Dynamical generation of decoherence: Universal scaling of decoherence factors

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Acknowledgement: Tanay Nag, IIT Kanpur, Kanpur
Dr. Shraddha Sharma, IIT Kanpur, Kanpur
Dr. Uma Divakaran, IIT PKD, Palakad
Dr. Victor Mukerjee, Weismann Inst., Israel
Prof. Sei Suzuki, Saitama , Japan

ICTP, 25th August, 2016

V. Mukherjee, S. Sharma and A. Dutta, Phys. Rev. B **86**, 020301 (R) (2012).
T, Nag, U. Divakaran and A. Dutta, Phys. Rev. B **86**, 020401(R) (2012)
S. Suzuki, T. Nag and A. Dutta, Phys. Rev. A **93**, 012112 (2016).

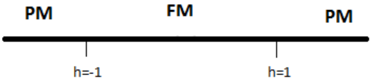
Outline of the talk

- introduction to models
- Slow quenching dynamics across Quantum critical points:

Defect in the final state: Kibble-Zurek Scaling

- Central Spin model and decoherence of the qubit.
- Driven environment and dynamics of decoherence
- Is there a universal scaling of the decoherence factor?
- Ground state quantum fidelity and finite size scaling
- Universal scaling of the decoherence factor

Quantum Phase Transitions: Transverse Ising Chain



$$H = -\sum_{<ij>} \sigma_i^x \sigma_{i+1}^x - h \sum_i \sigma_i^z$$

For $h > 1$, $\langle \sigma_i^x \rangle = 0$; Paramagnetic

For $h < 1$; $\langle \sigma_i^x \rangle \neq 0$; Ferromagnetic

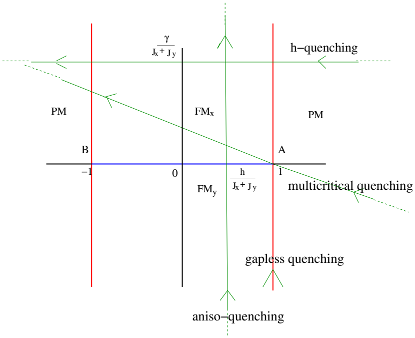
- Quantum critical point $\lambda = |h - 1| = 0$
- Diverging length Scale: $\xi \sim \lambda^{-\nu}$
- Diverging time Scale: $\xi_\tau \sim \xi^z$

Dutta, Aeppli, Chakrabarti, Divakaran, Rosenbaum and Sen, CUP (2015); Suzuki, Innoe and Chakrabarti, Springer (2013).

Transverse XY chain

$$H^{\mathrm{XY}} = -\sum_{i=1}^{N} \left[J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + h\sigma_i^z \right] \quad (1)$$

We shall set $J_x + J_y = 1$ $J_x - J_y = \gamma$.



Quenching across quantum critical point and the defect density

Change a parameter $\lambda(t) = t/\tau$ across the QCP at $\lambda = 0$

The defect density scales as $n \sim \frac{1}{\tau^{\nu d/(\nu z+1)}}$

$h(t) = 1 - t/\tau$; Cross QCPs with $\nu = z = 1 \longrightarrow n \sim \tau^{-1/2}$

Zurek, Dorner and Zoller, Phys. Rev. Lett. **95**, 1057 (2005); Polkovnikov, Phys. Rev. B **72**, 161201 (R), (2005)

Dziarmaga, Phys. Rev. Lett. **95**, 245701 (2005).); Damski, Phys. Rev. Lett. **95**, 035701 (2005).

Kolodrubetz, Clark, Huse, Phys. Rev. Lett. **109**, 015701 (2012), Chandran, Erez, Gubser and Sondhi, Phys. Rev. B **86**, 064304(2012).

The scaling is not conventional when quenched through

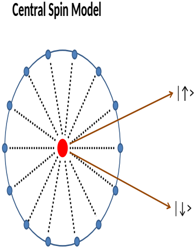
- The gapless phase: $n \sim \frac{1}{\tau^{1/3}}$
- The multicritical point: $n \sim \frac{1}{\tau^{1/6}}$

Mukherjee, Divakaran, Dutta, Sen, Phys. Rev. B (2007); Divakaran, Dutta and Sen, Phys. Rev. B (2008)

Pellegrini, Montangero, Santoro, Fazio, Phys. Rev. B **77** 140404 (2008); Caneva, Fazio, Santoro, Phys. Rev. B **76**, 144427 (2007)

Polkovnikov, *et al*, RMP (2011); Dziarmaga, Adv. in. Phys. (2011); Dutta *et al*, CUP (2015).

The central spin model and decoherence of a qubit



- A qubit coupled to a quantum critical many body system
- "Qubit" $\rightarrow$ a single Spin-1/2
- Environment $\rightarrow$ Quantum XY Spin chain
- A global coupling
- LE: Loss of phase information of the Qubit close to the QCP.

The Central Spin model

- A central spin globally coupled to an environment.
- We choose the environment to be Transverse XY spin chain

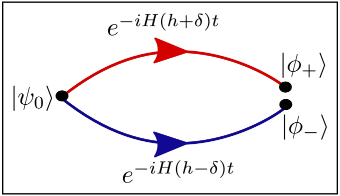
$$H = -J_x \sum_i \sigma_i^x \sigma_{i+1}^x - J_y \sum_i \sigma_i^y \sigma_{i+1}^y - h \sum_i \sigma_i^z$$

- and a global coupling $-\delta \sum_i \sigma_i^z \sigma_S^z$
- Qubit State: $|\phi_S(t=0)\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$
- The environment is in the ground state $|\phi_E(t=0)\rangle = |\phi_g\rangle$
- Composite initial wave function:

$$|\psi(t=0)\rangle = |\phi_S(t=0)\rangle \otimes |\phi_g\rangle$$

Quan *et al*, Phys. Rev. Lett. **96**, 140604 (2006).

Coupling and Evolution of the environmental spin chain



- At a later time t, the composite wave function is given by $|\psi(t)\rangle = c_1|\uparrow\rangle \otimes |\phi_+\rangle + c_2|\downarrow\rangle \otimes |\phi_-\rangle$.

$|\phi_\pm\rangle$ are the wavefunctions evolving with the environment Hamiltonian $H_E(h \pm \delta)$ given by the Schrödinger equation

$$i\partial/\partial t|\phi_\pm\rangle = H[h \pm \delta]|\phi_\pm\rangle.$$

- The coupling δ essentially provides two channels of evolution of the environmental wave function with the transverse field $h + \delta$ and $h - \delta$.

What happens to the central spin?

The reduced density matrix:

$$\rho_S(t) = \begin{pmatrix} |c_1|^2 & c_1 c_2^* d^*(t) \\ c_1^* c_2 d(t) & |c_2|^2 \end{pmatrix}.$$

- The decoherence factor (Loschmidt Echo)

$$D(t) = d^*(t)d(t) = |\langle \phi_+(t)|\phi_-(t)\rangle|^2$$

Overlap between two states evolved from the same initial state with different Hamiltonian

- $D(t) = 1$, pure state. $D(t) = 0$ Complete Mixing
- Coupling to the environment may lead to Complete loss of coherence
- Decay of Loschmidt echo

T. Gorin, T. Prosen, T. H. Seligman, M. Znidaric, Phys. Rep. **435**, 33-156 (2006);

- Enhanced decay close to a QCP

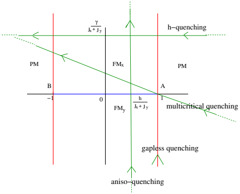
Quan *et al*, Phys. Rev. Lett. **96**, 140604 (2006).

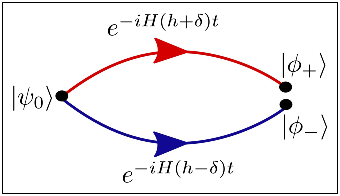
Loschmidt echo

various applications in quenched closed quantum systems

- Work Statistics (Gambassi and Silva)
- Dynamical Phase transitions (Heyl, Polkovnikov and Kehrein)
- Emergent thermodynamics is closed quantum systems (Dorner *et al*, Deffner and Lutz)
- $\cdots$

Ramped environment: dynamic generation of decoherence





Assume $h(t) = 1 - t/\tau$, driven spin chain environment

$$H_k^{\pm}(t) = 2\begin{pmatrix} h(t) \pm \delta + \cos k & \gamma \sin k \\ \gamma \sin k & -(h(t) \pm \delta + \cos k) \end{pmatrix}.$$

B. Damski, Quan and Zurek, Phys. Rev. A **83**, 062104 (2011).

The decoherence factor $D(t)$

$$|\phi^{\pm}(t)\rangle \;\; = \;\; \prod_k |\phi_k^{\pm}(t)\rangle = \prod_{k>0} \left[u_k^{\pm}(t)|0\rangle + v_k^{\pm}(t)|k,-k\rangle \right].$$

$$i\partial/\partial t \left(u_k^{\pm}(t), v_k^{\pm}(t) \right)^T = H_k^{\pm}(t) \left(u_k^{\pm}(t), v_k^{\pm}(t) \right)^T$$

with $\prod_k F_k(t) = \prod_k |\langle \phi_k^{+}(h(t)+\delta)|\phi_k^{-}(h(t)-\delta)\rangle|^2$,

$$D(t) = \exp\left[\frac{N}{2\pi} \int_0^{\pi} dk \;\; \ln F_k \right] \qquad (2)$$

where F_k can be written in terms of $u_k^{\pm}$ and $v_k^{\pm}$.

The question we address:

We assume $\delta \to 0$ and and work within the appropriate range of time;
λ is the driving parameter.

One finds: Far away from the critical point $\lambda = 0$

$$\ln D \sim (-t^2 L^d \delta^2 f(\tau))$$

What is the scaling of this function $f(\tau)$?

- Is that identical to the scaling of the defect density?

Not necessarily! Even for this integrable system!

How to Calculate $D(t)$?...

- Use the Landau-Zener transition formula:

$$p_k = |u_k|^2 = \exp(-2\pi\tau\gamma^2 \sin^2 k)$$

$$\begin{aligned} F_k(t) &= 1 - 4p_k(1-p_k)\sin^2(\Delta t) \\ &= 1 - 4\left[e^{-2\pi\tau\gamma^2 k'^2} - e^{-4\pi\tau\gamma^2 k'^2}\right]\sin^2(4\delta t) \end{aligned} \tag{3}$$

$\sin k$ has been expanded near the critical modes $k = \pi$, with $k' = \pi - k$ and we have taken the limit $\delta \to 0$.

B. Damski, Quan and Zurek, Phys. Rev. A **83**, 062104 (2011); Pollmann, Mukherjee, Green and Moore, Phys. Rev. E **81**, 020101(R) (2010)

How to calculate $D(t)$?

Assume $\delta \to 0$

$$\begin{aligned} D(t) &= \exp \frac{N}{2\pi} \int_0^\infty dk \\ &\quad \ln \left[1 - \left(e^{-2\pi\tau\gamma^2 k'^2} - e^{-4\pi\tau\gamma^2 k'^2}\right) 64\delta^2 t^2\right] \end{aligned}$$

Finally D is given by

$$D(t) \sim \exp\{-8(\sqrt{2}-1)N\delta^2 t^2/(\gamma\pi\sqrt{\tau})\}.$$

- $\ln D(t) \sim \tau^{-1/2}$

The same scaling as the defect density

Quenching through a critical line

Change $\gamma = t/\tau$ with $h = 1$. Quenched through the MCP

Modified CSM with interaction:

$$H_{SE} = -(\delta/2)\sum_i (\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y)\sigma_S^z$$

The coupling δ provides two channels of the temporal evolution of the environmental ground state with anisotropy $\gamma + \delta$ and $\gamma - \delta$. The appropriate two-level Hamiltonain

- The defect density in the final state $n \sim \tau^{-1/3*}$

U. Divakaran *et al*, Phys. Rev. B **78**, 144301 (2008).

$$D(t) \sim \exp\{-2^{14/3} N \delta^2 t^2/(3\pi\tau)\}.$$

- Scaling of ln $D(\sim \tau^{-1})$ is completely different!!

T, Nag, U. Divakaran and A. Dutta, Phys. Rev. B **86**, 020401(R) (2012).

Question we ask?

Is there universal scaling?

Recall the scaling of the fidelity susceptibility and finite size scaling

What happens in non-integrable models?

The ground state Quantum Fidelity

We consider the Hamiltonian

$$H(\lambda) = H_0 + \lambda H_I; \quad H(\lambda)|\psi_0(\lambda)\rangle = E_0|\psi_0(\lambda)\rangle$$

where $|\psi_0(\lambda)$ is the ground state wave function.

- λ is the driving term. The QCP is at $\lambda = 0$.
- The quantum fidelity: modulus of the overlap between two ground state corresponding to parameters λ and $\lambda + \delta$

$$F(\lambda, \delta) = |\langle\psi_0(\lambda)|\psi_0(\lambda + \delta)\rangle|$$

- Indicator of Quantum Criticality: Shows a dip close to it

Finite size scaling

Recall finite size scaling:

Close to the critical point: $L \ll \xi (\sim \lambda^{-\nu})$ Scaling with L

Away from the critical point: $L \gg \xi (\sim \lambda^{-\nu})$ Scaling with ξ

Smaller length scale dictates the scaling

Thermal phase transition:

Finite size scaling of the magnetic susceptibility

$$\chi(t, L) \sim |t|^{-\gamma} f\left(\frac{\xi}{L}\right); \qquad t \sim (T - T_c)$$

Away from the critical point : $f(x) \to \mathrm{const}$

$$\chi(t, L) \sim |t|^{-\gamma} \sim \xi^{\gamma/\nu}$$

Close to it: $\quad f(x) \to x^{-\gamma/\nu} \qquad \chi(t, L) \sim L^{\gamma/\nu}$

Fidelity susceptibility Approach

$\delta \to 0$ and small L

$$F(\lambda, \delta) = 1 - \frac{1}{2}\delta^2 L^d \chi_F(\lambda) + \cdots$$

Fidelity susceptibility $\chi_F = -\frac{2}{L^d} \lim_{\delta \to 0}(\ln F/\delta^2) = -\frac{1}{L^d}\partial^2 F/\partial\delta^2$

$$\delta^2 L^d \chi_F(\lambda) << 1$$

Enlist the length scales

The scales of the problem: $L, \xi \sim \lambda^{-\nu}, \delta^{-\nu}$

δ sets a length scale in the problem: $\delta^{-\nu}$

Set $\lambda = 0$ (at the QCP; more precisely $\xi \gg L$)

- $L \ll \delta^{-\nu}$; fidelity susceptibility approach is meaningful
- $L \gg \delta^{-\nu}$; fidelity susceptibility approach is NOT meaningful

$L \gg \delta^{-\nu}$ Fidelity in the thermodynamic limit

L is the largest length scale of the problem and δ is finite.

Rams and Damski,Phys. Rev. Lett. **106**, 055701 (2010)

Scaling of χ_F

Scaling of the fidelity susceptibility $\delta^{-\nu}$ is the largest

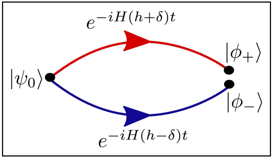
$$F = 1 - \frac{1}{2} L^d \delta^2 \chi_F + \cdots$$

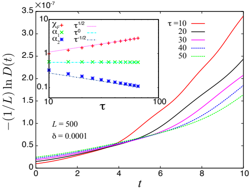
F is dimensionless: $\delta^{-\nu} \sim L$

- $\xi(= \lambda^{-\nu}) \gg L$; $\chi_F \sim L^{2/\nu - d}$ Close to the QCP
- $\xi(= \lambda^{-\nu}) \ll L$; $\chi_F \sim \lambda^{\nu d - 2}$ Away from the QCP.

Venuti and Zanardi Phys. Rev. Lett. **99**, 095701 (2007). De Grandi, Gritsev and Polkovnikov, Phys. Rev. **81**, 012303 (2010)

Universal scaling of the DF: early time limit $t = 0+$





• ESS Hamiltonian is quenched $\lambda = t/\tau$, with t starting from a large negative value $\lambda = h - 1$, t=0 is the QCP

• two channels of evolutions of the initial ground state of the ESS dictated by two Hamiltonians with parameters $\lambda + \delta$ and $\lambda - \delta$

In the limit, small δ and t; Not Gaussian decay

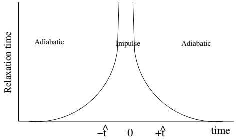
$$\frac{1}{L^d}\ln D(t) \approx -\left(\chi_F(\tau) + \alpha_1(\tau)t + \frac{1}{2}\alpha_2(\tau)t^2 + \cdots\right)\delta^2,$$

$$\chi_F(\tau) = -\frac{1}{\delta^2 L^d}\ln D(0), \quad \alpha_m(\tau) = -\frac{1}{\delta^2 L^d}\frac{d^m}{dt^m}(\ln D(t))\,|_{t=0},$$

How to arrive at the universal scaling? Dimensional Analysis

What is the characteristic length scale?

- Hamiltonian $H(\lambda) \Rightarrow \lambda = |h-1| = 0$ Linear driving $\lambda = t/\tau$.



How to find out $\hat{t}$? At $t = \hat{t}$,

relaxation time $\sim$ rate of driving $\implies \frac{1}{\lambda^{\nu z}} \sim \frac{\lambda}{\dot{\lambda}}$

- $\hat{t} \sim \tau^{\nu z/(\nu z+1)} \implies \hat{L} \sim \tau^{\nu/(\nu z+1)}$

$\hat{L}$ is the characteristic length scale of the problem

$$n \sim \frac{1}{\hat{L}^d} \sim \tau^{-\nu d/(\nu z+1)}.$$

Dimensional Analysis using $\hat{L}$

$\ln D(t)$ must be dimensionless

$$\frac{1}{L^d}\ln D(t) \approx -\left(\chi_F(\tau) + \alpha_1(\tau)t + \frac{1}{2}\alpha_2(\tau)t^2\right)\delta^2,$$

$t \sim \hat{L}^z$ and $\delta\ (\equiv \lambda) \sim \hat{L}^{-1/\nu}$

$$\chi_F(\tau) \sim \tau^{(2-d\nu)/(z\nu+1)}, \alpha_m(\tau) \sim \tau^{(2-d\nu-mz\nu)/(z\nu+1)} \quad (m = 1, 2)$$

Non-linear quenching: $\lambda(t) = -|\frac{t}{\tau}|^r \mathrm{sgn}(t)$ $\hat{L} \sim \tau^{r\nu/(r\nu z+1)}$

$$\chi_F(\tau) \sim \tau^{r(2-d\nu)/(rz\nu+1)}, \quad \alpha_m(\tau) \sim \tau^{r(2-d\nu-mz\nu)/(rz\nu+1)}$$

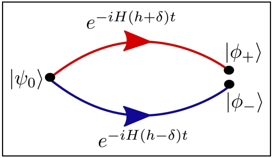
h-quenching: $d = 1, \nu = 1$ and $z = 1$

$$\chi_F(\tau) \sim \tau^{1/2}, \quad \alpha_1(\tau) \sim \tau^0, \quad \alpha_2(\tau) \sim \tau^{-1/2}$$
$$\chi_F(\tau) \sim \tau^{r/(r+1)}, \quad \alpha_1(\tau) \sim \tau^0, \quad \alpha_2(\tau) \sim \tau^{-r/(r+1)}$$

Suzuki, Nag and Dutta, Phys. Rev. A (2012).

$\chi_F(\tau)$: the generalised fidelity susceptibility

Not the ground state fidelity susceptibility



$\lambda = -t/\tau$ which stops at the critical point at $\lambda = t = 0$.

$$\ln D(\lambda = t = 0) = 2\ln(|\langle\phi_+(t=0)|\phi_-(t=0)\rangle|);$$

$$\ln(|\langle\phi_+(t=0)|\phi_-(t=0)\rangle|) \sim -\delta^2 L^d \chi_F(\tau).$$

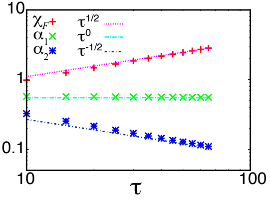
$$L^{2/\nu - d} \quad \text{length scale} \quad \hat{L} \sim \tau^{\nu/(\nu z+1)}$$

$$\chi_F(\tau) \sim \tau^{(2-\nu d)/(\nu z+1)}$$

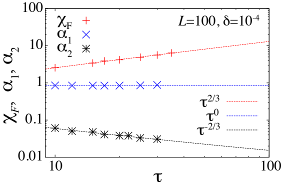
linear and non-linear quenching: numerical results

For linear quenching integrable models: exact analytical results

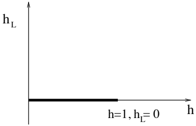
- $h - 1 = -\frac{t}{\tau}$



- Non-linear quenching: $h(t) - 1 = -|\frac{t}{\tau}|^r \mathrm{sign}(t)$; $r = 2$



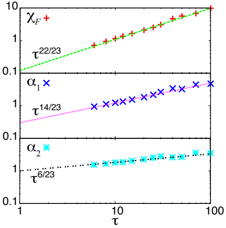
Beyond integrability

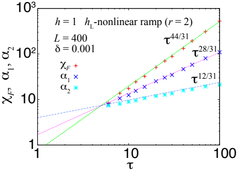


$$H_E^{\mathrm{L}} = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_L \sum_i \sigma_i^z - h \sum_i \sigma_i^x$$

$h = 1$ integrable critical point: apply $h_L \implies \nu = 8/15, z = 1$

$h_L = -t/\tau$ and Two channels: $h_L + \delta$ and $h_L - \delta$





late time limit: far away from the QCP

For integrable models:

$$\ln D \sim -t^2\delta^2 L^d \tilde{\alpha}_2(\tau)$$

- $\tilde{\alpha}_2$ has the same scaling as α_2
- linear quenching: $\tilde{\alpha}_2(\tau) \sim \tau^{(2-d\nu-2z\nu)/(z\nu+1)}$
- Non-linear quenching: $\tilde{\alpha}_2(\tau) \sim \tau^{r(2-d\nu-2z\nu)/(rz\nu+1)}$
- $\tilde{\alpha}_2 \sim$ defect density iff $\nu z = 1$

$$r = 1;\ h(t) = 1 - t/\tau;\ \tilde{\alpha}_2 \sim \tau^{-1/2}$$

$$r = 1,\ h = 1,\ \gamma = t/\tau,\ \tilde{\alpha}_2 \sim \tau^{-1}$$

Concluding comments

- There is a universal scaling of decoherence factors
- Follows from simple dimensional analysis with $\hat{L}$
- In the late time limit: scaling is identical to the defect when $\nu z = 1$
- Beyond the central spin model?

How to Calculate $D(t)$?

Use the integrable two-level nature of the environmental Hamiltonian.

Far away from the QCP ($|h(t)| \gg 1$ $(t \to +\infty)$)

$$|\phi_k(h+\delta)\rangle = u_k|0\rangle + v_k e^{-i\Delta^+ t}|k,-k\rangle$$

$$|\phi_k(h-\delta)\rangle = u_k|0\rangle + e^{-i\Delta^- t} v_k|k,-k\rangle$$

$$\Delta^+ = 4\sqrt{(h+\delta+1)^2 + \gamma^2 \sin k^2}$$
$$\Delta^- = 4\sqrt{(h-\delta+1)^2 + \gamma^2 \sin k^2},$$

are the energy of two excitations in $|k,-k\rangle$ when the transverse field is equal to $h+\delta$ and $h-\delta$, respectively.

Excitations occur only in the vicinity of QCPs

F. Pollman *et al*, Phys. Rev. E **81** 020101 (**R**) (2010).

How to Calculate $D(t)$?...

How does one know u_k and v_k?

- Use the Landau-Zener transition formula:

$p_k = |u_k|^2 = \exp(-2\pi\tau\gamma^2 \sin^2 k)$

$$\begin{aligned} F_k(t) &= |\langle \phi_k(h(t)+\delta)|\phi_k(h(t)-\delta)\rangle|^2 \\ &= \left| |u_k|^2 + |v_k|^2 e^{-i(\Delta^+ - \Delta^-)t} \right|^2, \end{aligned} \qquad (4)$$

In the vicinity of the quantum critical point at $h = 1$
$\Delta = (\Delta^+ - \Delta^-)/2$,

$$\begin{aligned} F_k(t) &= 1 - 4p_k(1-p_k)\sin^2(\Delta t) \\ &= 1 - 4\left[e^{-2\pi\tau\gamma^2 k'^2} - e^{-4\pi\tau\gamma^2 k'^2}\right]\sin^2(4\delta t) \end{aligned} \qquad (5)$$

$\sin k$ has been expanded near the critical modes $k = \pi$, with $k' = \pi - k$ and we have taken the limit $\delta \to 0$.

How to calculate $D(t)$?

Assume $\delta \to 0$

$$\begin{aligned} D(t) &= \exp \frac{N}{2\pi} \int_0^\infty dk \\ &\quad \ln \left[1 - \left(e^{-2\pi\tau\gamma^2 k'^2} - e^{-4\pi\tau\gamma^2 k'^2}\right) 64\delta^2 t^2\right] \end{aligned}$$

Finally D is given by

$$D(t) \sim \exp\{-8(\sqrt{2}-1)N\delta^2 t^2/(\gamma\pi\sqrt{\tau})\}.$$

- $\ln D_{non-ad} \sim \tau^{-1/2}$

The same scaling as the defect density

Non-linear Quenching

Non-linear Quenching: $h = 1 - \mathrm{sgn(t)(t/\tau)}^{\alpha}$

The scaling form $p_k = |u_k|^2 = G(k^2\tau^{2\alpha/(\alpha+1)})$

$$D(t) = \exp(-CN\delta^2 t^2/\tau^{\alpha/(\alpha+1)})$$

• $\ln D(t) \sim \tau^{-\alpha/(\alpha+1)}$

Quenching through a MCP

$$\ln D_{non-ad}(t) \sim (t - J_y\tau)^2/\tau^{1/6} \sim (J_x - J_y)\tau^{11/6}$$

• Quenching through Isolated critical points: $\ln D_{non-ad}(\tau) \sim n$

Is this scenario true in general?

Quenching through a critical line

Change $\gamma = t/\tau$ with $h = 1$. Quenched through the MCP

Modified CSM with interaction:

$$H_{SE} = -(\delta/2)\sum_i (\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y)\sigma_S^z$$

The coupling δ provides two channels of the temporal evolution of the environmental ground state with anisotropy $\gamma + \delta$ and $\gamma - \delta$. The appropriate two-level Hamiltonain

$$H_k^{\pm}(t) = 2\begin{pmatrix} (\gamma \pm \delta)\sin k & h + \cos k \\ h + \cos k & -(\gamma \pm \delta)\sin k \end{pmatrix}.$$

- The defect density in the final state $n \sim \tau^{-1/3*}$

Does that mean ln $D_{non-ad} \sim \tau^{-1/3}$?

* U. Divakaran *et al*, Phys. Rev. B **78**, 144301 (2008).

A completely different Scaling

$$F_k = 1 - 4(e^{-\pi\tau k^3/2} - e^{-\pi\tau k^3})\sin^2(4\delta kt)$$

- An exponential decay:

$$D_{non-ad}(t) \sim \exp\{-2^{14/3}N\delta^2 t^2/(3\pi\tau)\}.$$

- Scaling of ln $D_{non-ad}(\sim \tau^{-1})$ is completely different!!