Fictitious but Efficient Annealing Dynamics, and Role of Quantum Entanglement Therein

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Motivated by the idea of the annealing computations, we investigate a mechanism which drives an initial state into a ground state of a given Hamiltonian. Instead of natural dynamics in physics, however, we dare to introduce/consider a fictitious dynamics[1]

$$\frac{d}{ds}\hat{U}_s = -i\hat{\eta}_s\hat{U}_s, \quad \hat{U}_0 = \text{Identity Operator} \tag{1}$$

with

$$\hat{\eta}_s := -i\Big[\hat{H}, |\varphi_s\rangle\langle\varphi_s|\Big], \quad \text{and} \quad |\varphi_s\rangle := \hat{U}_s|\varphi_0\rangle. \tag{2}$$

On the fictitious dynamics, efficiency and mechanism of the convergence are demonstrated with examples. In particular, by applying the approach to Hamiltonian of Ising model, we show that the classical gradient descent mechanism interestingly appears as a classical approximation of the proposed dynamics, and that quantum entanglement plays an essential role to make the dynamics superior to the classical one.

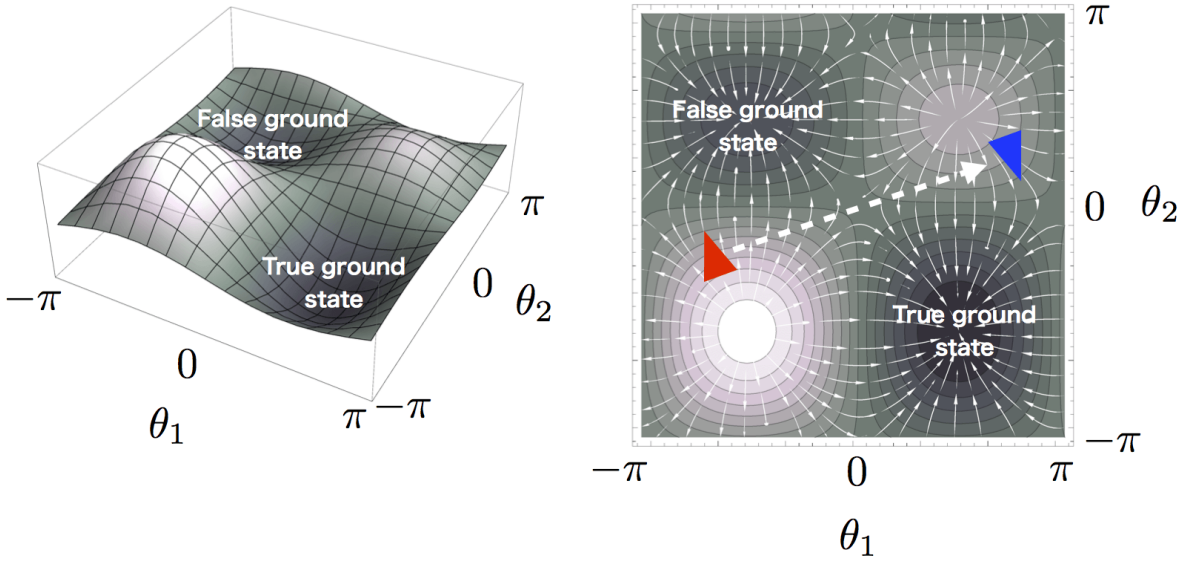

Figure 1: The left figure shows the potential (or energy landscape) defined as $H(\theta_1, \theta_2) := \sum_{i=1}^{2} h_i \sin\theta_i + \sum_{i<j} J_{ij} \sin\theta_j \sin\theta_i$. In the right figure, the white solid arrows represent the gradient descent forces by the potential. Besides the forces, the dynamics generates an "entangled component (represented by the downward blue triangle)" in a symmetrical position with respect to the origin. The force at the position drives the component to the true ground state whereas the force at the position of original state does not.

[1] K. Imafuku and S. Kawabata, manuscript in preparation.