On the quantum spin glass transition on the Bethe lattice

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work done in collaboration with T. Parolini, S. Pilati, A. Scardicchio

Some known facts about entanglement and quantum computation:

- slightly entangled computations are easy to simulate classically
- quantitative relations between entanglement and QAA performances
- no systematic studies of entanglement are known for large systems

Main Goal:

we want a benchmark for the entanglement dynamics of an ideal (*i.e.* perfectly adiabatic) run of the QAA.

The Model

Model: Ising Spin Glass in a Transverse Field

$$H[J_{ij}] = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x \qquad J_{ij} \sim \mathrm{unif}\{\pm 1\}$$

on a Regular Random Graph.

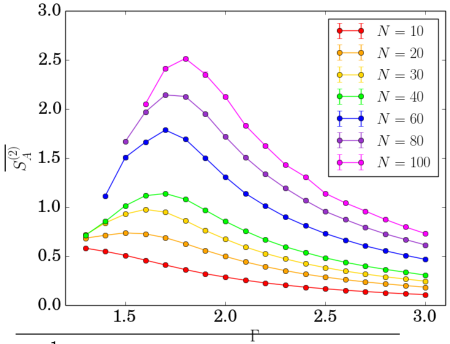
Why is it interesting?

- D-Wave machine's native optimization problem.
- Expander structure should give volume entanglement.
- It is known to enter a glassy phase at small Γ
- Bethe lattice is the thermodynamic limit of RRG.

Goal: compute the (disordered-averaged) Rényi 2 entanglement entropy of the ground state on the $T = 0$ line

$$S_A^{(2)} = -\log \mathrm{Tr}(\rho_A^2)$$

Methods: Quantum Monte Carlo (PIMC replica approach[1])



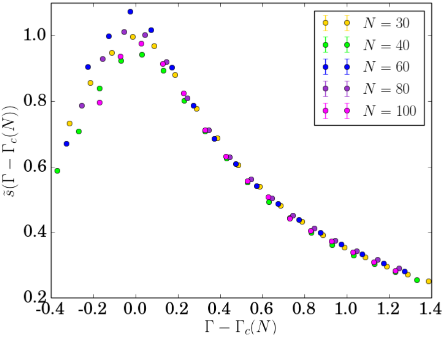
1. all curves attain a peak
2. value at the peak grows with N
3. position of the peak shifts with N

[1] Humeniuk, Roscilde, Phys. Rev. B **86**,235116 (2012)

Numerical Results: Entanglement

Finite-Size Scaling Ansatz:

$$\begin{aligned} S(N,\Gamma) &= (aN+b)s(\Gamma-\Gamma_c(N)) \\ \Gamma_c(N) &= \Gamma_c + \Delta\Gamma/N \end{aligned}$$



- $S_A^{(2)} \propto N$ for all Γ
- peak at $\Gamma_c \approx 1.84$

Quantum version of the Fisher Information:

- It's a measure of the distiguishability of ρ from $e^{-i\theta\hat{O}}\rho e^{i\theta\hat{O}}$
- can be used to derive multipartite entanglement lower bounds

$$F[\rho;\hat{O}]/N \geq k \Rightarrow \rho \text{ is } k\text{-party entangled}$$

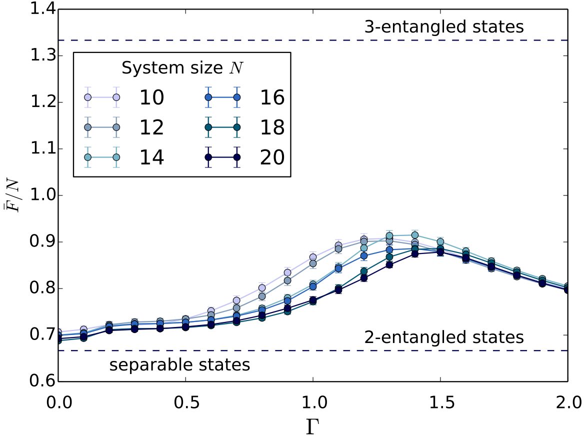
- For a pure state $|\psi\rangle$:

$$F[\psi;\hat{O}] = 4(\langle\psi|\hat{O}\hat{O}|\psi\rangle - \langle\psi|\hat{O}|\psi\rangle^2) = 4\,\mathrm{Var}(\hat{\mathrm{O}})$$

- We compute a "total spin" Q.F.I. (average of the magnetization along axes x, y, z) of the ground state, *i.e.* we fix an *a priori* reasonable choice for $\hat{O}$.

Numerical Results: Quantum Fisher Information

$$\overline{F}/N \to O(1)$$



- **Facts:**
  - Volume-law entanglement ($S_A^{(2)} \propto N$)
  - Constant multipartite entanglement ($k = 2$)
- **Interpretation:** G.S. is made up of pairs of entangled spins. Volume entanglement due to the expander properties of the RRG.

This is good news for D-Wave: no need to create and maintain globally-entangled states!

To be continued...

Further questions:

- Entanglement peaks are associated to critical points of QPT. Is our Rényi peak (*i.e.* critical point) the same as the critical point of the glassy transition?
- Might I interest you in a mean-field (quasiparticle) theory from perturbation series in J that tries to explain the numerics?
- Is there a MBL phase transition?

Look for me at the poster presentation!