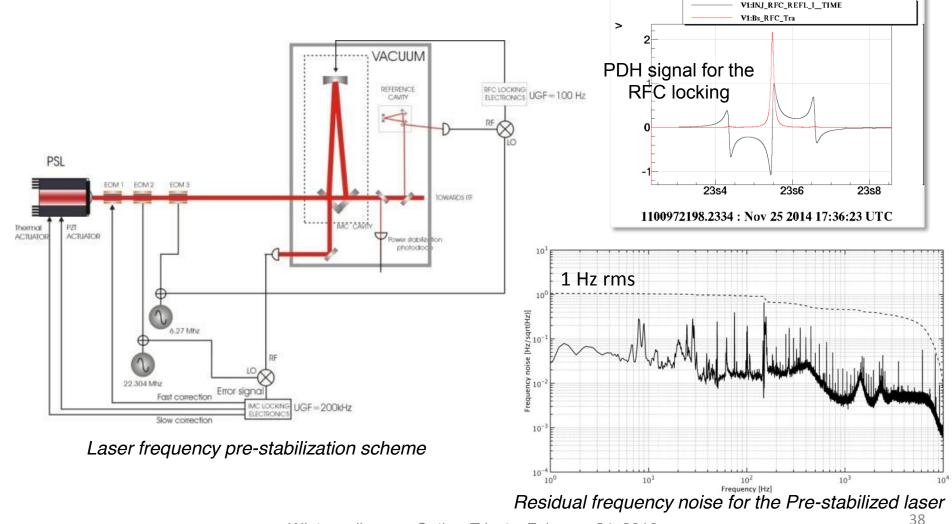
Reference:

[1] O. Palashov, D. Zheleznov, A. Voitovich, V. Zelenogorsky, E. Kamenetsky, E. Khazanov, R. Martin, K. Dooley, L. Williams, A. Lucianetti, V. Quetschke, G. Mueller, D. Reitze, D. Tanner, E. Genin, B. Canuel, and J. Marque, High-vacuum compatible high-power Faraday isolators for gravitational-wave interferometers, JOSA B, Vol. 29, Issue 7, pp. 1784-1792 (2012).



Laser frequency pre-stabilization





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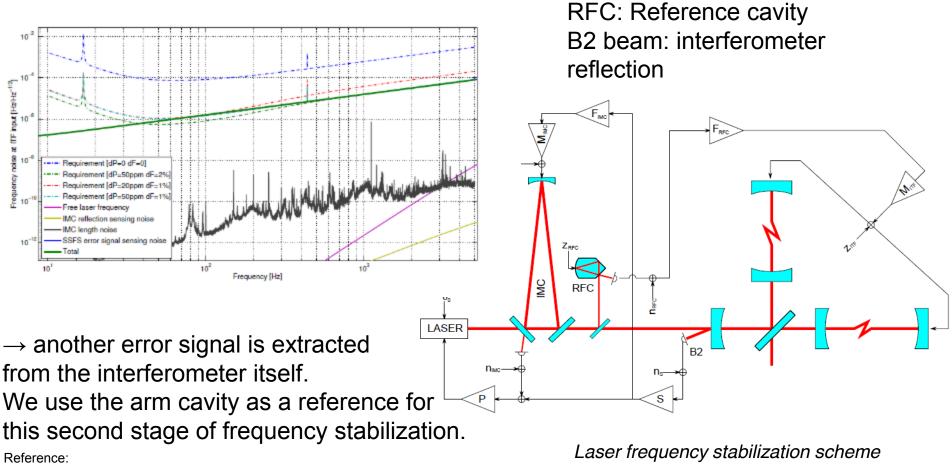
E. Genin



Laser frequency second stage of frequency stabilization



To achieve the sensitivity required, we should get a relative stability of the laser frequency better δv/v than 10⁻²¹ (the long term drift of the frequency is not that important for us). (v=300 THz)
IMC: Input Mode Cleaner



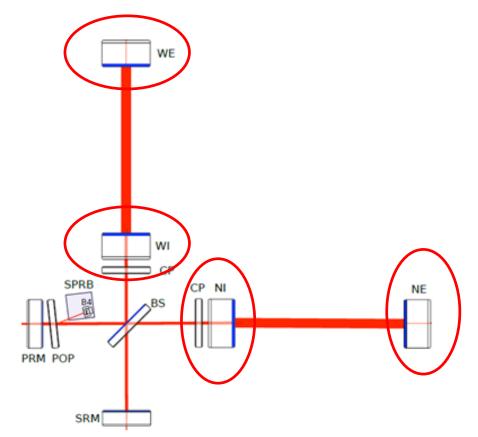
The Virgo collaboration, Laser with an in-loop relative frequency stability of 10^{-21} on a 100-ms time scale, PHYSICAL REVIEW A 79, 053824 , 2008.



The Interferometer optics



Arm cavities optics are the most critical and demanding in term of roughness, and surface figures in general.





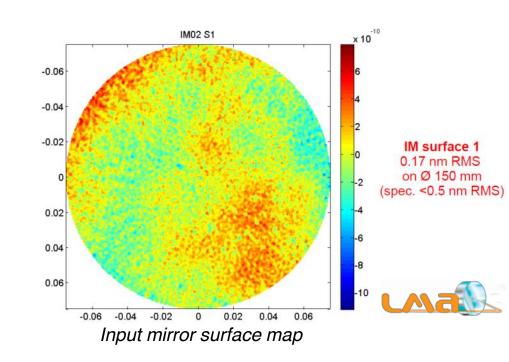
The Interferometer optics



- All the main optics of the interferometer has been realized under the supervision of Laboratoire des Matériaux avancés (Lyon, France).
- A suitable material (Suprasil 3002) has been selected as substrate: low-absorption of NIR light (0.3ppm/cm), good uniformity (Dn<5.10⁻⁷). Heraeus (EU) produced all the substrates.
- □ The polishing has been carried out by ZYGO company (US)



Example of a 3 km arm cavity input mirror (350 mm in diameter, 200 mm thick)



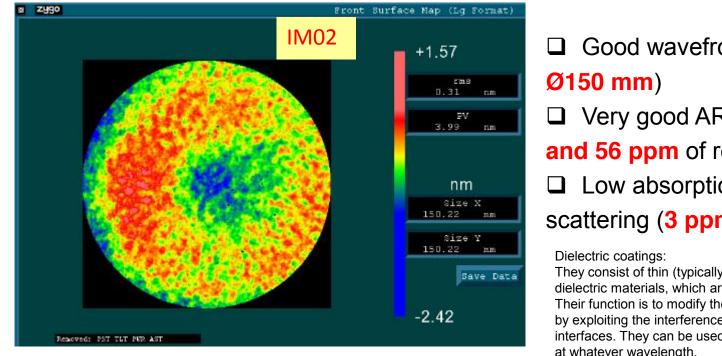
Credits: L. Pinard (LMA) Winter college on Optics, Trieste, February 24, 2016 *E. Genin*



The Interferometer optics



The coatings have been realized by Laboratoire des Matériaux avancés.



Example of Input mirror surface map after coating (Credits: L. Pinard (LMA)).

Good wavefront (0.31 nm RMS on Ø150 mm)
 Very good AR coating : 32 ppm and 56 ppm of reflectivity
 Low absorption (0.2 ppm) and scattering (3 ppm)

They consist of thin (typically sub-micron) layers of transparent dielectric materials, which are deposited on a substrate. Their function is to modify the reflective properties of the surface by exploiting the interference of reflections from multiple optical interfaces. They can be used to get whatever reflectivity at whatever wavelength.

□ LMA is able to achieve the best coatings in the world for laser interferometry.

 \rightarrow LMA has coated Advanced Virgo and Advanced Ligo mirrors.





Mirror aberrations



□ Mirror aberrations (cold and thermal defects) can spoil the sensitivity of the interferometer

□ Mechanisms worsening the sensitivity

□ **Mode mismatch** – beam intensity profile and phase don't match that of the resonator

□ Scattering – the cavity beam is scattered off by the surface roughness.

□ **Frequency splitting** – modes of the same order see a different overall radius of curvature, and their resonance frequencies result to be different.

□ Principle of thermal correction

□ Use an auxiliary heat source to induce controlled thermal effects in the optics and therefore correct the beam phase aberrations

$$OPL = S_0 + n\Delta d + d\Delta n_T + d\Delta n_E$$

Thermoelastic deformation $\Delta d \propto \alpha \Delta T$
Thermorefractive effect $\Delta n_T = \frac{dn}{dT}\Delta T$
Elastooptic effect $\Delta n_E \approx -\alpha(1+\nu)n\Delta T$

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Credits: A. Allocca (INFN)



Thermal compensation devices ((O)) VIRG

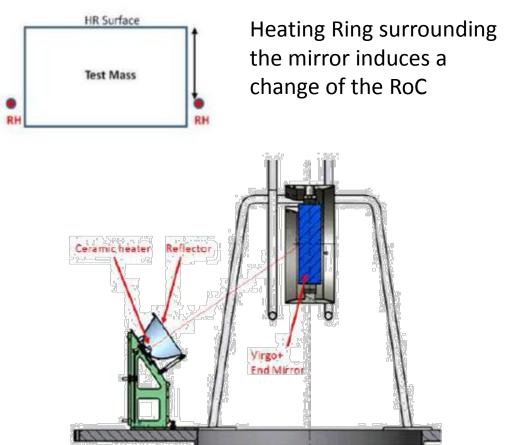


Correct the radius of curvature with Ring Heater...



□ ...or with CHRoCC

Heat projection on the mirror rear face to induce a change of the RoC

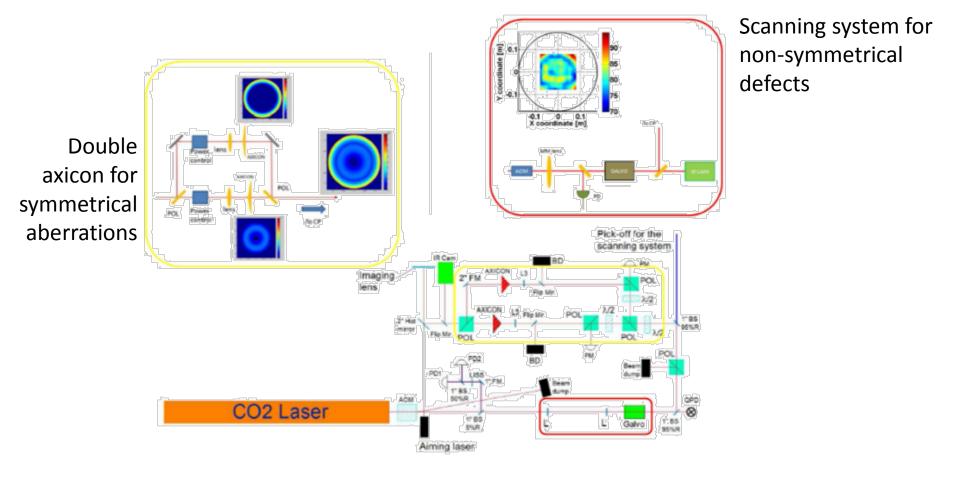




Thermal compensation devices



□ Correct high spatial frequency defects with CO2 laser...





Thermal compensation devices



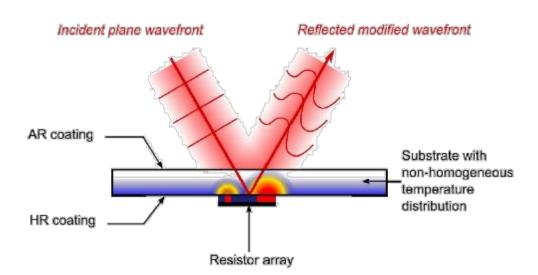
Reduce mode mismatch: thermally deformable mirrors

Array of resistors attached to the rear surface of the mirror inducing a change of temperature inside the substrate

Change of the substrate refractive index



Change of the beam OPL



 B. Canuel, R. Day, E. Genin, P. La Penna and J. Marque, "Wavefront aberration compensation with thermally deformable mirror", Class. Quantum Grav. 29, 085012 (2012)
 M. Kasprzack, B. Canuel, F. Cavalier, R. Day, E. Genin, et al.. Performance of a thermally deformable mirror for correction of low-order aberrations in laser beams. Applied Optics, OSA, 2013, 52, pp.2909-2916.

Credits: A. Allocca (INFN)

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LSC Useful links to optical simulations (((O))) V RG used for the design of GW detectors

□ Modal codes:

□ FINESSE (Frequency domain INterferomEter Simulation SoftwarE), Developped at GEO600 by Andreas Freise. <u>http://www.gwoptics.org/finesse/</u>.

MIST, developped at Virgo/Ligo by Gabriele Vajente <u>https://sourceforge.net/projects/optics-mist/files/</u>

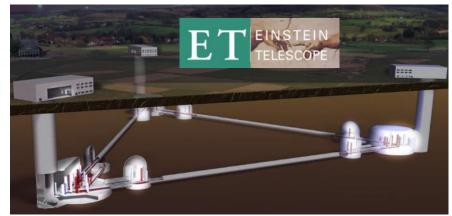
☐ FFT-based codes:

- □ SIS (with FOG inside), developped at Ligo/Virgo by Hiro Yamamoto and Richard day
- OSCAR, developped at GEO by Jerome Degallaix <u>http://www.mathworks.com/matlabcentral/fileexchange/20607-oscar</u>
- \rightarrow Those simulation tools are useful for the design of optical cavities and inteferometers.



Conclusion and next steps ((O)) VIRGO

- Advanced Virgo optics have been produced and are being installed.
- □ The laser and the input optics systems have been installed and are working since more than 1.5 year.
- → For the moment, almost all are workforce is working on a unique goal have the current interferometer operational and able to detect GW.
- □ Perspectives: Develop new components for future GW detectors (Einstein telescope for example <u>http://www.et-gw.eu/</u>) or US Lungo (40 km-long arms)
 - □ Components optimized for other wavelength: 1.55 um or 2 um
 - □ Improvement the coating uniformity/ reflectivity
 - Test new materials such as silicon at cryogenic temperature



Visit Virgo website https://www.virgo-gw.eu/scientists.html





Extra slides



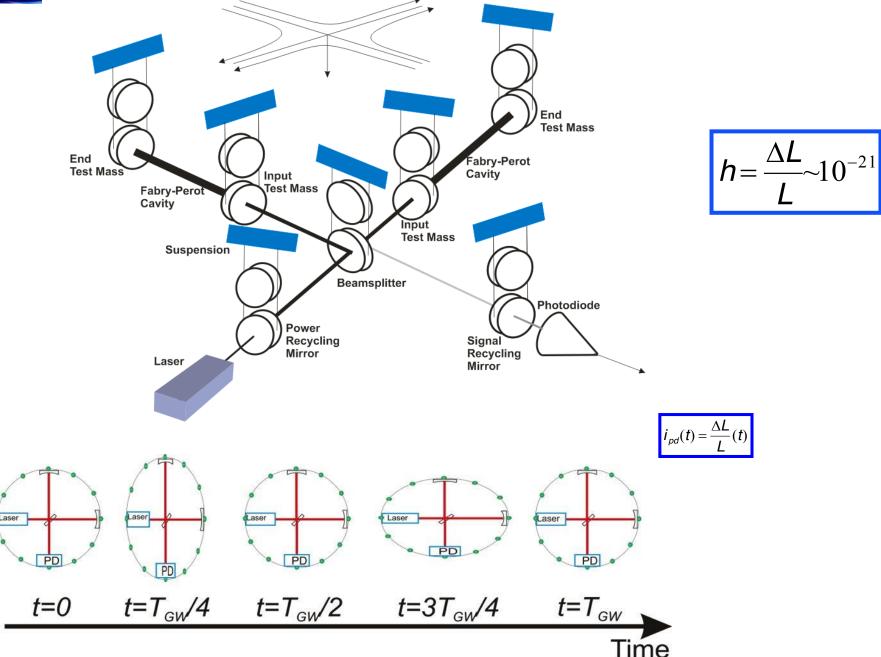
A bit of history ((O))/IRG

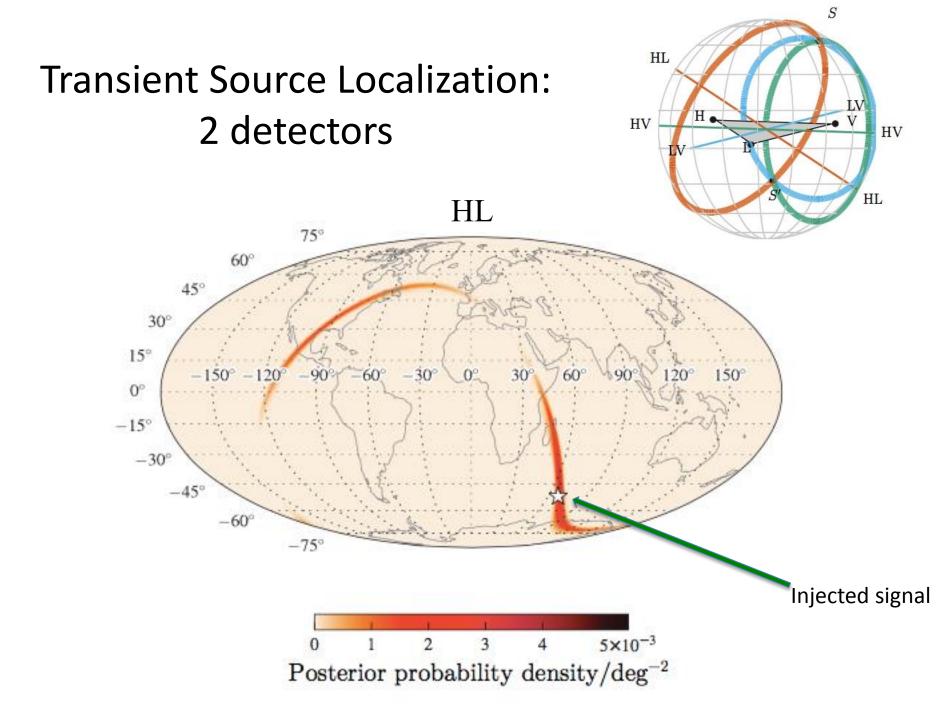
The LIGO project was approved in 1992 and inaugurated in 1999. Built at a cost of almost 3x10⁸ \$, LIGO was the largest single enterprise ever undertaken by the foundation. It started the operation in 2002.

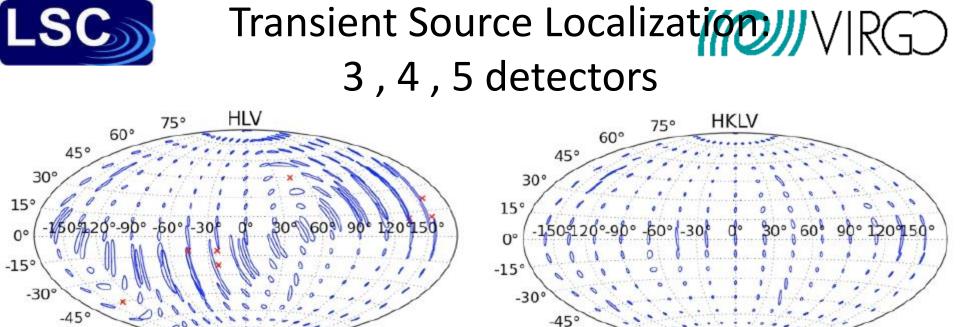
- ❑ VIRGO was formally proposed in 1989 and approved in 1993. The construction was divided in two step: it started in 1996 and then completed in 2003. The first science run is date 2007. The total investment done by CNRS and INFN was almost 8 x 10⁷ \$.
- □ **GEO600** was proposed in 1994. Since September 1995 this British-German GW detector was under construction. The first science run was performed in 2002. In 2013 Squeezing light was used over one complete year!
- First attempt to exchange data and mix the data analysis groups started in 2004. The formal MoU of data sharing and common analysis among GEO-LIGO-VIRGO was signed in 2007.



Special features to improve the sensitivity $|R(\tau)$







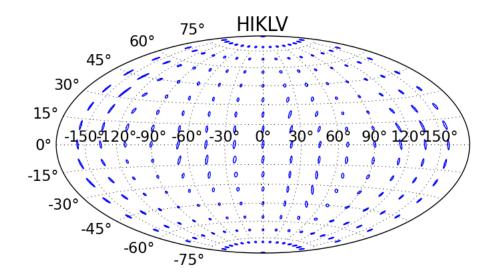


-60

-75°

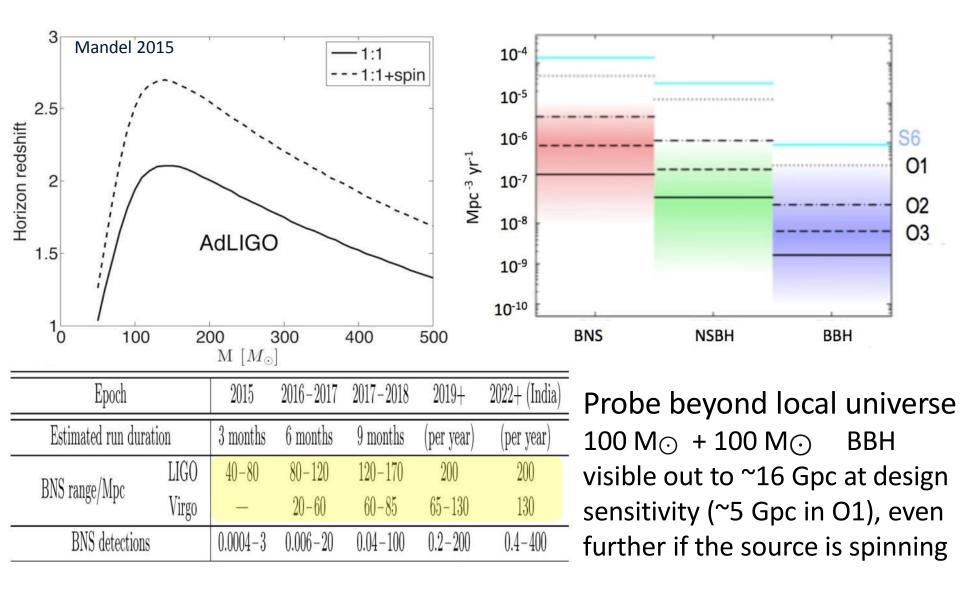
-60

-75°



LSC Compact Coalescing

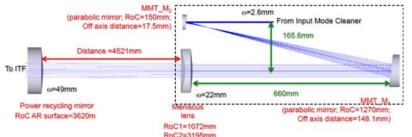
Detection perspectives with advanced detectors Phys. Rev D85 (2012) 082002





Optical design activities: (O)/VRG High magnification beam expander/reducer

Due to the large laser beam and the limited space available, we had to design an original and compact design for the launching telescope for Advanced Virgo. This is a catadioptric system.



Applications:

- Astronomy (Laser guide stars)
- Whatever experiment which need a high magnification compact laser beam expander
- → This design has been chosen by the AdV Project for the interferometer input and output telescopes.
- Optimization has been made keeping in mind the compactness and the lowest possible aberrations (in particular spherical aberrations compensation was required as well as low astigmatism).
- A complete tolerancing study has been carried out to define the requirements on the mechanics and on the optics and to determine to actuators needed to adjust its alignment while under vacuum.
- Scattered light has been studied to determine the requirements on optics surface errors and on baffling.

Ref.: B. Canuel, E. Genin, G. Vajente, J. Marque, Displacement noise from back scattering and specular reflection of input and output optics in advanced GW detectors, Optics Express, Vol. 21, Issue 9, pp. 10546-10562 (2013).

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AdV launching telescope

((O))/VIRGD

Meniscus lens

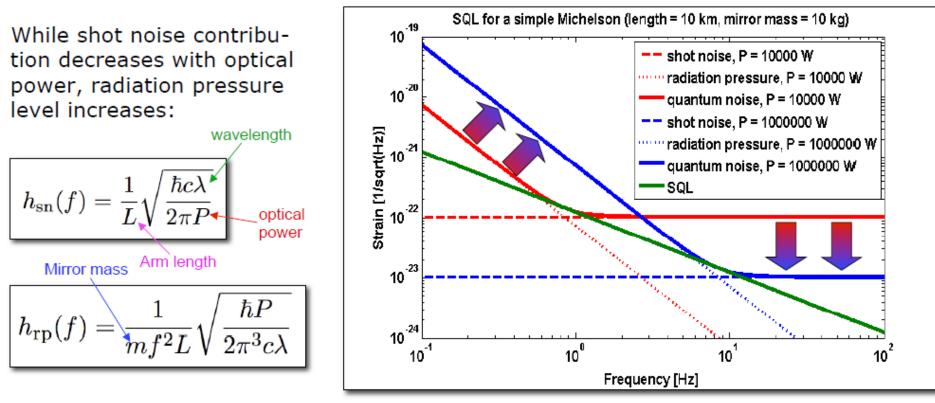
1st parabolic mirror

2nd parabolic mirror

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Standard quantum noise limit



The SQL is the minimal sum of shot noise and radiation pressure noise. Using a classical quantum measurement the SQL represents the lowest achievable noise. V.B. Braginsky and F.Y. Khalili: Rev. Mod. Phys. 68 (1996)