Phase transitions in quantum error correction

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Quantum Error Correction

Inevitable task for scalable quantum computers

Deep connection between
physics and quantum information

Quantum Annealing

Theoretical aspect of Quantum Annealing
as quantum computation

Error correction

Quantum speedup

Rapid progress in experiments

D Wave 2X

In this talk:

Optimisation: find the lowest energy state
Error: excitation during QA

Problem:

Quantum phase transitions
(Landau-Zener)

Annealing time $t \sim (\Delta E)^{-2}$

Suppressing phase transition is important for both
error correction and "quantum speedup"

Understand phases from computational point of view

Optimisation problem ⟶ Physical Hamiltonian

one-to-many mapping

Programable error correction methods

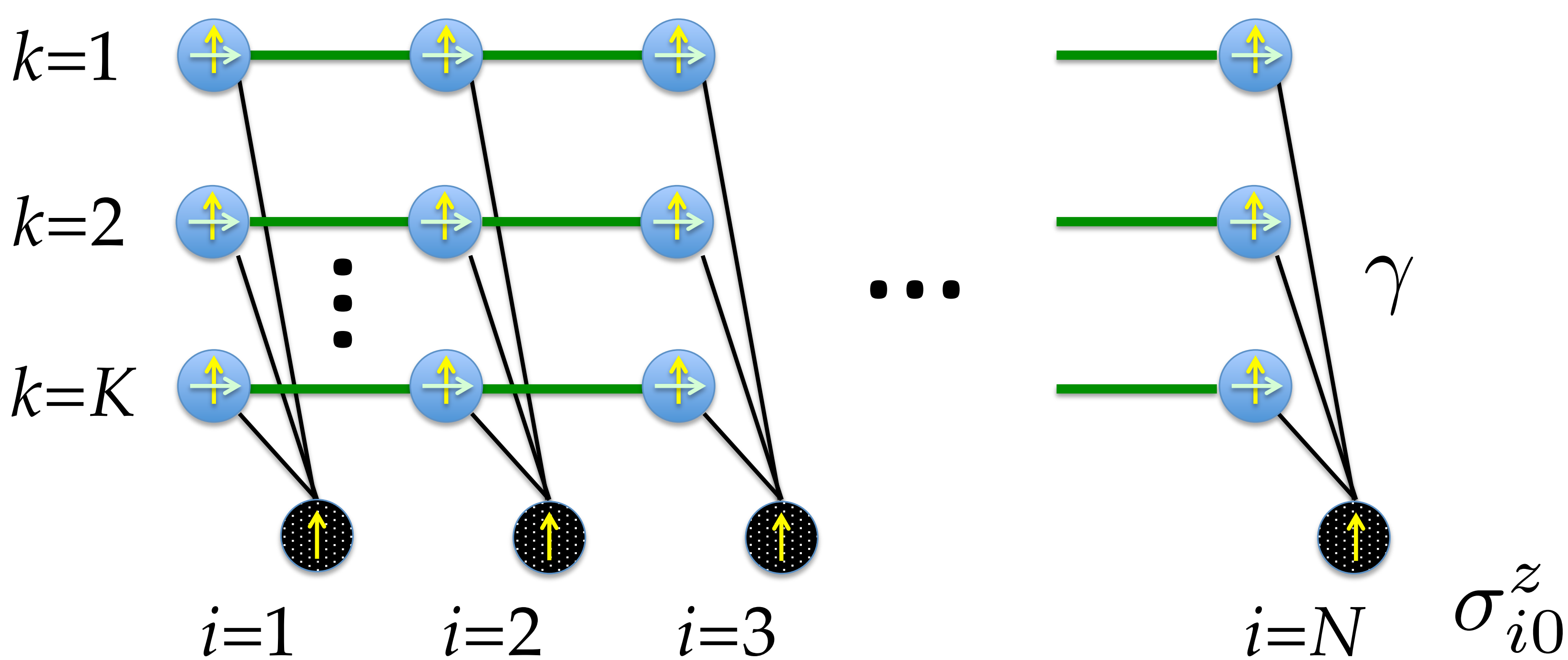
1) Penalty Quantum Annealing Correction

2) Nested Quantum Annealing Correction

Penalty Quantum Annealing Correction

[Pudenz-Albash-Lidar]

$$H_Z = -\sum_{(ij),k} J_{ij}\sigma^z_{ik}\sigma^z_{jk} - \sum_{i,k} h_i \sigma^z_{ik} - \gamma \sum_{i,k} \sigma^z_{ik}\sigma^z_{i0}$$



Problem qubits

Penalty qubits

Repetition code

$$|0\rangle \rightarrow |00\cdots 0\rangle$$
$$|1\rangle \rightarrow |11\cdots 1\rangle$$

D-wave machine results:
significant improvement in success rate

Meal field analysis

Infinite range model

$$H = -N\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_{iz}^{k}\right)^{p} - \Gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ix}^{k} - \gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{iz}^{k}\sigma_{iz}^{0}.$$

Partition function: $Z = \mathrm{Tr}(e^{-\beta H})$

Suzuki-Trotter decomposition

$$e^{A+B} = \lim_{M\to\infty}\left(e^{A/M}e^{B/M}\right)^{M}$$

Order parameter $m = \frac{1}{N}\left\langle\sum_{i=1}^{N}\sigma_i^z\right\rangle$

Free energy: $F = -\frac{1}{\beta N}\log Z$

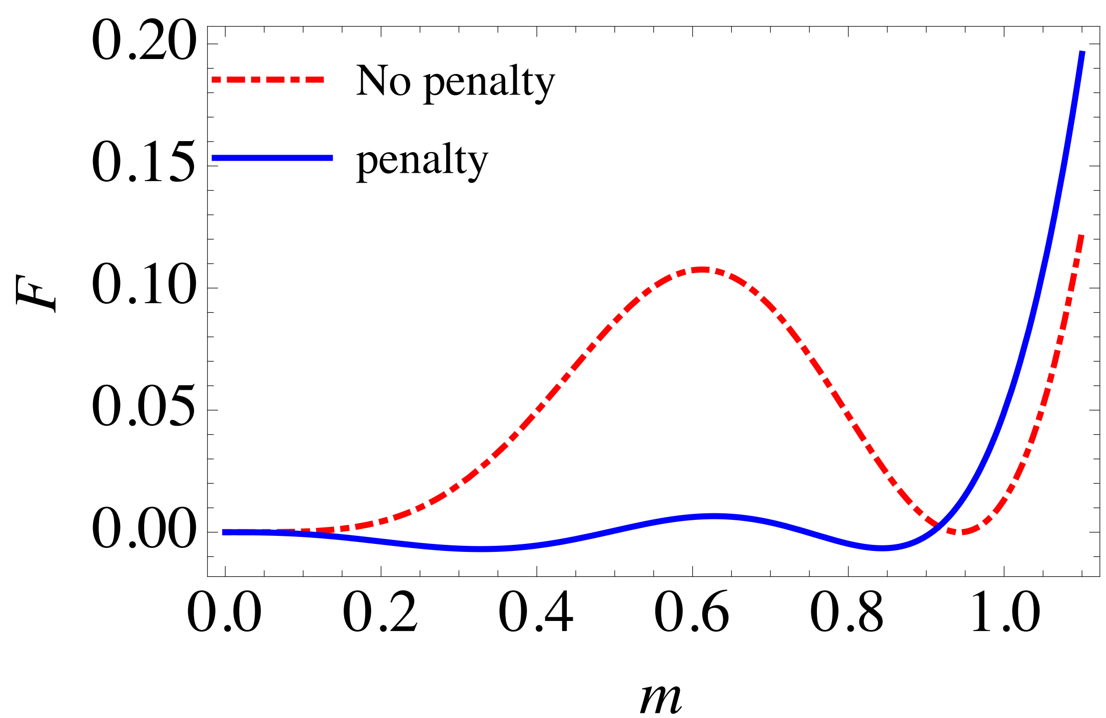
$$F = (p-1)m^p - \frac{1}{\beta K}\log\left(\left[2\cosh(\beta\sqrt{v_-^2+\Gamma^2})\right]^K + \left[2\cosh(\beta\sqrt{v_+^2+\Gamma^2})\right]^K\right)$$

$$v_\pm(m) = pm^{p-1} \pm \gamma$$

penalty qubit behaves as a random external field

Zero temperature

Free energy at critical point

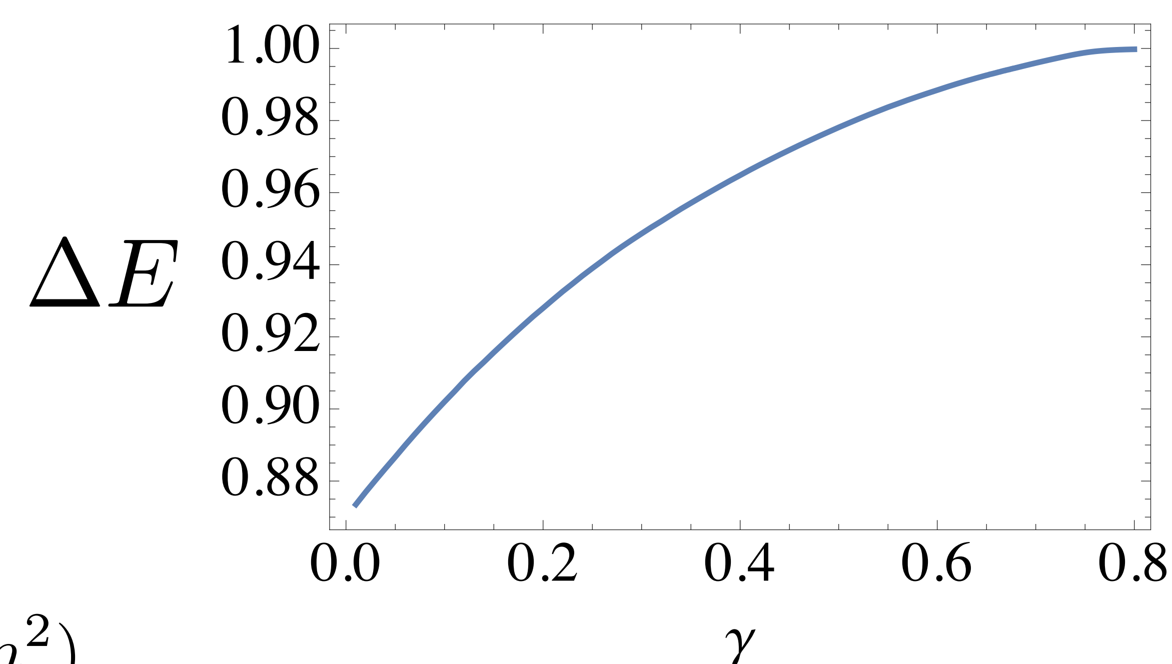


$$F/K = -\sqrt{\gamma^2+\Gamma^2} - \frac{2\gamma}{\sqrt{\gamma^2+\Gamma^2}}|m| + \mathcal{O}(m^2)$$

Energy Gap

$\Delta E \simeq$ Instanton solution

$$\simeq \exp\left(-\int_0^\beta \mathcal{L}dt\Big|_{\substack{m(0)=m_{\mathrm{small}}\\ m(\beta)=m_{\mathrm{large}}}}\right)$$



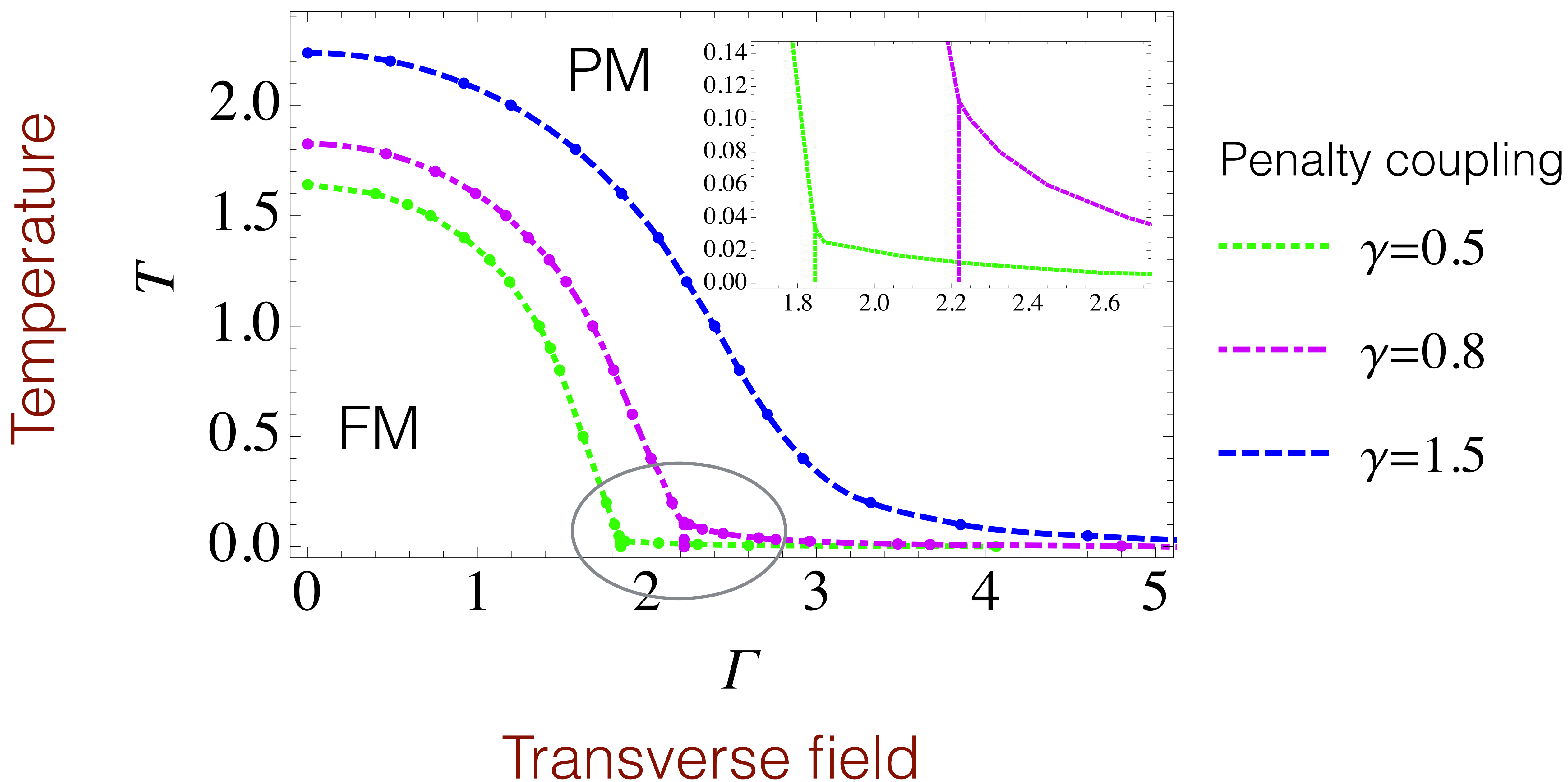
Penalty strength

Penalty term weakens/removes first order phase transitions

Finite Temperature

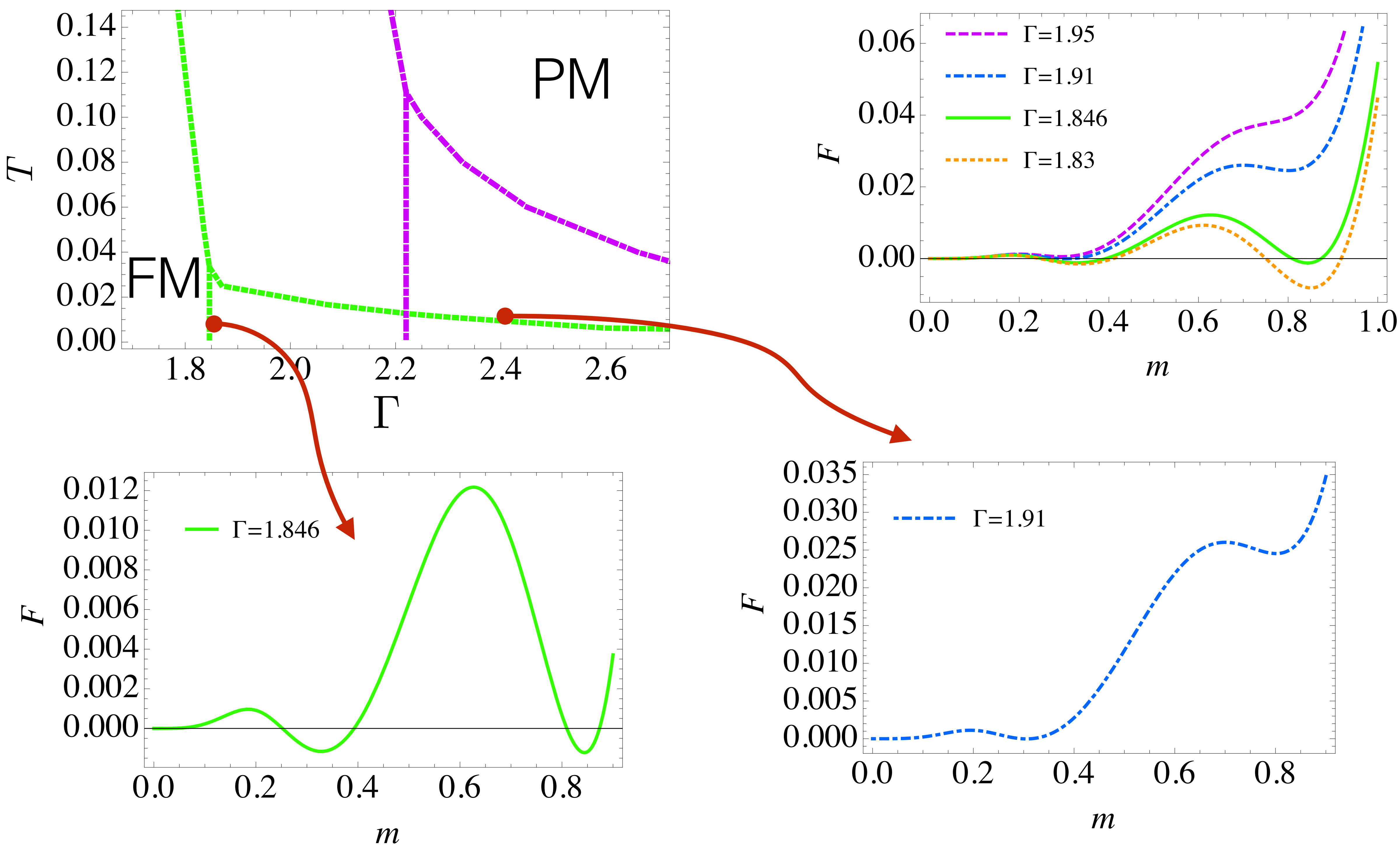
Phase diagram

$$H = -N \sum_{k=1}^{K} \left(\frac{1}{N} \sum_{i=1}^{N} \sigma_{iz}^{k} \right)^{p} - \Gamma \sum_{k=1}^{K} \sum_{i=1}^{N} \sigma_{ix}^{k} - \gamma \sum_{k=1}^{K} \sum_{i=1}^{N} \sigma_{iz}^{k} \sigma_{iz}^{0}.$$



penalty coupling: $\gamma = 0.5$

1st order phase transition exists at zero T



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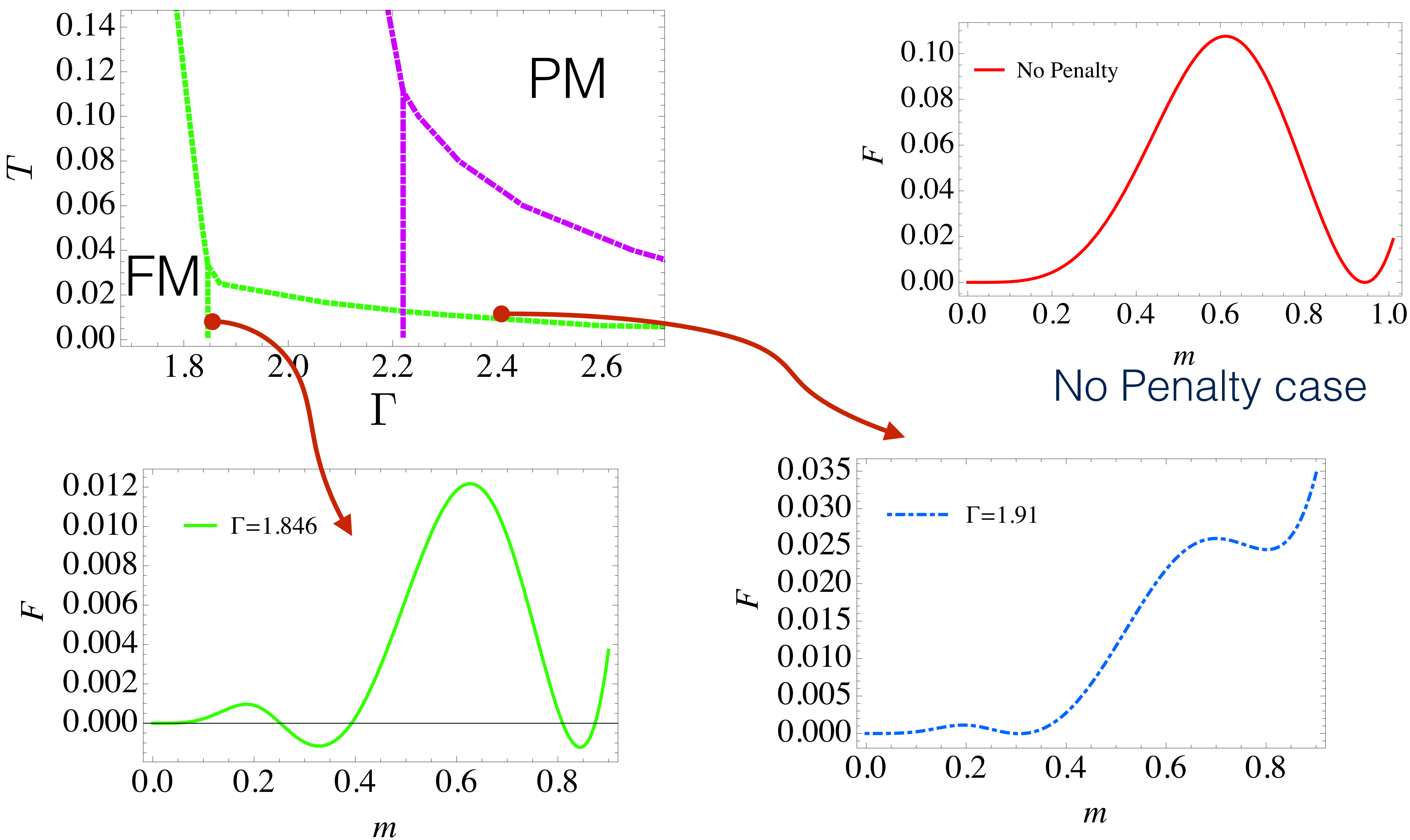
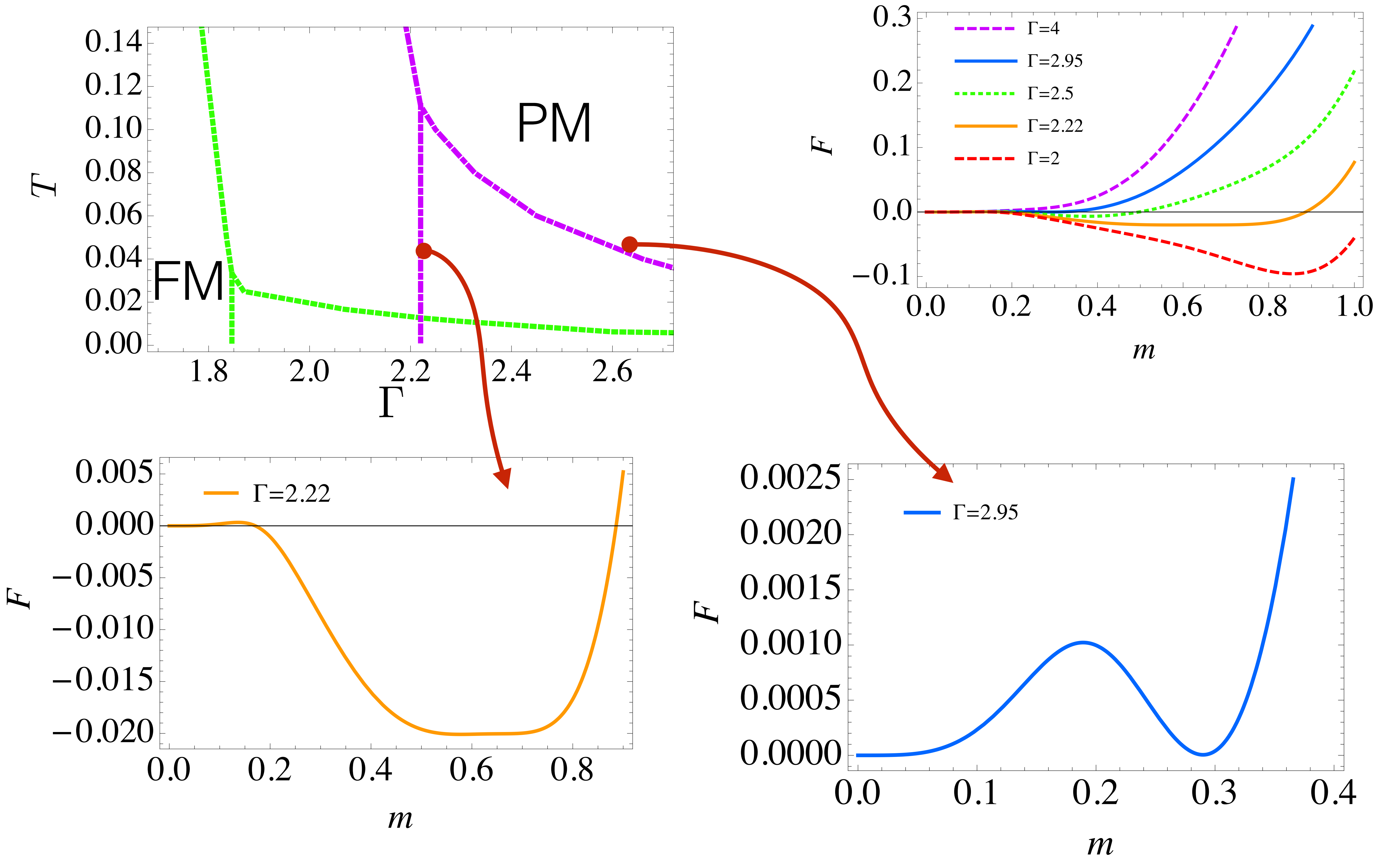


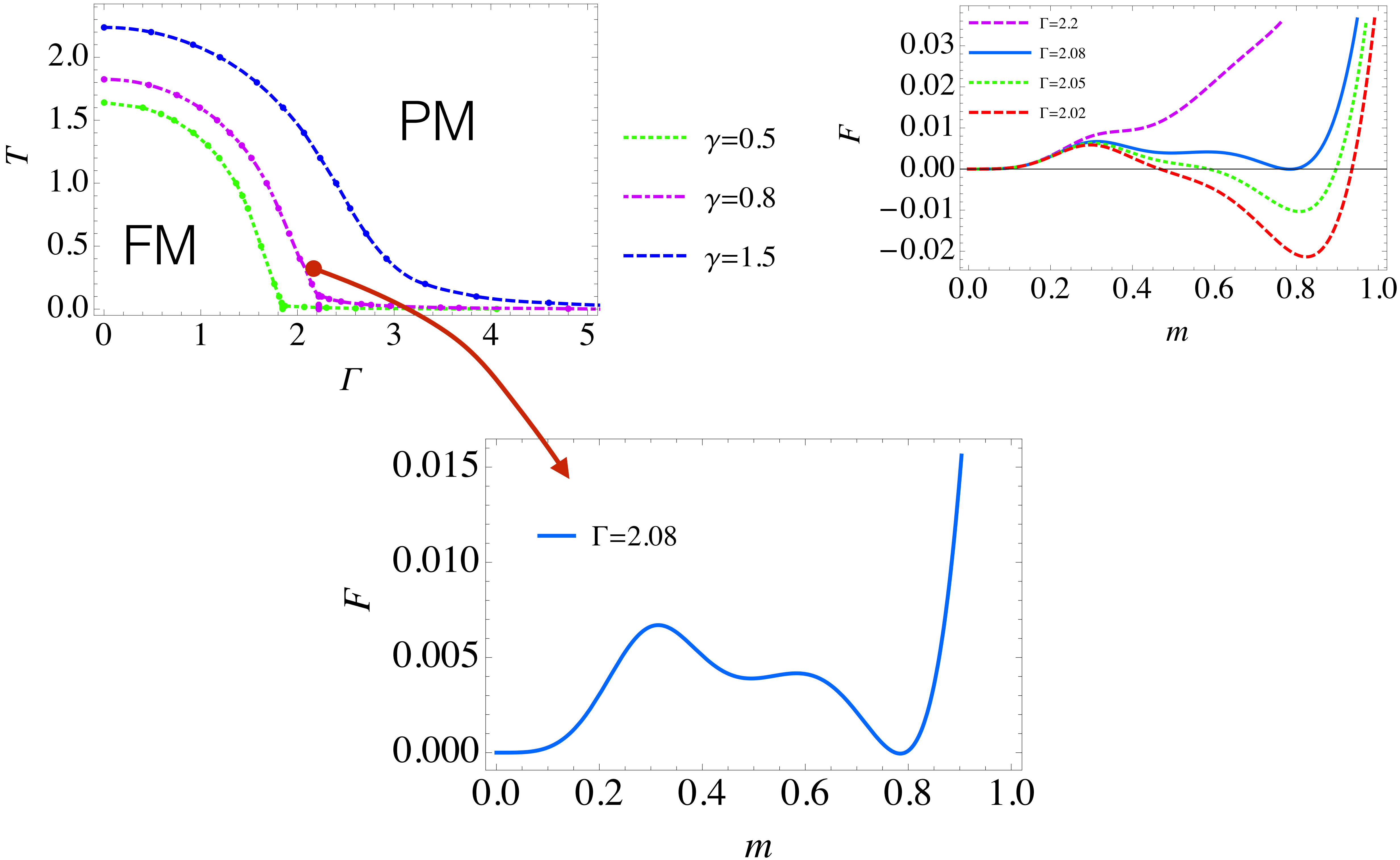
Difficult problem → multiple easier problems

penalty coupling: $\gamma = 0.8$

phase transition disappears at zero T



Higher temperature

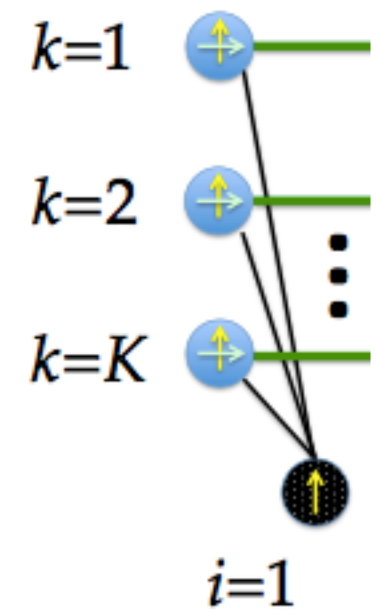


temperature reduces error correction effect

Transverse field in penalty qubits

Microscopic Hamiltonian

$$H = -N \sum_{k=1}^{K} \left(\frac{1}{N} \sum_{i=1}^{N} \sigma_{ik}^{z} \right)^{p} - \Gamma \sum_{k=1}^{K} \sum_{i=1}^{N} \sigma_{ik}^{x} - \gamma \sum_{k=1}^{K} \sum_{i=1}^{N} \sigma_{ik}^{z} \sigma_{i0}^{z} - \epsilon\Gamma \sum_{i=1}^{N} \sigma_{i0}^{x}$$



Effective Hamiltonian

$$H_0 = \sum_{k=1}^{K} H_k, \quad H_k = -pm^{p-1}\sigma_z^k - \gamma\sigma_z^0\sigma_z^k - \Gamma\sigma_x^k$$

$$V = -\epsilon\Gamma\sigma_x^0$$

$$H = H_0 + V$$

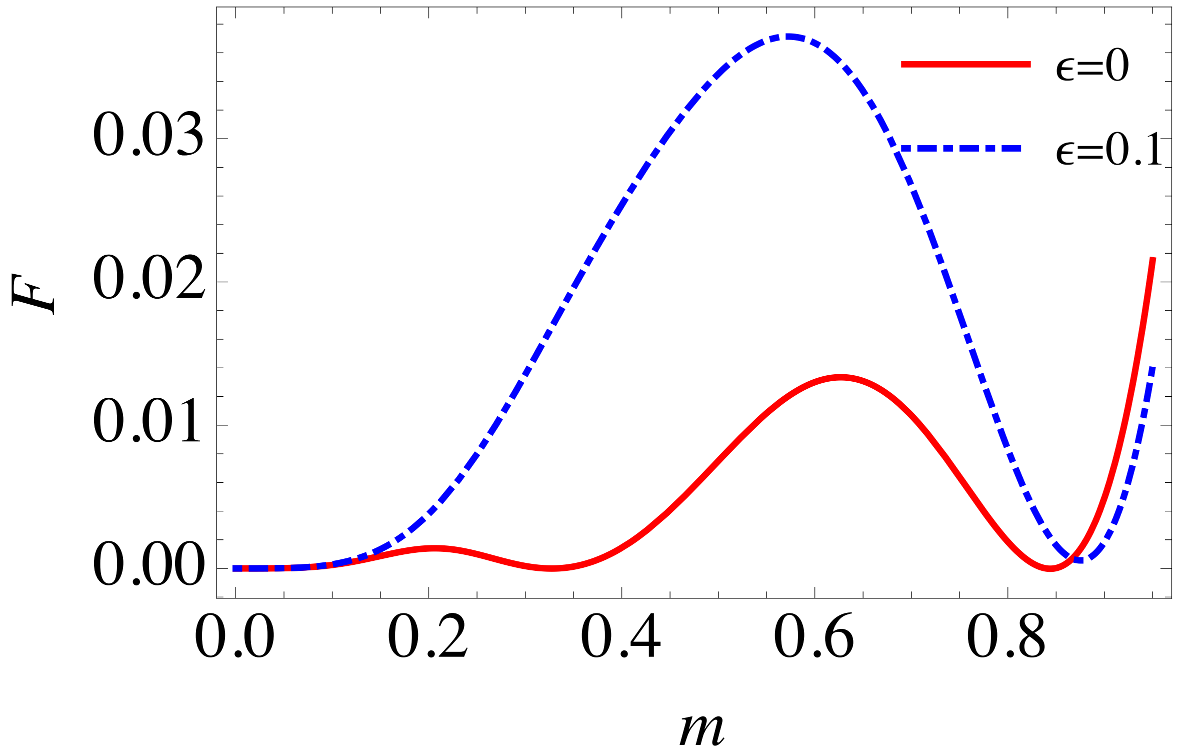
Transverse field in penalty qubits

First order phase transition

$$F = (p-1)m^p - \frac{1}{K\beta}\log\left[\mathrm{Tr}\exp(\beta H)\right]$$

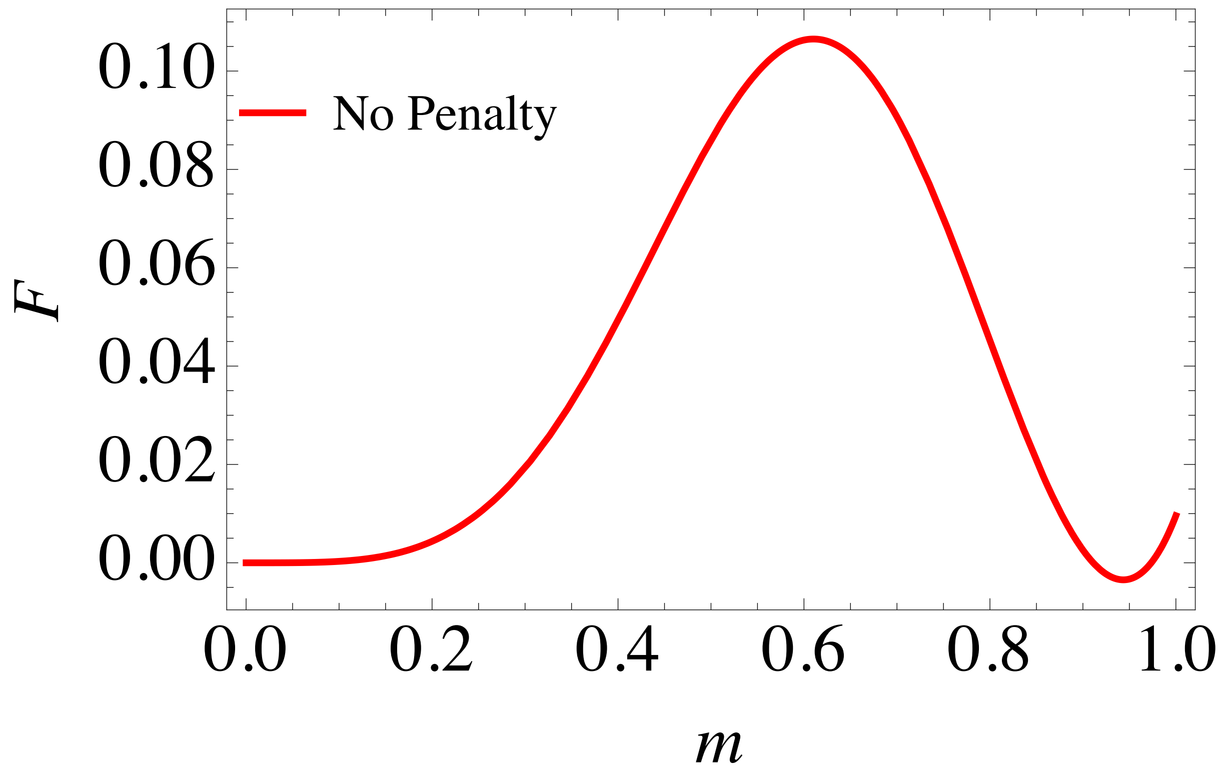
Numerical results

Penalty



$$\gamma = 0.5, \quad T = 0.03$$

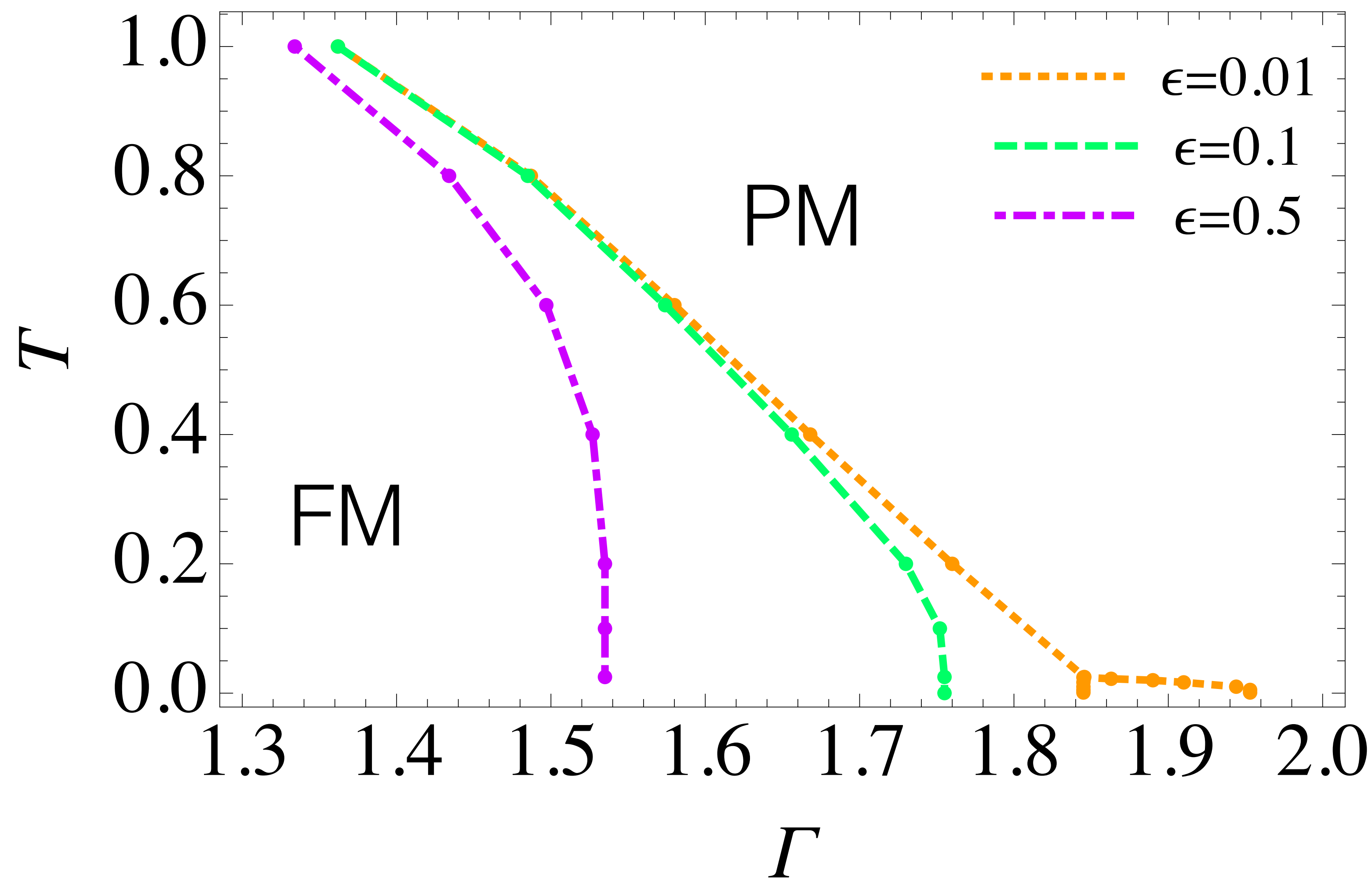
No penalty



$$\gamma = 0, \quad T = 0.03$$

Phase diagram

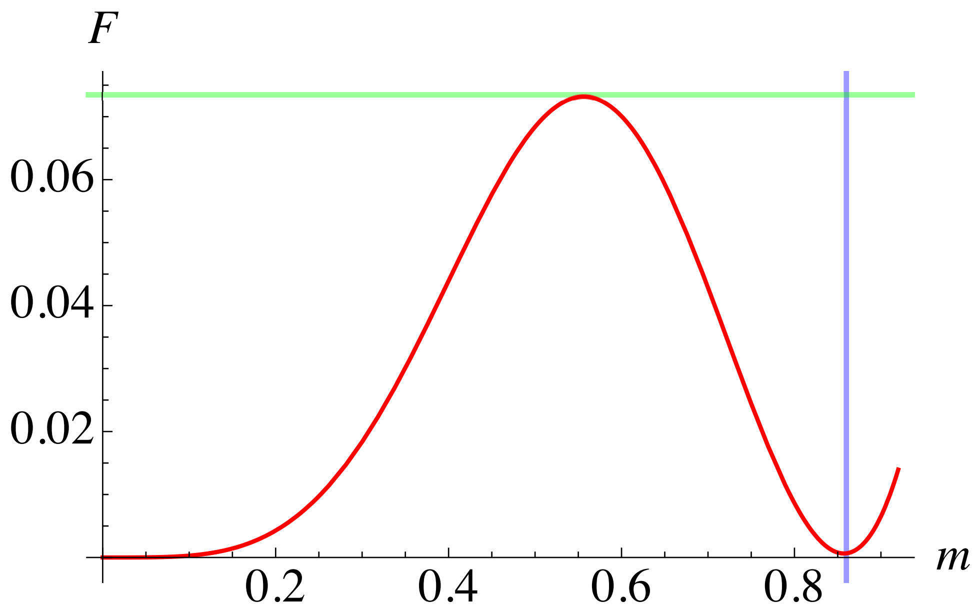
(1st order phase transition)



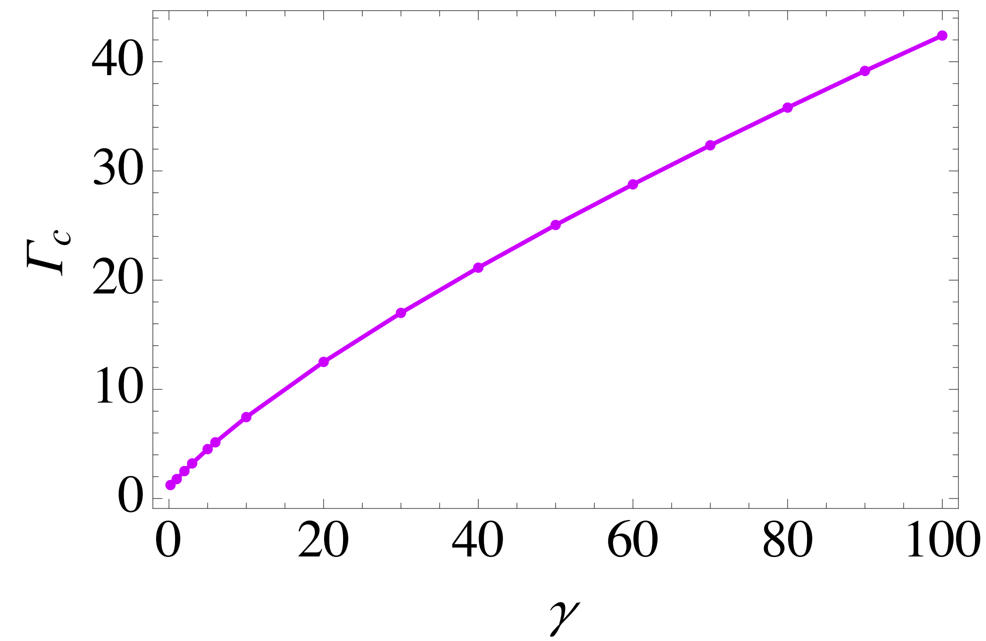
Driver Transverse field

transverse field in penalty reduces error correction effect

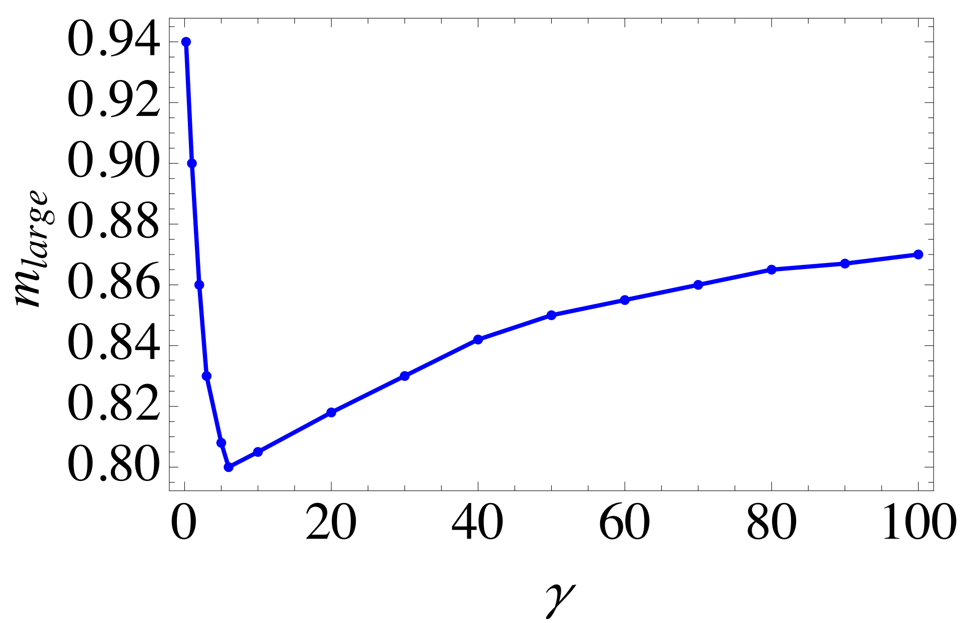
Optimal value of penalty coupling in the presence of penalty transverse field



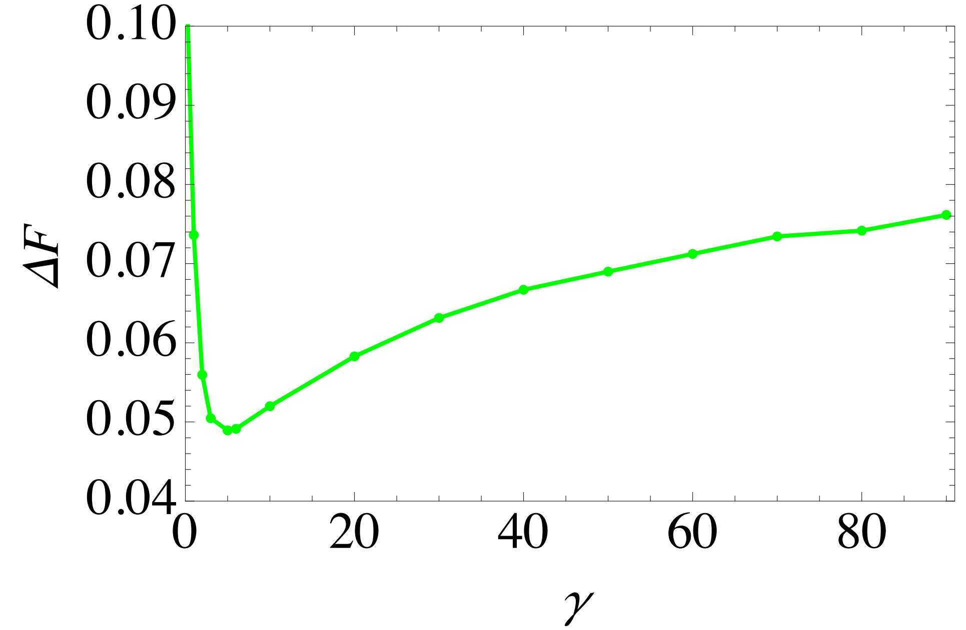
Free energy



Critical value



width

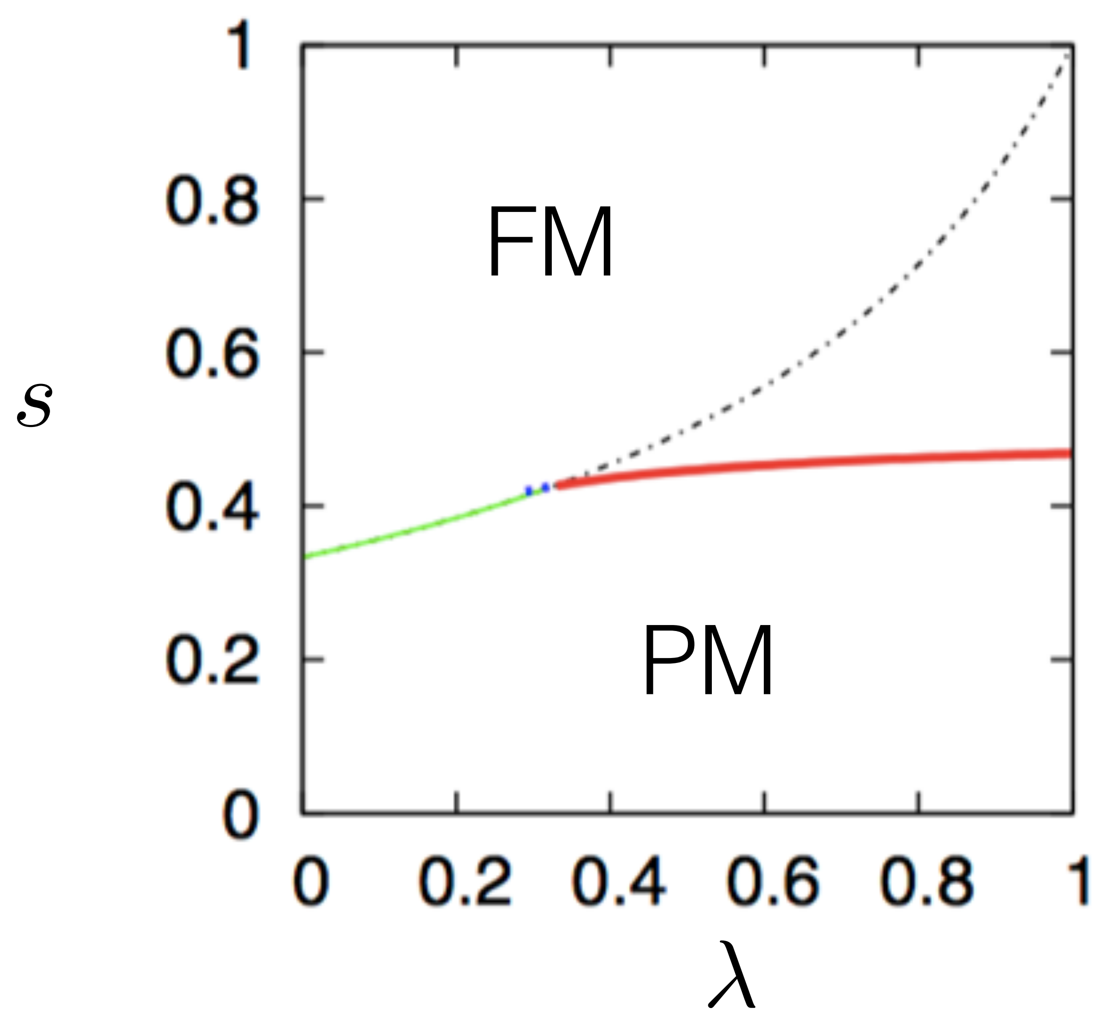


height

Non-stoquastic driver Hamiltonian

[Seki-Nishimori]

$$H = s\lambda\left[-N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p\right] + s(1-\lambda)\left[N\left(\frac{1}{N}\sum_i \sigma_i^x\right)^2\right] + (1-s)\left[-\sum_i \sigma_i^x\right]$$



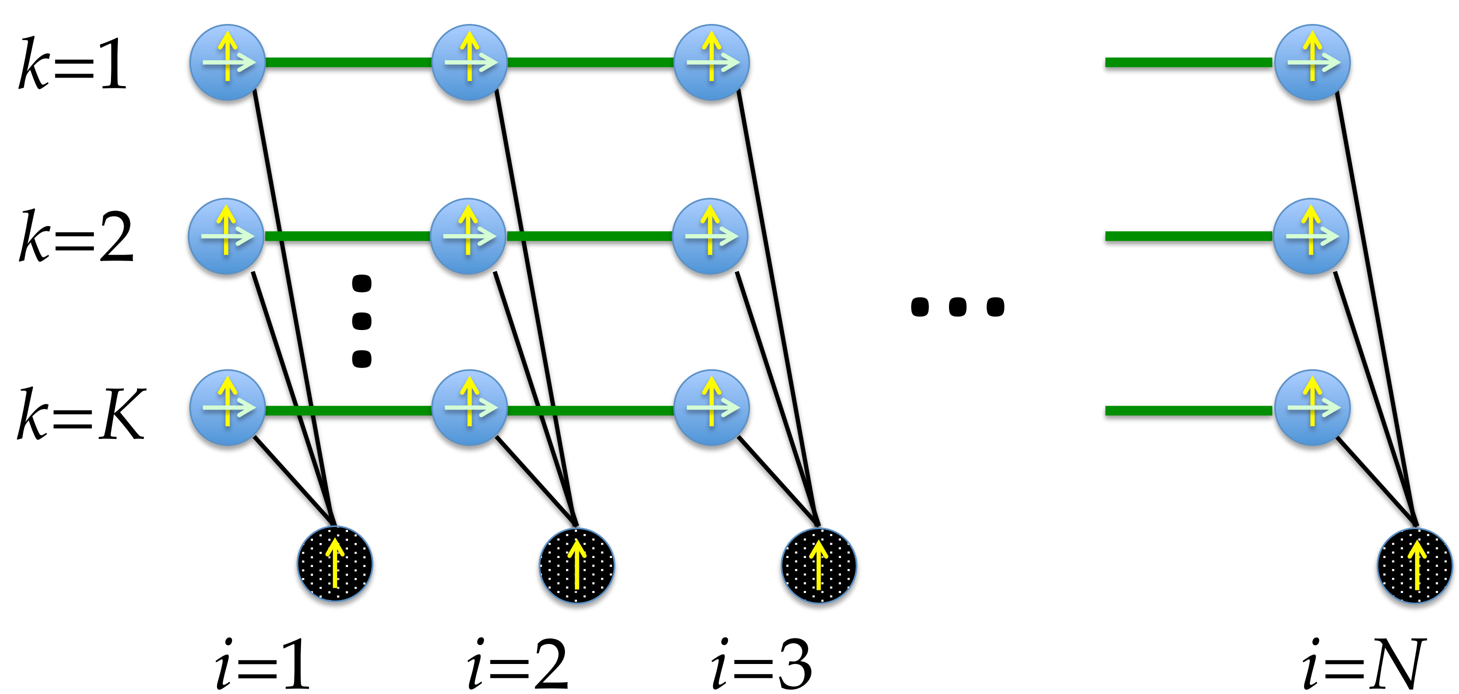
XX driver Hamiltonian in penalty QAC

$$H = -N\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_{ik}^{z}\right)^{p} - \gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{z}\sigma_{i0}^{z}$$

$$- \Gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{x} - \epsilon K\Gamma\sum_{i=1}^{N}\sigma_{i0}^{x} + \eta K\sum_{i=1}^{N}\left(\frac{1}{K}\sum_{k=1}^{K}\sigma_{ik}^{x}\right)^{q} + \lambda\sum_{i=1}^{N}\sum_{k=1}^{K}\sigma_{ik}^{x}\sigma_{i0}^{x}.$$

Problem XX

Penalty XX

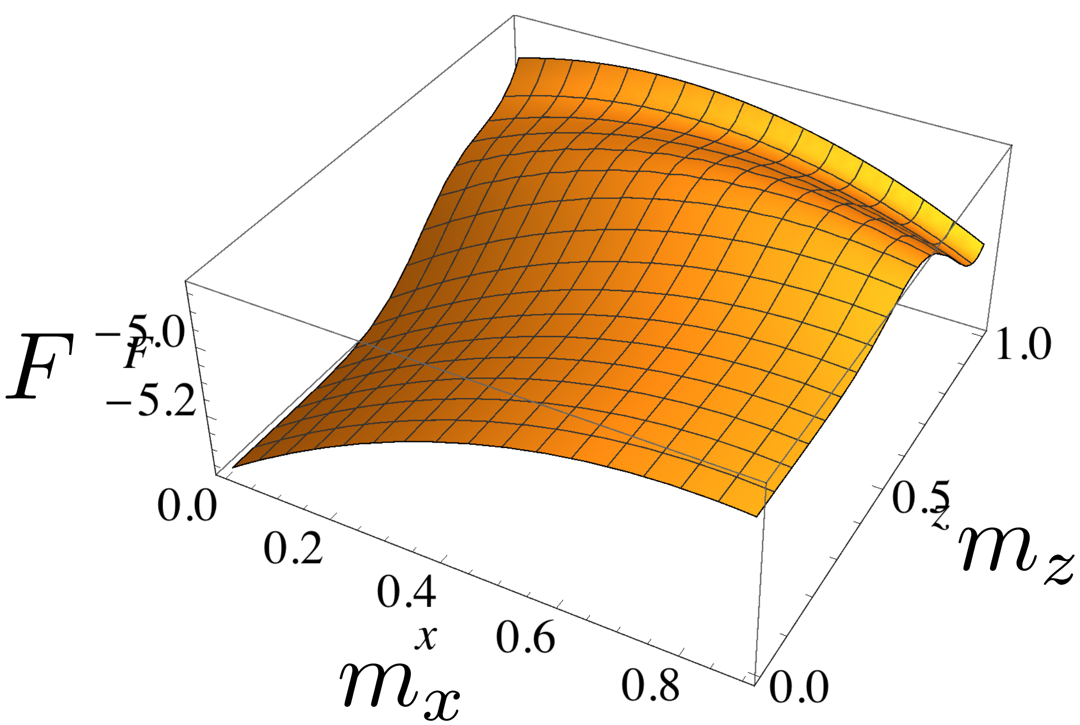


Free energy

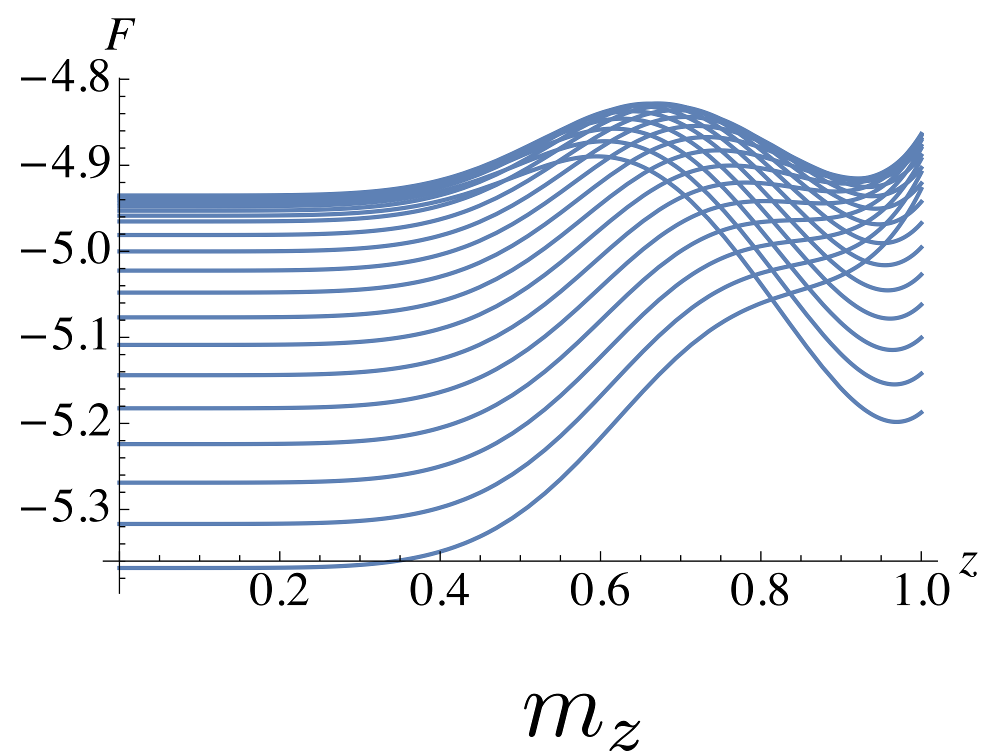
Order parameters $m_x = \langle \sigma^x \rangle \quad m_z = \langle \sigma^z \rangle$

$$F = \sum_k \frac{1}{K}(p-1)(m_k^z)^p - (q-1)\frac{\eta}{N}\sum_i (m_i^x)^q$$
$$-\frac{1}{\beta N K}\log \mathrm{Tr}\exp\left(\sum_{ik}\beta p (m_k^z)^{p-1}\sigma_{ik}^z + \sum_{ik}\beta(\Gamma - 2\eta m_i^x)\sigma_{ik}^x + \beta\gamma\sum_{ik}\sigma_{ik}^z\sigma_{i0}^z + \beta\epsilon K\Gamma\sum_i \sigma_{i0}^x - \beta\lambda\sum_{ik}\sigma_{ik}^x\sigma_{i0}^x\right)$$

Free energy



Free energy (projection)



w/ problem XX

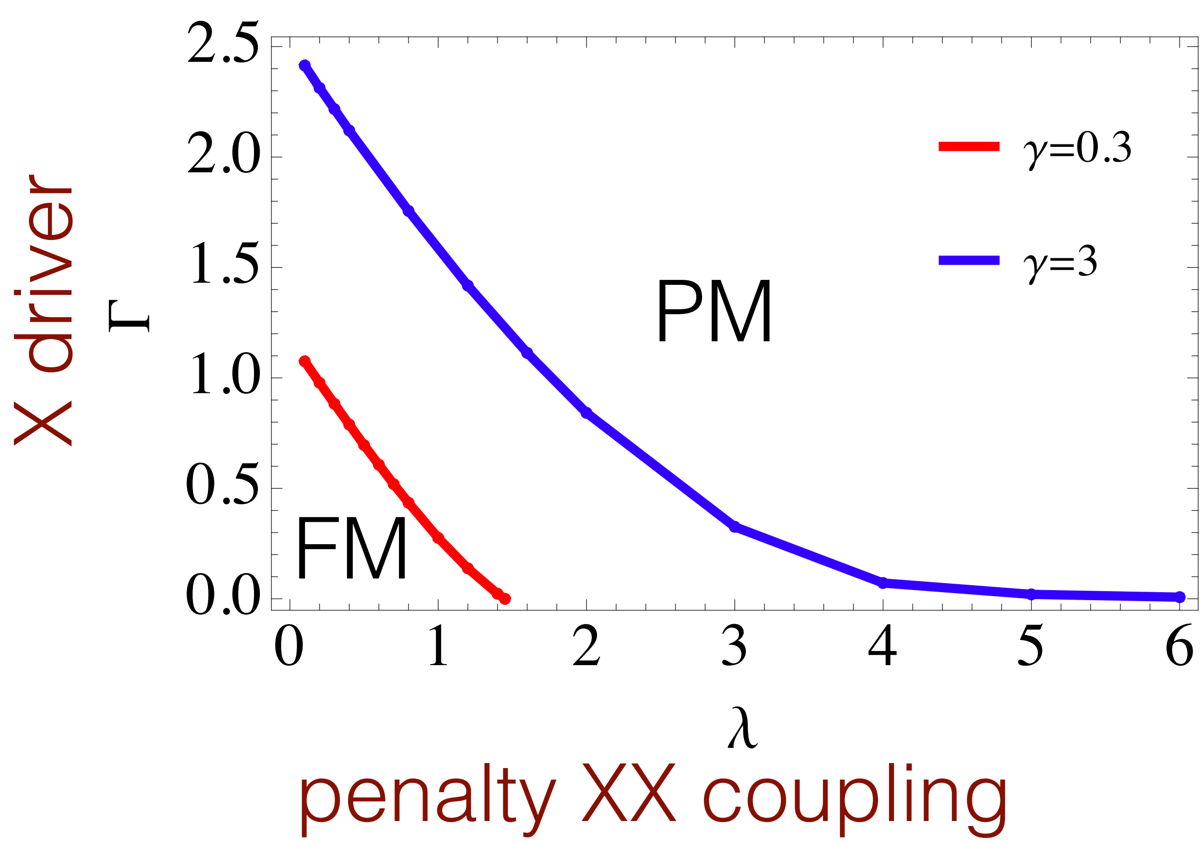
$$p = 6, \gamma = 3, \Gamma = 3.1, \epsilon = 1, \eta = 0.6, \lambda = 0,$$

Penalty XX driver

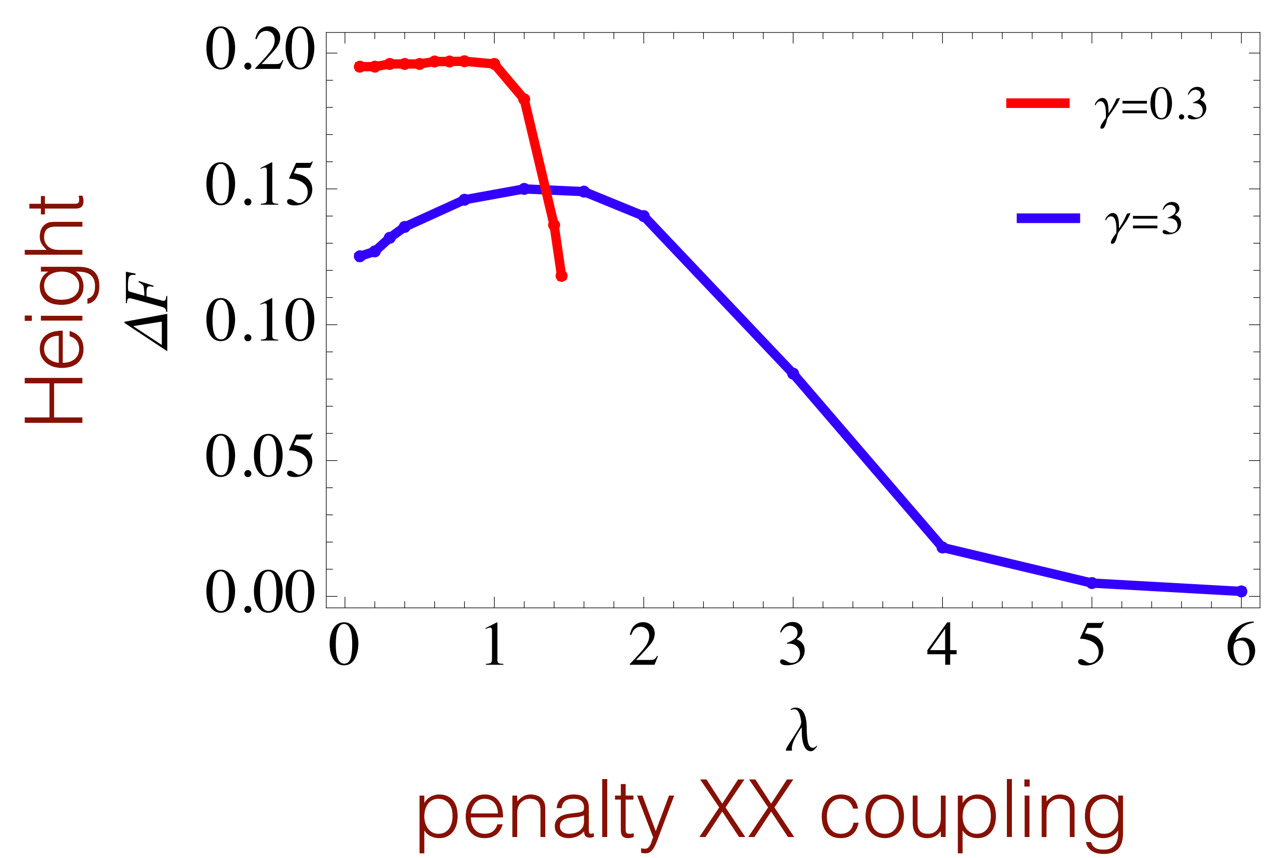
$$H = -N\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_{ik}^{z}\right)^{p} - \gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{z}\sigma_{i0}^{z}$$
$$-\Gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{x} - \epsilon K\Gamma\sum_{i=1}^{N}\sigma_{i0}^{x} + \eta K\sum_{i=1}^{N}\left(\frac{1}{K}\sum_{k=1}^{K}\sigma_{ik}^{x}\right)^{q} + \lambda\sum_{i=1}^{N}\sum_{k=1}^{K}\sigma_{ik}^{x}\sigma_{i0}^{x}.$$

Problem XX Penalty XX

Phase diagram



Height of potential barrier

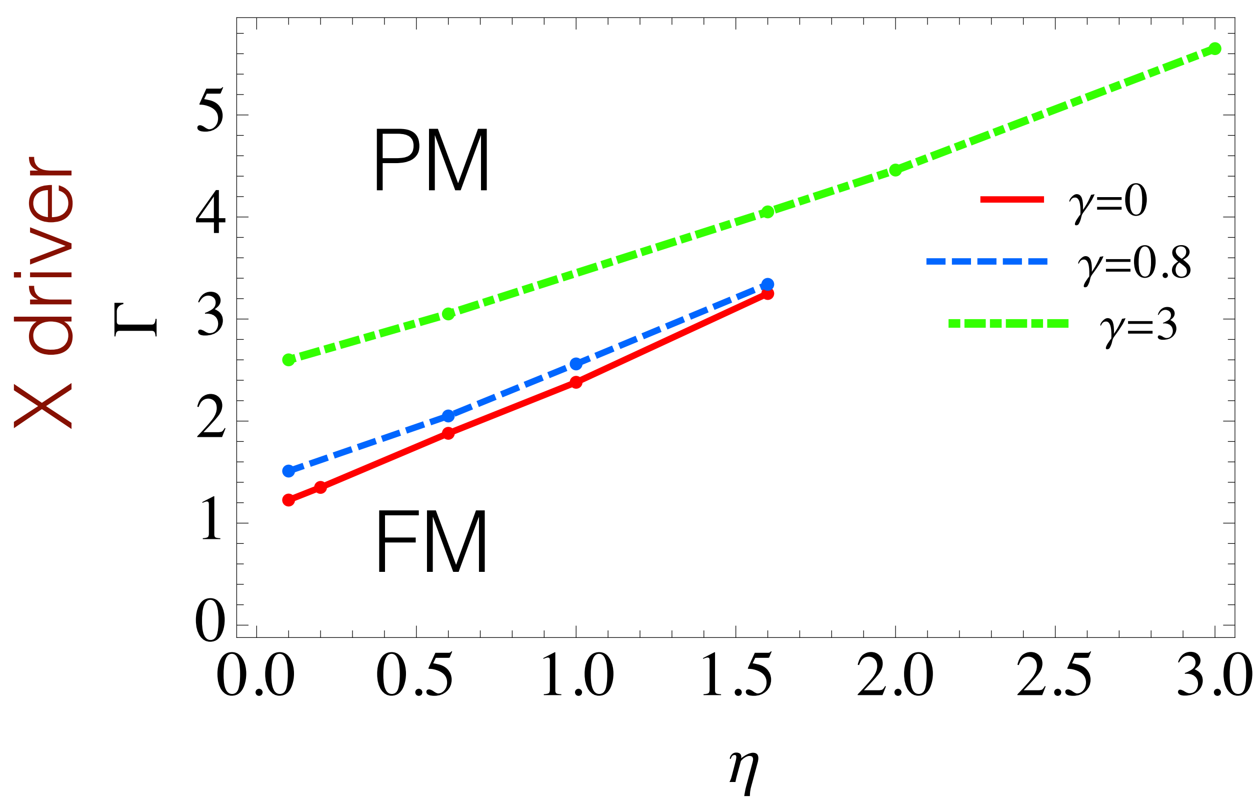


Problem XX driver

$$H = -N\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_{ik}^{z}\right)^{p} - \gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{z}\sigma_{i0}^{z} - \Gamma\sum_{k=1}^{K}\sum_{i=1}^{N}\sigma_{ik}^{x} - \epsilon K\Gamma\sum_{i=1}^{N}\sigma_{i0}^{x} + \eta K\sum_{i=1}^{N}\left(\frac{1}{K}\sum_{k=1}^{K}\sigma_{ik}^{x}\right)^{q} + \lambda\sum_{i=1}^{N}\sum_{k=1}^{K}\sigma_{ik}^{x}\sigma_{i0}^{x}.$$

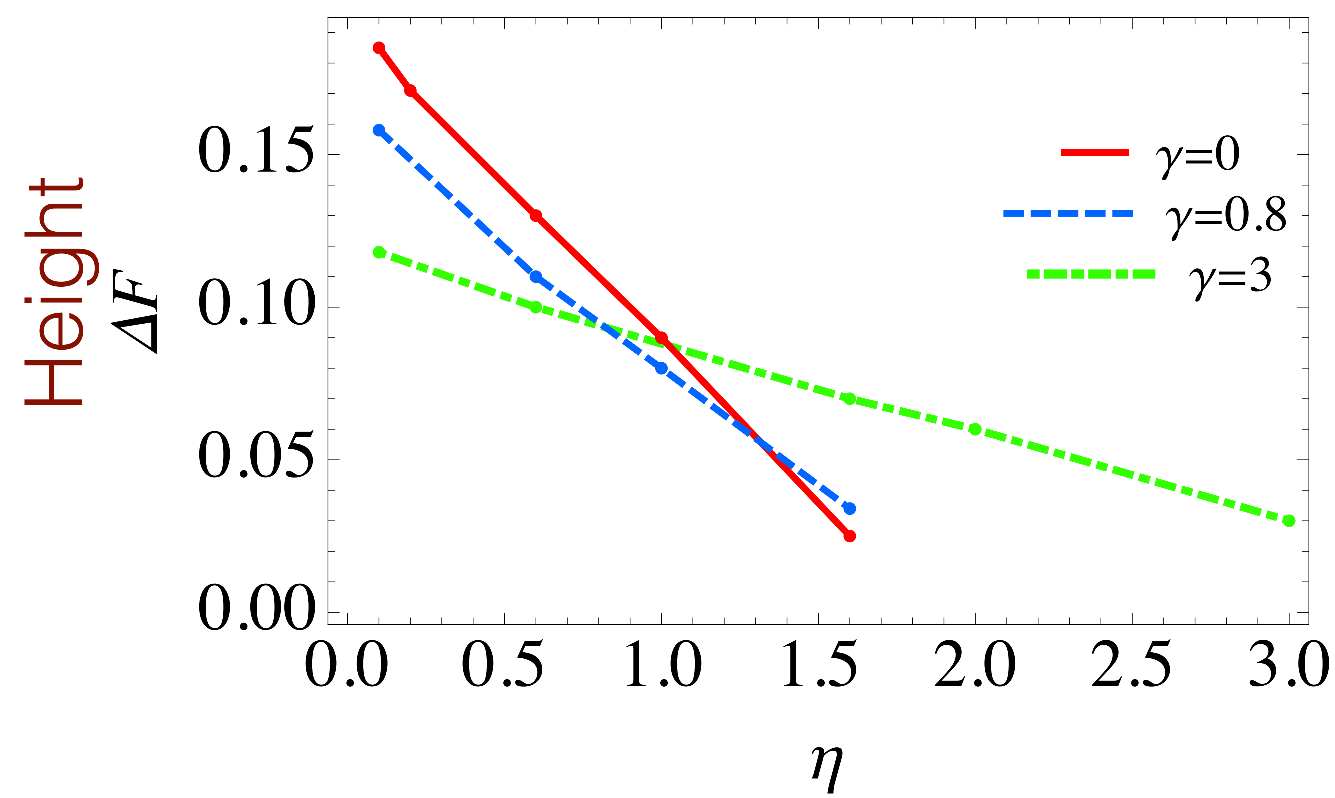
Problem XX Penalty XX

Phase diagram



Problem XX coupling

Height of potential barrier

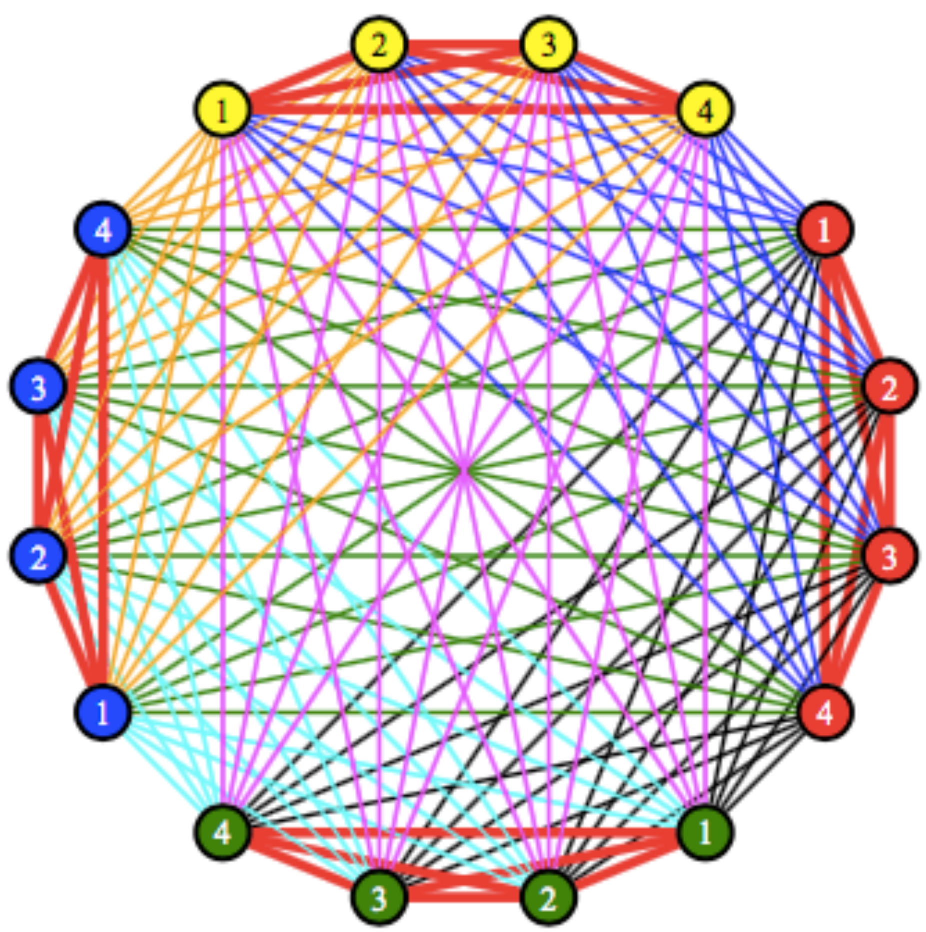


Problem XX coupling

Nested Quantum Annealing Correction

[Vinci-Albash-Lidar]

$$H_Z = -\sum_{(i,j)(k,k')} J_{(i,k),(i,k')}\sigma^z_{i,k}\sigma^z_{j,k'} - \sum_{i,k} h_{(i,k)}\sigma^z_{i,k}$$



$$J_{(i,k),(i,k')} = \gamma, \quad \forall k \neq k',$$
$$h_{(i,k)} = K h_i, \quad \forall k, i$$
$$J_{(i,k),(j,k')} = J_{ij}, \quad \forall k, k' \; i \neq j$$

D-wave machine results:
significant improvement in success rate

Mean Field Analysis

$$H = -\frac{J}{N}\left(\sum_i \sum_{c_i} \sigma^z_{i,c_i}\right)^2 - \gamma \sum_i \left(\sum_{c_i} \sigma^z_{i,c_i}\right)^2 - \Gamma \sum_{i,c_i} \sigma^x_{i,c_i}$$

Order parameter $$m_i = \left\langle \frac{1}{K} \sum_k \sigma^z_{ik} \right\rangle$$

Free energy: $$F = JK^2m^2 + \frac{\gamma K^2}{N} \sum_{i=1}^{N} m_i^2 - \frac{K}{\beta N} \sum_{i=1}^{N} \log\left[2\cosh\left(\beta\sqrt{4K^2(Jm+\gamma m_i)^2 + \Gamma^2}\right)\right]$$

Scaling property: $F \to FK^2$

$$J \to JK^2, \quad \gamma \to \gamma K^2, \quad \Gamma \to \Gamma K. \qquad T \to T/K^2$$

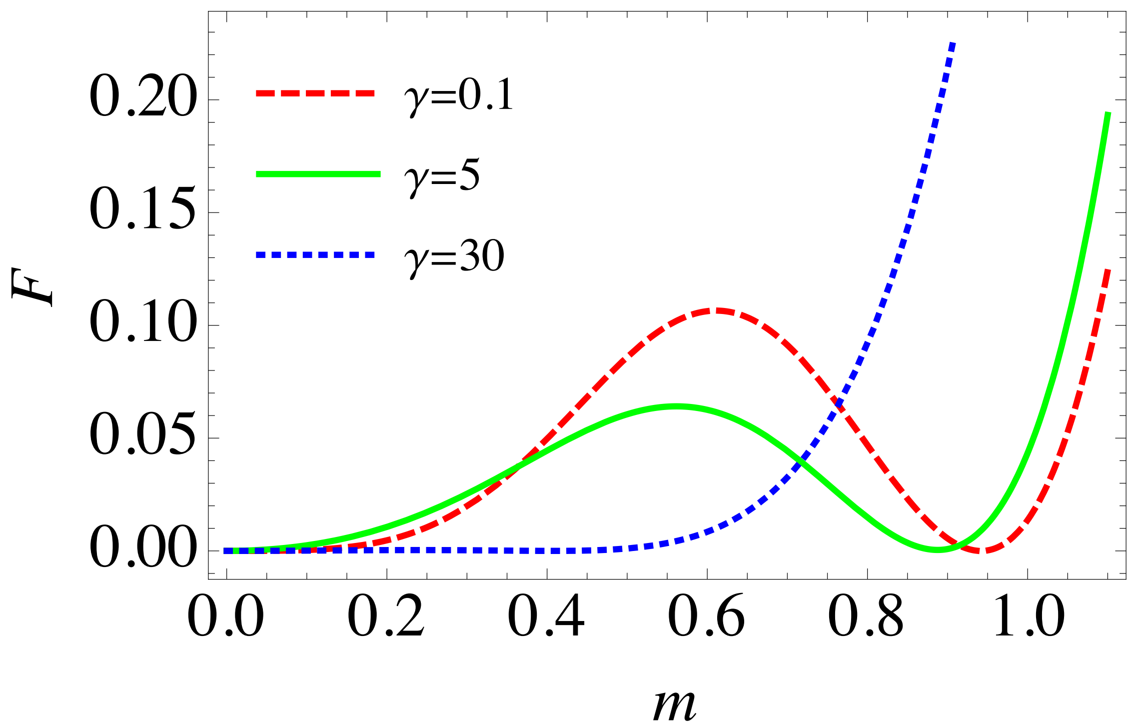
(Penalty QAC: $T \to T/K$)

Efficiently reduce the effective temperature

First order phase transition

All-to-all interaction

$$H = -\frac{J}{N^3}\left(\sum_{i\neq j\neq t\neq l}\sum_{k_i,k_j,k_t,k_l}\sigma^z_{i,k_i}\sigma^z_{j,k_j}\sigma^z_{t,k_t}\sigma^z_{l,k_l}\right) - \gamma\sum_i\left(\sum_{k_i}\sigma^z_{i,k_i}\right)^2 - \Gamma\sum_{i,k_i}\sigma^x_{i,k_i}$$



- Order parameter $m = \langle\sigma^z\rangle$
- Coupling within logical qubit γ

Phase transition is significantly weakened if penalty coupling is quadratic

Summary

Error correction and Quantum speedup are closely related to quantum phase transitions

We considered two possibilities:

Penalty and Nested Quantum Annealing Correction

$$\text{problem} \rightarrow H_Z$$

XX driver Hamiltonian

$$H_X$$

In both cases, the first order phase transitions can be significantly weakened.

Need more systematic understanding of phase transitions