Quantum Tunneling & Ergodicity in Quantum Spin Glasses

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Work done in collaboration with:

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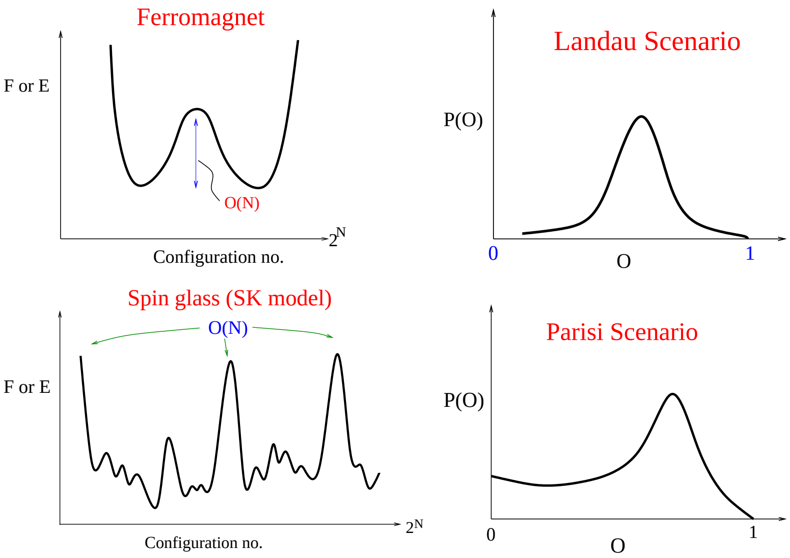
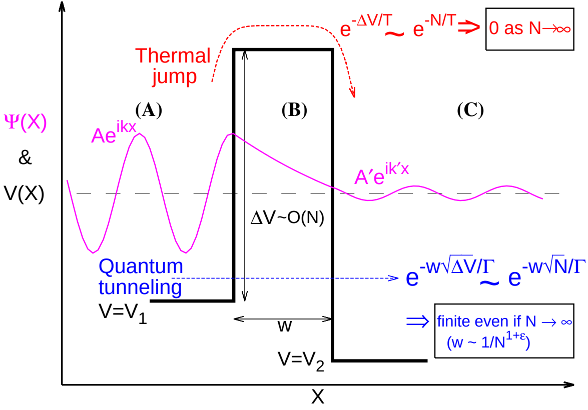
Ferromagnet vs. Spin glass energy landscapes



Tunneling picture



- Here $k = \sqrt{E - V_1} \simeq k' = \sqrt{E - V_2}$ & $V_1 \simeq V_2 = 0$
- Tunneling probability, $\mid \frac{A'}{A} \mid^2 \sim e^{-\kappa w}$;

  $\kappa = \sqrt{\Delta V - E} \sim \sqrt{N}(1 - \Gamma)^{\frac{1}{2}}$

Ref: P. Ray, B. K. Chakrabarti, and A. Chakrabarti Phys. Rev. B. **39**, 11828 (1989)

V. N. Smelyanskiy et al., arXiv:1204.2821 (2012).

Quantum Sherrington-Kirkpatrick spin glass model

$$H = H_0 + H_I;\ H_0 = -\sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z;\ H_I = -\Gamma\sum_{i=1}^{N}\sigma_i^x$$

J_{ij} are the interactions between two spins at i and j and follow the distribution

$$\rho(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{\frac{1}{2}} \exp\left(\frac{-NJ_{ij}^2}{2J^2}\right)$$

Γ is the transverse field term.

Motivation: Study of critical behavior of the model for the entire phase diagram in $\Gamma - T$ plane

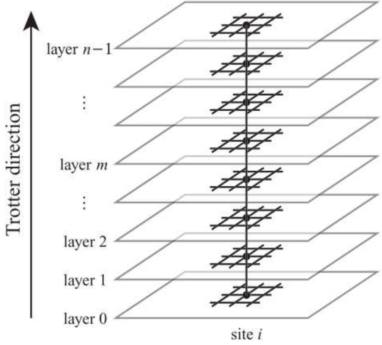
Refs: S. Suzuki, J.-i. Inoue, and B. K. Chakrabarti, Springer, Heidelberg (2013); A. Dutta, G. Aeppli, B. K. Chakrabarti, U. Divakaran, T. Rosenbaum and D. Sen, Cambridge Univ. Press, Delhi (2015).

Suzuki-Trotter formalism

The effective classical Hamiltonian

$$H_{eff} = -\sum_{n=1}^{M}\sum_{i<j}\frac{J_{ij}}{M}\sigma_i^n\sigma_j^n - \sum_{i=1}^{N}\sum_{n=1}^{M}\frac{1}{2\beta}\log \coth\frac{\beta\Gamma}{M}\sigma_i^n\sigma_i^{n+1}$$

where $\sigma_i^n = \pm 1$ is the classical Ising spin and β is the inverse of temperature T



Monte Carlo simulation: order parameter and Binder cumulant

Consider the replica overlap q, with $\overline{\langle q \rangle}$ as a order parameter

$$q = \frac{1}{NM} \sum_{i=1}^{N} \sum_{n=1}^{M} (\sigma_i^n(t))^{\phi} (\sigma_i^n(t))^{\theta}$$

where $(\sigma_i^n)^{\phi}$ and $(\sigma_i^n)^{\theta}$ are the spins of two different replicas ϕ and θ corresponding to the same realization of disorder

Define the average Binder cumulant as

$$g = \frac{1}{2}\left[3 - \overline{\left(\frac{\langle q^4 \rangle}{(\langle q^2 \rangle)^2}\right)}\right]$$

Ref: K. Binder and A.P. Young, Rev. Mod. Phys. **58**, 801 (1986); M. Guo, R. N. Bhatt, and D. A. Huse, Phys. Rev. Lett. **72**, 4137 (1990)

Scaling of the Binder cumulant

Near critical point g scales as

$$g = g(L/\xi, M/L^z)$$

where $L \rightarrow$ linear size of the system, $z \rightarrow$ dynamical exponent and $\xi \rightarrow$ correlation length, scales as $\xi \sim (T - T_c)^{-\nu_T}$ or $(\Gamma - \Gamma_c)^{-\nu_\Gamma}$ with exponents ν_T or ν_Γ

Hence close to critical region g can be written as

$$g \sim g((T - T_c)N^{x_T}, \frac{M}{N^{z/d_c}}) \text{ or } g((\Gamma - \Gamma_c)N^{x_\Gamma}, \frac{M}{N^{z/d_c}})$$

where $x_T = \frac{1}{\nu_T d_c}$ and $x_\Gamma = \frac{1}{\nu_\Gamma d_c}$ with $L = N^{1/d_c}$, $d_c \rightarrow$ effective dimension of the system

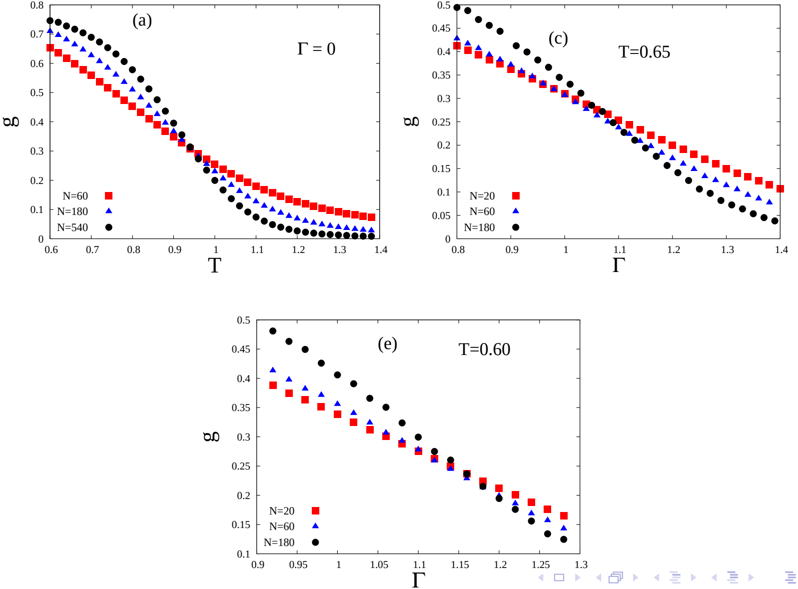
Monte Carlo simulation

- To simulate H_{eff} we take system sizes $N = 20, 60, 180$ and consider $d_c = 6$ and $z = 4 \Rightarrow$ associated with classical S-K model
- keep M/L^z fixed, $M = 10, 21, 43$ for system sizes $N = 20, 60, 180$ respectively
- Equilibrium time $\equiv 75000$ Monte Carlo steps and 25000 steps for thermal averaging

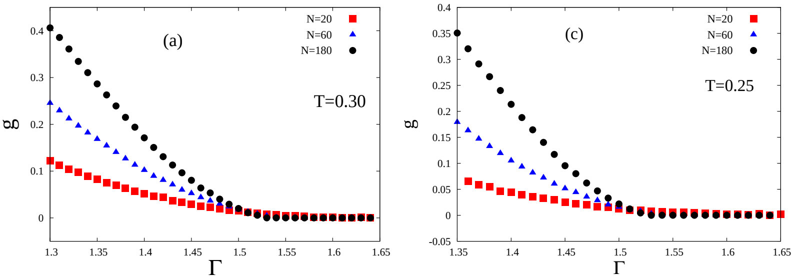
The intersection of the g vs. Γ curves for different N (keeping M/L^z fixed) gives the estimate of values of Γ_c and critical Binder cumulant g_c

Ref: A. Billoire and I. A. Campbell, Phys. Rev. B. **84**, 054442 (2011).

Binder cumulant: high T



Binder cumulant: low T

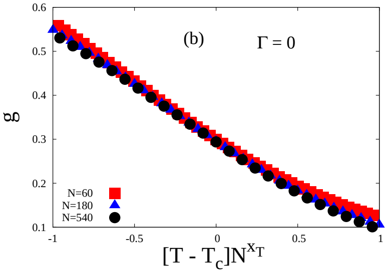


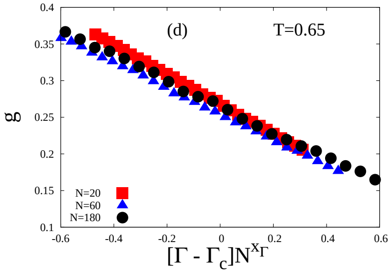
Starting from the classical S-K model at $\Gamma = 0$ to $T = 0.50$ ($\Gamma \simeq 1.30$), g_c takes a constant value 0.22 ± 0.02

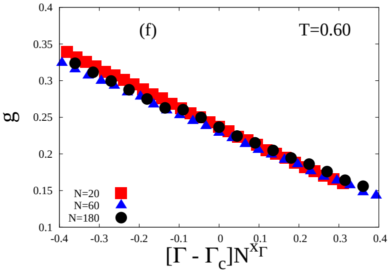
Whereas in the range $T = 0.35$ ($\Gamma \simeq 1.45$) to $T = 0.20$ ($\Gamma \simeq 1.54$), $g_c \simeq 0$

Does this crossover in g_c indicate something interesting?.....

Collapses of g curves

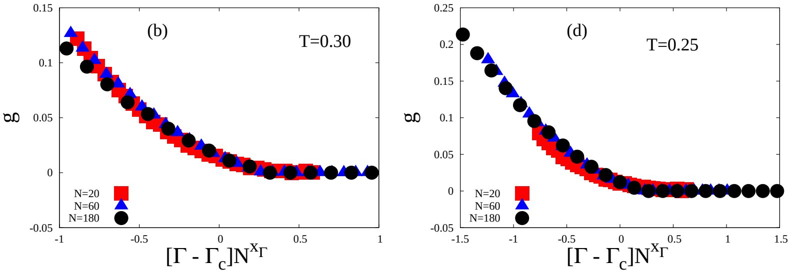






$x_T = x_\Gamma = 0.31 \pm 0.02$

Collapses of g curves contd.



Do not get good collapses with $d_c = 6$ and $z = 4$ for any x_Γ

Repeat our simulation with $d_c = 8$ and $z = 2$ (values correspond to quantum S-K model)

Collapse of g curves for $x_\Gamma = 0.50 \pm 0.02$

How does the critical behavior of g look like at $T = 0$?

Refs: D. Lancaster and F. Ritort, J. Phys. A: Math. Gen, **30**, L41 (1997); N. Read, S. Sachdev and J. Ye, Phys. Rev. B. **52**, 384 (1995)

Exact diagonalization: Lanczos method

$$H = H_0 + H_I; \; H_0 = -\sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z; \; H_I = -\Gamma\sum_{i=1}^{N}\sigma_i^x$$

- Construct the Hamiltonian in spin basis states i.e., the eigenstates of the spin operators (σ_i^z, $i = 1, .., N$)
- $|\psi_n\rangle = \sum_{\alpha=0}^{2^N-1} a_\alpha^n|\varphi_\alpha\rangle$, where $|\varphi_\alpha\rangle$ are the eigenstates of the Hamiltonian H_0 and $a_\alpha^n = \langle\varphi_\alpha|\psi_n\rangle$

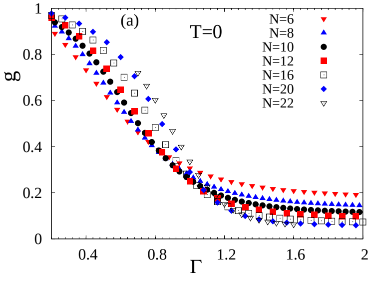
$$g = \frac{1}{2}\Big[3 - \overline{\Big(\frac{Q_4}{(Q_2)^2}\Big)}\Big]$$

where the various moments can be calculated using

$$Q_k = \frac{1}{N^k}\sum_{i_1}^{N}\ldots\sum_{i_k}^{N}\langle\psi_0|\sigma_{i_1}^z\ldots\sigma_{i_k}^z|\psi_0\rangle^2$$

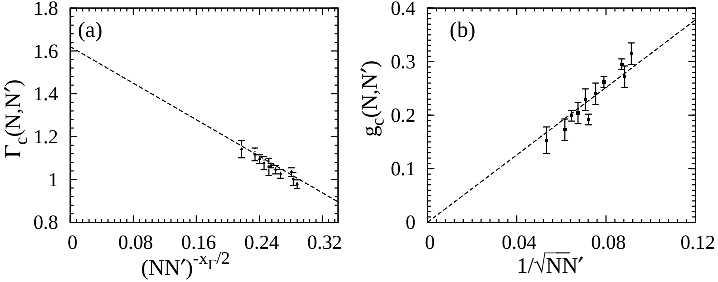
Ref: P. Sen, P. Ray, and B. K. Chakrabarti, arXiv:cond-mat/9705297 (1997)

Zero-temperature results



The larger system sizes intersect at higher values of Γ signifying finite size effect of the system. Binder cumulant curves for different system sizes (N) with an estimated $\Gamma_c^e = 1.62$ and exponent $x_\Gamma = 0.5$

Zero-temperature results contd.

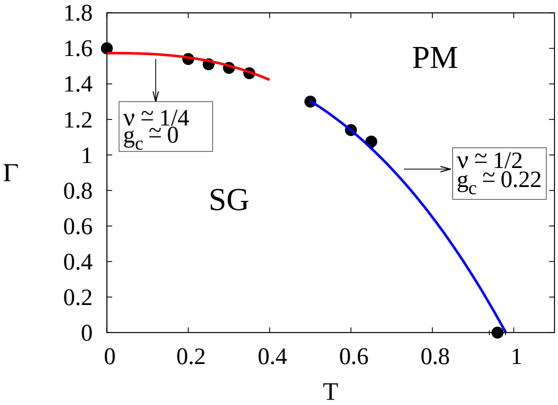


The extrapolated value of $\Gamma_c(N, N')$ is 1.62 ± 0.03 in the limit of $N, N' \to \infty$ and the best fit value of the scaling exponent x_Γ is 0.51.

These values are consistent with that obtained from collapse of g curves for different N

g_c takes nearly a zero value in the limit of $N, N' \to \infty$

Phase diagram

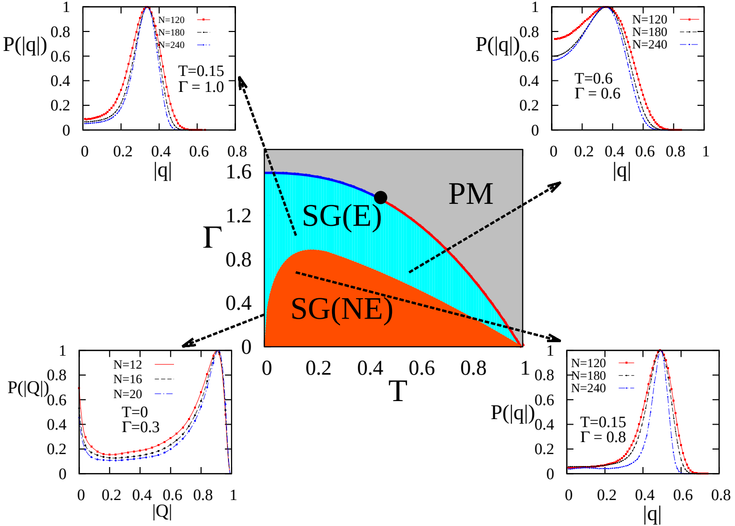


correlation length exponent ν is obtained from $x_\Gamma = x_T = 1/d_c\nu$ with $d_c = 6$ and 8 for classical and quantum cases respectively

Ref: S. Mukherjee, A. Rajak, and B. K. Chakrabarti, Phys. Rev. E. **92**, 042107 (2015)

N. Y. Yao et al., arXiv:1607.01801 (2016).

Possible ergodic region of the spin glass phase in the quantum SK model: Tunneling effect



(Work in progress; in collaboration with S. Mukherjee and A. Rajak)

Thank You