# Hamiltonian engineering for many-body quantum systems by shortcuts to adiabaticity

**Kazutaka Takahashi**

Department of Physics, Tokyo Institute of Technology, Tokyo 152–8551, Japan

We study the engineering of the optimal time-dependent Hamiltonian by using the method of shortcuts to adiabaticity. This method allows us to control the adiabatic quantum states with a finite speed. There exist several formulations of the method and we focus on the inverse engineering of the Hamiltonian. The existence of the Lewis–Riesenfeld dynamical invariant is the essential ingredient of the method and we can actually show the optimality of the driving by using the minimum action principle [1].

We apply the inverse engineering to quantum spin systems. By using the equation for the dynamical invariant, we design the optimal annealing schedule. The schedule is determined for a given adiabatic passage. It can be obtained without solving the differential equations. By using this method, we can discuss the efficiency of the annealing schedule.

We consider several possible applications.

- Transverse Ising model

$$\hat{H}(t) = f(t)\left(-\sum_{i,j=1}^{N} J_{ij}\hat{S}_i^z\hat{S}_j^z - \sum_{i=1}^{N} h_i\hat{S}_i^z\right) - \Gamma(t)\sum_{i=1}^{N}\hat{S}_i^x.$$

  For mean-field systems, the equation for the dynamical invariant takes a simple form and we easily find the time dependence of the coupling functions $f(t)$ and $\Gamma(t)$. We discuss general properties of the annealing schedule which are applicable to non-mean-field systems.

- One-dimensional quantum XX model

$$\hat{H}(t) = -\sum_{i=1}^{N} J_i(t)\left(\hat{S}_i^x\hat{S}_{i+1}^x + \hat{S}_i^y\hat{S}_{i+1}^y\right) - \sum_{i=1}^{N} h_i(t)\hat{S}_i^z.$$

  The isotropic XX model is solved by using the Jordan-Wigner transformation. We discuss how the inverse engineering is applied when we have the phase transitions [2]. We also solve the anisotropic XX model by using the solutions of the Toda equations [3]. We show that the results from the classical nonlinear integrable systems, such as the KdV equations and Toda equations, are directly applied to quantum adiabatic dynamics. For example, we can realize the adiabatic control of spin-state transfer by using the soliton solutions of the Toda equations.

[1] K. Takahashi, *How fast and robust is the quantum adiabatic passage*, J. Phys. A **46**, 315304 (2013).
[2] K. Takahashi, *Transitionless quantum driving for spin systems*, Phys. Rev. E **87**, 062117 (2013).
[3] M. Okuyama and K. Takahashi, *From classical nonlinear integrable systems to quantum shortcuts to adiabaticity*, arXiv 1603.01053.