Expansion in large coordination number for the quantum Ising model

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Abstract— Quantum lattice systems are encountered in many field of physics: solid states, ultracold gases, quantum optics, and metamaterials. For this class of systems, we establish a set of hierarchy equations describing the non equilibrium time evolution of the n-site spatial correlation reduced density matrices and solve it iteratively through a 1/Z expansion where Z is the coordination number. We focus on the particular study of quench dynamics of the quantum Ising model.

1. INTRODUCTION

The quantum lattice systems are encountered in many field of physics: solid states, ultracold gas and quantum metamaterials such as photonic band gap materials. The typical examples are the Bose or Fermi Hubbard models, the array of quantum dots or Josephson junctions, or the Heisenberg spin model. More recently, the Jaynes-Cumming-Hubbard model describing the interaction of light with two-level atoms (Rydberg atoms) in a lattice has attracted considerable interests since it displays interesting phenomena such as coulomb blockade or polaritons-like excitations. An accurate modelling of these many body systems may help to understand many open problems encountered for example in high temperature superconductivity, oxytronic, topological insulator or graphene. With these lattice models, we wish to describe the non equilibrium phenomena resulting from a sudden external perturbation, for example, those in the cold gas resulting from a quench from a insulating state to a superfluid state or those in a oxyde material resulting from a brutal change in the electronic density of state caused by the radiation of a laser or electronic scattering.

In this paper, we investigate these lattice systems from a general point of view using the $1/Z$ expansion method where Z is the coordination number. This method developed by us [1, 5] assumes that the coordination number Z is large and that the next order corrective terms $1/Z^l$ give decreasingly smaller contributions to the lattice system dynamics. This formalism provides a general framework of hierachical equations for n-sites reduced density matrices allowing to systematically determine the equilibrium properties such as the ground states [7, 8] but also to describe the non equilibrium dynamics [1, 5]. Below are the aspects under active and/or future considerations.

2. THE 1/Z EXPANSION IN LATTICE GASES IN THERMODYNAMIC EQUILIBRIUM: BOSONS, FERMIONS AND SPIN

We are currently working on the Bose-Hubbard model that describes cold atoms trapped in lattice sites in order to test the accuracy of $1/Z$ expansion techniques [1, 6, 5, 4]. Comparisons with other methods (exact diagonalisation, perturbation techniques, Monte-Carlo, ...) allow to assess the convergence of the expansion. Recently, we achieved this expansion at finite temperature [7] and showed that this method describes the entire crossover region from weak coupling (superfluid regime) to strong coupling (Mott insulating regime).

The Heisenberg model for the spin magnet has been described with this formalism [3]. Recently, we show the convergence of the expansion for the determination of the quantum transition point in the quantum Ising model [8].

3. NON EQUILIBRIUM ASPECTS OF A LATTICE BOSON GAS

The 1/Z expansion method can be applied to study non equilibrium physical phenomena. We are currently studying the behavior of quantum fluctuations in cold gases by changing suddenly the parameters of the hamiltonian (sweeping). One of the typical example is the Kibble-Zurek mechanism where a sudden quench towards a phase transition to superfluid phase results in the domain formation. In one of our work, these domain formations originates from quantum fluctuations of an initial the Mott phase suddenly swept in the superfluid regime [1]. A similar study has been done for a quench within a Mott phase with a an explicit description of the relaxation towards a prethermalized state [5]. Another example is the Sauter-Schwinger mechanism that predicts the production of electron-positron pairs that appear when a strong electric field is applied to the quantum vacuum. By analogy, such a tilt (bias force) has been introduced in the Bose lattice in the Mott-insulating phase and creates instead doublon-holon pairs [2].

4. QUANTUM ISING MODEL

In our last work [8], we use the 1/Z expansion to describe the ground state and the quench dynamics of the quantum Ising model in one, two and three dimensions. Our method reproduces quite well the physics of this model such as the quantum phase transition between the paramagnetic and ferromagnetic phases or also the excitation spectrum. Such a model happens to describe the dynamics of array of Josephson junctions such as squids perturbed by an external magnetic field under the conditions of long decoherence time. Such structures are designed nowadays by the D-Wave company and are potentially promising for adiabatic quantum computations such as annealing. Numerous studies have currently been achieved in order to check the adiabaticity during the annealing and to assess whether the D-wave devices preserve its quantum characteristics. Our recent developments [8] may provide more insights on the understanding of the quantum fluctuations generated by these devices.

5. CONCLUSIONS

From the examples above, we illustrate how the large coordination number expansion method reveals to be useful for an understanding of the quantum phenomena involved in quantum lattice devices such as matematerials. The last recent developments have brought new insights on the efficiency and the powerfulness of the method.

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