Trieste, 24/08/2016

Inhomogeneous quasi-adiabatic driving of quantum critical dynamics in disordered spin chains

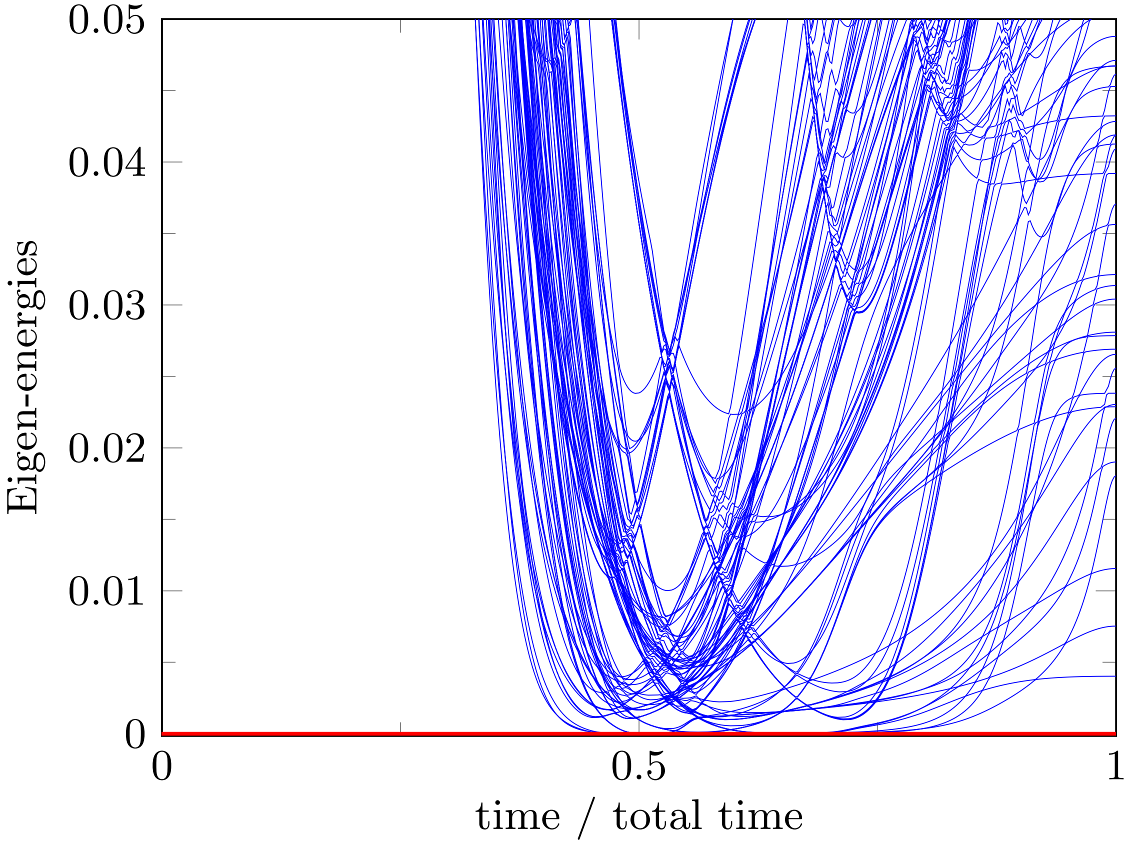
Marek M. Rams
@Jagiellonian University, Cracow

In collaboration with:
Masoud Mohseni @ Google
Adolfo del Campo @ University of Massachusets

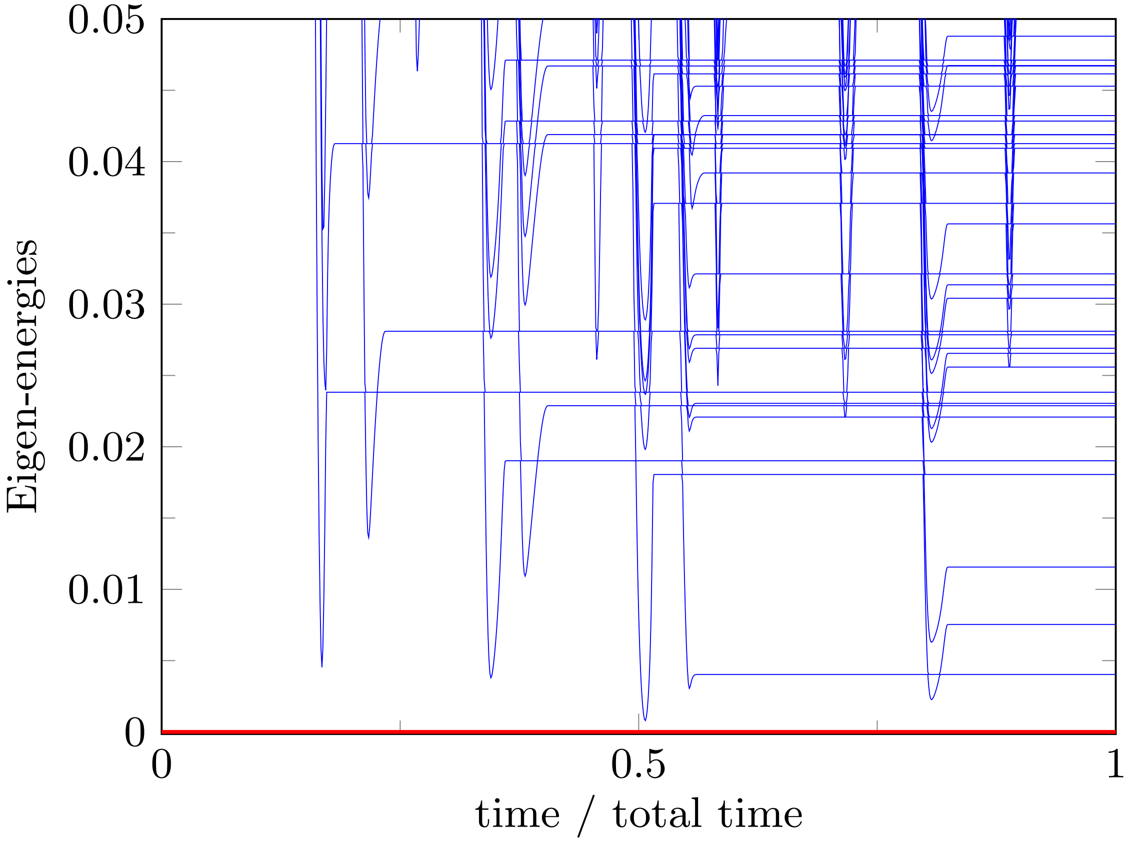
Motivation: Adiabatic quantum optimization

$$H(t) = H_C(t) + H_P(t)$$

$$|\psi(t_0)\rangle \longrightarrow |\psi(t_f)\rangle$$



$|\psi(t)\rangle$ gets highly excited



$|\psi(t)\rangle$ stays in low energy sector

Can spatially inhomogeneous driving field help?

Rams, Mohseni, del Campo, arxiv:1606.07740

Mohseni, del Campo, Rams '16 in prep.

Outline:

- Introduction to Kibble-Zurek mechanism
- Beyond KZM: spatial inhomogeneity and quantum phase transition
- Universality and suppressing defects in random Ising model
- Next steps

Quantum critical point (second order)

$$H(g) = gH_C + H_P$$

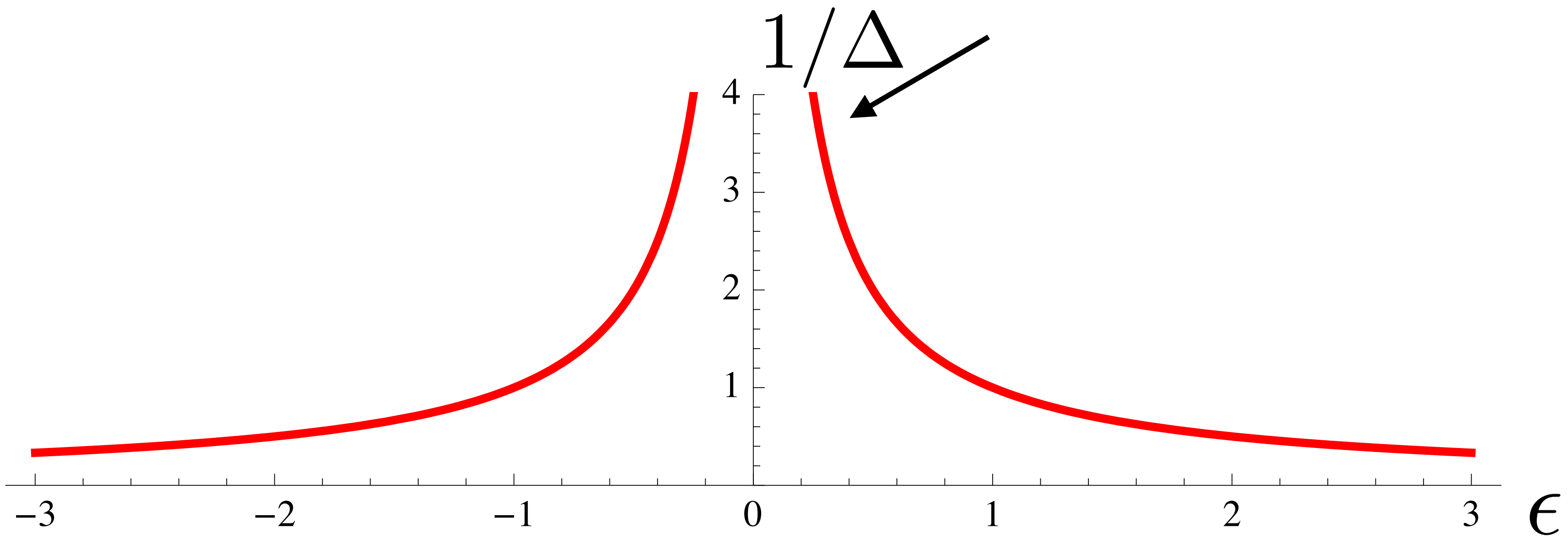
g control parameter

g_c critical point

Δ characteristic energy scale

Distance from the critical point: $\epsilon = (g - g_c)/g_c$

relaxation time diverges at the continuos critical point



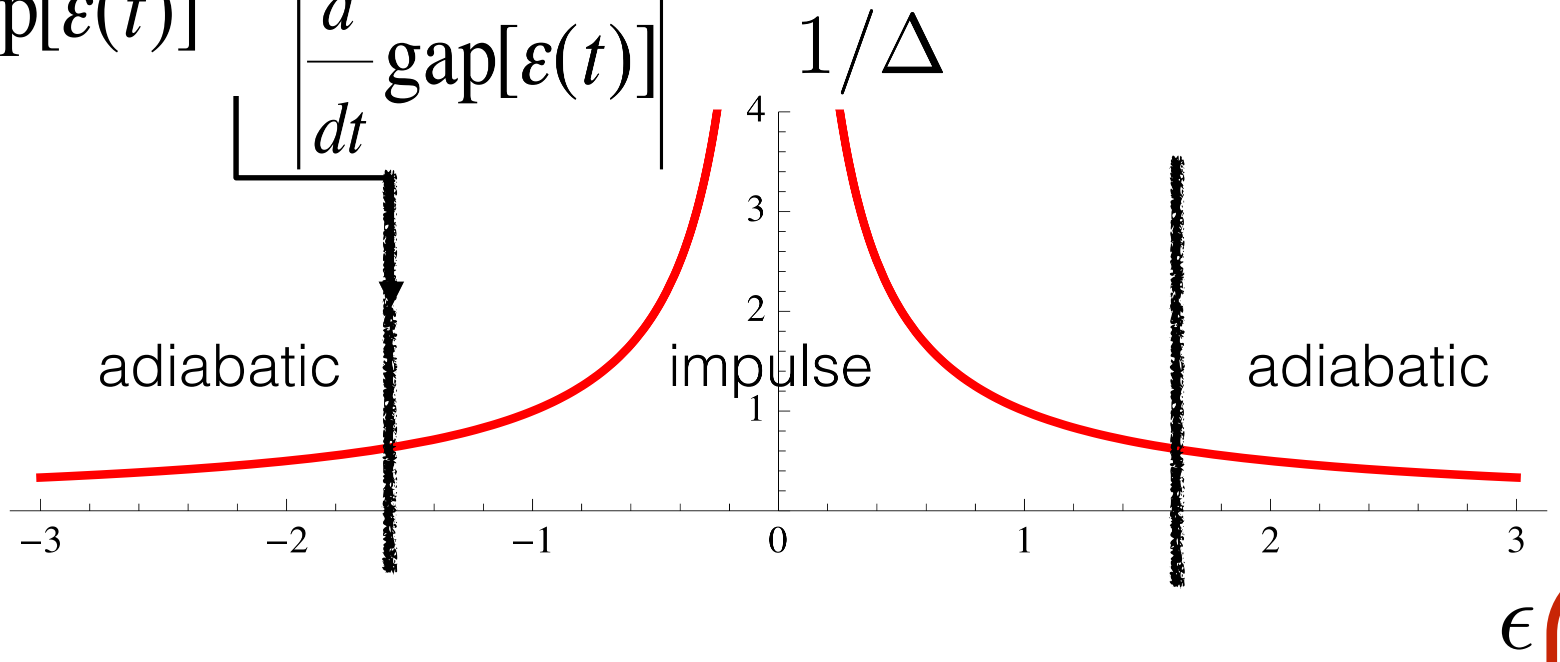
$$\Delta \sim |\epsilon|^{z\nu}$$

$$\xi \sim |\epsilon|^{-\nu}$$

Dynamical transition (Kibble-Zurek mechanism)

Close to the the critical point expand: $\epsilon(t) = \frac{t}{\tau_Q} + O(t^2)$

$$\frac{1}{\text{gap}[\varepsilon(t)]} \sim \frac{\text{gap}[\varepsilon(t)]}{\left|\frac{d}{dt}\text{gap}[\varepsilon(t)]\right|}$$



Characteristic KZ scales:

$$\hat{\xi} \sim \tau_Q^{\nu/(1+z\nu)}$$

$$\hat{t} \sim \tau_Q^{z\nu/(1+z\nu)}$$

Density of defects:

$$d_{ex} \sim \hat{\xi}^{-d} \sim \tau_Q^{-d\nu/(1+z\nu)}$$

Classical systems:

Kibble '76. Zurek '85.

Quantum systems:

Zurek, Dorner, Zoller, '05.

Dziarmaga, '05. Polkovnikov '05.

Dynamical scaling hypothesis:

Kolodrubetz, Clark, Huse '12.

Chandran, Erez, Gubser, Sondhi '12

Disorder has a profound effect

E.g. Random Ising spin chain $H(t) = -\sum_{n=1}^{N} J_n \sigma_n^z \sigma_{n+1}^z + g_n(t)\sigma_n^x$

- Critical point in the universality class of infinite disorder fixed point
- Griffiths phase surrounding the critical point

$$\Delta_{min} \sim e^{-CN^{1/2}}$$

$$\Delta(\epsilon) \sim |\epsilon|^{1/|\epsilon|} \longrightarrow z \simeq 1/2|\epsilon| \xrightarrow{\epsilon \to 0} \infty$$

Fisher '95

Young, Rieger '96

Fisher, Young '98

Dynamical transition: $\epsilon(t) = \frac{t}{\tau_Q} + O(t^2)$

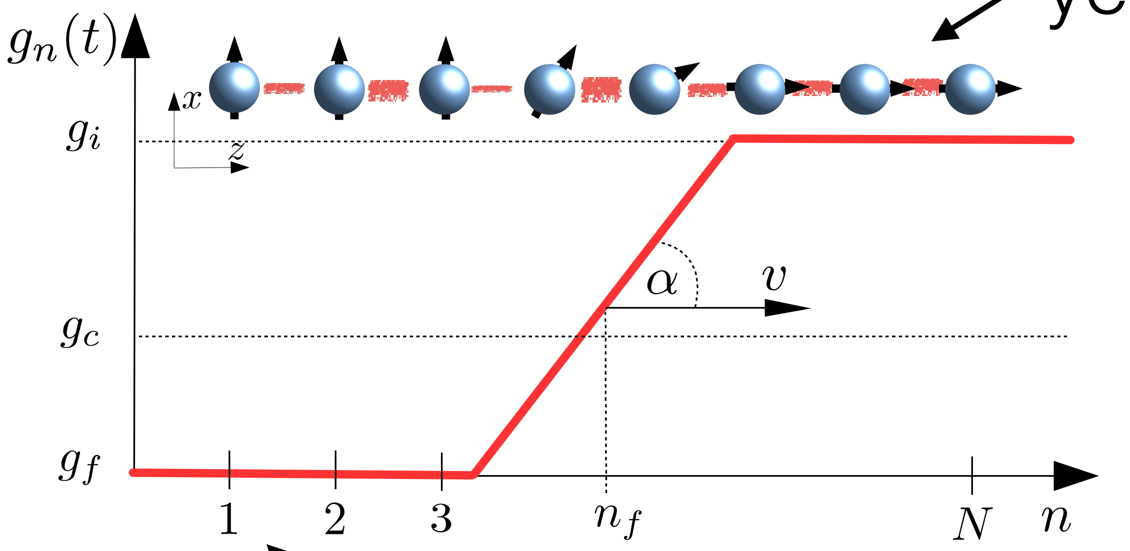
Density of defects: $d_{ex} \sim (\log \tau_Q)^{-2}$

Dziarmaga '06

Caneva, Fazio, Santoro '07

Suzuki '11

Can spatial inhomogeneity help?



yet to experience phase transition

already experienced phase transition

Taking the system gradually from one phase to another part after part

Driving field is both time and space dependent

$$H(t) = -\sum_{n=1}^{N} [g_c + \epsilon(n,t)]\sigma_n^x + J_n \sigma_n^z \sigma_{n+1}^z$$

e.g.: $\epsilon(n,t) = \tanh(\alpha(n - vt))$

linearize close to critical point

$$\epsilon(n,t) \approx \alpha(n - vt)$$

velocity of the front

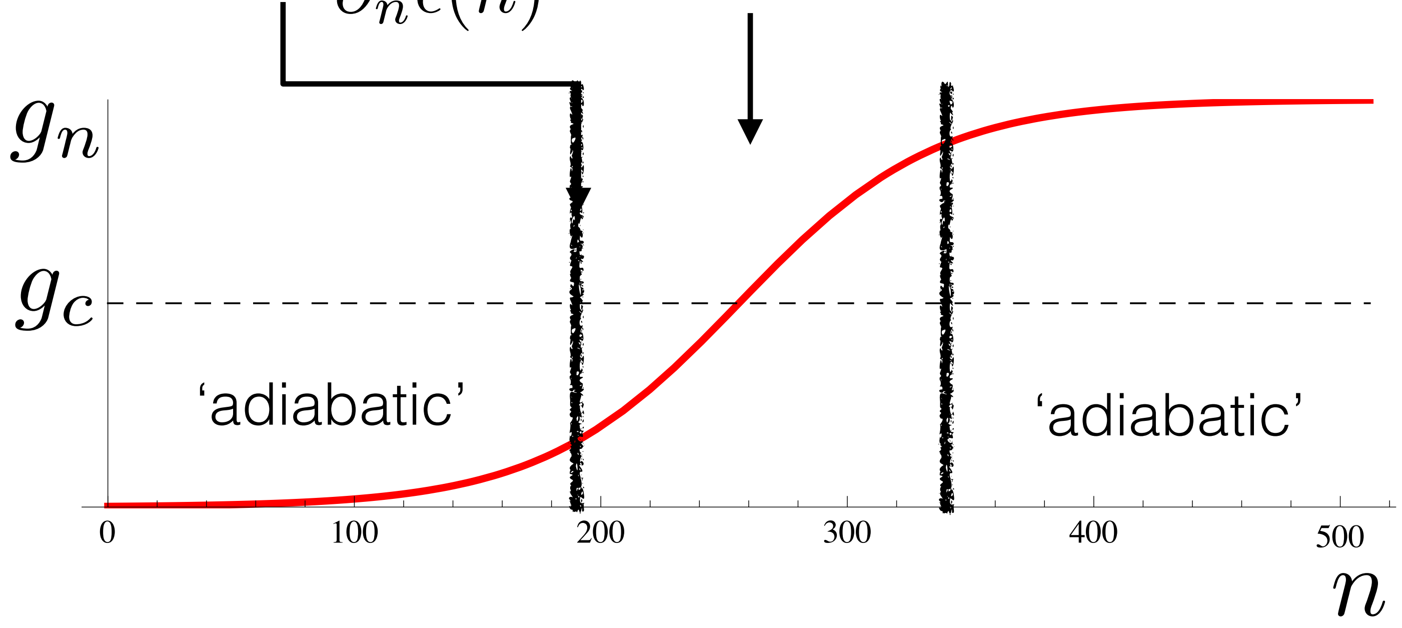
slope

position in space

Inhomogeneous Kibble-Zurek mechanism (no disorder)

$$\xi(\epsilon(n)) \sim \frac{\epsilon(n)}{\partial_n \epsilon(n)}$$

system cannot adjust locally to changing field



penetration depth into symmetric phase

$$\hat{\xi} \sim \alpha^{-\nu/(1+\nu)}$$

system effectively is governed by gapped Hamiltonian

$$\hat{\Delta} \sim \hat{\xi}^{-z} \sim \alpha^{z\nu/(1+\nu)}$$

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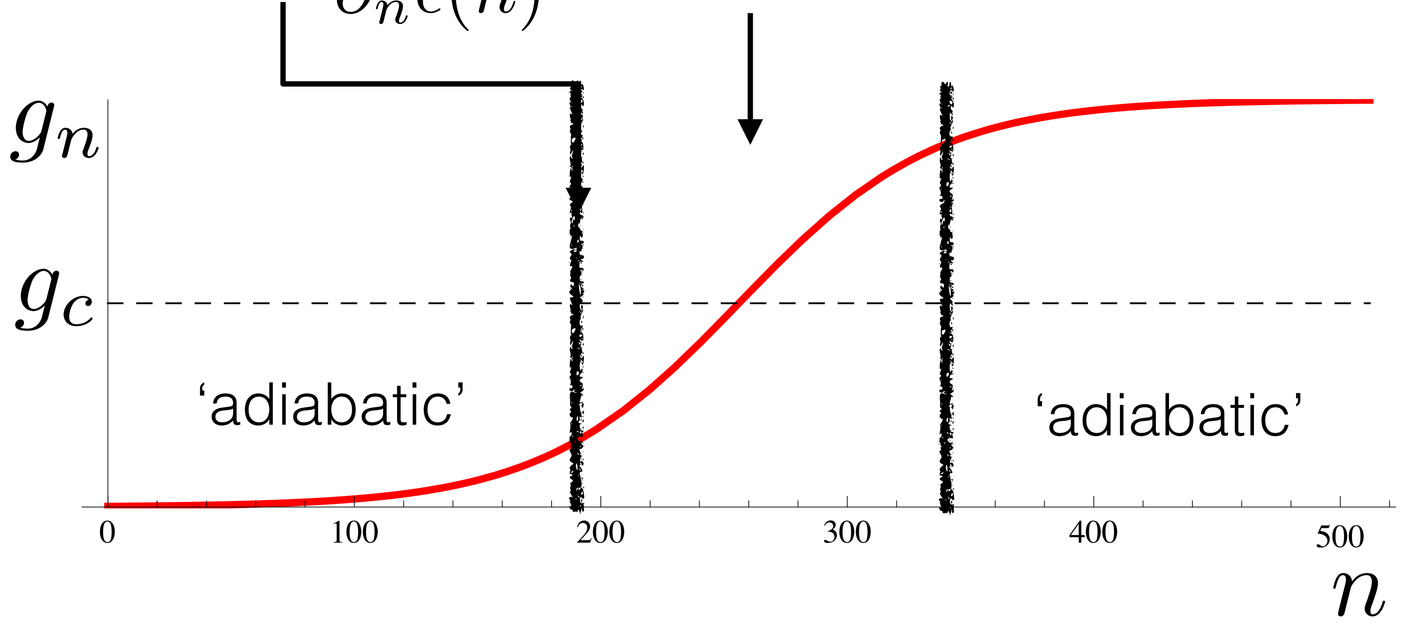


penetration depth into symmetric phase $\hat{\xi} \sim \alpha^{-\nu/(1+\nu)}$

system effectively is governed by gapped Hamiltonian $\hat{\Delta} \sim \hat{\xi}^{-z} \sim \alpha^{z\nu/(1+\nu)}$

$\hat{\Delta} \xrightarrow{\alpha \to 0} 0$

$\hat{\xi} \xrightarrow{\alpha \to 0} \infty$

trade-off — optimal α ?

↔

$\hat{\Delta} \xrightarrow{\alpha \to 1} \mathrm{const}$

$\hat{\xi} \xrightarrow{\alpha \to 1} 0$

Inhomogeneous Kibble-Zurek mechanism (no disorder)

When spatial inhomogeneity is relevant?

$$v_t \sim \frac{\hat{\xi}}{\hat{\tau}} \sim \alpha^{(z-1)\nu/(1+\nu)}$$

$v \gg v_t$ → irrelevant, system gets excited as for homogeneous case

$v \ll v_t$ → relevant, excitations are suppressed

Classical systems:	Quantum systems:	Ion chains:
Kibble, Volovik ’97	Zurek, Dorner ’08	Pyka et al. ’13
Dziarmaga, Laguna, Zurek ‘99	Dziarmaga, Rams ‘10	del Campo et al ’10

What happens in the presence of disorder?

adiabatic condition

local gap

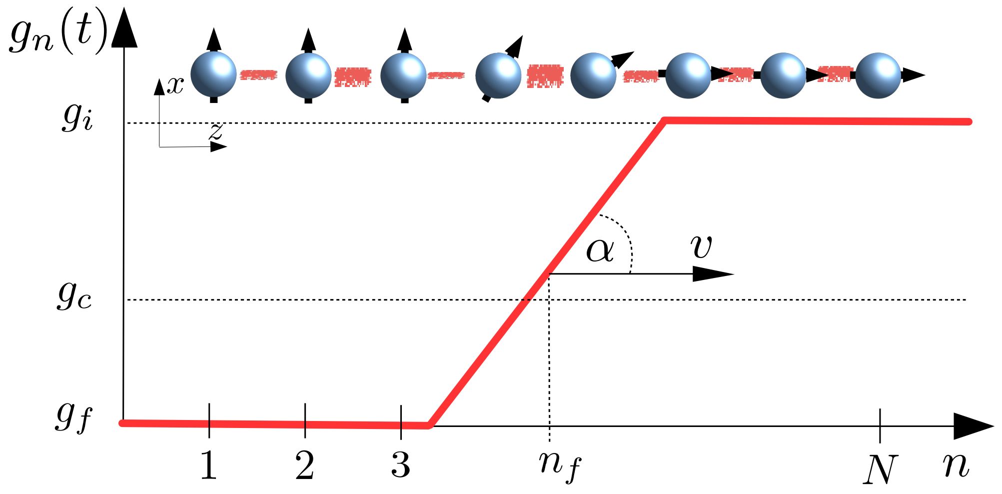
position of the front

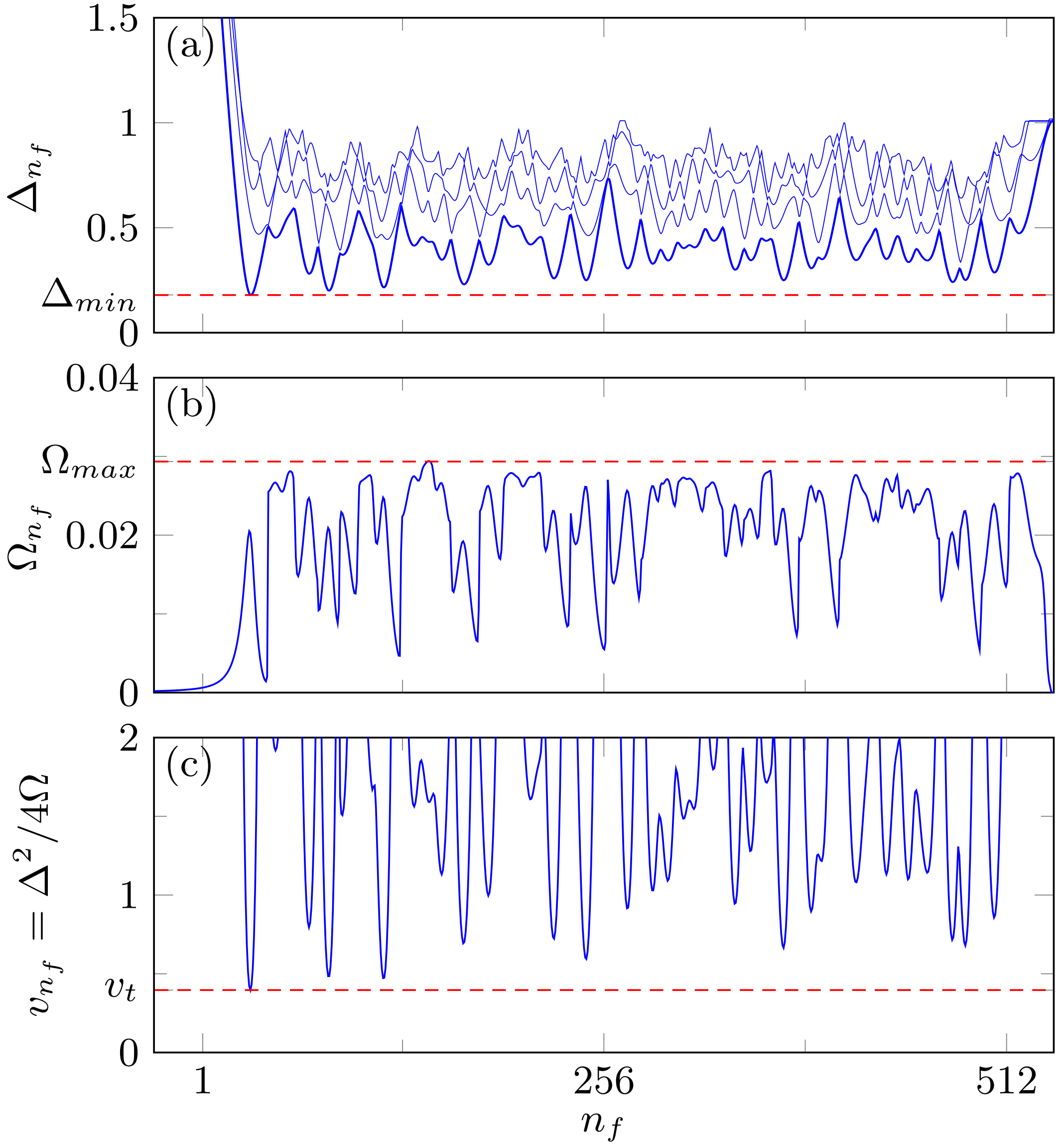
$$v < v_t = \frac{\Delta^2(n_f)}{4|\langle 0,t|\frac{dH}{dn_f}|1,t\rangle}$$

mixing term $\Omega(n_f)$

$$H(t) = -\sum_{n=1}^{N} J_n\sigma_n^z\sigma_{n+1}^z + g_n(t)\sigma_n^x$$

$$J_n \in [0.5, 1.5]$$





$\alpha = 1/32$

$N = 512$

What happens in the presence of disorder?

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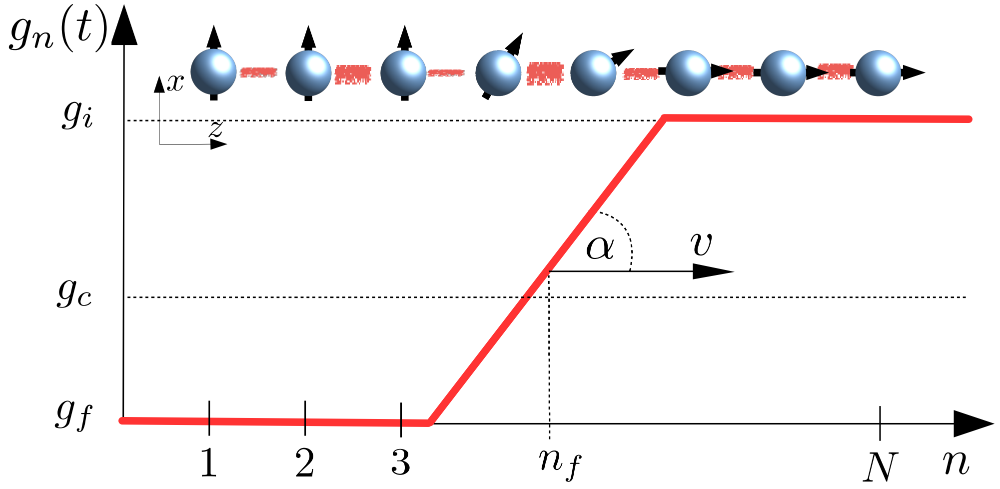
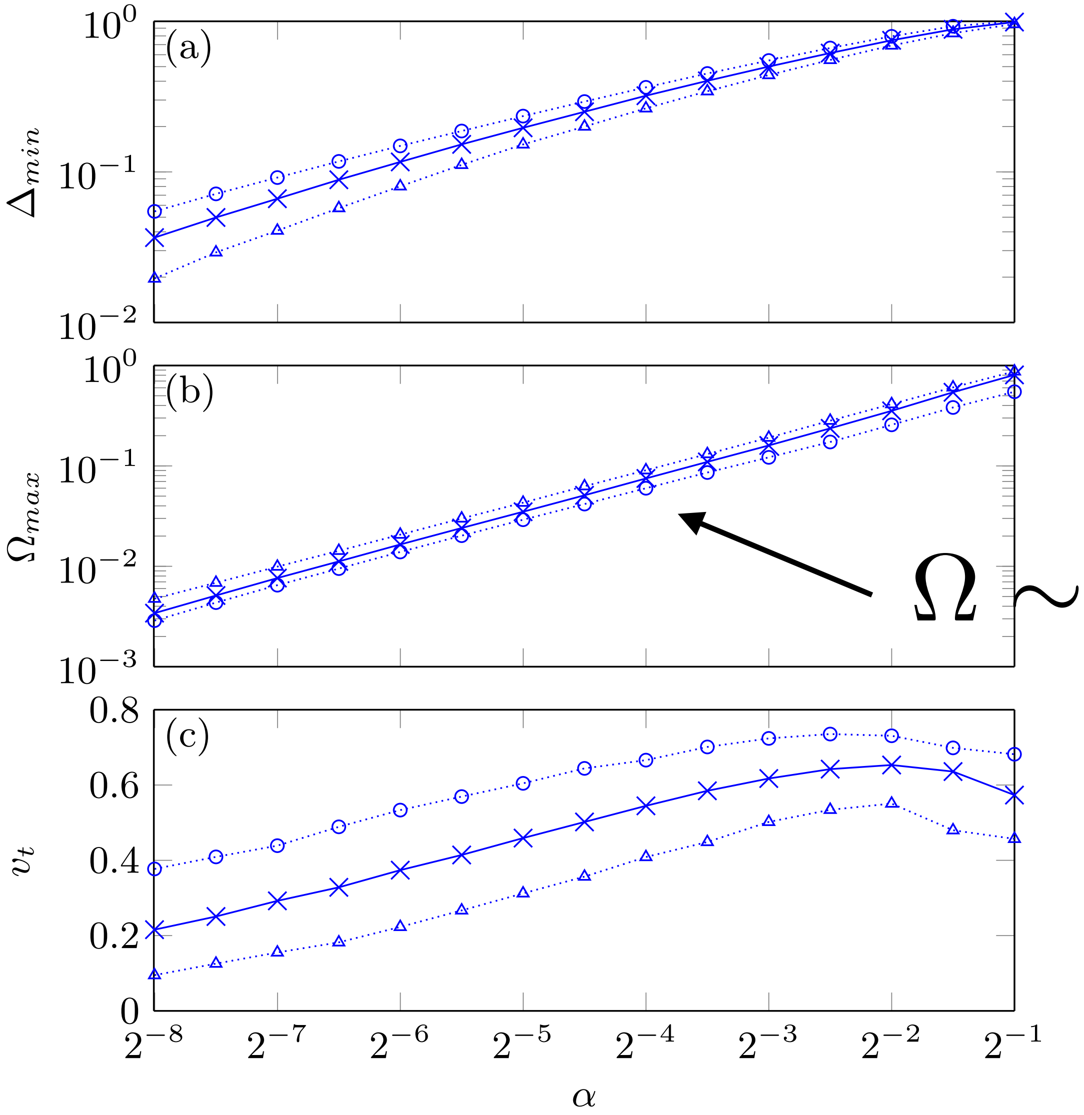




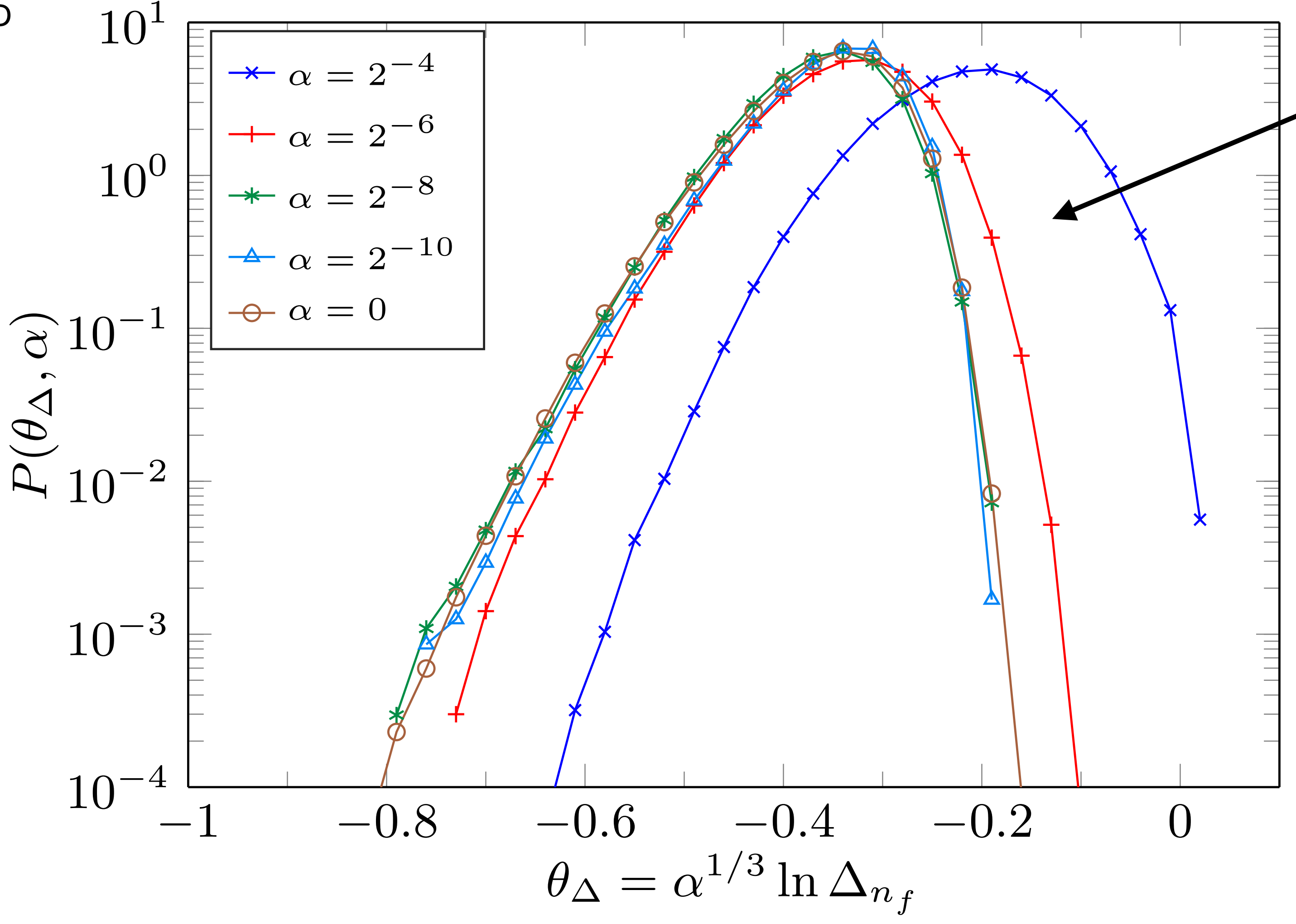
$\Omega \sim \alpha^{1.1}$

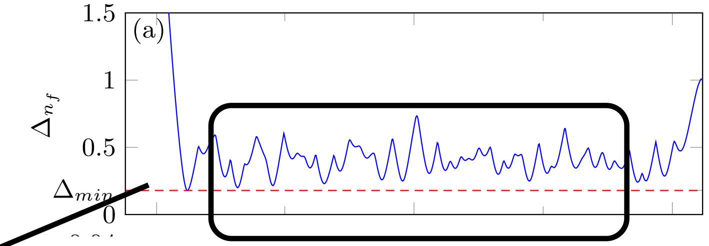
Universal scaling of the gap

$$\theta_\Delta = N^{-1/2}\log\Delta \xrightarrow{\text{conjecture}} \theta_{\Delta_{n_f}} = \hat{\xi}^{-1/2}\log\Delta_{n_f}$$

Young, Rieger '96

Fisher, Young '98





$J_n \in [0.5, 1.5]$

$N = 512$

homogeneous

$\Delta \sim e^{-CN^{1/2}}$

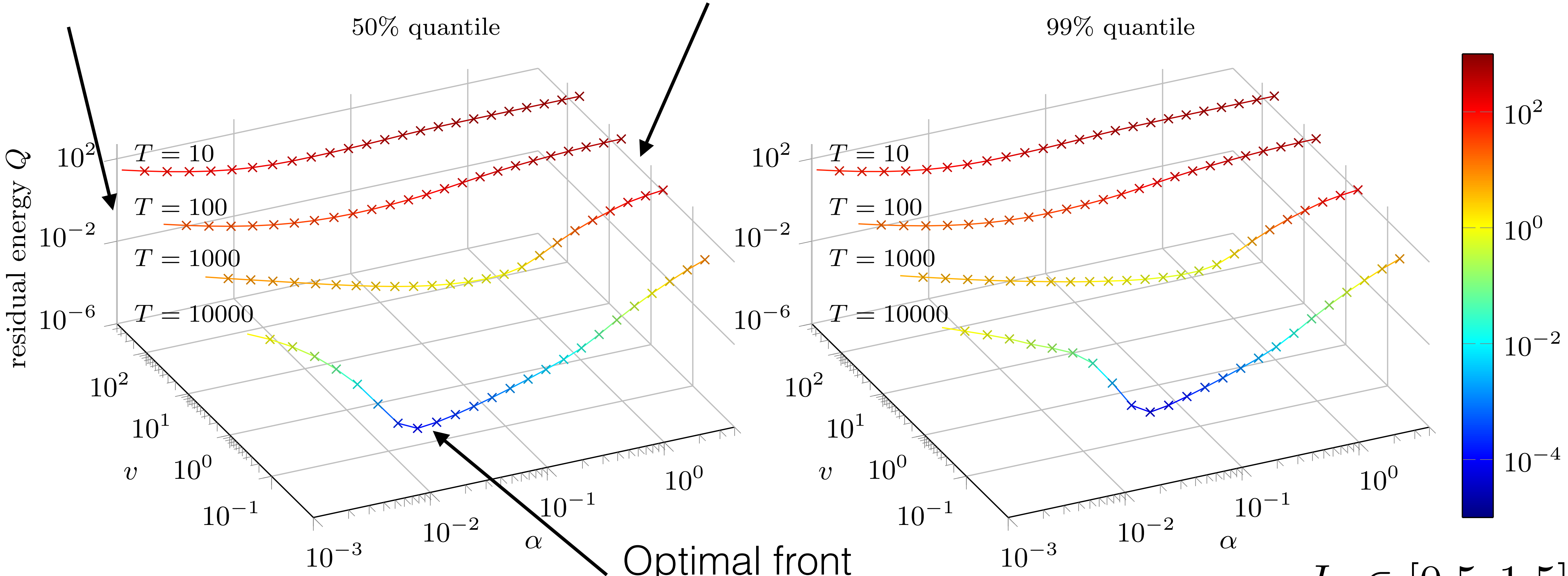
inhomogeneous

$\Delta \sim e^{-C'\alpha^{-1/3}}$

Suppressing defects in random Ising model

Homogeneous

One spin at a time



Optimal front

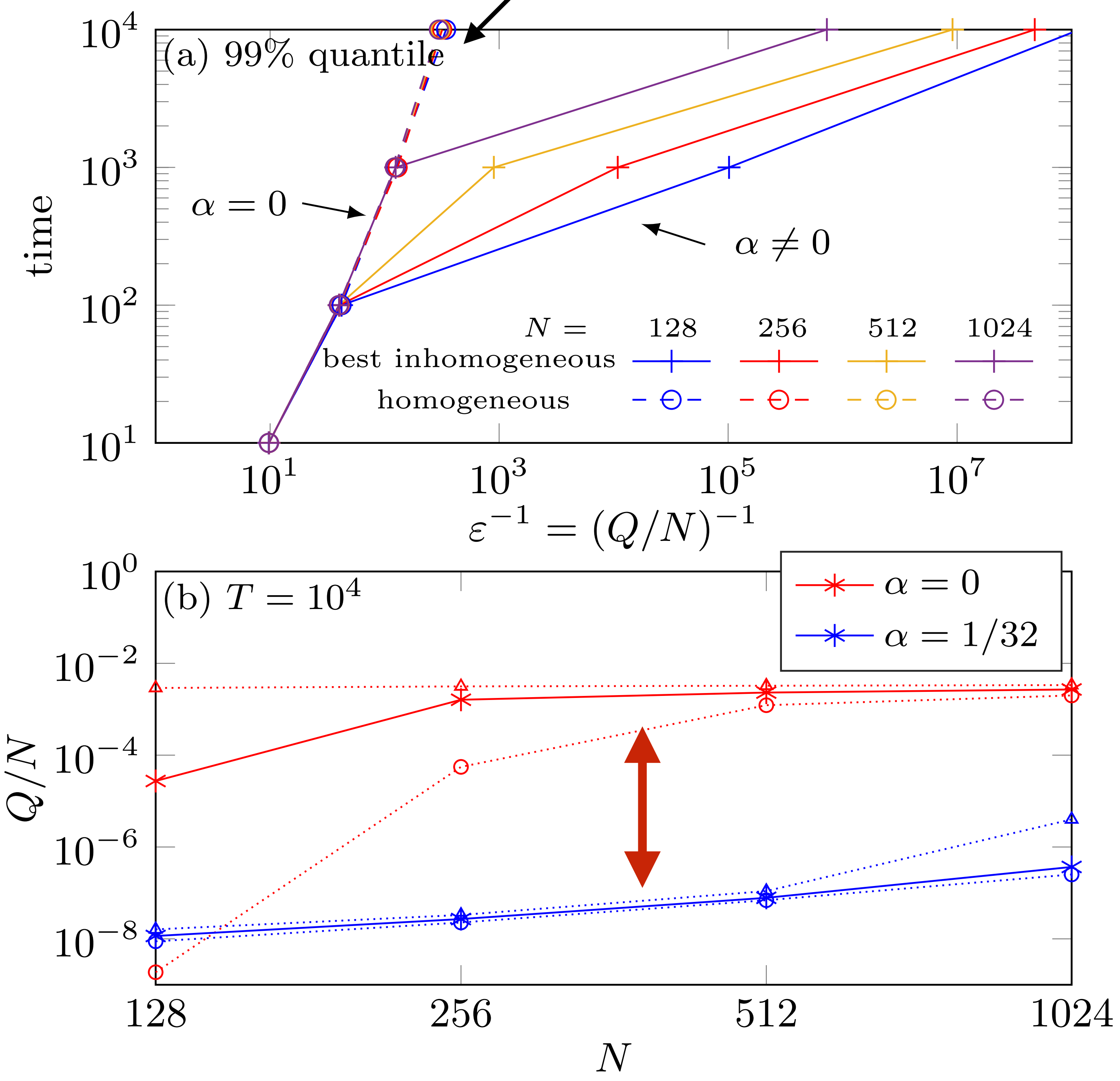
$$J_n \in [0.5, 1.5]$$

$$N = 512$$

$J_n \in [0, 2]$ → qualitatively the same picture

Suppressing defects in random Ising model

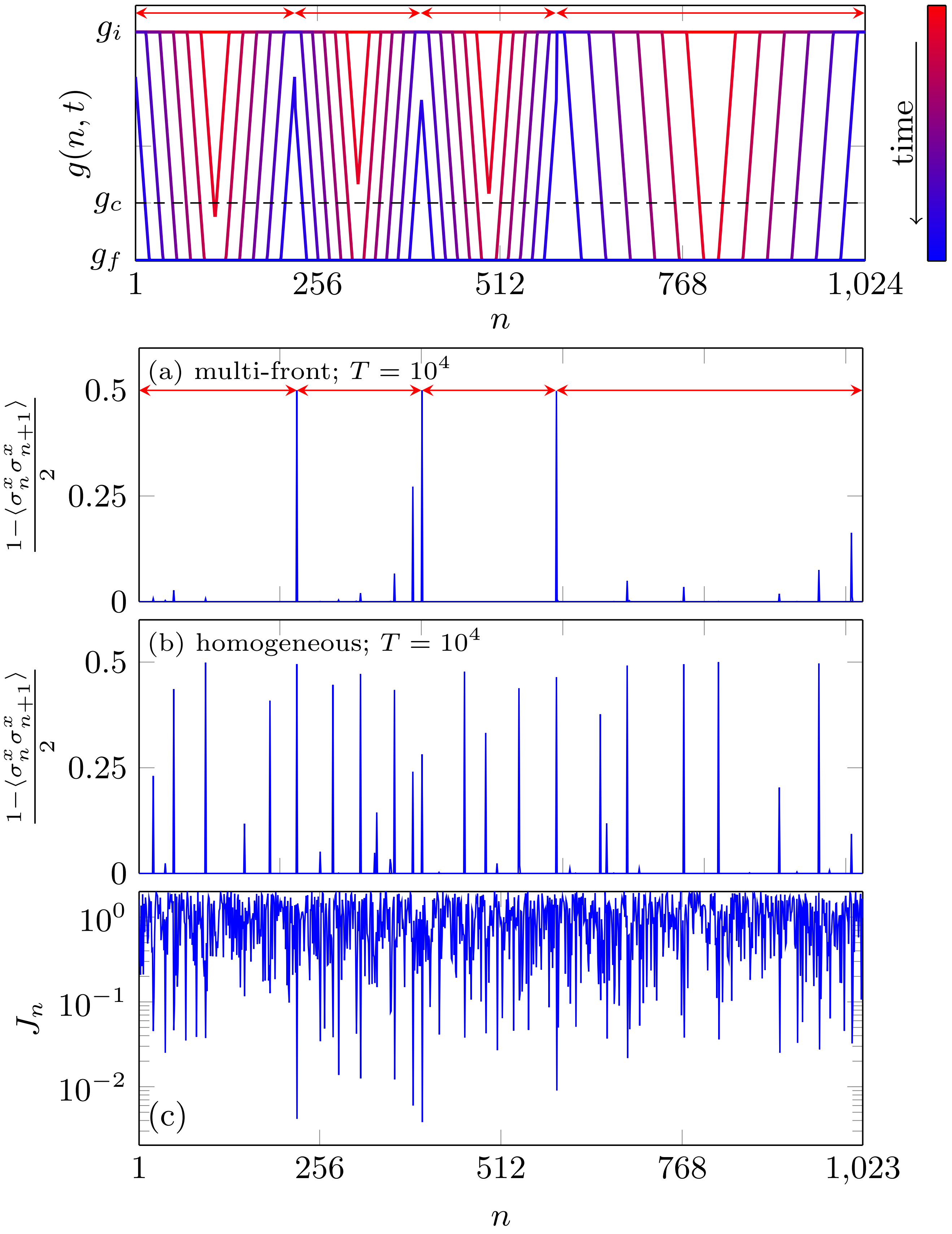
$$Q/N \sim (logT)^{-2}$$



$$J_n \in [0.5, 1.5]$$

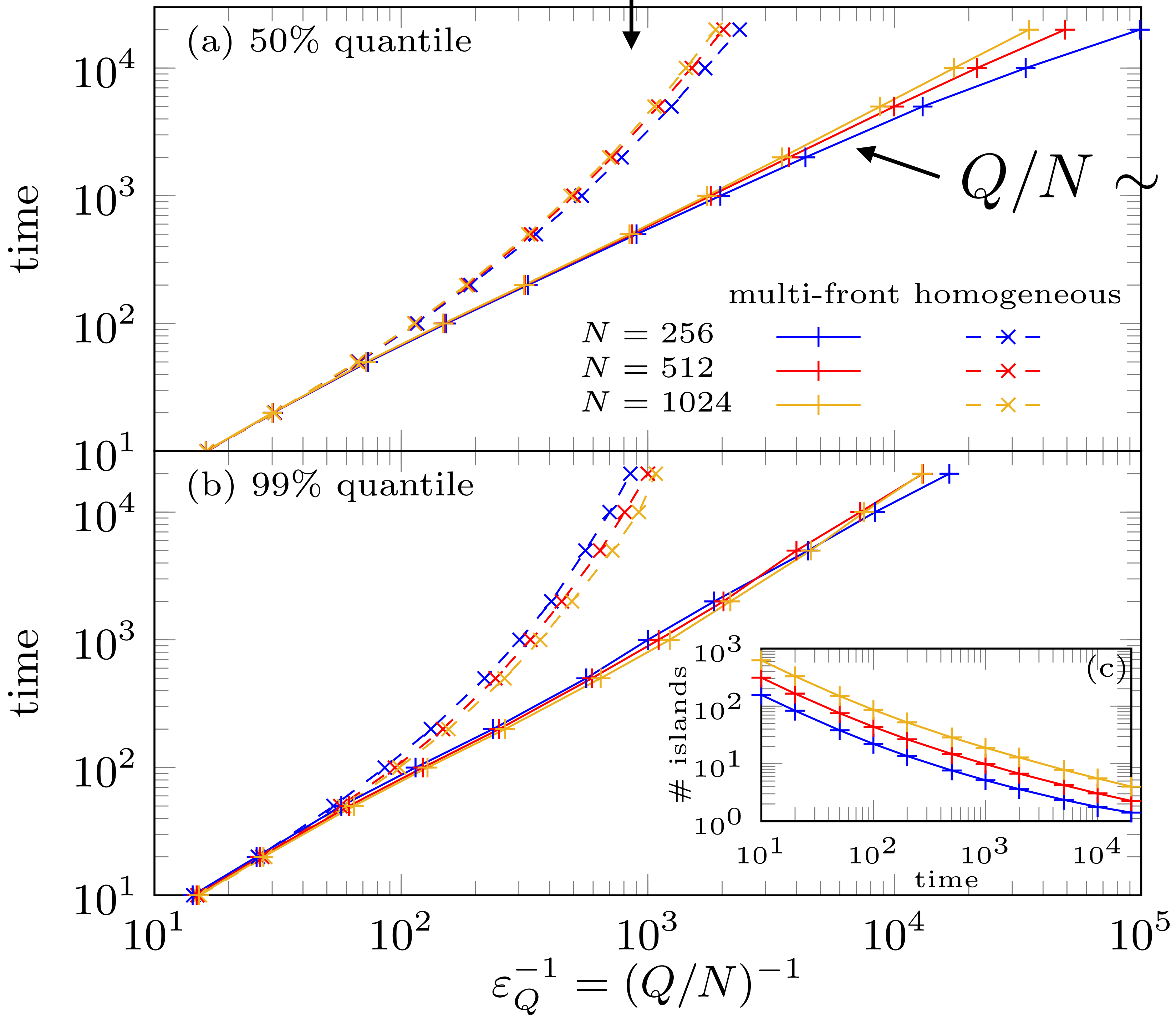
Multiple fronts at the same time

Mohseni, del Campo, Rams, ’16 in prep



$$J_n \in [0, 2]$$

Multiple fronts at the same time

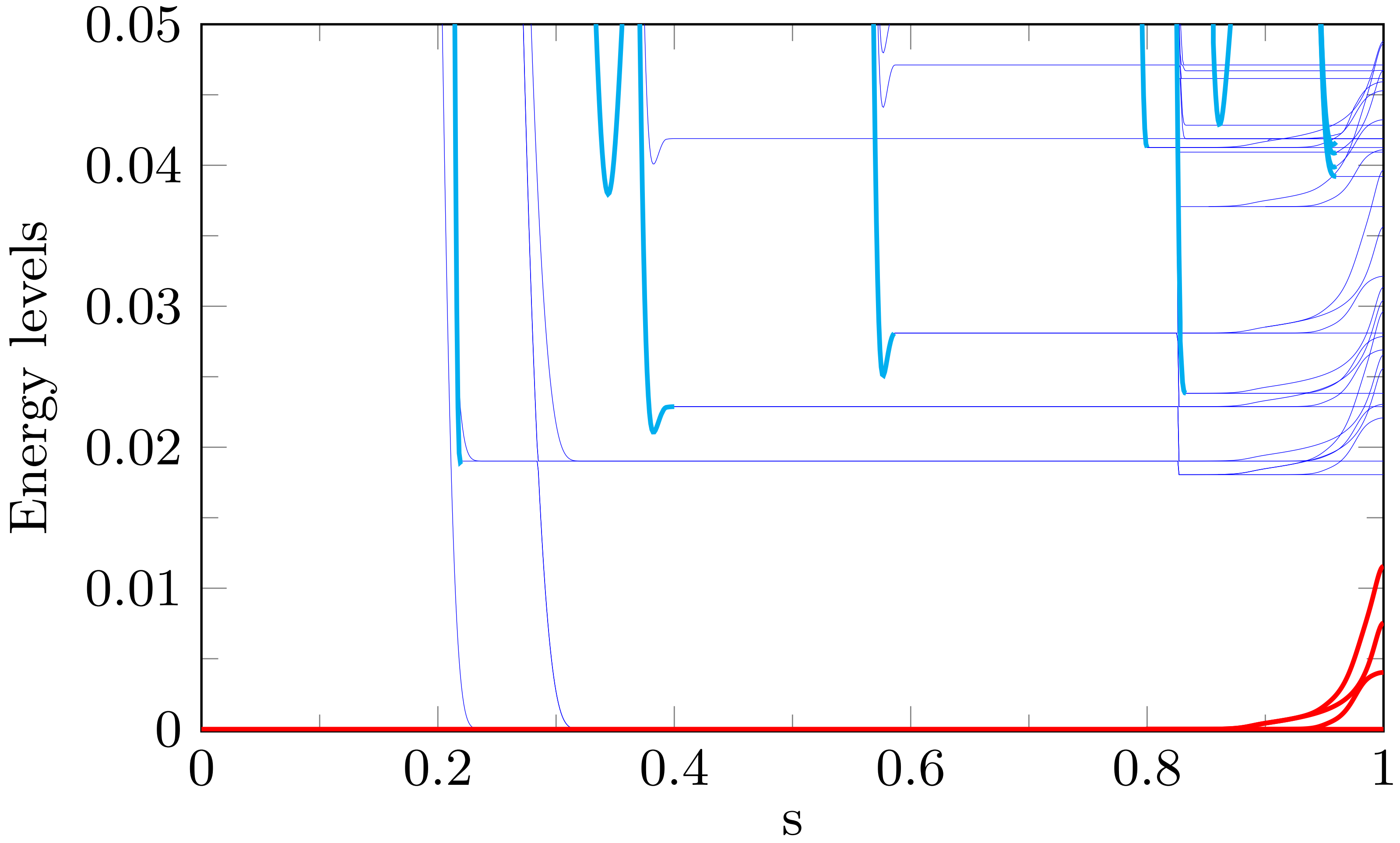


$Q/N \sim (\log T)^{-\gamma}$

$Q/N \sim 1/T$

$J_n \in [0, 2]$

Multiple fronts at the same time



Summarizing:

It is possible to go beyond the standard Kibble-Zurek prediction e.g. by inhomogeneous driving (here in disordered spin chain)

We observed universal scaling of the gap with the shape of the front for random Ising chain

Outline:

Generalized multi-front control Hamiltonians

Application to 2d systems (currently underway at Google Quantum AI team, preliminary results presented by Masoud Mohseni at AQC 2016).

Thank you!

Rams, Mohseni, del Campo, arxiv:1606.07740

Mohseni, del Campo, Rams '16 in prep.