Optical lattice atomic clocks

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La seconde

est la durée de 9 192 631 770 périodes de la radiation correspondant à la transition entre les deux niveaux hyperfins de l'état fondamental de l'atome de césium 133.

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Who needs a better time?

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Satellite navigation

Who needs a better time?

Satellite navigation

Telecommunications

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Satellite navigation

Telecommunications

Deep space missions

Who needs a better time?

Satellite navigation

Transport

Telecommunications

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E-commerce, banking,

stock exchange

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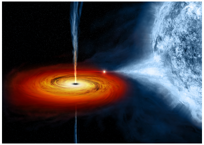
Fundamental research



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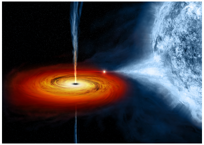
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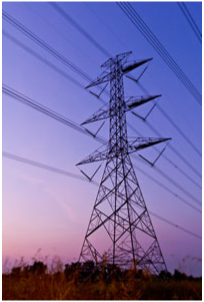
Electric power

smart grid

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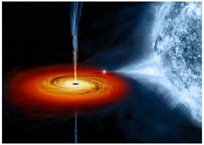
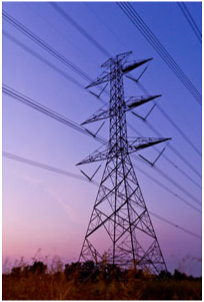
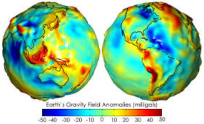


Electric power

smart grid



Geodesy



Worldwide market

Global GNSS market size: 250 billion € in 2016

source: European GNSS Agency (GSA), 4rd Market Report (2015)

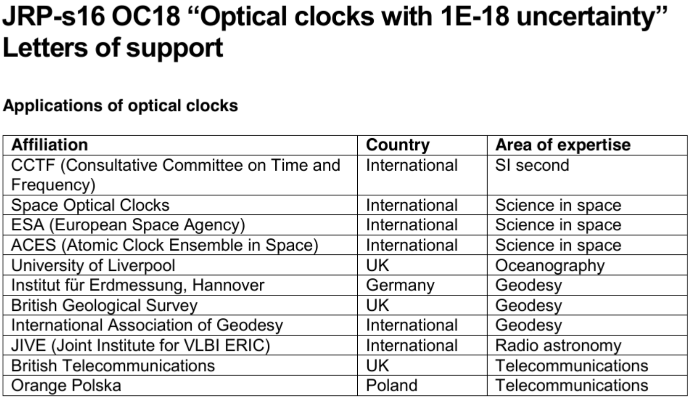
Telecommunications: one trillion $ market by 2019

source: Global Wireless Infrastructure Market 2013 Forecast to Industry Size, Shares, Strategies, Trends, and Growth 2019

Smart electrical grid: 400 billion € by 2020

source: GTM Research, Global Smart Grid Technologies and Growth Markets, 2013-2020

Direct applications

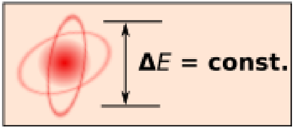


JRP-s16 OC18 “Optical clocks with 1E-18 uncertainty” Letters of support

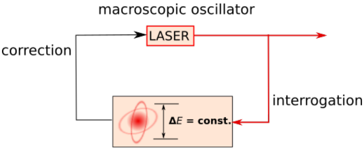
Applications of optical clocks

Affiliation	Country	Area of expertise
CCTF (Consultative Committee on Time and Frequency)	International	SI second
Space Optical Clocks	International	Science in space
ESA (European Space Agency)	International	Science in space
ACES (Atomic Clock Ensemble in Space)	International	Science in space
University of Liverpool	UK	Oceanography
Institut für Erdmessung, Hannover	Germany	Geodesy
British Geological Survey	UK	Geodesy
International Association of Geodesy	International	Geodesy
JIVE (Joint Institute for VLBI ERIC)	International	Radio astronomy
British Telecommunications	UK	Telecommunications
Orange Polska	Poland	Telecommunications

Atomic clock

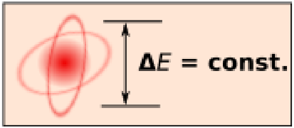


Atomic clock diagram



- Ideal clock: a signal with stable and universal frequency.
- Energy levels of unperturbed atoms are stable and universal.

Atomic clock



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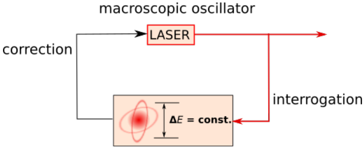


$$\omega(t) = \omega_{ef} * (1 + \varepsilon + y(t))$$

ε -fractional offset of frequency $y(t)$ - fractional fluctuations of frequency

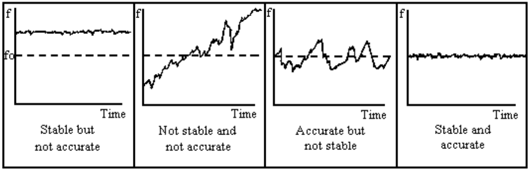
Accuracy

- uncertainty of ε

Stability

- statistical properties of $y(t)$, characterized by the Allan deviation $\sigma_y(\tau)$

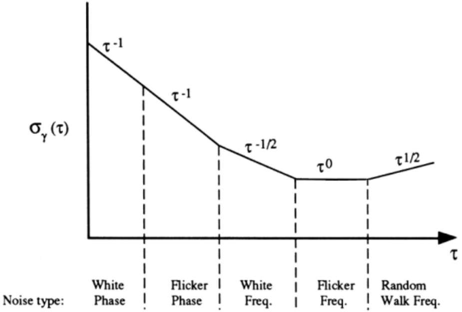
Accuracy and stability



source: nist.gov

Allan deviation

$$\sigma_f(\tau) = \sqrt{\frac{1}{2}\sum_n \frac{\left(f_{n+1,\tau} - f_{n,\tau}\right)^2}{n}}$$



source: nist.gov

f_n is a set of frequency offset measurements that consists of individual measurements, f_1, f_2, f_3, and so on and the data are equally spaced in segments τ seconds long.

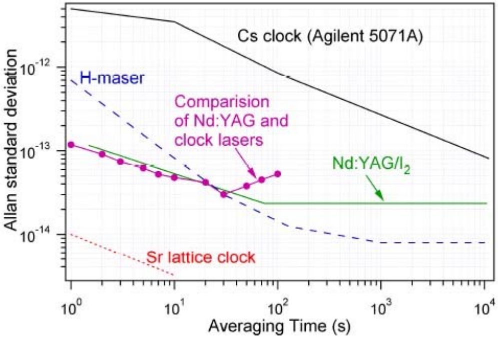
Time standards

1. Atomic fountains, **accuracy** of $\sim 10^{-16}$
2. Commercial caesium clocks, with good long term **stability** $\sim 10^{-15}$ over few months and **accuracy** of $\sim 10^{-13}$
3. Hydrogen masers: 1.4 GHz hyperfine structure transition in atomic hydrogen. Much better short-time **stability** than any commercial caesium clock: $\sim 10^{-15}$ over few hours

Caesium clocks and hydrogen masers

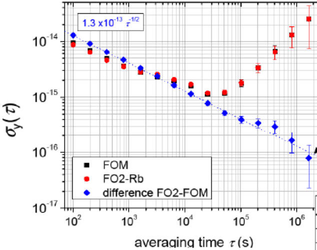
HP5071A caesium clock and VCH-1005 hydrogen maser in the Central Office of Measures in Poland

Caesium clocks and hydrogen masers



Feng-Lei Hong et al. Opt. Express 13, 5253-5262 (2005)

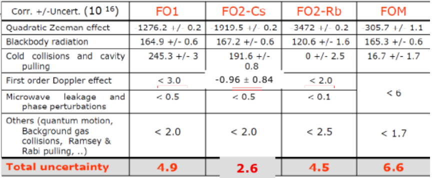
Atomic fountain



Resolution 6x10^{-17} at 50 days (assuming white noise)

ν(FO2-Rb) (2007) =6 834 682 610.904 309 (8) Hz

Total uncertainty 1.1x10^{-15}



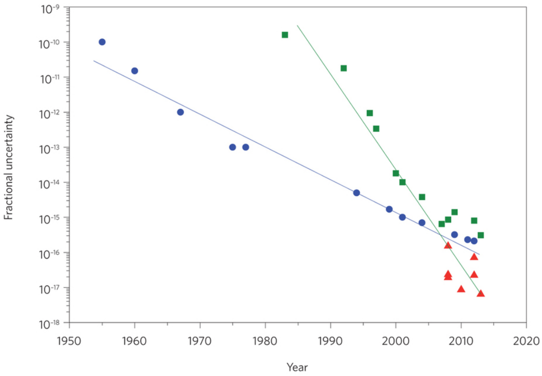
Corr. +/-Uncert. (10 $^{-16}$)	FO1	FO2-Cs	FO2-Rb	FOM
Quadratic Zeeman effect	-1276.2 +/- 0.2	-1919.5 +/- 0.2	-3472 +/- 0.2	-305.7 +/- 1.1
Blackbody radiation	164.9 +/- 0.6	167.2 +/- 0.6	120.6 +/- 1.6	165.3 +/- 0.6
Cold collisions and cavity pulling	245.3 +/- 3	191.6 +/- 0.8	0 +/- 2.5	16.7 +/- 1.7
First order Doppler effect	< 3.0	-0.96 ± 0.84	< 2.0	< 6
Microwave leakage and phase perturbations	< 0.5	< 0.5	< 0.1	
Others (quantum motion, Background gas collisions, Ramsey & Rabi pulling, ..)	< 2.0	< 2.0	< 2.5	< 1.7
Total uncertainty	**4.9**	**2.6**	**4.5**	**6.6**

Best accuracy: 2.6x10^{-16}

Real-time control of collision shift with adiabatic passage: Phys. Rev. Lett. 89, 233004 (2002)

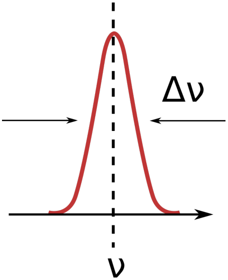
Source: Systèmes de Références Temps-Espace

Microwave clocks vs optical clocks



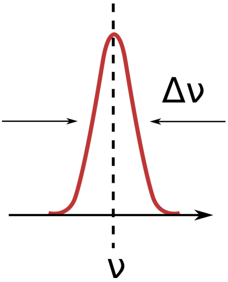
Helen Margolis, Nature Physics 10, 82-83 (2014)

Microwave clocks vs optical clocks



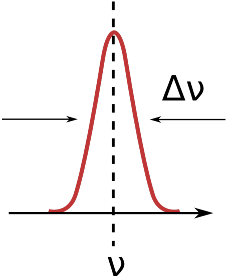
Quality of the clock: $Q = \frac{\nu}{\Delta\nu} \times \frac{S}{N}$

Microwave clocks vs optical clocks



Quality of the clock: $Q = \frac{\nu}{\Delta\nu} \times \frac{S}{N} \sim \nu T \times \frac{S}{N}$

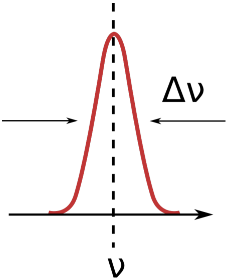
Microwave clocks vs optical clocks



Quality of the clock: $Q = \frac{\nu}{\Delta\nu} \times \frac{S}{N} \sim \nu T \times \frac{S}{N}$

Stability of an atomic clock: $\sigma_y(\tau) \sim \frac{\sigma_{spect}}{Q}\sqrt{\frac{T_c}{\tau}}$

Microwave clocks vs optical clocks



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Stability of an atomic clock: $\sigma_y(\tau) \sim \frac{\sigma_{spect}}{Q}\sqrt{\frac{T_c}{\tau}}$

Quantum Shot Noise limitation: $\sigma_y(\tau) = \frac{1}{\pi Q} \times \frac{1}{N_{at}} \times \sqrt{\frac{T_c}{\tau}}$

Optical clocks: trapped ions and neutral atoms

Ion traps: Ions trapped in the Paul trap by the RF field

- Trap is perturbed only slightly ⇒ excellent accuracy 3×10^{-18}

  N. Huntemann et al. Phys. Rev. Lett. 116, 063001 (2016)
- Good stability ($3 \times 10^{-15}/\sqrt{\tau}$), but restricted by Quantum Shot Nose - only 1 ion.

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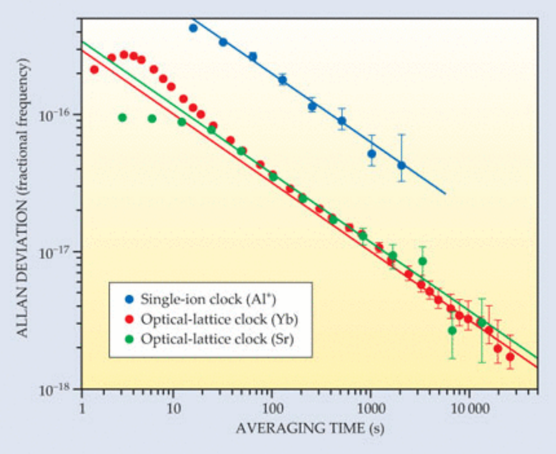
Neutral atoms: Optical lattice

- A trap with a hight-intensity light ⇒ high perturbation, though well under control. $2 * 10^{-18}$

  T.L. Nicholson et al. Nature Communications, 6, 6896 (2015)
- High number of atoms (10^4)⇒ high stability possible. $1.8 \times 10^{-16}/\sqrt{\tau}$ down to 2×10^{-18}

Optical clocks: trapped ions and neutral atoms



A. G. Smart, Phys. Today 67, 3, 12 (2014)

A transition good for optical clock

Natural linewidth of an electric dipole (E1) transition is typically bigger than 1 MHz ⇒ Q below 10^8

A transition good for optical clock

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Clock transition should be:

- Narrow (forbidden)
- Mostly insensitive to external fields.

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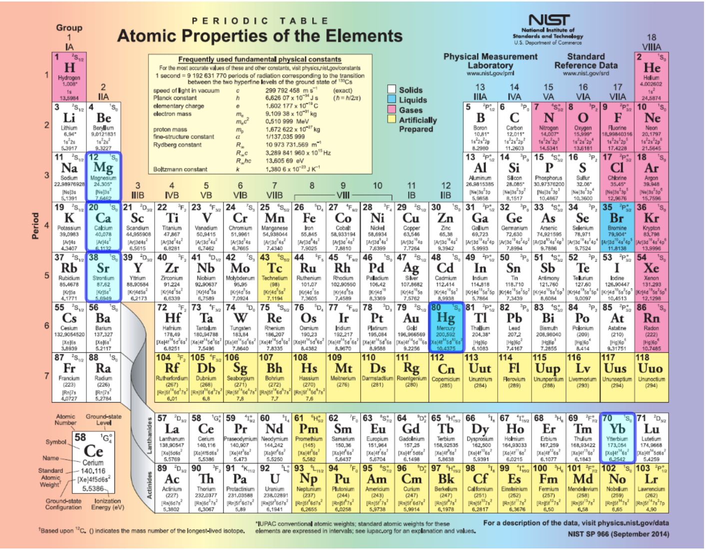
Possible candidates:

- two-photon transitions and higher order electric transitions (quadrupole, octupole . . .)
- a low energy nuclear transition (still sought at ^{229}Th)
- an intercombination transition

Alkaline-earth and alkaline-earth like atoms/ions



PERIODIC TABLE

Atomic Properties of the Elements

NIST National Institute of Standards and Technology U.S. Department of Commerce

Physical Measurement Laboratory www.nist.gov/pml

Standard Reference Data www.nist.gov/srd

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.626 07 x 10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.602 177 x 10^{-19} C	
electron mass	m_e	9.109 38 x 10^{-31} kg	
	m_ec^2	0.510 999 MeV	
proton mass	m_p	1.672 622 x 10^{-27} kg	
fine-structure constant	α	1/137.035 999	
Rydberg constant	R_∞	10 973 731.569 m^{-1}	
	$R_\infty c$	3.289 841 960 x 10^{15} Hz	
	$R_\infty hc$	13.605 69 eV	
Boltzmann constant	k	1.380 6 x 10^{-23} J K^{-1}	

☐ Solids
☐ Liquids
☐ Gases
☐ Artificially Prepared

Period	Group 1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIII	9 VIII	10 VIII	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 H Hydrogen 1.008*																	2 He Helium 4.002602
2	3 Li Lithium 6.94*	4 Be Beryllium 9.0121831											5 B Boron 10.81*	6 C Carbon 12.011*	7 N Nitrogen 14.007*	8 O Oxygen 15.999*	9 F Fluorine 18.99840316	10 Ne Neon 20.1797
3	11 Na Sodium 22.98976928	12 Mg Magnesium 24.305*											13 Al Aluminum 26.9815385	14 Si Silicon 28.085*	15 P Phosphorus 30.97376200	16 S Sulfur 32.06*	17 Cl Chlorine 35.45*	18 Ar Argon 39.948
4	19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.955908	22 Ti Titanium 47.867	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938044	26 Fe Iron 55.845	27 Co Cobalt 58.933194	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.630	33 As Arsenic 74.921595	34 Se Selenium 78.971	35 Br Bromine 79.904*	36 Kr Krypton 83.798
5	37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90584	40 Zr Zirconium 91.224	41 Nb Niobium 92.90637	42 Mo Molybdenum 95.95	43 Tc Technetium (98)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.90550	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.760	52 Te Tellurium 127.60	53 I Iodine 126.90447	54 Xe Xenon 131.293
6	55 Cs Cesium 132.9054520	56 Ba Barium 137.327		72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.084	79 Au Gold 196.966569	80 Hg Mercury 200.592	81 Tl Thallium 204.38*	82 Pb Lead 207.2	83 Bi Bismuth 208.98040	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)
7	87 Fr Francium (223)	88 Ra Radium (226)		104 Rf Rutherfordium (267)	105 Db Dubnium (268)	106 Sg Seaborgium (271)	107 Bh Bohrium (272)	108 Hs Hassium (270)	109 Mt Meitnerium (276)	110 Ds Darmstadtium (281)	111 Rg Roentgenium (280)	112 Cn Copernicium (285)	113 Uut Ununtrium (284)	114 Fl Flerovium (289)	115 Uup Ununpentium (288)	116 Lv Livermorium (293)	117 Uus Ununseptium (294)	118 Uuo Ununoctium (294)

Lanthanides	57 La Lanthanum 138.90547	58 Ce Cerium 140.116	59 Pr Praseodymium 140.90766	60 Nd Neodymium 144.242	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.500	67 Ho Holmium 164.93033	68 Er Erbium 167.259	69 Tm Thulium 168.93422	70 Yb Ytterbium 173.054	71 Lu Lutetium 174.9668
Actinides	89 Ac Actinium (227)	90 Th Thorium 232.0377	91 Pa Protactinium 231.03588	92 U Uranium 238.02891	93 Np Neptunium (237)	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)

†Based upon ^{12}C. () indicates the mass number of the longest-lived isotope.

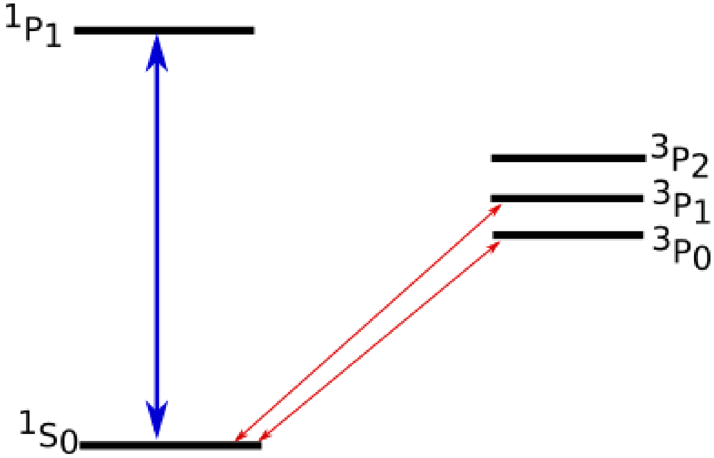
*IUPAC conventional atomic weights; standard atomic weights for these elements are expressed in intervals; see iupac.org for an explanation and values.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2014)

Source: NIST

Alkaline-earth and alkaline-earth like atoms/ions



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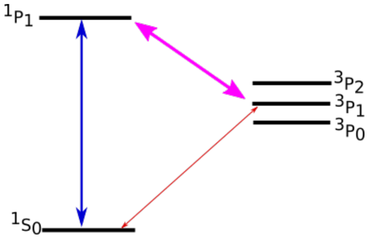
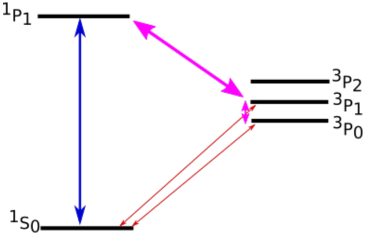


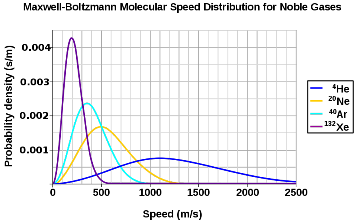
- Forbidden $^1S_0 - {}^3P_1$ transition:
  - Fine structure interaction

Alkaline-earth and alkaline-earth like atoms/ions



- Forbidden $^1S_0 - {}^3P_1$ transition:
    - Fine structure interaction
- Double forbidden $^1S_0 - {}^3P_0$ transition:
    - Fermions: hyperfine interaction
    - Bosons: quenching by a static B field

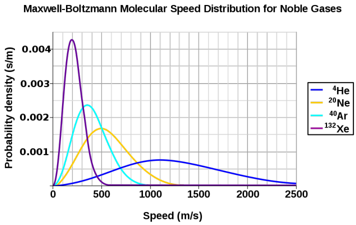
Quality killers: Doppler shift and recoil shift



source: Pdbailey at Wikipedia

- $f(v) \sim \frac{v^2}{T^{3/2}} e^{-\frac{mv^2}{2kT}}$

Quality killers: Doppler shift and recoil shift



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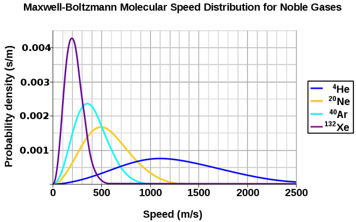
- $f(v) \sim \frac{v^2}{T^{3/2}} e^{-\frac{mv^2}{2kT}}$
- Doppler shift: $\nu = \nu_0 \left(1 + \frac{v}{c}\right)$

Quality killers: Doppler shift and recoil shift



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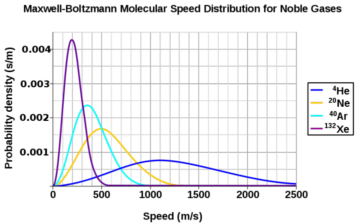

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- $P_\nu(\nu)d\nu \sim \sqrt{\frac{1}{T^2\nu_0^2}} \exp\left(-\frac{mc^2(\nu-\nu_0)^2}{2kT\nu_0^2}\right) d\nu$

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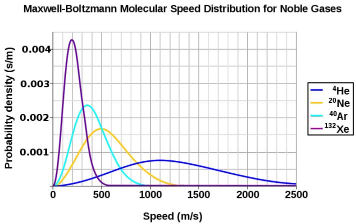

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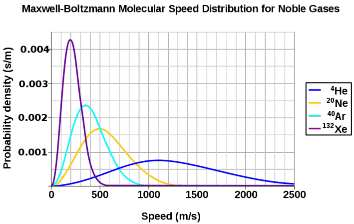
Quality killers: Doppler shift and recoil shift



source: Pdbailey at Wikipedia

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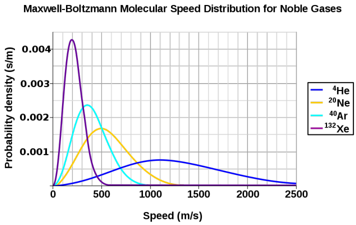

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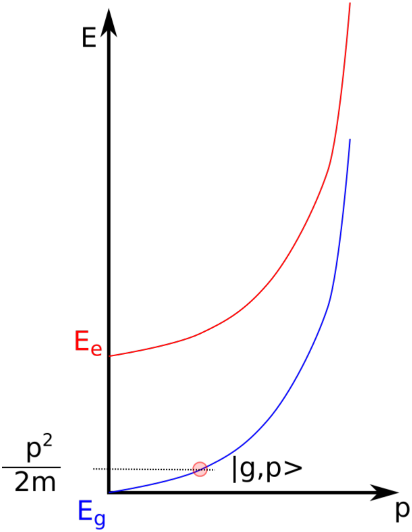
Sub-Doppler spectroscopy, or cool down atoms. . .

Quality killers: Doppler shift and recoil shift



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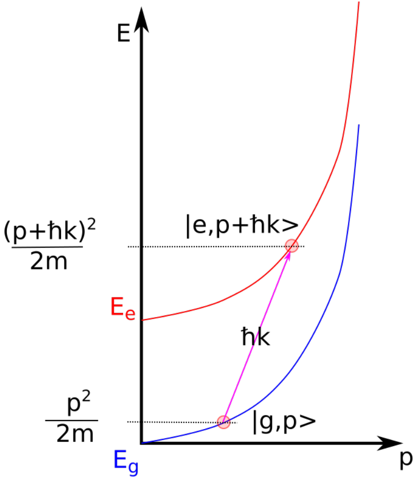

Two-level atom in EM field

Unperturbed Hamiltonian of the atom: $H_0 = \hbar\omega_g \left|g\right\rangle \left\langle g\right| + \hbar\omega_e \left|e\right\rangle \left\langle e\right|$

Zero point energy can be chosen at will: $H_0 = \hbar\omega_0 \left|e\right\rangle \left\langle e\right|$

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$\Longrightarrow H_{int} = \frac{\hbar}{2}\Omega(\vec{r})e^{-\hat{\imath}\omega t}\left(\left|e\right\rangle\left\langle g\right| + \left|g\right\rangle\left\langle e\right|\right) + \frac{\hbar}{2}\Omega^{*}(\vec{r})e^{\hat{\imath}\omega t}\left(\left|g\right\rangle\left\langle e\right| + \left|e\right\rangle\left\langle g\right|\right)$

where $\Omega(\vec{r}) = -2d\cdot\vec{E_0}e^{-\phi(\vec{r})}/\hbar$ is the so-called Rabi frequency.

Two-level atom in EM field

After rotating wave approximation ($|\Delta| = |\omega - \omega_0| \ll \omega_0$):

$$H_{int}^{RWA} = \frac{\hbar}{2}\Omega(\vec{r})e^{-\hat{\imath}\omega t}\left|e\right\rangle\left\langle g\right| + \frac{\hbar}{2}\Omega^*(\vec{r})e^{\hat{\imath}\omega t}\left|g\right\rangle\left\langle e\right|$$

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Using the abbreviation for the generalized Rabi frequency: $\tilde{\Omega} = \sqrt{\Delta^2 + \Omega^2}$

the probability of finding the atom in the upper level state is:

$$P_e(t) = \frac{\Omega}{\tilde{\Omega}}\sin^2(\tilde{\Omega}t/2).$$

$$P_e(\omega) \sim \frac{\sin^2(\omega t/2)}{(\omega t/2)^2}.$$

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Doppler shift across the atomic velocity distribution broadens the line shape into a Gaussian or Voigt line shape.

Two-level atom in EM field in a harmonic oscillator

In a harmonic potential the atomic motion is not a continuous variable, but is restricted to the quantized motional states $|n\rangle$ of the system.

$\Longrightarrow$ New term in Hamiltonian: $H_{osc} = \hbar\omega_h \left(a^\dagger a + \frac{1}{2}\right)$.

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Modified Rabi rate of transitions in two-level atom from $|g\rangle\,|n\rangle$ state to $|e\rangle\,|m\rangle$ state:

$$\Omega_{mn} = \tilde{\Omega}\langle m|e^{\hat{\imath}\eta\left(a+a^\dagger\right)}|n\rangle = \tilde{\Omega}e^{-\eta^2/2}\sqrt{\frac{n_<!}{n_>!}}\eta^{|m-n|}L_{n_<}^{|m-n|}(\eta^2)$$

where $\eta = \sqrt{\frac{\hbar k^2}{2m\omega_h}} = \frac{kx_0}{\sqrt{2}}$ is the so-called Lamb-Dicke parameter.

$\eta \ll 1$ required!

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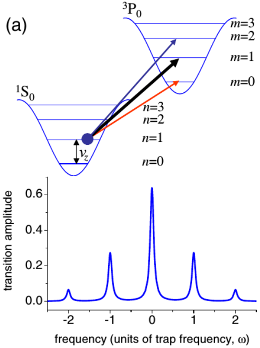

source: Ludlow et al. Rev. Mod. Phys. 87, 637 (2015)

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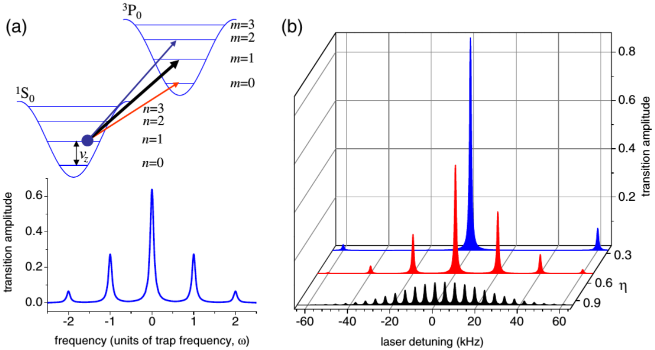

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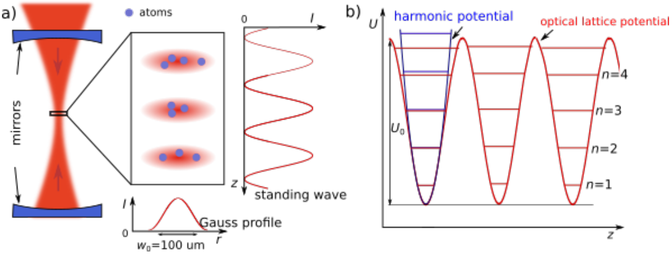
Optical lattice



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In the classical Lorentz Oscillator model of an atom
$(\ddot{x} + \Gamma_\omega \dot{x} + \omega_0^2 x = -eE(t)/m_e)$

$\alpha = 6\pi\varepsilon_0 c^3 \frac{\Gamma/\omega_0^2}{\omega_0^2 - \omega^2 - \hat{\imath}(\omega^3/\omega_0^2)\Gamma}$, where $\Gamma = (\omega_0/\omega)^2 \Gamma_\omega$.

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After rotating wave approximation ($|\Delta| = |\omega - \omega_0| \ll \omega_0$):

$$U_{dip} = \frac{3\pi c^2}{2\omega_0^3}\frac{\Gamma}{\Delta} I(\vec{r})$$

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- The interaction potential U_{dip} is proportional to the light intensity $I(\vec{r})$ and its sign depends on the detuning.
  - $\Delta < 0$ — red detuned — force directed to potential maxima
  - $\Delta > 0$ — blue detuned — force directed to potential minima
- $\Gamma_{sc}/U_{dip} \propto 1/\Delta \Longrightarrow$ large detuning are better for us

Optical lattice

The most basic optical lattice trap consists of two focused, counter-propagating red-detuned laser beams ($\vec{k}$ and $-\vec{k}$)

Intensity in the direction of propagation (z) becomes a standing wave:

$$I(z) = \left|\vec{E_1}(z,t) + \vec{E_2}(z,t)\right|^2 = 2\left|E_0 \cos(\omega t)\cos(kz)\right|^2 \approx E_0^2 \cos^2(kz)$$

and the potential $U(z) \sim \cos^2(kz)$

Wannier-Stark regime

One-dimensional optical lattice:

$$U(r,z) = U_o\left(1 - e^{-2r^2/w_0^2}\cos^2(kz)\right)$$

Two-level atoms in the lattice probed by the clock laser (ω_p, k_p) can be described by:

$$\hat{H} = \hbar\omega_{eg}|e\rangle\langle e| + (\hbar\Omega\cos(\omega_p t - k_p\hat{z})|e\rangle\langle g| + H.C.) + \hat{H}_{ext},$$
$$\hat{H}_{ext} = \frac{\hbar^2\hat{\kappa}^2}{2m} + U_0(1 - e^{-2(\hat{x}^2+\hat{y}^2)/w_0^2}\cos^2(k\hat{z}))$$

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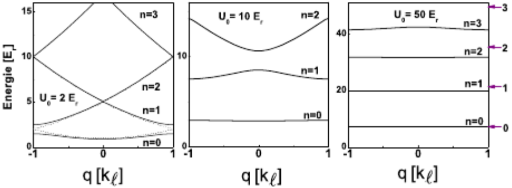
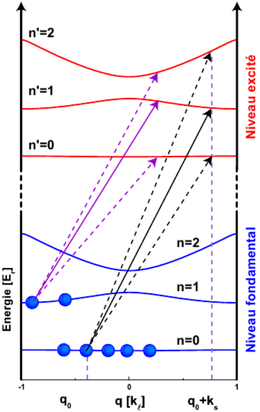
$\Rightarrow$ Bloch states!

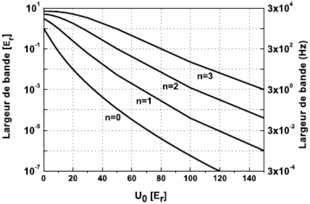


source: PhD thesis Rodolphe Le Targat

Wannier-Stark regime

⇒ Bloch states ⇒ tunneling between sites ⇒ no longer in L-D regime.





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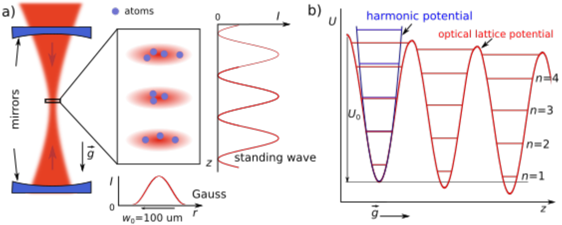
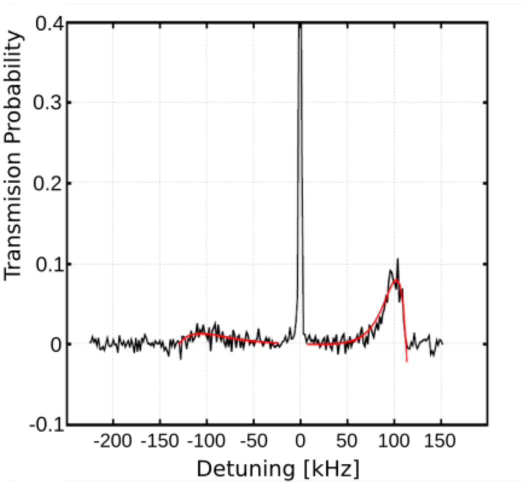
Wannier-Stark regime

Solution: vertical lattice

$$\hat{H}_{ext} = \frac{\hbar^2\hat{\kappa}^2}{2m} + U_0(1 - e^{-2(\hat{x}^2+\hat{y}^2)/w_0^2}\cos^2(k\hat{z})) + mg\hat{z}$$



Resolved sidebands in ^{88}Sr optical lattice clock



Lattice depth

Typical depth of the lattice:

few tens of μK.

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Atoms have to be cooled down to tens of μK before loading into the lattice . . .

Laser cooling

The basic idea - absorption and spontaneous emission

$$\Delta\vec{p} = \sum \hbar\vec{k}_{abs} - \sum \hbar\vec{k}_{em} = N\hbar\vec{k}_{abs} - 0$$

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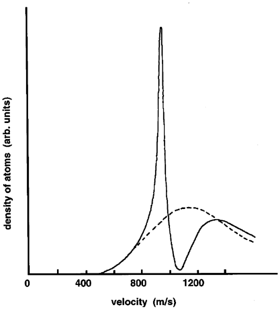
Example:

Na atoms, $\lambda = 590$ nm, $m = 23$, $v = 600$ m/s at $T = 400$ K.
Absorption of 1 photon $\Longrightarrow \Delta v = \hbar k/m = 3$ cm/s.
$\Longrightarrow$ it takes ~20 000 photons to stop.
If $I = 6$ mW/cm^2, then the atomic beam will stop in 1 ms at 0.5 m.
Deceleration: 10^6 m/s^2.

Laser cooling

A complication: atoms will go off resonance due to the Doppler shift.



source: WD. Phillips, Rev. Mod. Phys., 70, 721, (1998)

Laser cooling

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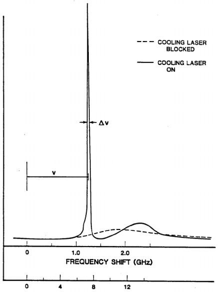
Solutions:

- Tune the frequency of the laser.
- Tune the transition frequency in atoms.

Laser cooling

A complication: atoms will go off resonance due to the Doppler shift.

"Chirping" the laser frequency:



source: WD. Phillips et al., j. Opt. Soc. Am. B, 2, 1751 (1985)

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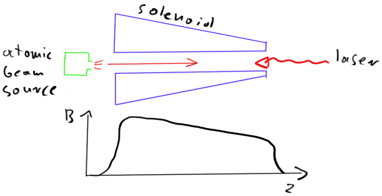
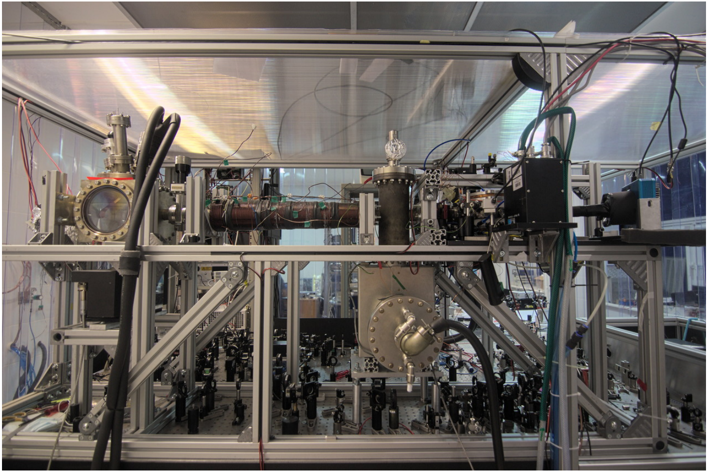
Zeeman slower

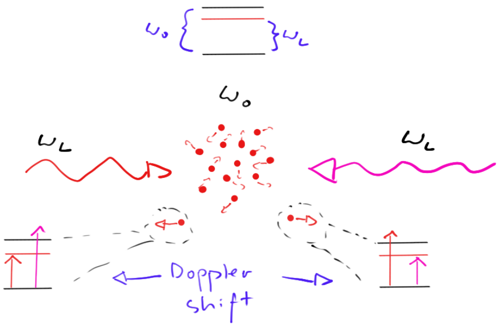


Laser cooling — Zeeman slower



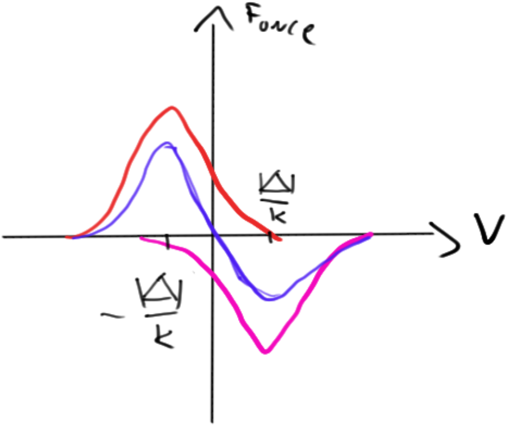
Laser cooling — Optical molasses

Two counter propagating, red-detuned laser beams with frequency $\omega < \omega_0$



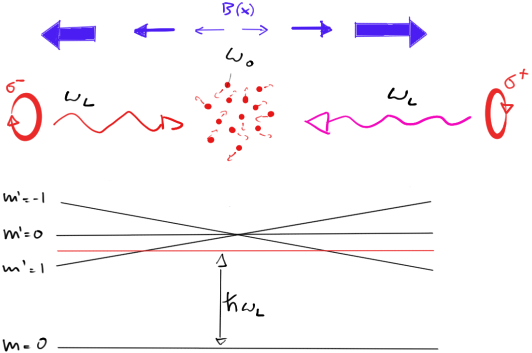
Atoms are more in resonance with counter-propagating beam. Atoms loose energy when emitting

Laser cooling — Optical molasses



For low velocities $F \propto v \implies$ cooling.

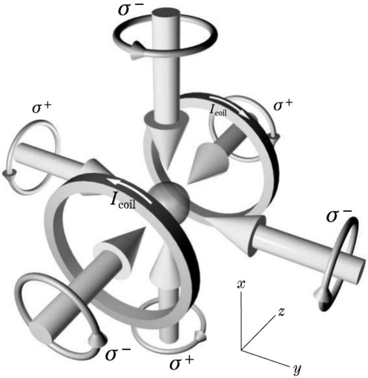
Laser cooling — Magneto-Optical trap



Force which depends on position $F \propto x$

$\Longrightarrow$ Magneto-Optical Trap

Laser cooling — Magneto-Optical trap



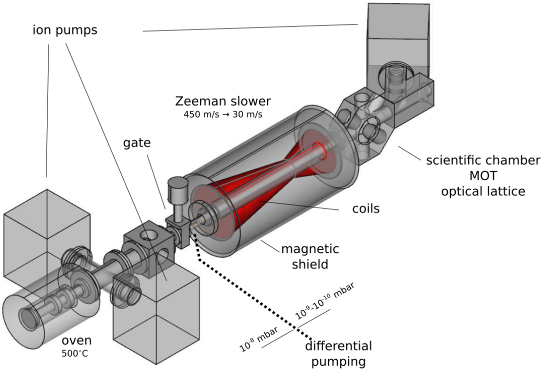
source: TM Brzozowski PhD Thesis

Cooling limit — In emissions of photons $\langle v^2 \rangle \neq 0$

$\Longrightarrow$ Doppler limit: $K_B T = \hbar\Gamma/2$

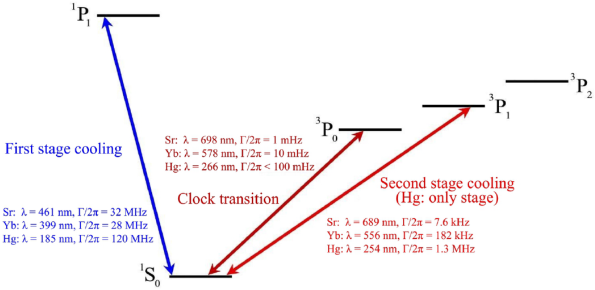
Vacuum System

And now back to Optical Lattice Clocks. . .



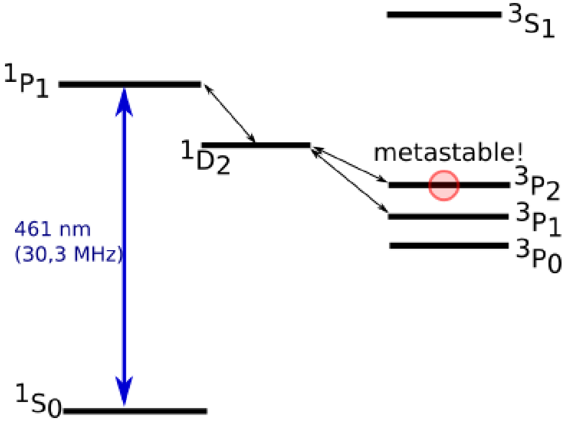
source: P. Morzynski PhD Thesis

Alkaline earth-like



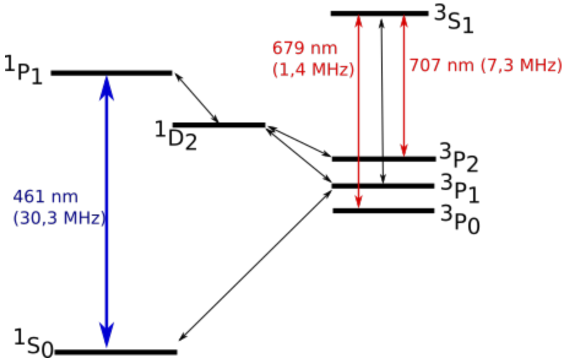
Two stages of magneto-optical trapping needed.

Alkaline earth-like



Real atoms in vast majority are not two-level atoms. . .

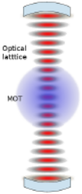
Alkaline earth-like

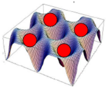


Solution:

Repumping lasers.

Lamb-Dicke regime

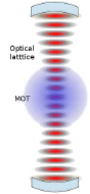


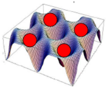


Lamb-Dicke regime: $\sqrt{\frac{\omega_{rec}}{\omega_V}}\sqrt{(n+1)} \ll 1$

Potential with depth $10E_{rec}$ or more is needed to neglect motions of atoms.

Lamb-Dicke regime



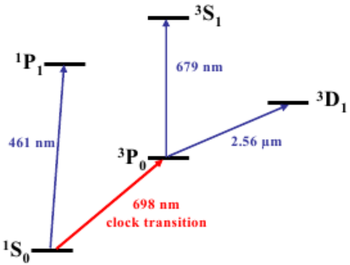


Lamb-Dicke regime: $\sqrt{\frac{\omega_{rec}}{\omega_V}}\sqrt{(n+1)} \ll 1$

Potential with depth $10E_{rec}$ or more is needed to neglect motions of atoms.

Strong trapping light

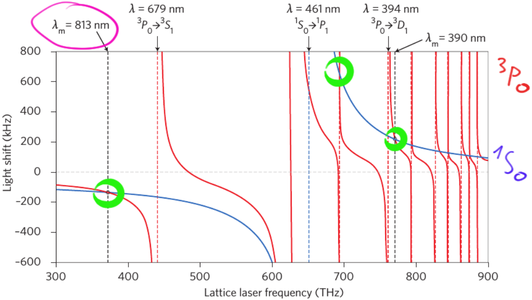
- huge light shift (AC Stark shift), at least several tens of kHz,
- polarisation dependent effects



Magic

$$U = -\frac{1}{4}\alpha(\epsilon, \omega_L)E^2$$

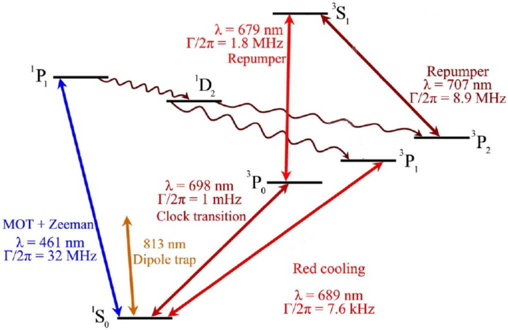
Solution: "Magic" wavelength



Source: Katori, Nature Photonics 5, 203, (2011)

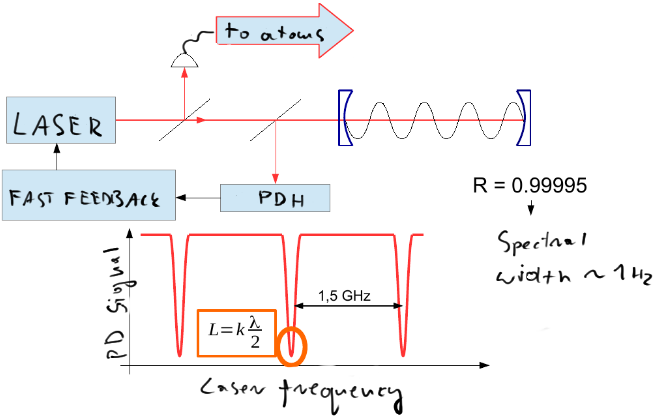
where polarizabilites, i.e. AC-Stark shifts, are equal for both clock states

Lasers for Sr optical lattice clock

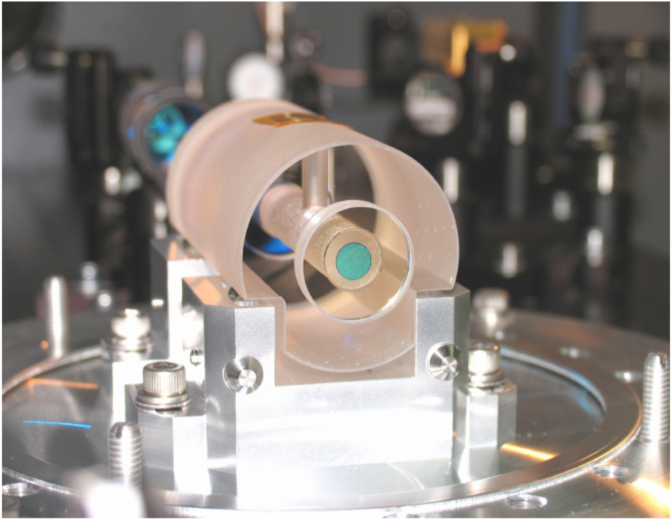


source: PG. Westergaard PhD Thesis

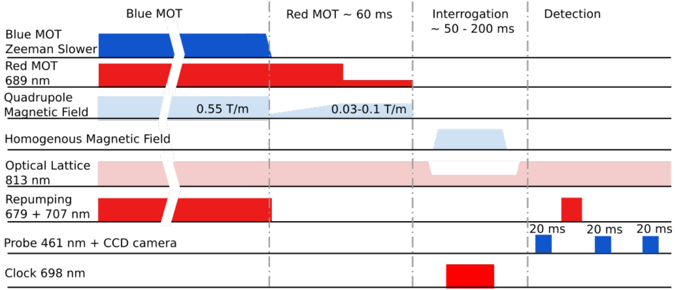
Clock laser


to atoms
LASER
FAST FEEDBACK
PDH
R = 0.99995
Spectral width ~ 1 Hz
PD Signal
1,5 GHz
$L=k\frac{\lambda}{2}$
Laser frequency

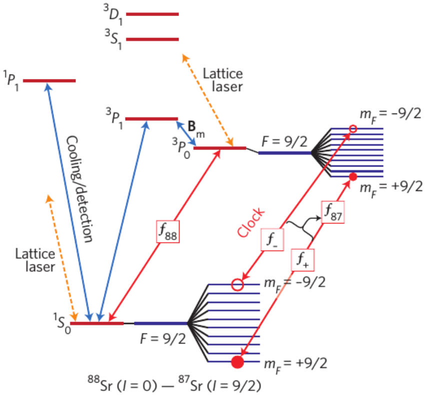
Ultra-stable cavity



Timing scheme of one cycle in ^{88}Sr clock



Bosons vs fermions



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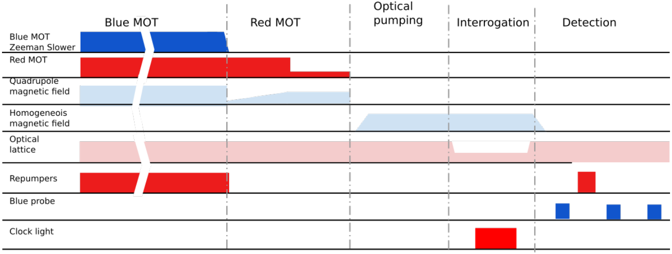
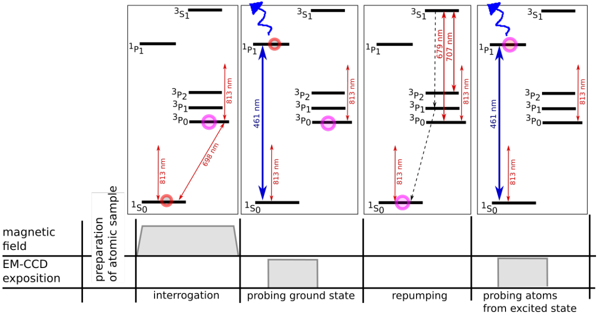
Timing scheme of one cycle in ^{87}Sr clock

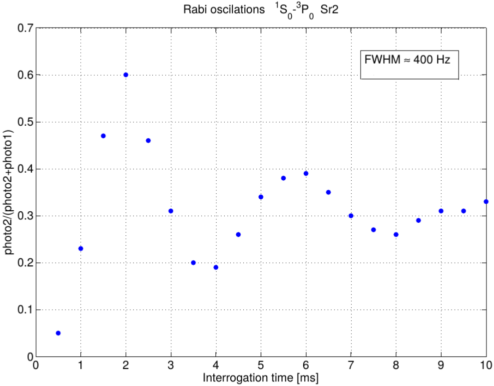


Interrogation and detection



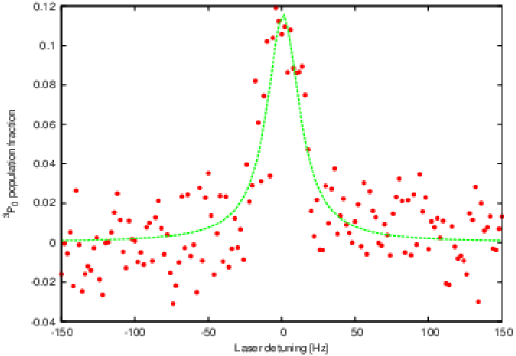
Fluorescence imaging with electron shelving technique probes directly the probability of the transition $P_c = \frac{N_e}{N_e+N_g}$

Detection



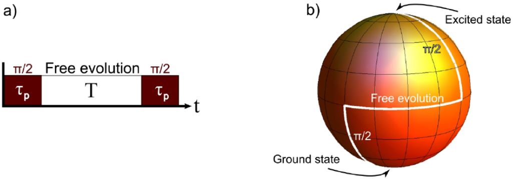
Remember Rabi oscillations?

Rabi interrogation



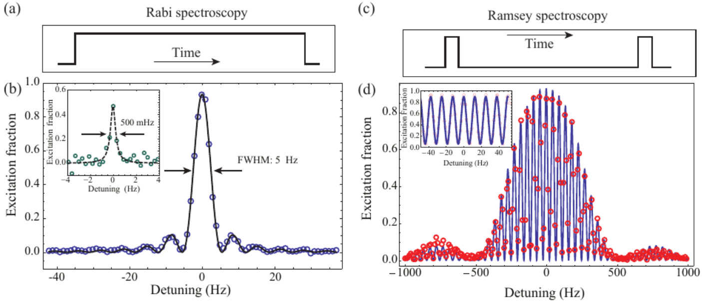
Single clock laser π pulse.

Ramsey interrogation



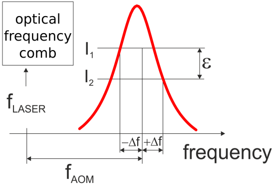
source: PG. Westergaard PhD Thesis

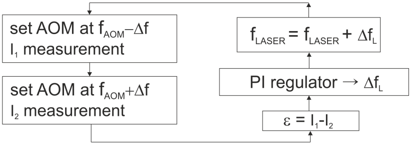
Ramsey interrogation



source: MN Bishof PhD Thesis

Lock to the clock line





set AOM at $f_{AOM}-\Delta f$
I_1 measurement

set AOM at $f_{AOM}+\Delta f$
I_2 measurement

$\varepsilon = I_1 - I_2$

PI regulator → Δf_L

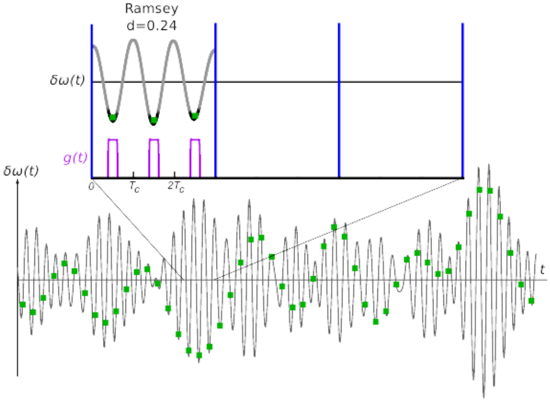
$f_{LASER} = f_{LASER} + \Delta f_L$

Stability

- noises of an oscillator (laser)
- noises of detection
- quantum projection noise

Dick effect

Dead time in each clock cycle leads to degrading the long term clock stability due to fast fluctuation of clock laser



source: PG Westergaard et al., IEEE Trans. Ultrason., Ferroelect., Freq. Control, 57, 623 (2010)

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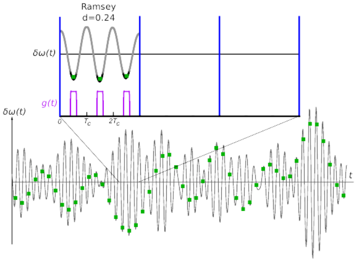
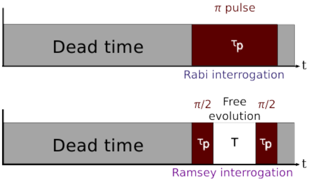


Depends on:

- S(f): noise of the laser
- d: Duty cycle
- $g(t)$: type of interrogation used



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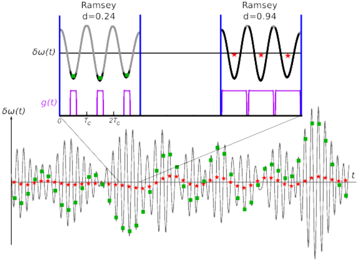

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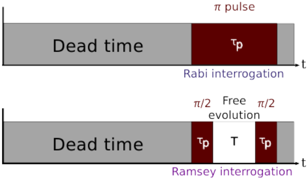

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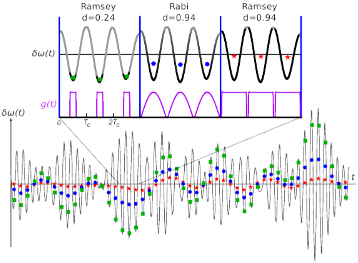

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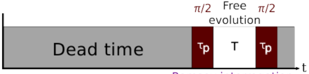
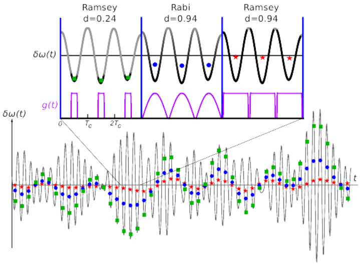
Rabi interrogation



Ramsey interrogation

Dick effect

Dead time in each clock cycle leads to degrading the long term clock stability due to fast fluctuation of clock laser



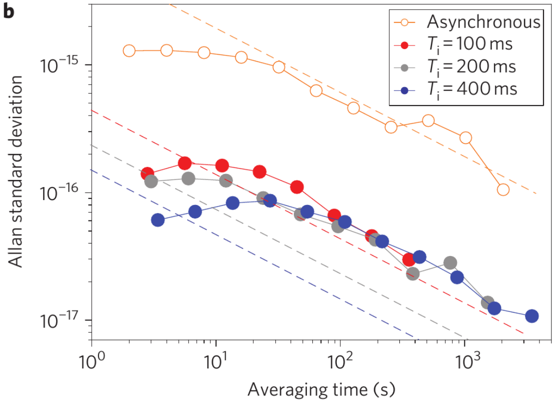
Depends on:

- S(f): noise of the laser
- d: Duty cycle
- $g(t)$: type of interrogation used

Limitation of the fractional Allan variance:

$$\sigma_L^2(\tau) = \frac{1}{\tau}\sum_{m=1}^{\infty}\left|\frac{g_m}{g_0}\right|^2 S(m/T_c)$$

Dick effect - synchronous vs asynchronous comparison



source: M. Takamoto et al. Nat. Photon. 5, 288, ((2011)

Detection noise

How the fluctuations of the clock transition frequency transfer into fluctuation of the locked clock laser

- An example - Ramsey interrogation, $\pi/2$ impulses τ_d long, separated by T
- Probability of transition at frequency ν is $p(\nu) = \frac{1}{2}\left(1 + cos\left(2\pi T\left((\nu - \nu_0)\right)\right)\right)$
- Fluctuations of the probability measurements δp and fluctuations of transition frequency $\delta\nu_0$ are connected by: $\delta p = \pi T \delta\nu_0$
- For any measurement $\delta p = \pi \int_{cycle} g(t)\delta\nu_0(t)dt$, where $g(t)$ is an atomic sensivity function
- δp is applied as an error signal to the PI lock of the laser. Therefore any noise in probability measurement induce noise of the frequency of the locked laser $\delta P/\pi T_{cycle} < g(t) >$

Quantum projection noise

- A two-level system prepared as a linear superposition $|\psi\rangle = c_A|A\rangle + c_B|B\rangle$
- We detect if the system is in $|A\rangle$ or $|B\rangle$
- Probability of the system in state $|A\rangle$ is equal to $p_A = |c_A|^2$
- Measurements can be predicted in certainty only when $c_A = 0$ or $c_B = 0$

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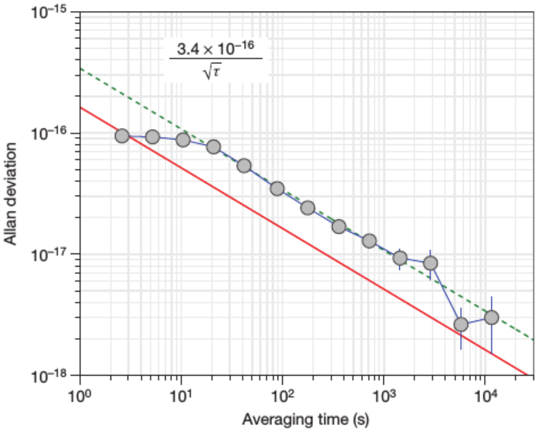
- Measurements can be predicted in certainty only when $c_A = 0$ or $c_B = 0$
- N independent atoms, N_A i N_B are in $|A\rangle$ and $|B\rangle$ states, respectively
- $P(N_B, N, p_B) = \frac{N!}{N_B!(N!-N_B!)}(p_B)^{N_B}(1-p_B)^{N-N_B}$
- variance of the binomial distribution: $\sigma^2 = Np_B(1-p_B)$

State-of-the art - above QPN



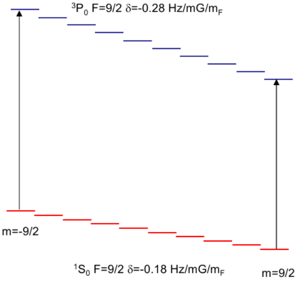
source: BJ Bloom et al. Nature 506, 71, (2014)

Accuracy

How well we can control shifts due to environment

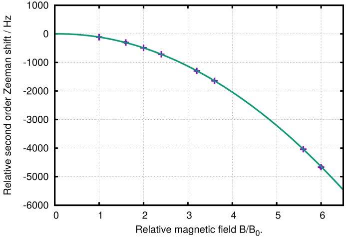
Zeeman effect: 1 and 2 order

linear



Optical pumping + interleaved measurements $m_F = \pm 1/2$

quadratic

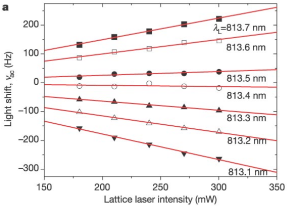


source: P. Morzynski et al. Sci Rep, 5, 17495 (2015)

Measurement with different value of total magnetic field and interpolation to zero

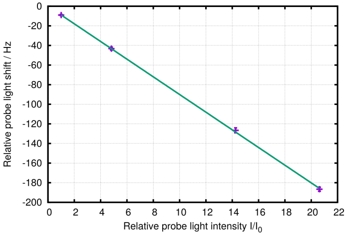
Linear AC Stark shift

Lattice light



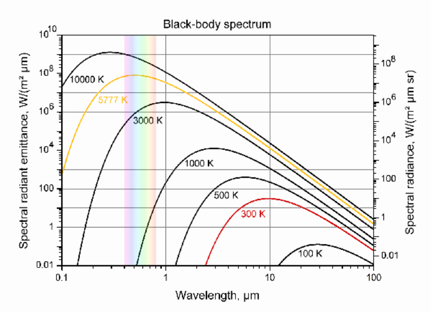
source: M. Takamoto et al Nature 435, 321 (2005)

Probe light



source: P. Morzynski et al. Sci Rep, 5, 17495 (2015)

Black body radiation



$$\Delta E_{|j>} = -\frac{1}{4h\varepsilon_0\pi^3c^3}\int_0^\infty \alpha_{|j>}(\omega)\frac{\omega^3}{e^{\hbar\omega/k_BT}-1}d\omega$$

Room temperature $\Rightarrow \lambda \sim 10\mu m$

^{87}Sr: $\Delta\nu_{|^1S_0>\rightarrow|^3P_0>,E1} = -2.354 \pm 0.032Hz$

Black body radiation

How to fight?

- Fractional correction from BBR shift at 300 K
  - Sr: -5.5×10^{-15}
  - Yb: -2.6×10^{-15}
  - Hg: -1.6×10^{-16}

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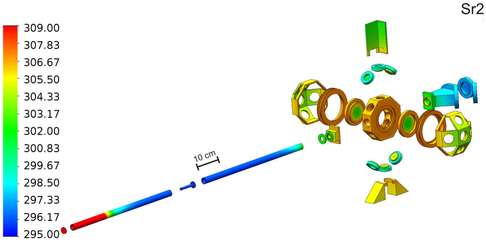
- Measure surroundings as good as possible and do the ray-tracing



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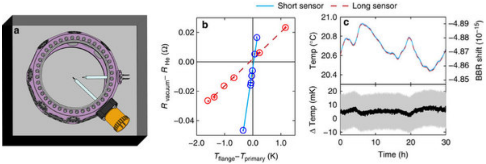

source: TL Nicholson et al Nat. Commun.6, 6896 (2015)

Black body radiation

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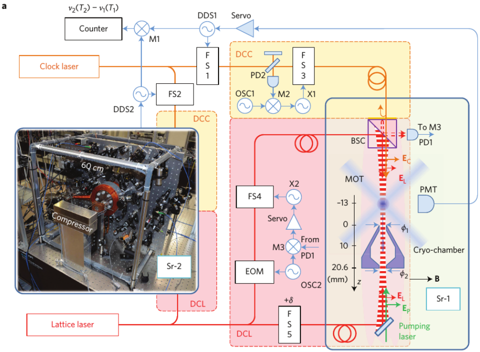
- Fractional correction from BBR shift at 300 K
  - Sr: -5.5×10^{-15}
  - Yb: -2.6×10^{-15}
  - Hg: -1.6×10^{-16}
- Measure surroundings as good as possible and do the ray-tracing
- Cryogenic environment



source: I. Ushijima Nat. Photon. 9, 185, (2015)

Higher-order light shifts

Accuracy of $10^{-18} \leftrightarrow$ light-shift control better than 10^{-8}

Higher-order light shifts

Accuracy of 10^{-18} ↔ light-shift control better than 10^{-8}

- hyperpolarisability
  $\Delta\nu = -\frac{1}{4h}\Delta\alpha(\omega, \boldsymbol{e})E^2 - \frac{1}{16h}\Delta\gamma(\omega, \boldsymbol{e})E^4$

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- three contributions of electrical dipol polarisability $\alpha(\omega, \boldsymbol{e})$: scalar, vector and tensor
  $\Delta\nu_{\alpha}^{E1} = \left(\Delta\kappa^{s} + \Delta\kappa^{v}\xi m_F \sin\psi + \Delta\kappa^{t} f(\theta, \psi, \xi)\left[3m_F^2 - F(F-1)\right]\right) U_0$

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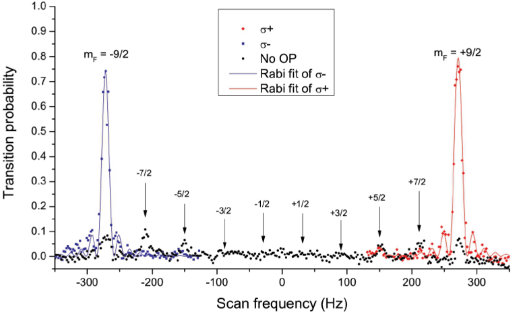
- higher multipoles besides $E1$
  $\Delta\nu^{M1/E2} = \zeta(n + 1/2)\sqrt{U_0}$

Other effects

- Line pulling



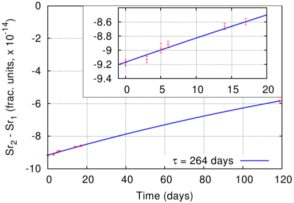
Other effects

- Line pulling
- Collisions
  In fermions cold-cold collisions are limited, since *s*-wave scattering is forbidden (possible *p*-wave scattering of $^3P_0$ states)

Other effects

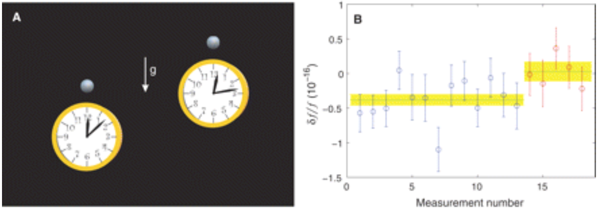
- Line pulling
- Collisions
- DC Stark shift

  Static charges on dielectrics inside the vaccum



Can be high (up to $\sim$ 40 Hz), but easy to remove.
(IEEE TUFFC 59, 411, 2012)

Gravitational redshift



Clock raised by 33 cm.