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Nonadiabatic cavity QED effects with superconducting qubit-resonator nonstationary systems

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Outline

- **Motivation:** cavity QED nonstationary effects
- **Basic idea:** dynamical Lamb effect via tunable qubit-photon coupling
- **Theoretical model:** parametrically driven Rabi model beyond RWA, energy dissipation
- **Results:** system dynamics; a method to enhance the effect; interesting dynamical regimes
- **Summary**

Motivation-1

Superconducting circuits with Josephson junctions

- Quantum computation (qubits)
- A unique platform to study cavity QED nonstationary phenomena

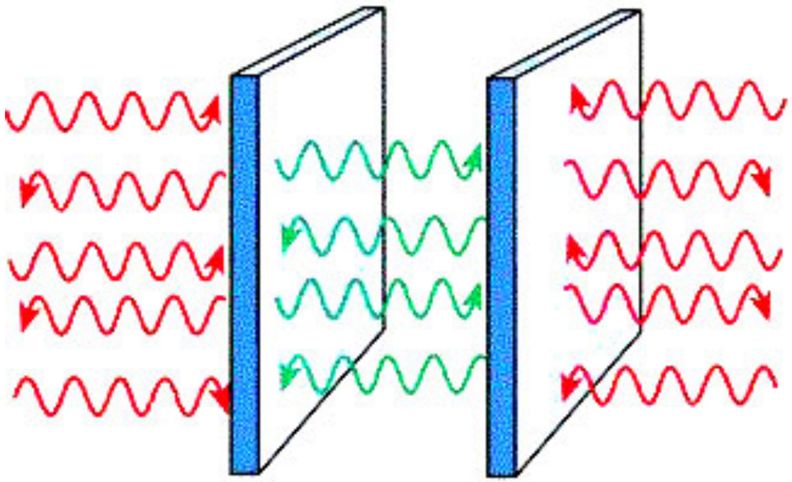
First observation of the dynamical Casimir effect – tuning boundary condition for the electric field via an additional SQUID

C. M. Wilson, G. Johansson, A. Pourkabirian, J. R. Johansson, T. Duty, F. Nori, P. Delsing, Nature (2011).

Initially suggested as photon production from the "free" space between two moving mirrors due to zero-point fluctuations of a photon field

Motivation-2

Static Casimir effect (1948)

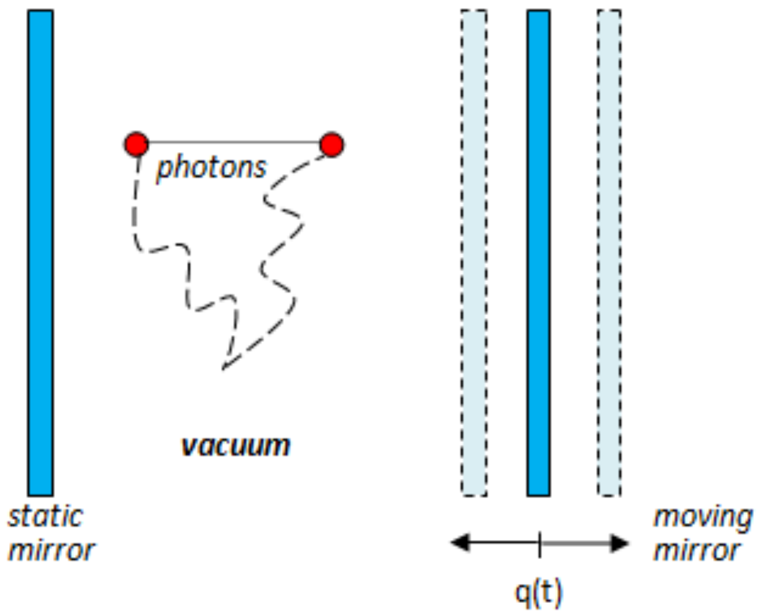


Two conducting planes attract each other
due to vacuum fluctuations (zero-point energy)

Motivation-3

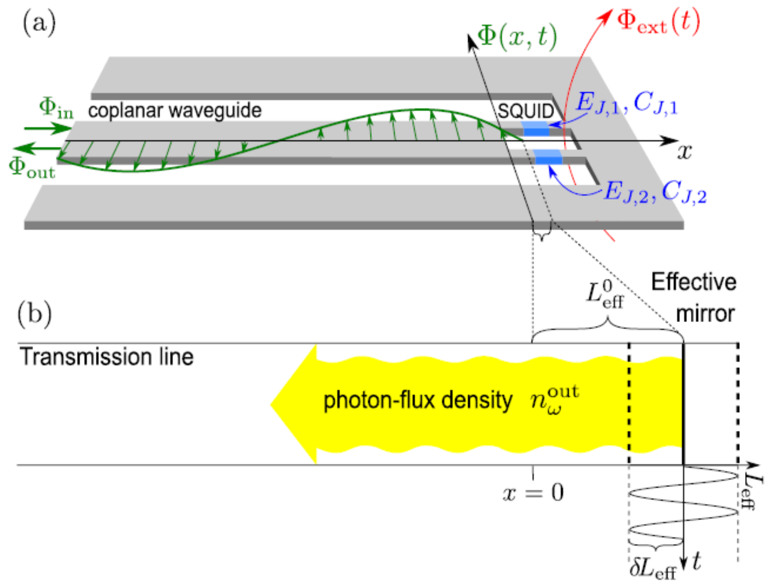
Dynamical Casimir effect

Prediction (Moore, 1970)



- Generation of photons
- Difficult to observe in experiments with massive mirrors
- Indirect schemes are needed

First observation (2011)



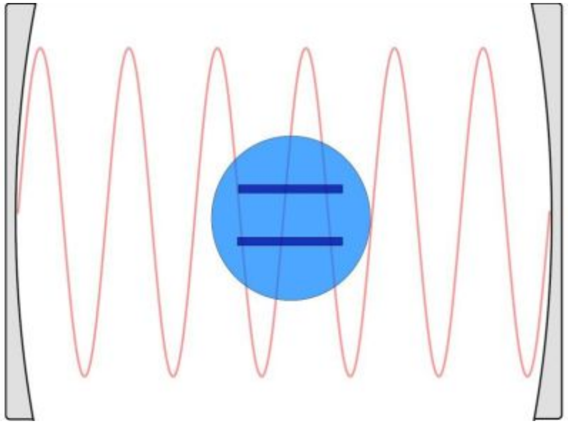
- *Tuning an inductance*
- Superconducting circuit system: high and fast tunability!

C. M. Wilson, G. Johansson, A. Pourkabirian, J. R. Johansson, T. Duty, F. Nori, P. Delsing, Nature (2011).

Motivation-4

Natural atom in a nonstationary cavity

N. B. Narozhny, A. M. Fedotov, and Yu. E. Lozovik,
Dynamical Lamb effect versus dynamical Casimir effect, PRA (2001).



Two channels of atom excitation

(nonadiabatic modulation):

- Absorption of Casimir photons
- Nonadiabatical modulation of atomic level Lamb shift:

"Shaking" of atom's dressing.

New nonstationary QED effect: dynamical Lamb effect

Motivation-5

Static Casimir effect ---> Dynamical Casimir effect
Lamb shift of atom's energy levels ---> Dynamical Lamb effect

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- *The first mechanism due to absorption of Casimir photons is dominant*
- *How can the dynamical Lamb effect be enhanced with respect to other mechanisms of atom excitation?*

Our major aim is to explore the dynamical Lamb effect in superconducting circuit systems taking into account their outstanding tunability and flexibility.

Basic idea-1

Dynamically tunable qubit-resonator <u>coupling</u>

- In contrast to the optical system with a natural atom, it is possible to dynamically tune also an effective photon-qubit coupling

Already implemented both for flux qubits and transmons

No Casimir photons!

- A lot of potential applications: decoupling qubit and resonator
- Understanding of the role played by the dynamical Lamb effect is of a practical importance.

Phys. Rev. A 91, 063814 (2015)

Theoretical model-1

Rabi model beyond the rotating wave approximation

(one mode photon field)

$$H = \omega a^\dagger a + \frac{1}{2}\epsilon(1+\sigma_3) + V,$$

photons (GHz) qubit coupling

$$V = g(a + a^\dagger)(\sigma_- + \sigma_+)$$

Numerical and analytical approaches

Lindblad equation:

$$\partial_t \rho(t) - \Gamma(\rho(t)) = -i[H(t), \rho(t)]$$

$$\Gamma[\rho] = \kappa(a\rho a^\dagger - \{a^\dagger a, \rho\}/2) + \gamma(\sigma_-\rho\sigma_+ - \{\sigma_+\sigma_-, \rho\}/2) + \gamma_\varphi(\sigma_z\rho\sigma_z - \rho)$$

Dissipation in a qubit >> dissipation in a cavity

Theoretical model-2

- Qubit-photon coupling:

$$V = V_1 + V_2$$

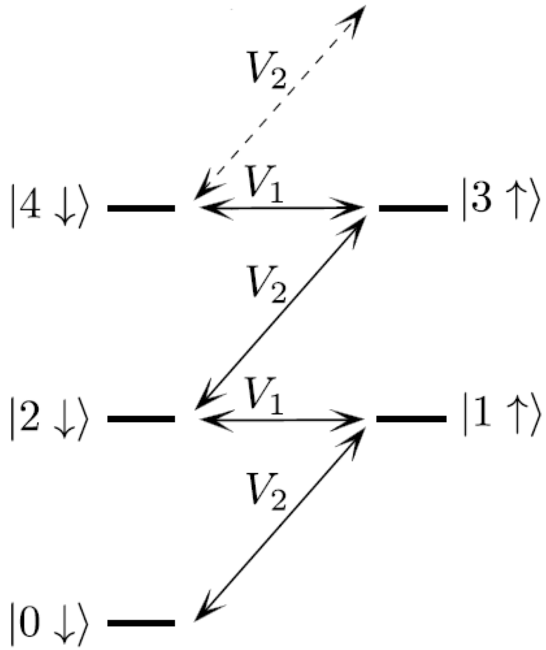
$$V_1 = g(a\sigma_+ + a^\dagger\sigma_-)$$

RWA, conserves excitation number.

$$V_2 = g(a^\dagger\sigma_+ + a\sigma_-)$$

Counterrotating wave term.
Responsible for the dynamical Lamb effect

Structure of bare energy levels (resonance)

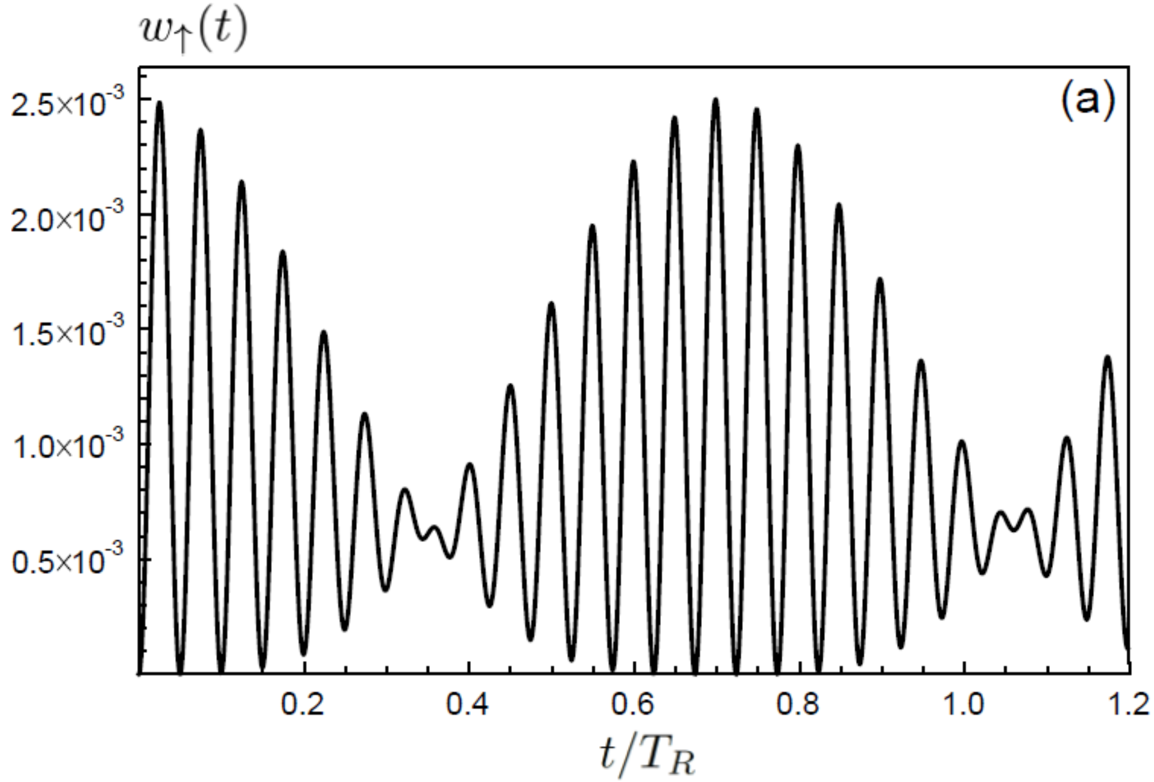


Results-1: single instantaneous switching

Qubit excited state population due to the dynamical Lamb effect

$$w_{\uparrow}(t) = \frac{g^2}{8\omega^2}\left(3 + \cos 2\sqrt{2}gt - 4\cos 2\omega t \cos \sqrt{2}gt\right)$$

in a full resonance, no dissipation

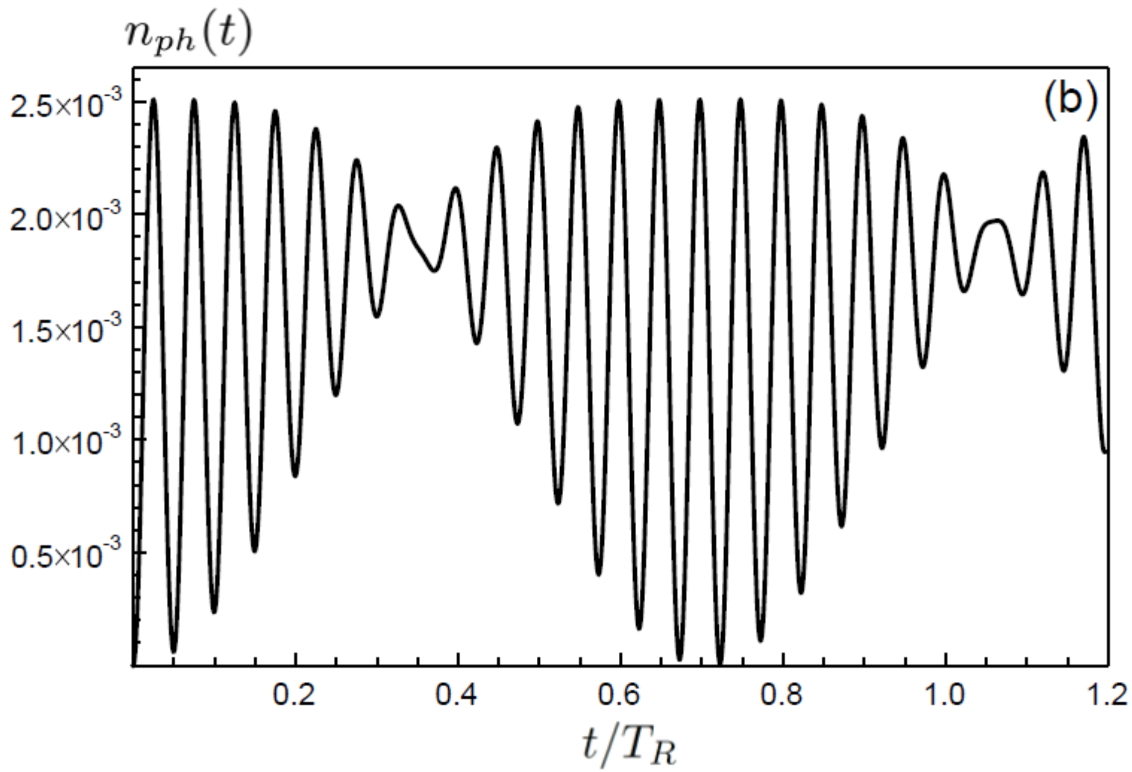


Good news: the excitation is possible.

Bad news: <u>the effect is weak.</u>

Results-2: single instantaneous switching

Number of generated photons

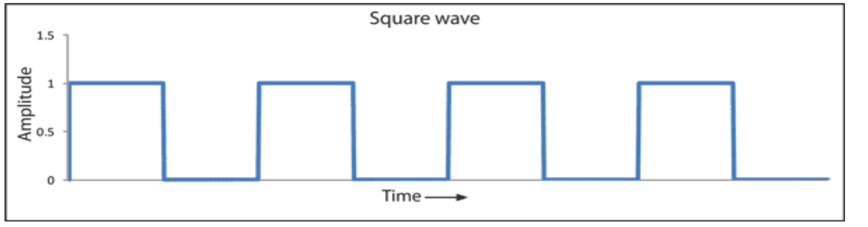


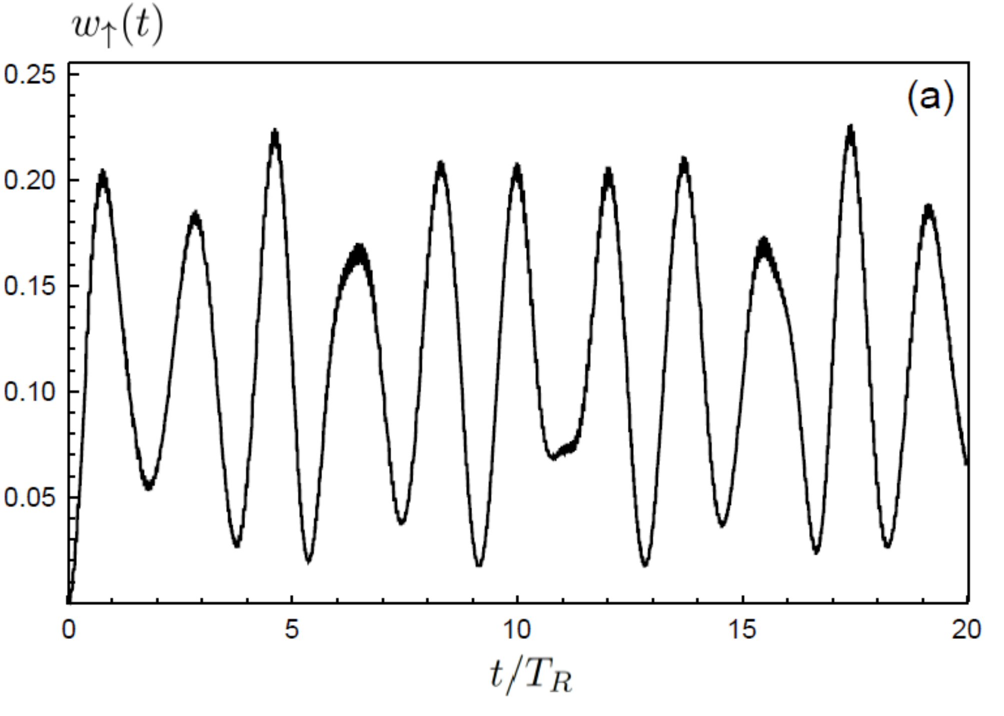
An excellent agreement between an analytical treatment and numerics
The effect, however, is weak. Strong coupling regime?

Results-3: how to enhance the effect

$g(t) = g\theta(\cos 2\omega t)$ -- parametric driving

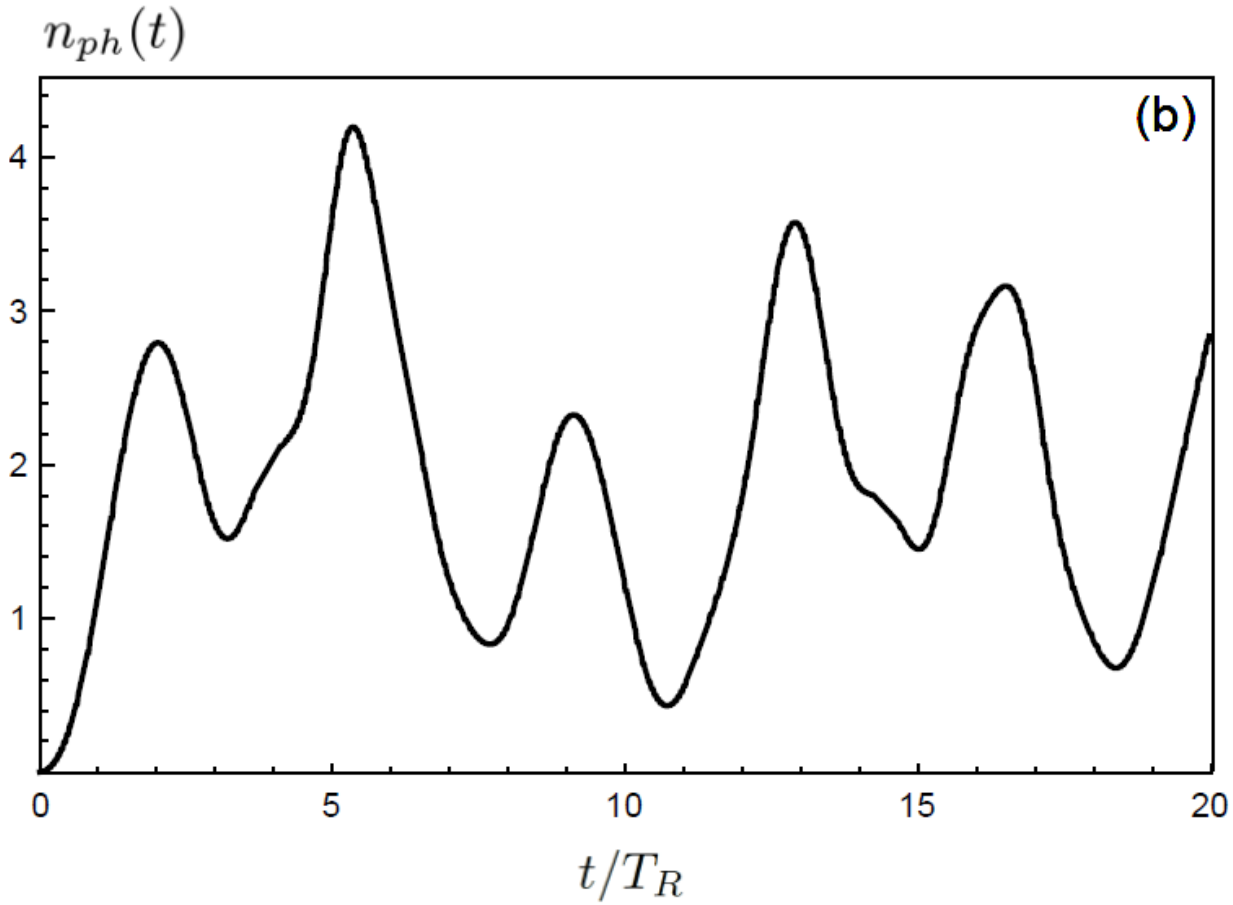
$$w_{\uparrow}(t) \simeq \frac{2}{\pi^2}(1 - \cos\sqrt{2}gt)$$





Huge enhancement of qubit excited state population

Results-4: parametric driving



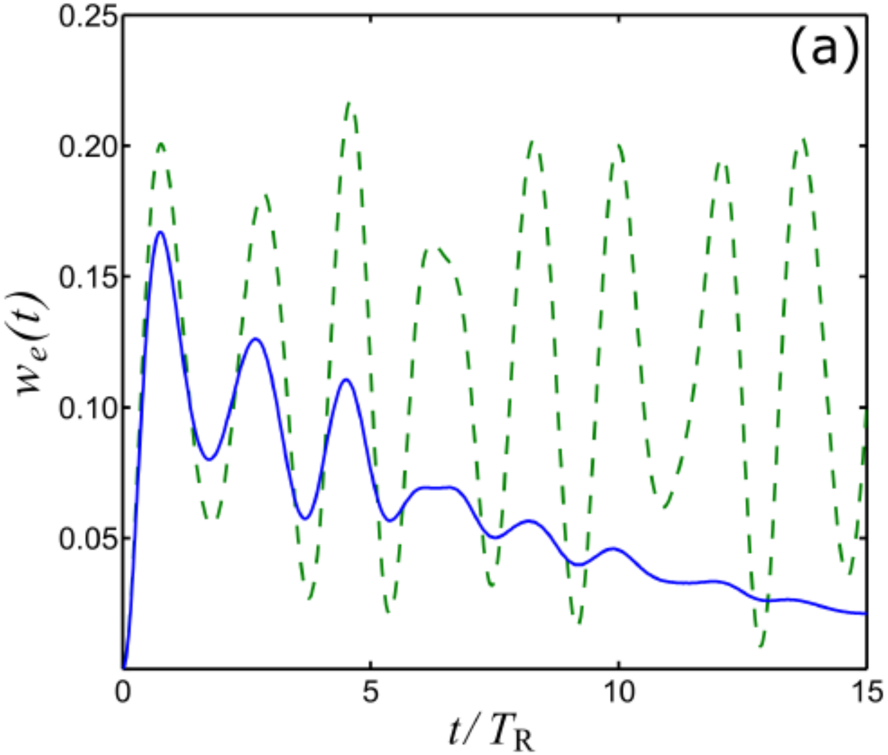
Mean number of generated photons is also enhanced

Results-5: energy dissipation vs driving

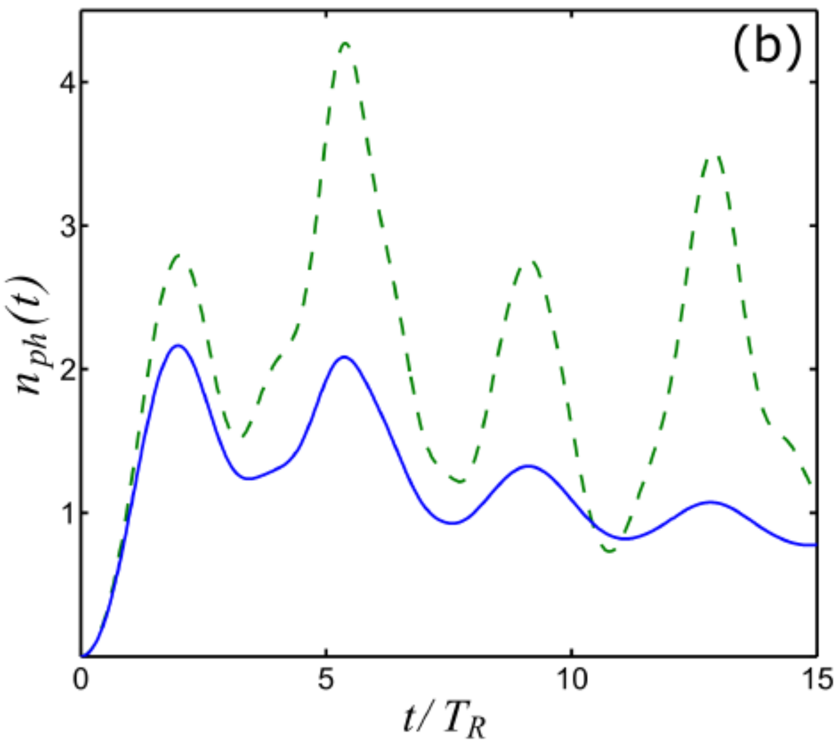
Dissipation is of importance: Qubit relaxation is opposite to the dynamical Lamb effect

g (t) is not modulated too much (not sign-alternating)

Qubit excited state population



Mean photon number



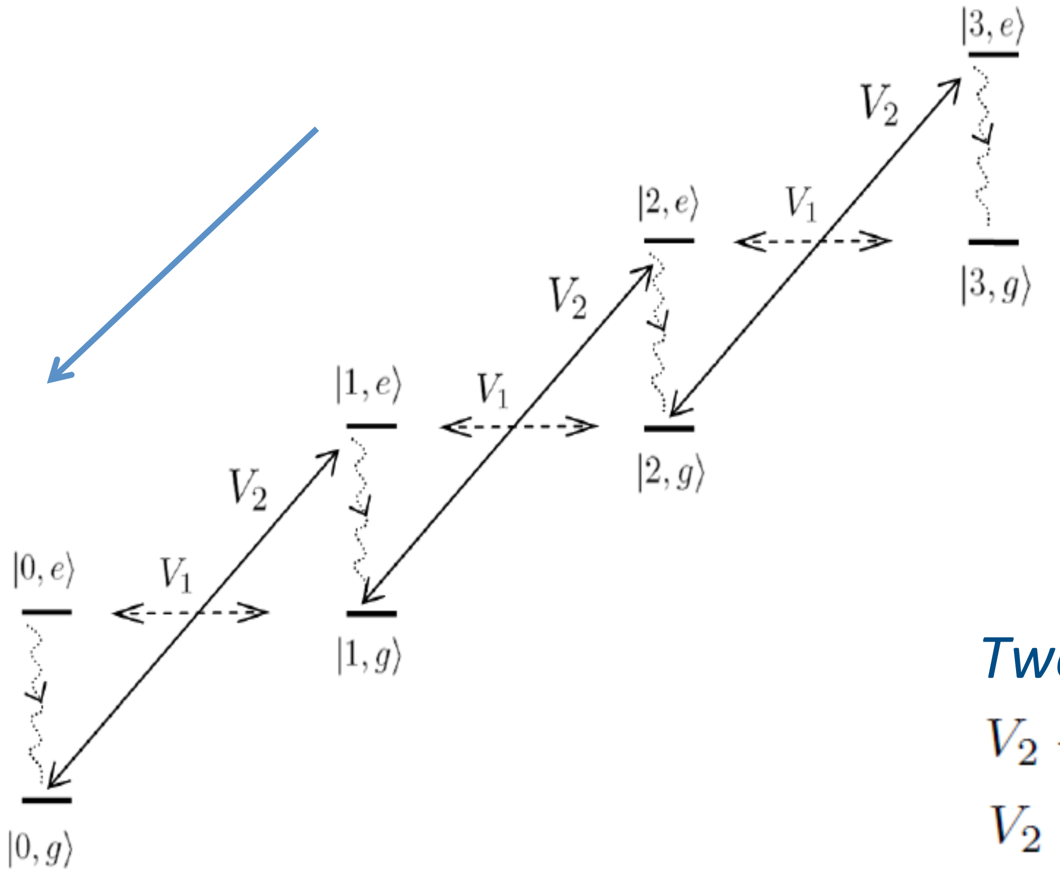
No cavity dissipation for the moment…

No effect at long time

Phys. Rev. A 93, 063845 (2016)

Results-6: energy dissipation vs driving

Ladder of bare energy states



Two processes:

$$V_2 - \gamma - V_1 - \gamma$$

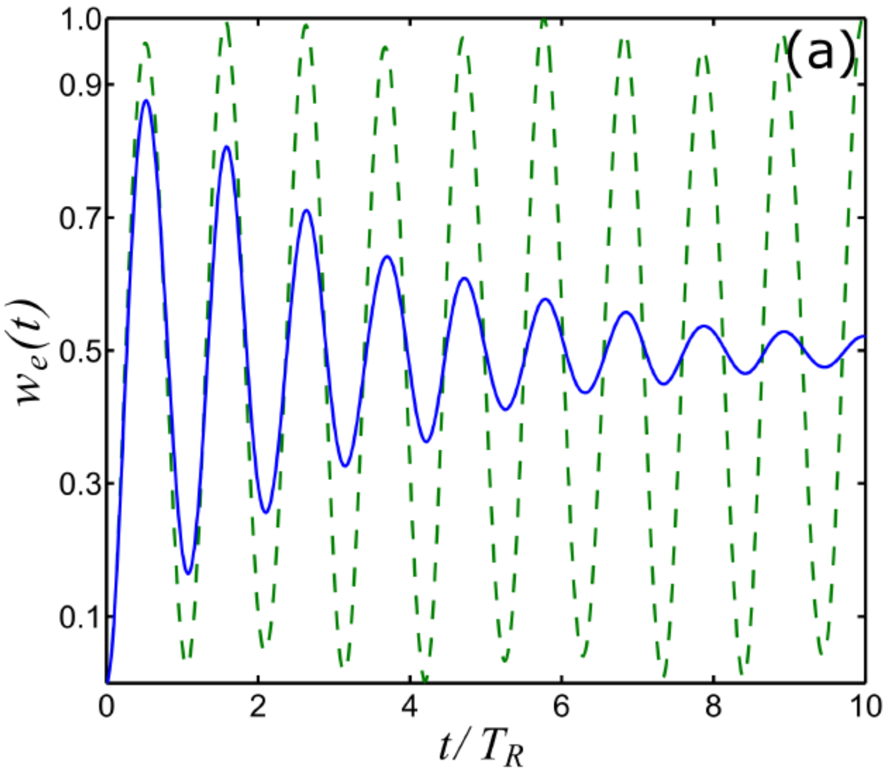
$$V_2 - \gamma - V_2$$

Energy dissipation brings the driven system down

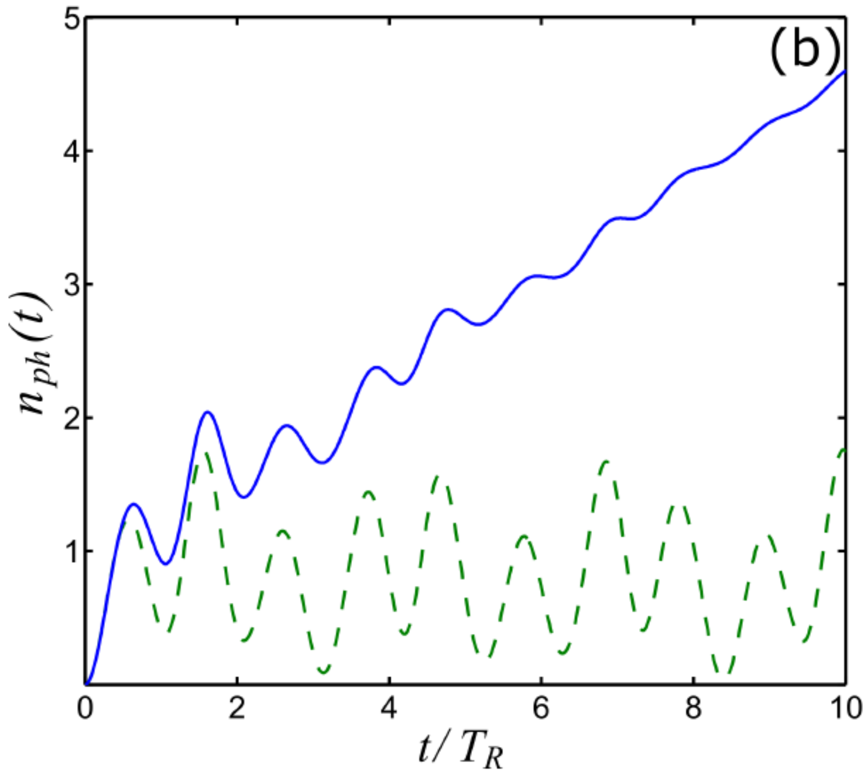
Results-7: energy dissipation assisting driving

g (t) is sign-alternating

Qubit excited state population



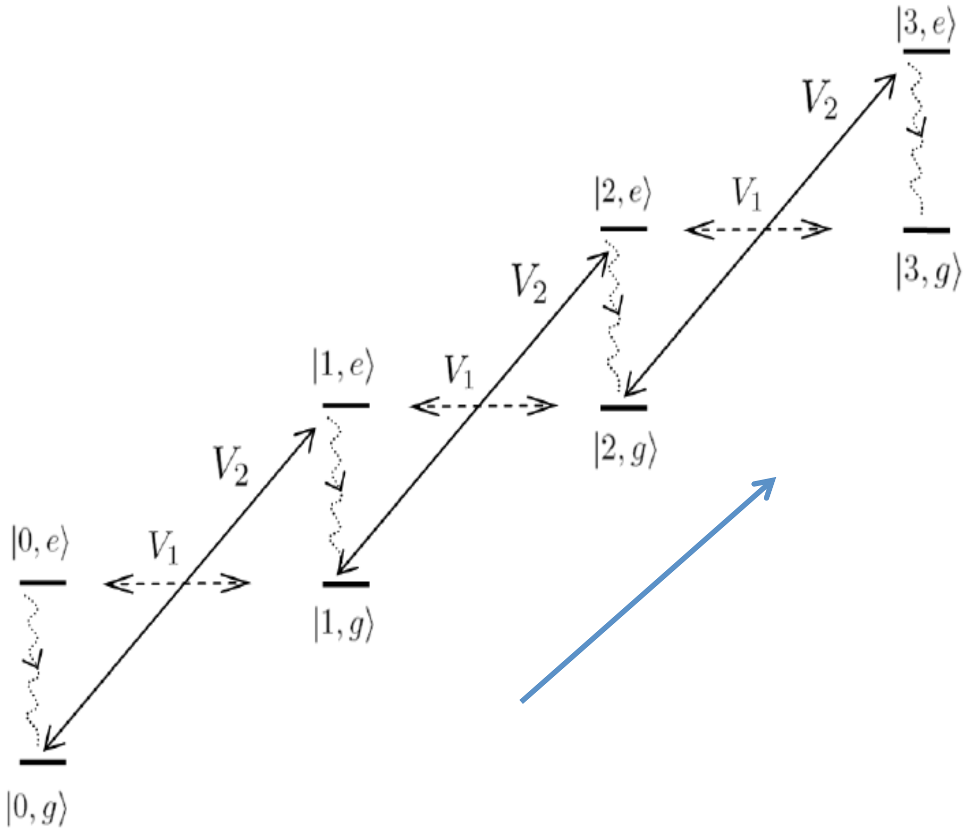
Mean photon number



Qubit excitation survives at long time

Results-8: energy dissipation assisting driving

Ladder of bare energy states



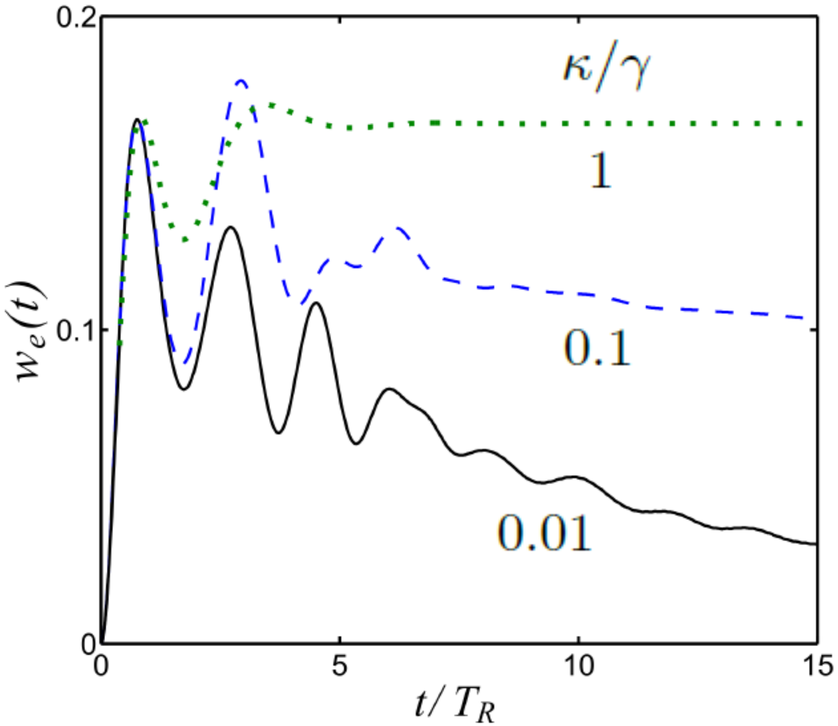
Two processes:

$$V_2 - \gamma - V_1 - \gamma$$
$$V_2 - \gamma - V_2$$

- Energy dissipation brings the driven system UP
- New channel of photon generation with assistance of dissipation
- This dynamical regime does not exist in a dissipationless system

Results-9: cavity relaxation

Qubit excited state population



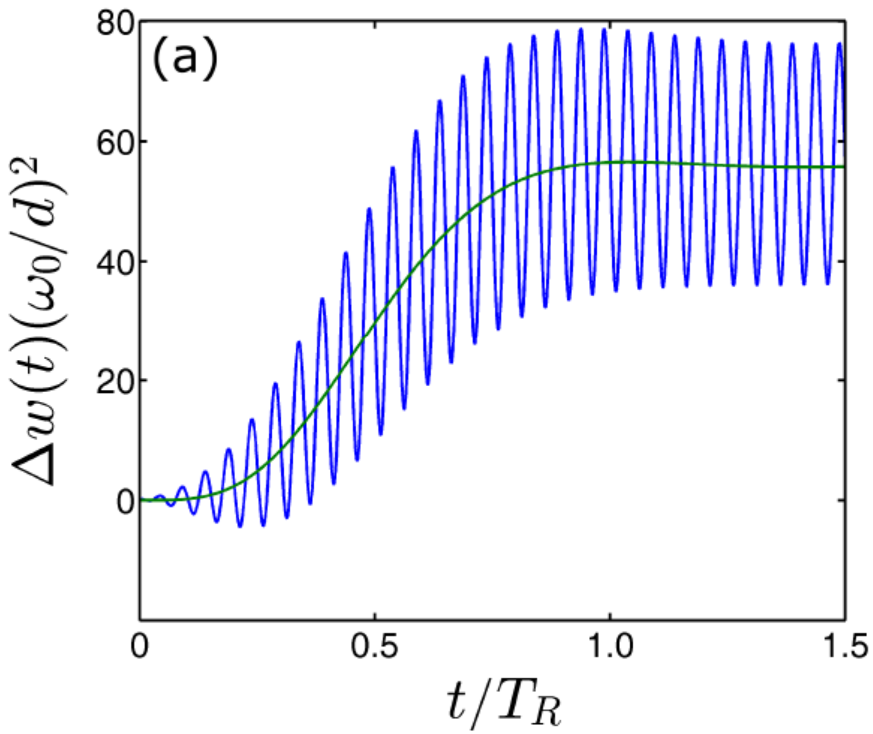
In a certain domain of parameters the effect can be enhanced

Results-10: rotating wave vs counterrotating wave physics

$$H(t) = H_0(t) + H_{Cas}(t) + V.$$

$$H_{Cas}(t) = i\hbar \frac{\partial_t \omega(t)}{4\omega(t)} (a^2 - a^{+2}).$$

$$\omega(t) = \omega_0 + d\cos(\Omega t)$$



Counterrotating wave physics near the resonance, provided *parametric periodic modulation of cavity frequency* is applied

arxiv: 1607.03054

Summary

- Dynamical Lamb effect can be observed in tunable-coupling superconducting qubit-resonator systems
- Parametric pumping as an efficient method to enhance the effect
- New interesting dynamical regimes due to the energy relaxation in a qubit which can amplify photon generation from vacuum with the help of the dynamical Lamb effect
- Counterrotating wave physics near the resonance under the periodic parametric driving

Phys. Rev. A 91, 063814 (2015); Phys. Rev. A 93, 063845 (2016); arxiv: 1607.03054

- Resonator frequency– 10 GHz
- g – 1-100 MHz
- Decoherence – 1-30 MHz or smaller in new transmons
- Quality factor 10^4
- Resonator size - centimeter
- Bifurcation oscillators, Josephson ballistic interferometers, 1 picosecond

$$\mathbf{E}(t) = \mathscr{E}\widehat{\varepsilon}(a\, e^{-i\nu t} + a^{+}\, e^{i\nu t}),$$

$$X_1 = \frac{1}{2}(a + a^{+}),$$

$$[X_1, X_2] = \frac{i}{2}$$

$$X_2 = \frac{1}{2i}(a - a^{+}).$$

С помощью этих операторов выражение (2.6.5) можно записать как

$$\mathbf{E}(t) = 2\mathscr{E}\widehat{\varepsilon}(X_1 \cos\nu t + X_2 \sin\nu t), \qquad (2.6.10)$$

т. е. эрмитовые операторы X_1 и X_2 можно рассматривать как амплитуды двух квадратур поля, имеющих разность фаз, равную $\pi/2$. Согласно (2.6.9), соотношение неопределенностей этих амплитуд имеет вид

$$\Delta X_1 \Delta X_2 \geqslant \frac{1}{4}. \qquad (2.6.11)$$

Условием сжатого состояния является выполнение неравенства

$$(\Delta X_i) < \frac{1}{4} \quad (i = 1 \text{ или } 2). \qquad (2.6.12)$$

$$(\Delta X_1)^2 = \langle\alpha|X_1^2|\alpha\rangle - (\langle\alpha|X_1|\alpha\rangle)^2$$

Дипольное приближение

В рамках второго подхода мы рассматриваем атом как электрический диполь с дипольным моментом $\wp \equiv e\mathbf{r}$. Здесь $\mathbf{r}$ обозначает координату электрона относительно протона. Вновь напоминаем дипольное приближение: напряжённость электрического поля $\mathbf{E}$ электромагнитной волны оптического диапазона не меняется заметным образом на

размере атома. Следовательно, диполь обладает потенциальной энергией

$$H_{\mathbf{r}\cdot\mathbf{E}} \equiv -e\mathbf{r} \cdot \mathbf{E}(\mathbf{R}, t) \tag{14.1}$$

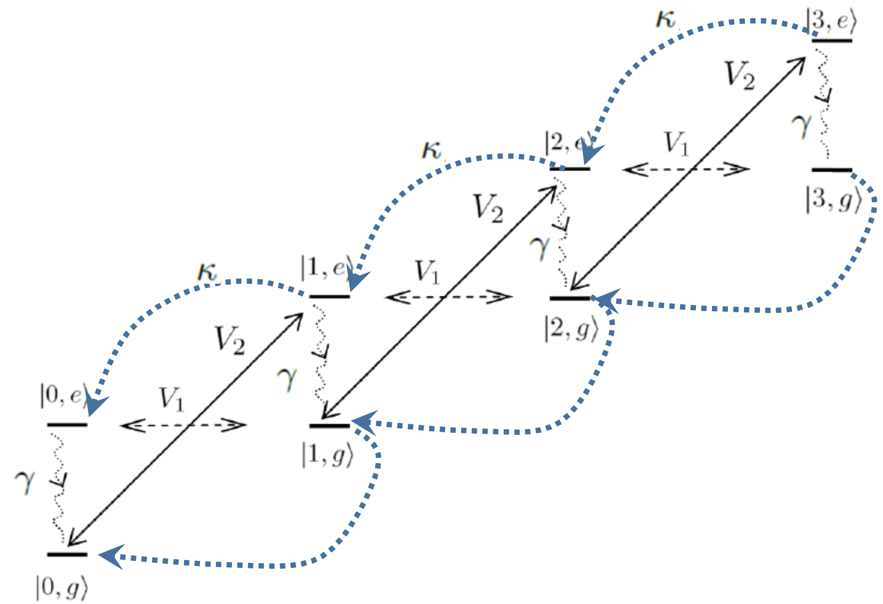
в электрическом поле $\mathbf{E}(\mathbf{R}, t)$, взятом в точке с координатой $\mathbf{R}$ центра инерции.

Некоторые пояснения

Релаксация в полости **усиливает эффект возбуждения** кубита!

Релаксация полости очевидно **ограничивает** рост фотонов, переводя систему в стационарное состояние, чем лучше полость, тем больше фотонных состояний может заселиться.

Качественно усиление населенности можно объяснить появлением нового канала «прихода» в возбужденное состояние.



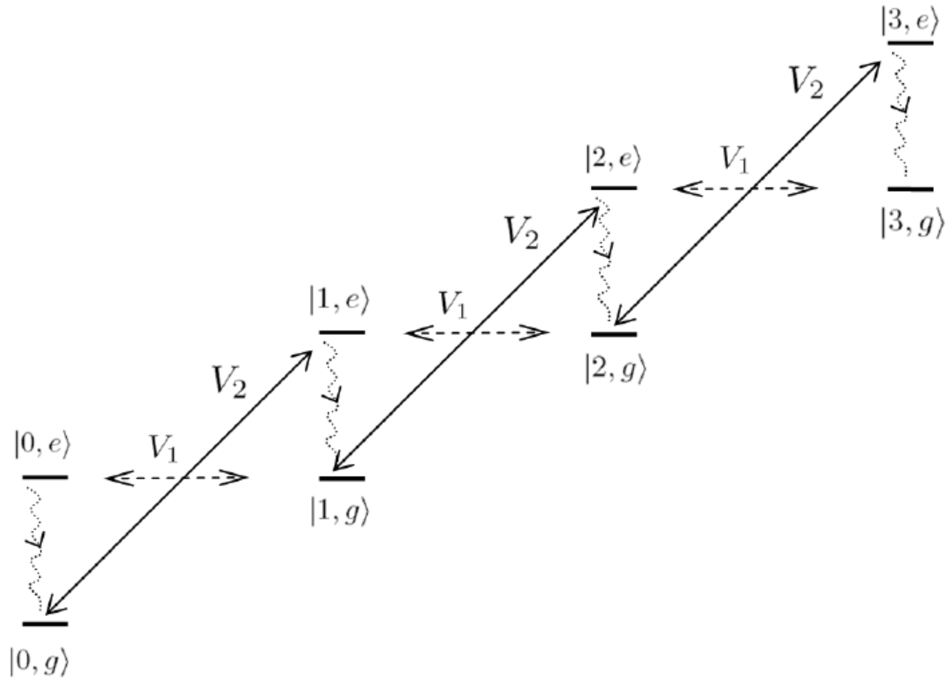
Система уравнений

Основная система уравнений для элементов матрицы плотности с учетом затухания кубита:

$$
\begin{aligned}
i\dot{\rho}^{gg}_{m,n} &= \rho^{gg}_{m,n}\omega(n-m) + i\gamma\rho^{ee}_{m,n} + g(t)\left(\sqrt{n}\rho^{ge}_{m,n-1} + \sqrt{n+1}\rho^{ge}_{m,n+1} - \sqrt{m}\rho^{eg}_{m-1,n} - \sqrt{m+1}\rho^{eg}_{m+1,n}\right)\\
i\dot{\rho}^{ee}_{m,n} &= \rho^{ee}_{m,n}\left[\omega(n-m) - i\gamma\right] + g(t)\left(\sqrt{n}\rho^{eg}_{m,n-1} + \sqrt{n+1}\rho^{eg}_{m,n+1} - \sqrt{m}\rho^{ge}_{m-1,n} - \sqrt{m+1}\rho^{ge}_{m+1,n}\right)\\
i\dot{\rho}^{eg}_{m,n} &= \rho^{eg}_{m,n}\left[\omega(n-m) - \varepsilon - i\gamma/2\right] + g(t)\left(\sqrt{n}\rho^{ee}_{m,n-1} + \sqrt{n+1}\rho^{ee}_{m,n+1} - \sqrt{m}\rho^{gg}_{m-1,n} - \sqrt{m+1}\rho^{gg}_{m+1,n}\right)\\
i\dot{\rho}^{ge}_{m,n} &= \rho^{ge}_{m,n}\left[\omega(n-m) + \varepsilon - i\gamma/2\right] + g(t)\left(\sqrt{n}\rho^{gg}_{m,n-1} + \sqrt{n+1}\rho^{gg}_{m,n+1} - \sqrt{m}\rho^{ee}_{m-1,n} - \sqrt{m+1}\rho^{ee}_{m+1,n}\right)
\end{aligned}
$$

Резонанс:

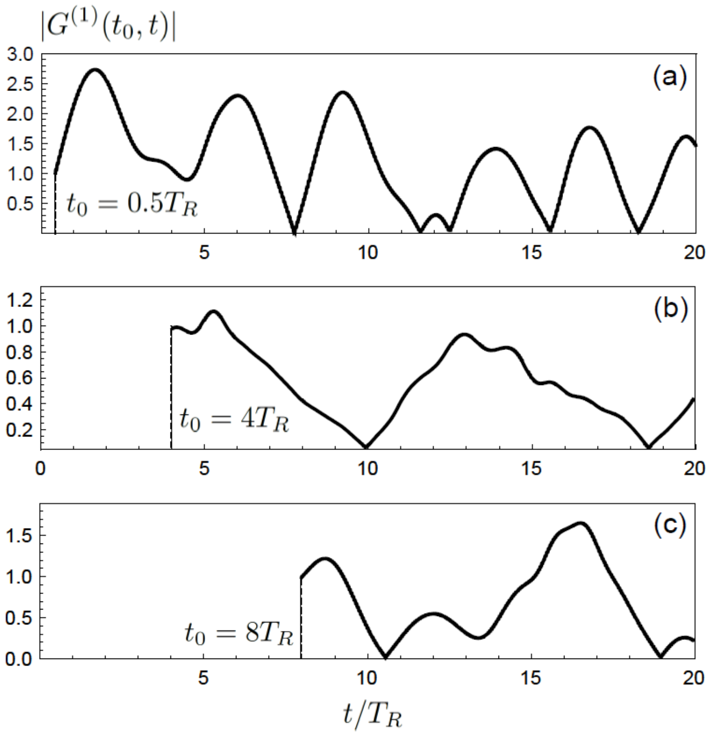
$\omega = \varepsilon$



Results-5: parametric driving

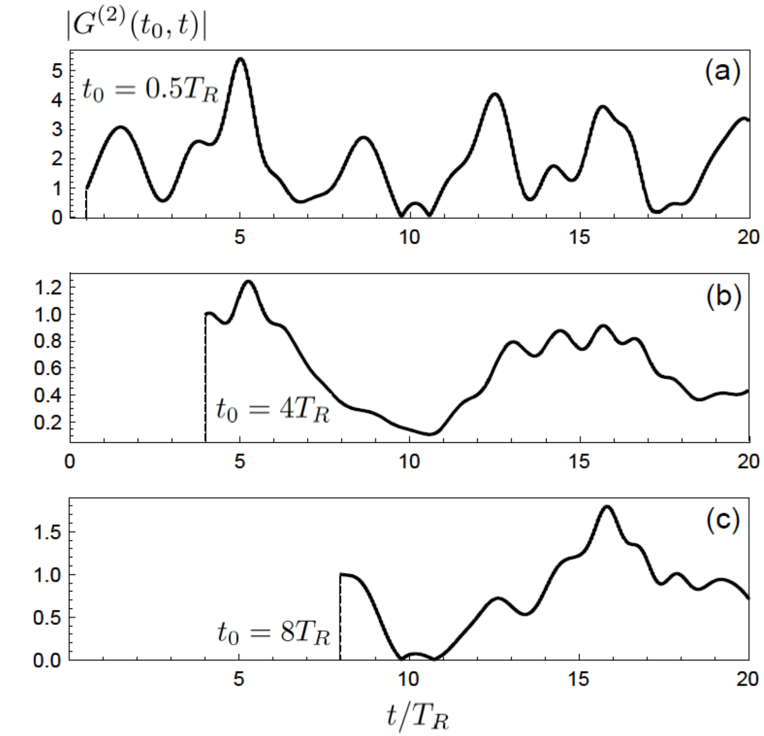
First-order photon correlation function

$$G^{(1)}(t_0, t) = \frac{\langle a^\dagger(t_0) a(t) \rangle}{\langle a^\dagger(t_0) a(t_0) \rangle}$$



Second-order photon correlation function

$$G^{(2)}(t_0, t) = \frac{\langle a^\dagger(t_0) a(t_0) a^\dagger(t) a(t) \rangle}{\langle a^\dagger(t_0) a(t_0) a^\dagger(t_0) a(t_0) \rangle}$$



Results-6: parametric driving

Universal behavior

$$w_{\uparrow}(t) \simeq \frac{2}{\pi^2}(1 - \cos\sqrt{2}gt)$$

- Qualitatively correct result, but quantitatively not so good.
- Universality, however, does exist

Results-7: parametric driving

Squeezing

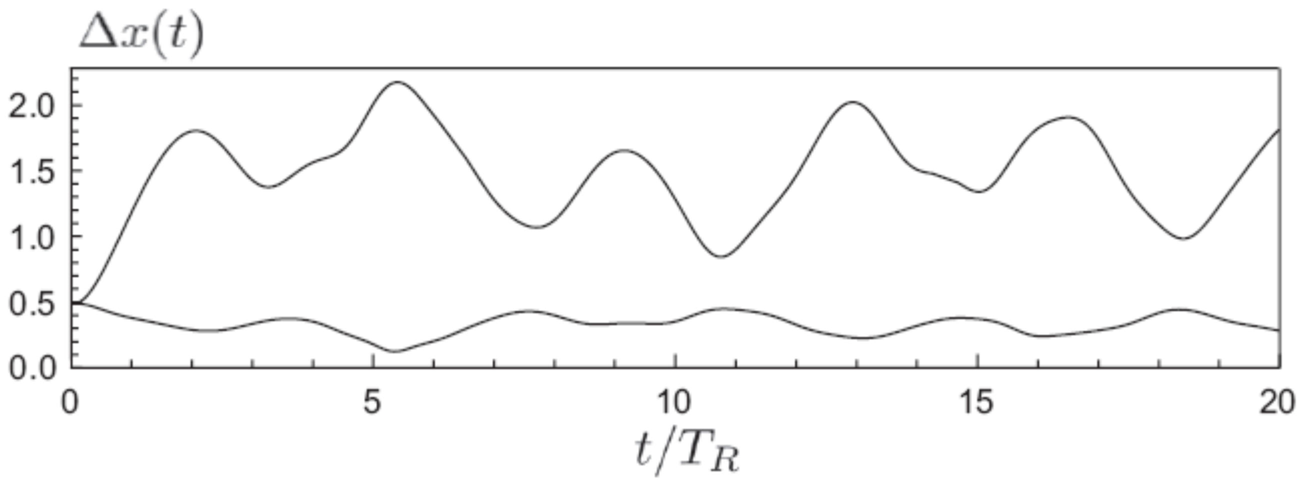


FIG. 4. Numerical results for the squeezing parameter $\Delta x(t)$ at $\omega/g = 20$ within 20 Rabi periods after the beginning of the parametric driving. Only lower and upper envelope curves are shown, while $\Delta x(t)$ experiences fast oscillations between them.

$$\Delta x \equiv \frac{1}{2}\sqrt{\langle (a + a^\dagger)^2 \rangle - \langle a + a^\dagger \rangle^2}$$

$$\mathbf{E}(t) = \mathscr{E}\widehat{\varepsilon}(a\, e^{-i\nu t} + a^+\, e^{i\nu t}),$$

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