Quantum annealing speedup over simulated annealing on random Ising chains

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Model – Ising chain

$$\boxed{H_{\mathrm{cl}} = -\sum_{j=1}^{L} J_j \sigma_j \sigma_{j+1}}$$

CLASSICAL CASE

$$\frac{\partial P(\sigma,t)}{\partial t} = \sum_j W_{\sigma,\overline{\sigma}^j} P(\overline{\sigma}^j,t) - \sum_j W_{\overline{\sigma}^j,\sigma} P(\sigma,t)$$ Glauber M.E.

- Symmetrization of transition matrix W
- Heat-bath choice for W

$$-\frac{\partial}{\partial t}|\psi(t)\rangle = \widehat{H}_{\mathrm{SA}}(t)|\psi(t)\rangle$$ Imaginary time Schrödinger equation

$$\widehat{H}_{\mathrm{SA}}(t) = -\sum_j \Gamma_j^{(0)} \hat{\sigma}_j^x + \sum_j \Gamma_j^{(2)} \hat{\sigma}_{j-1}^z \hat{\sigma}_j^x \hat{\sigma}_{j+1}^z - \sum_j \Gamma_j^{(1)} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{\alpha}{2} L$$

Model – Ising chain

$$\boxed{H_{\rm cl} = -\sum_{j=1}^{L} J_j \sigma_j \sigma_{j+1}}$$

QUANTUM CASE

$$\widehat{H}_{\rm QA}(t) = -\sum_j J_j \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z - \Gamma(t) \sum_j \hat{\sigma}_j^x$$

Transverse field Ising chain

$$\xi \frac{\partial}{\partial t} |\psi(t)\rangle = \widehat{H}_{\rm QA}(t) |\psi(t)\rangle \qquad \boxed{\begin{array}{ll} \xi = i & (\mathrm{RT}) \\ \xi = -1 & (\mathrm{IT}) \end{array}}$$

Real time and imaginary time

Schrödinger equation

Methods – Fermionization

$\widehat{H}_{\mathrm{SA}}(t)$ $\widehat{H}_{\mathrm{QA}}(t)$

Jordan-Wigner transformation

$$\widehat{H}(t) = \begin{pmatrix} \hat{\mathbf{c}}^\dagger & \hat{\mathbf{c}} \end{pmatrix} \begin{pmatrix} \mathbf{A}(t) & \mathbf{B}(t) \\ -\mathbf{B}(t) & -\mathbf{A}(t) \end{pmatrix} \begin{pmatrix} \hat{\mathbf{c}} \\ \hat{\mathbf{c}}^\dagger \end{pmatrix}$$

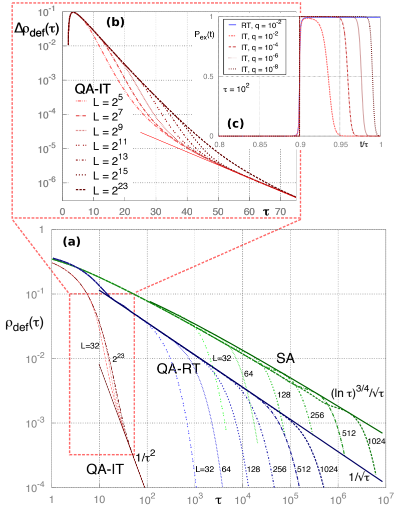
<u>BCS form</u>

$$|\psi(t)\rangle = \mathcal{N}(t)\ \exp\left(\frac{1}{2}\sum_{j_1 j_2} \mathbf{Z}_{j_1 j_2}(t)\hat{c}^\dagger_{j_1}\hat{c}^\dagger_{j_2}\right)|0\rangle$$

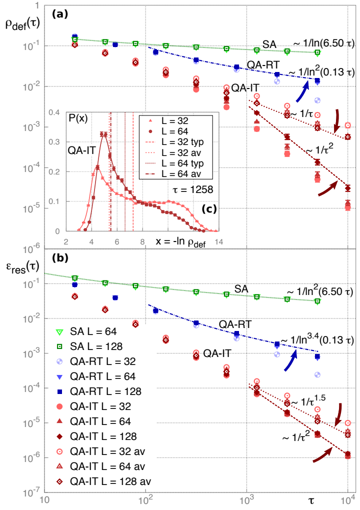
$$\boxed{\xi\,\dot{\mathbf{Z}} = 2\Big(\mathbf{A}\cdot\mathbf{Z} + \mathbf{Z}\cdot\mathbf{A} + \mathbf{B} + \mathbf{Z}\cdot\mathbf{B}\cdot\mathbf{Z}\Big)}$$

Results – Density of defects and residual energy

Ordered case



Disordered case



Thank you for your attention