Geometric and counter-diabatic optimization of quantum annealing protocols

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Outline

1. Perturbative non-adiabatic response for real time and imaginary time dynamics.
2. Imaginary time quantum annealing using QMC (with C. De-Grandi, C.-W. Liu, A. Sandvik)
3. Geometric optimization of fidelity in annealing protocols (with T. Souza and M. Tomka).
4. Counter-diabatic driving: idea, local CD driving - variational formulation, application to many-particle systems (with D. Sels).
5. Machine learning for optimization of quantum annealing (in progress, by M. Bukov and P. Mehta).

What is the moving frame and what is behind these transformations?

$$i\hbar\partial_t|\psi\rangle = H(\vec{\lambda}(t))|\psi\rangle$$

Let us do a unitary transformation to a co-moving frame, diagonalizing the instantaneous Hamiltonian

$$|\Psi\rangle = U^\dagger(\vec{\lambda})|\psi\rangle, \quad U^\dagger H U = \mathrm{diag}(E_1, E_2, \ldots)$$

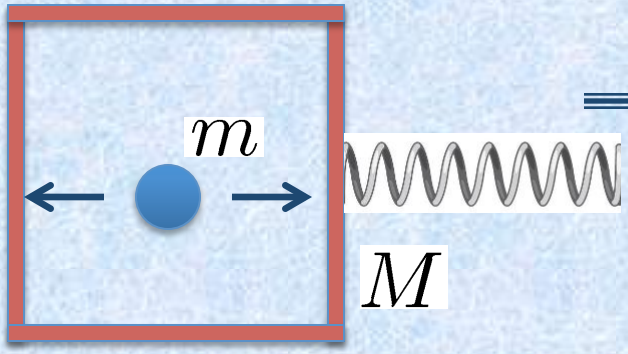
$$i\hbar\partial_t|\Psi\rangle = (U^\dagger H U - i\hbar U^\dagger d_t U)|\psi\rangle = (U^\dagger H U - \dot{\lambda}_a \mathcal{A}_a)|\Psi\rangle$$

$\mathcal{A}_\alpha = i\hbar U^\dagger \partial_{\lambda_\alpha} U, \ \mathcal{A}_\alpha^\dagger = \mathcal{A}_\alpha$ - gauge potential

Gauge potentials are Hamiltonians in parameter space:

$$i\hbar\partial_{\lambda_\alpha}|\psi(\vec{\lambda})\rangle = -\mathcal{A}_\alpha|\psi\rangle, \ \mathcal{A}_\alpha = i\hbar U^\dagger \partial_{\lambda_\alpha} U, \ \mathcal{A}_\alpha^\dagger = \mathcal{A}_\alpha$$

Classical Hamiltonian systems: gauge potentials – generators of canonical transformations.



$$H = \frac{p^2}{2m} + V(x - X_0(t))$$

$$U = \exp\left[-\frac{i}{\hbar} p X_0(t)\right] \qquad \mathcal{A}_{X_0} = i\hbar U^\dagger \partial_{X_0} U = p$$

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = H - \dot{X}_0 p$$

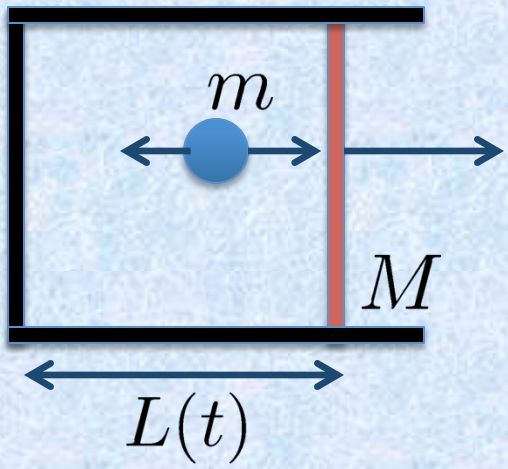
Galilean Transformation

Compute leading correction to the energy due to the Galilean term (consider the ground state)

$$\Delta E_1 = \langle 0| - \dot{X}_0 p|0\rangle = 0$$

$$\Delta E_2 = \dot{X}_0^2 \sum_{n\neq 0} \frac{\langle 0|p|n\rangle\langle n|p|0\rangle}{E_n - E_0} = \dot{X}_0^2 \hbar^2 \sum_{n\neq 0} \frac{\langle n|\partial_X|0\rangle^2}{(E_n - E_0)} = m\dot{X}_0^2 \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{16n^2}{(4n^2-1)^3} = \frac{m\dot{X}_0^2}{2}$$

Recover the mass term as the leading non-adiabatic correction to the energy.



Dilations

Moving frame

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = \frac{p^2}{2mL^2(t)} - \dot{L}\mathcal{D}$$

Dilation operator $\mathcal{D} = \frac{px + xp}{2L}$

Can absorb L^2 into time dilatation: $dt = L^2 d\tau, \quad H \to L^2 H$

Leading non-adiabatic correction.

$$\Delta E_2 = \dot{L}^2 \sum_{m \neq n} \frac{\langle n|\mathcal{D}|m\rangle\langle m|\mathcal{D}|n\rangle}{E_m - E_n} = m\dot{L}^2 \frac{16}{\pi^2} \sum_{m \neq m} \frac{n^2 m^2}{(m^2 - n^2)^3} = \frac{m\dot{X}_0^2}{2}\left(\frac{1}{3} - \frac{1}{2\pi^2 n^2}\right)$$

Recover “quantum” dilatation mass: the classical (massive spring) result plus an additional quantum correction.

Geometric structure of the ground state manifold

Hamiltonian: $\mathcal{H} = \mathcal{H}(\vec{\lambda})$. Ground state wave-function: $\psi_0 = \psi_0(\vec{\lambda})$.

Consider the following change $\vec{\lambda} \to \vec{\lambda} + \delta\vec{\lambda}$

$$||\delta\psi_0||^2 \equiv 1 - |\langle\psi_0(\vec{\lambda})|\psi_0(\vec{\lambda}+\delta\vec{\lambda})\rangle|^2 = \chi_{\alpha\beta} d\lambda_\alpha d\lambda_\beta$$

$\chi_{\alpha\beta}$ - geometric tensor (Provost, Vallee, 1980)

$$\chi_{\alpha\beta} = \langle 0|\overleftarrow{\partial_\alpha}\partial_\beta|0\rangle - \langle 0|\overleftarrow{\partial_\alpha}|0\rangle\langle 0|\partial_\beta|0\rangle = \langle\partial_\alpha\psi_0|\partial_\beta\psi_0\rangle_c;$$

Insert a complete basis of states and observe

$$\chi_{\alpha\beta} = \langle\mathcal{A}_\alpha\mathcal{A}_\beta\rangle_c, \quad \mathcal{A}_\alpha = i\partial_\alpha = iU^\dagger\partial_\alpha U$$

Geometric tensor is the covariance matrix of the gauge potential. Can be measured as a dynamical response.

Berry curvature. Defines the effective magnetic field

$$F_{\alpha\beta} = -i(\chi_{\alpha\beta} - \chi_{\beta\alpha}) = -i\langle 0|[\mathcal{A}_\alpha, \mathcal{A}_\beta]|0\rangle = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

Hall response, topological invariants, Coriolis forces, Lorentz forces,…

Symmetric part of the geometric tensor

$$g_{\alpha\beta} = \frac{1}{2}(\langle \partial_\alpha \psi | \partial_\beta \psi \rangle_c + \langle \partial_\beta \psi | \partial_\alpha \psi \rangle_c) = \frac{1}{2}(\chi_{\alpha\beta} + \chi_{\beta\alpha}) = \frac{1}{2}\langle 0|\mathcal{A}_\alpha \mathcal{A}_\beta + \mathcal{A}_\beta \mathcal{A}_\alpha\rangle_c$$

Metric tensor. Defines the Riemannian metric structure on the manifold of ground states, fidelity susceptibility, Fisher information.

Defines leading non-adiabatic corrections to work fluctuations

$$\delta W^2 = \dot{\lambda}_\alpha g_{\alpha\beta} \dot{\lambda}_\beta$$

g also defines mass renormalization in the classical (high temp.) limit.

$$W = \Delta E = \frac{1}{2}\dot{\lambda}_\alpha \kappa_{\alpha\beta} \dot{\lambda}_\beta, \quad \kappa_{\alpha\beta} = \frac{1}{k_b T} g_{\alpha\beta}$$

Non-adiabatic response in imaginary time (can do Monte-Carlo)

$$\partial_\tau |\psi\rangle = -H(\vec{\lambda}(\tau))|\psi\rangle$$

Instantaneous moving frame

$$\partial_\tau |\Psi\rangle = (-U^\dagger H U - U^\dagger d_\tau U)|\psi\rangle = (-U^\dagger H U + i\dot{\lambda}_a \mathcal{A}_a)|\Psi\rangle$$

Can use similar linear response theory as in real time.

Generic off-diagonal observables

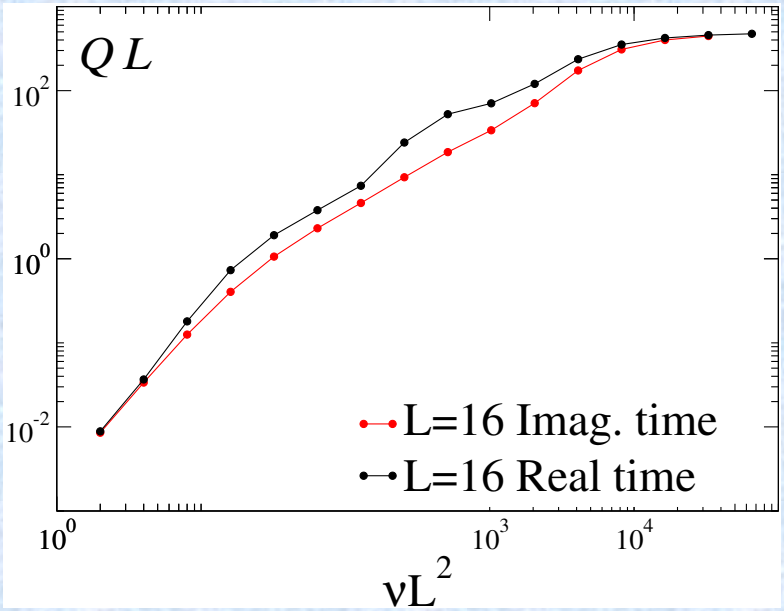
$$\langle -\partial_\alpha H\rangle \approx \langle -\partial_\alpha H\rangle_0 - 2g_{\alpha\beta}\dot{\lambda}_\beta$$

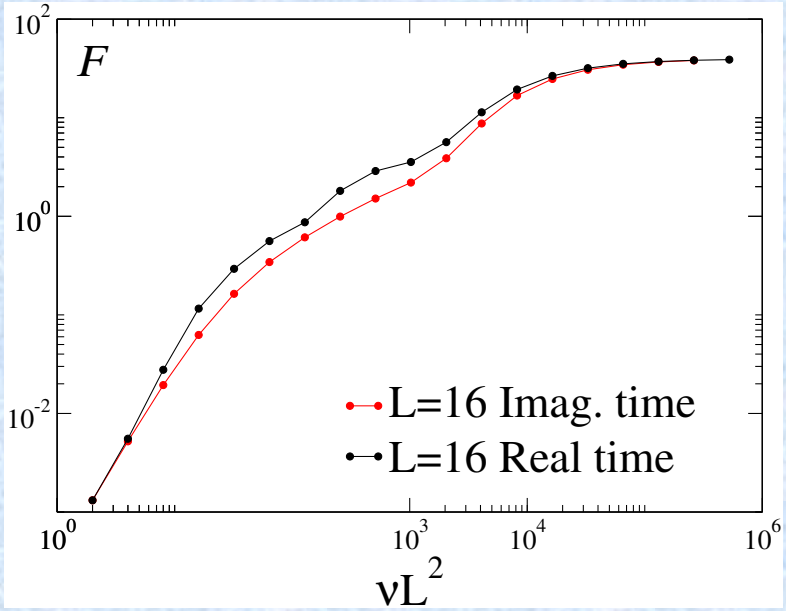
Can easily compute the metric tensor without calculating overlaps

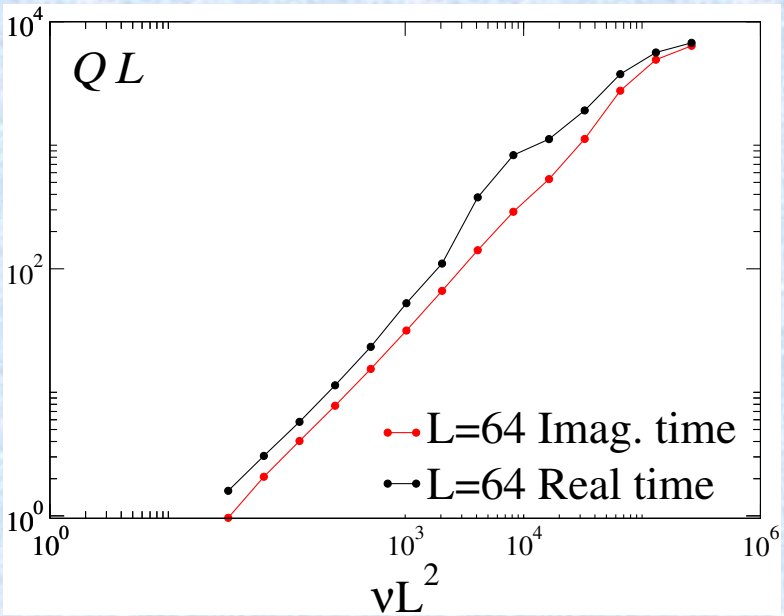
Diagonal observables: same as in real time

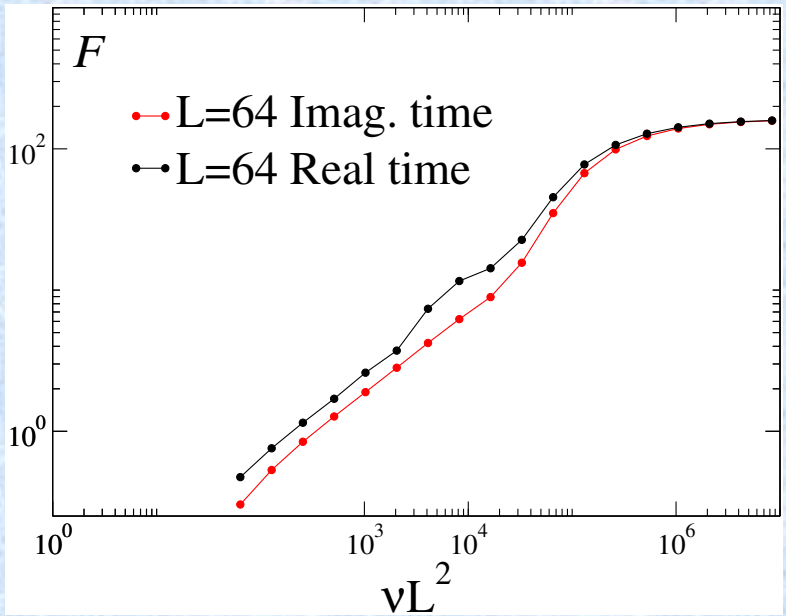
$$\delta W^2 = \dot{\lambda}_\alpha g_{\alpha\beta}\dot{\lambda}_\beta \qquad W = \Delta E = \frac{1}{2}\dot{\lambda}_\alpha \kappa_{\alpha\beta}\dot{\lambda}_\beta$$

Comparison of the excess energy and the log-fidelity for quenching to the critical point for RT and IT dynamics. Asymptotically agree in fast and slow limits.









Application to Quantum Annealing

$$H_0 = \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \sigma_i^z \sigma_j^z, \qquad (\sigma_i^z = \pm 1)$$

Introduce an auxiliary quantum term and slowly anneal it to zero

$$H_1 = h \sum_{i=1}^{N} \sigma_i^x = h \sum_{i=1}^{N} (\sigma_i^+ + \sigma_i^-).$$

$$H = sH_0 + (1-s)H_1 \quad s = vt$$

In the adiabatic limit follow the ground state.

Both thermal (simulated) annealing and quantum annealing have problems in glass phases. Hopes are that quantum annealing can be more efficient.

Application to a random graph model

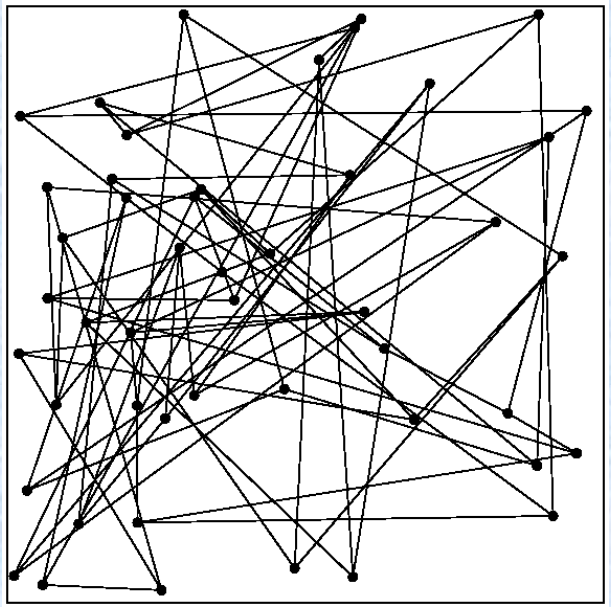


FIG. 1: A typical 3-regular graph with $N = 64$ spins.

Phase transition to a glass phase at finite temperature

$$T_c = -2\ln^{-1}[1 - 2/(1 + \sqrt{2})]$$

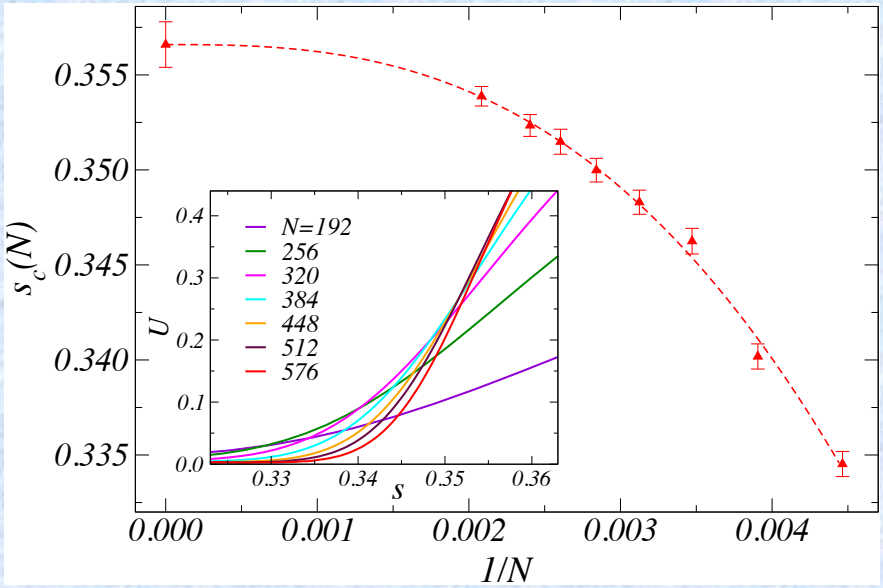
L. Zdeborová and F. Krzakała, Phys. Rev. E 76, 031131 (2007).

Quantum transition to a glass phase at $s_c \approx 0.37$

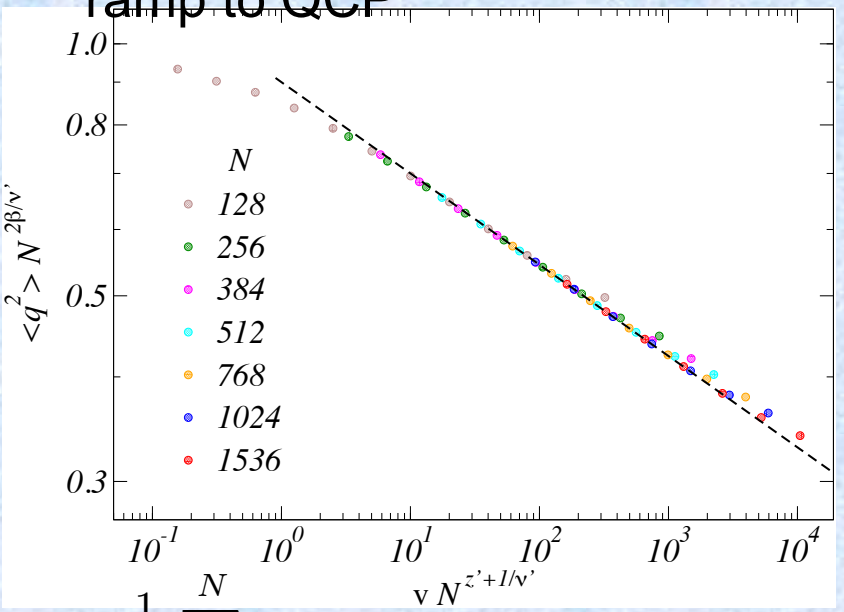
E. Farhi, et. al., Phys. Rev. A 86, 052334 (2012).

Imaginary time quantum annealing vs thermal annealing

Finding Critical Point



Extracting dynamical exponent, ramp to QCP



$$q = \frac{1}{N}\sum_{i=1}^{N} \sigma_i^z(1)\sigma_i^z(2).$$

$$z' + 1/\nu' \approx 1.3$$

Classical annealing

$$z' + 1/\nu' = 1$$

$$\langle q^2 \rangle = N^{2\beta/\nu'} f\left[L(s - s_c)^{\nu'}, N^{z'+1/\nu'} v\right]$$

For quenching to the critical point thermal annealing is more efficient.

Quantum annealing in IT is not quantum simulated annealing.
Opposite results in 1D: T. Zanco G. Santoro, 2016

Geometric approach to annealing

(M. Tomka, T. Souza, A.P. 2016)

Generalization of geometric suppression of dissipation in classical dissipative systems (D. Sivak and G. Crooks 2012)

Idea: maximize instantaneous fidelity along the path (time average of the energy variance, time component of the metric).

$$\mathcal{L}(\vec{\lambda}) = \int_{\vec{\lambda}_i}^{\vec{\lambda}_f} ds = \int_0^{t_f} \sqrt{g_{\mu\nu}\dot{\lambda}^\mu \dot{\lambda}^\nu}\, dt$$

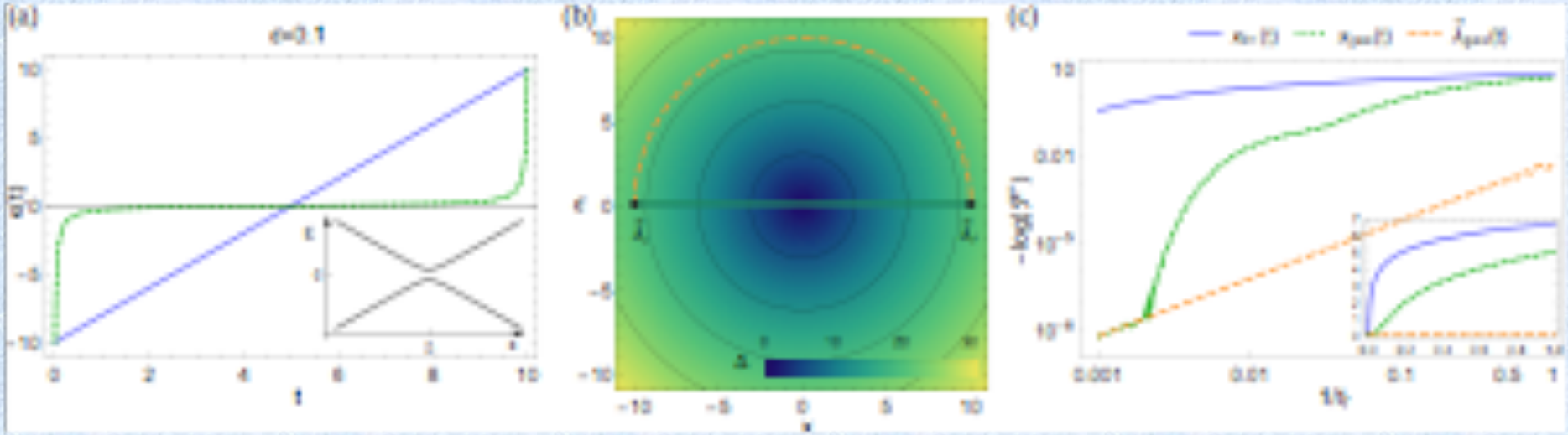
Solution: geodesic protocol defined by the Fubini-Study metric.

$$\ddot{\lambda}^\mu + \Gamma^\mu_{\nu\rho}\dot{\lambda}^\nu \dot{\lambda}^\rho = 0,$$

Avoids small gaps and most relevant perturbations; slows down near minimal gaps (constant energy variance along the path).

Application to LZ problem

Final fidelity vs. annealing time

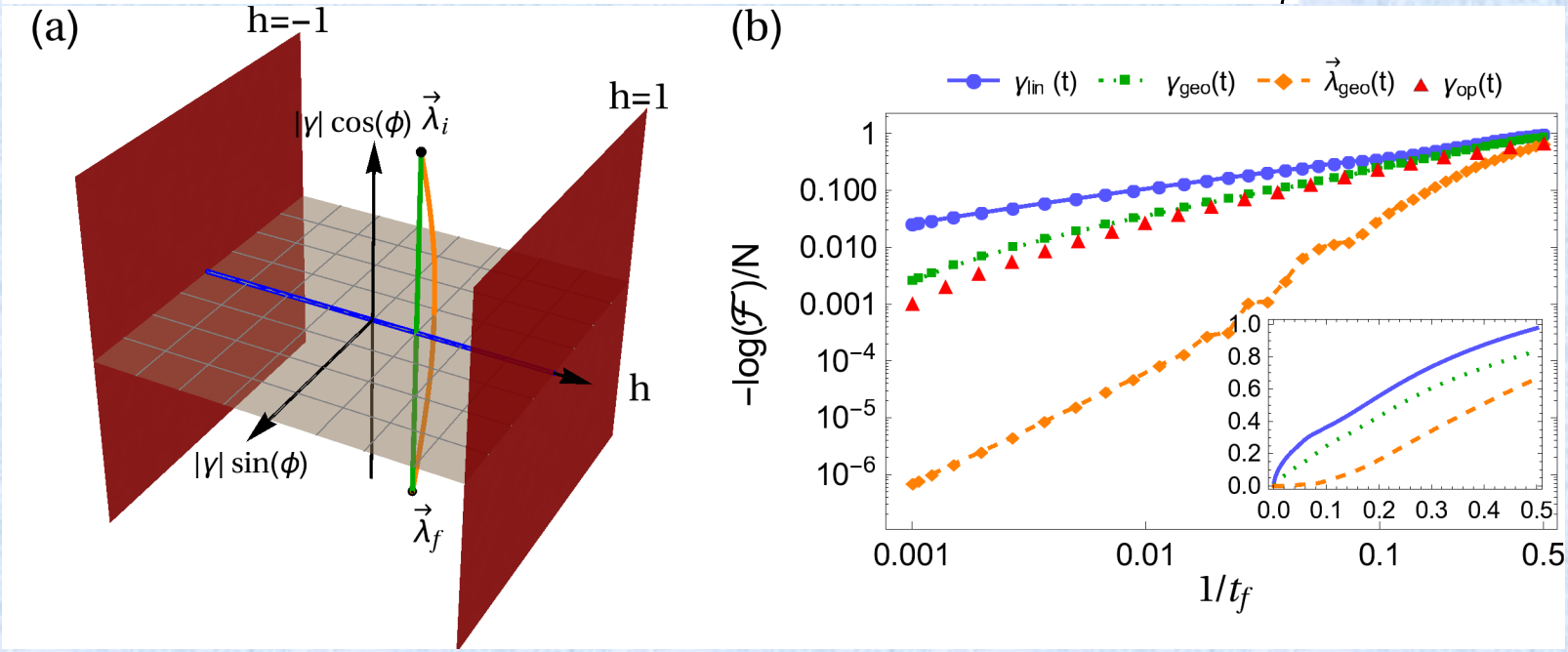


Degenerate geodesic solutions (problem too simple).
Geodesic solutions strongly outperform the naïve protocol.

Metric tensor can be measured (computed numerically) without need to do tomography (diagonalize Hamiltonian).

Application to XY spin chain

$$H_{XY} = -\sum_{j=1}^{N} \left[\frac{1+\gamma}{2} \sigma_j^x \sigma_{j+1}^x + \frac{1-\gamma}{2} \sigma_j^y \sigma_{j+1}^y + h\,\sigma_j^z \right]$$



Final fidelity: (i) naïve protocol, (ii) geodesic protocol, (iii) optimal power law protocol (R. Barankov, A.P. 2008), (iv) geodesic protocol which avoids QCP in an optimum way.

Beyond adiabatic response. Shortcuts to adiabaticity.

(M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+), also Bloch-Siegert shifts in NMR).

Moving frame Hamiltonian

$$\tilde{H}_m = \tilde{H} - \dot{\lambda}\tilde{\mathcal{A}}_\lambda, \quad \tilde{H} = \mathrm{diag}(E_1, E_2, \ldots)$$

Idea: introduce counter-adiabatic (CA) term

$$\tilde{H} \to \tilde{H} + \dot{\lambda}\tilde{\mathcal{A}}_\lambda \quad \tilde{H}_m = \tilde{H}$$

Moving frame follow eigenstates of $\tilde{H}$. Back to the lab frame:

$$H_{CD} = H + \dot{\lambda}\mathcal{A}_\lambda$$

CD driving intuitively:

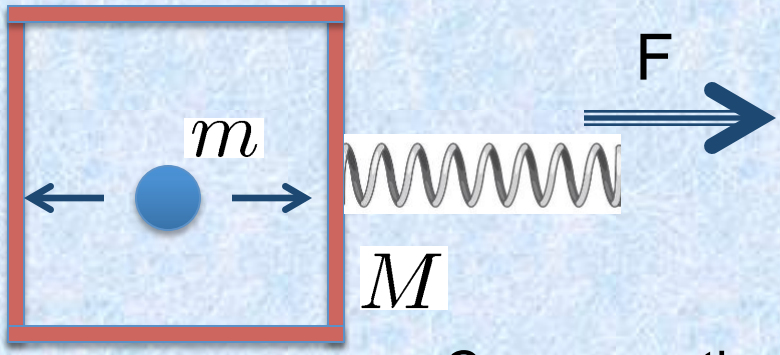
- Have to introduce extra parameters
- Do not follow instantaneous ground state
- Use only local (physical) counter terms, i.e. do not address individual water molecules

No CD term

CD term

Beyond adiabatic response. Shortcuts to adiabaticity.

(M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+)



$$H = \frac{p^2}{2m} + V(x - X_0(t))$$

Suppose we want to move a box in space without exciting a particle inside (without heating).

Can move the box slowly but it takes time. If move too slow will likely decohere due to a bath.

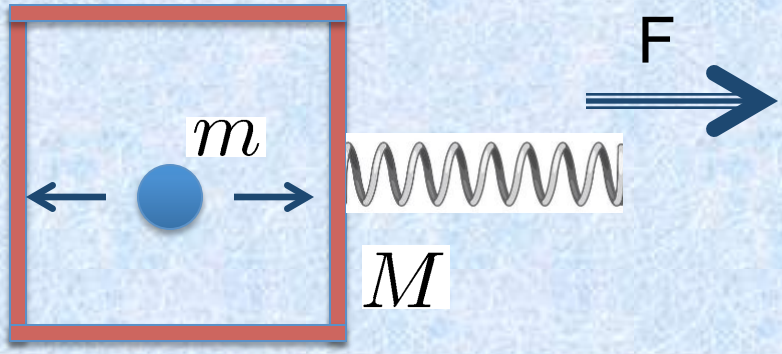
Recall a moving Hamiltonian

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = \frac{p^2}{2m} + V(x) - \dot{X}_0 p$$

Can compensate the last term by adding the counter term

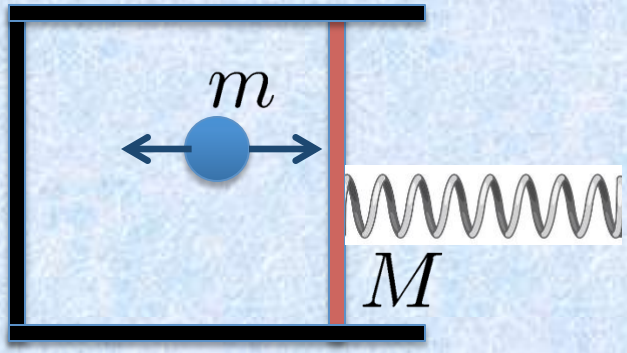
$$H \to H + \dot{X}_0 p$$

The moving frame the Hamiltonian is diagonal (time-independent). Can move arbitrarily fast. This is not what the waiter does!



$$
\begin{aligned}
H &= \frac{p^2}{2m} + V(x - X_0) + \dot{X}_0\, p \\
&= \frac{(p + m\dot{X}_0)^2}{2m} + V(x - X_0) - \frac{m\dot{X}_0^2}{2} \\
&\sim \frac{p^2}{2m} + V(x - X_0) - mx\ddot{X}_0.
\end{aligned}
$$

CD (counter-diabatic) term is simply a linear potential proportional to the acceleration (gravitational field).



$$\mathcal{D}: \quad x \to Lx,\ p \to p/L$$

$$
\begin{aligned}
H &= \frac{p^2}{2m} + \frac{1}{L^2} V(x/L) + \dot{L}\,\frac{px + xp}{2L} \\
&\sim \frac{p^2}{2m} + \frac{1}{L^2} V(x/L) - \frac{mx^2\ddot{L}}{L}.
\end{aligned}
$$

CD term is a harmonic potential (Deffner, Jarzynski, Del Campo 2014).

Finding adiabatic gauge potentials in complex systems

(important for CD driving, geodesics, Chern numbers, metric,…)

1. Through the unitary: $\mathcal{A}_\lambda = i(\partial_\lambda U)U^\dagger$
   Exact but not useful as we do not know the unitary.

2. Through the matrix elements of the instantaneous eigenstates:

$$\langle n|\mathcal{A}_\lambda|m\rangle = i\frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n}$$

   Same problem and hard to connect to local physical operators. Problem of small denominators in chaotic systems unless have special symmetries like Galilean invariance (related issues in classical chaotic systems Jarzynski 1997).

3. Need to find another root for finding approximate local adiabatic gauge potentials.

Recall definition of the moving frame as the one diagonalizing H

$$\tilde{H}(\lambda) = U^{\dagger}(\lambda) H(\lambda) U(\lambda)$$

Differentiate with respect to λ (moving derivative)

$$\partial_\lambda \tilde{H}(\lambda) = U^{\dagger}(\lambda) \partial_\lambda H(\lambda) U(\lambda) + \frac{i}{\hbar}[\tilde{\mathcal{A}}_\lambda, \tilde{H}]$$

By construction $[\partial_\lambda \tilde{H}, \tilde{H}] = 0$: gauge potential eliminates off-diagonal terms in the conjugate force

Go back to the lab frame (remove tildes), insert Planck's constant

$$\boxed{[i\hbar\partial_\lambda H - [\mathcal{A}_\lambda, H], H] = 0}$$

Classical systems

$$\boxed{\{\partial_\lambda H - \{\mathcal{A}_\lambda, H\}, H\} = 0}$$

Many-particle (non-interacting) systems

$$H = -J\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \sum_j V(j,\lambda) c_j^\dagger c_j$$

$$H_{CD} = H + \dot{\lambda}\mathcal{A}_\lambda$$

$$[i\partial_\lambda H - [\mathcal{A}_\lambda, H], H] = 0$$

It is clear that

$$\mathcal{A}_\lambda = i\sum_{i\neq j} \alpha_{ij} c_i^\dagger c_j, \quad \alpha_{ij} = -\alpha_{ji}$$

Gauge potential is imaginary, in general long range, hopping

Exact solution for a constant electric field $V_j = \lambda \sum_j j c_j^\dagger c_j$

$$\mathcal{A}_\lambda = -i\frac{J}{\lambda^2}\sum_j (c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$$

CD term is the current operator

Counter-diabatic Hamiltonian (set J=1)

$$H_{CD} = -\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \lambda \sum_j j c_j^\dagger c_j - \frac{i}{\lambda^2} \sum_j (c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$$

Can eliminate complex hopping by the gauge (Pierls) transformation

$$H_{CD} = -\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \frac{\lambda(t)}{\sqrt{1+\dot{\mu}^2}} \left(1 - \frac{\mu\ddot{\mu}}{1+\dot{\mu}^2}\right) \sum_j j c_j^\dagger c_j, \quad \mu = 1/\lambda$$

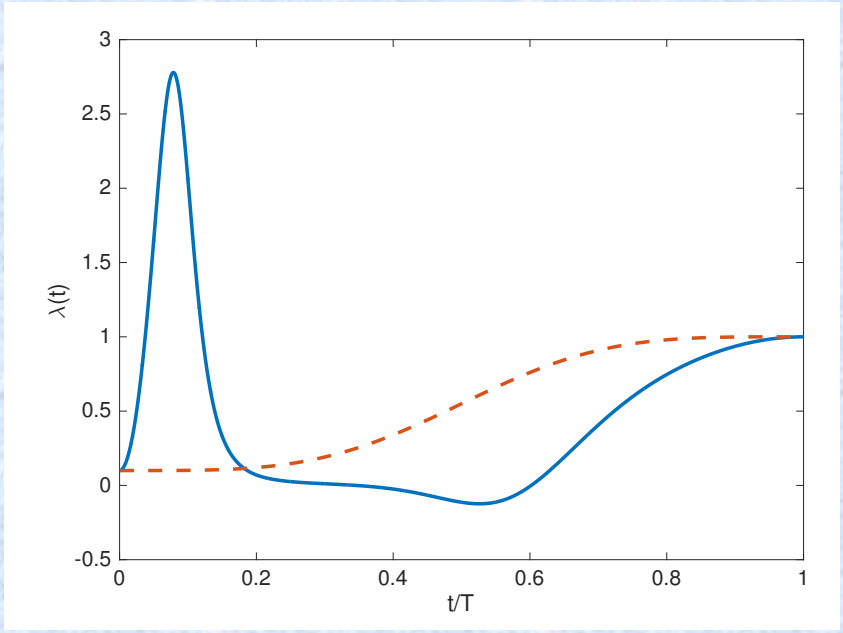
CD protocol depends on the rate and the acceleration (universal result).
Renormalization of hopping is absorbed into time dilations.

Counter-adiabatic vs. naïve protocols

$$\lambda(t) = \lambda_0 + (\lambda_f - \lambda_0) \sin^2\left(\frac{\pi}{2} \sin^2\left(\frac{\pi t}{2T}\right)\right)$$

$$\lambda_0 = 0.1, \ \lambda_f = 1$$

As time T increases CD and naive protocols approach each other.



Beyond the linear potential

$$[i\partial_\lambda H - [\mathcal{A}_\lambda, H], H] = 0$$

The exact solution exists in terms of long-range imaginary hopping. Hard to implement, can not gauge away. The situation is even worth for interacting systems (get many-particle interactions).

Need to find an approximate local solution

$$G = \partial_\lambda H + i[\mathcal{A}^*_\lambda, H]$$

Treat the gauge potential as a variational function:

$$\mathcal{A}^*_\lambda = i \sum_j \alpha_j (c^\dagger_j c_{j+1} - c^\dagger_{j+1} c_j)$$

Minimize norm of *G*. This talk: trace norm. Can use norm with UV cutoff, GS norm, finite temperature norm etc.

$$\mathcal{D}(\mathcal{A}^*_\lambda) = \mathrm{Tr}(G^2) \quad G = \partial_\lambda H + i[\mathcal{A}^*_\lambda, H]$$

Advantages of the trace norm: easy to find analytically, Wick's theorem applies to any Hamiltonian. Works both for the ground and excited states.

$$G = \sum_j \left(\partial_\lambda V_j - 2J(\alpha_j - \alpha_{j-1})\right) c^\dagger_j c_j$$

$$+ J\sum_j (\alpha_j - \alpha_{j-1})(c^\dagger_{j+1}c_{j-1} + c^\dagger_{j-1}c_{j+1}) + \sum_j (V_{j+1} - V_j)\alpha_j (c^\dagger_{j+1}c_j + c^\dagger_j c_{j+1}).$$

Result of the minimization

$$-3\Delta\alpha + (\nabla_j V)^2 \alpha = \nabla_j(\partial_\lambda V)$$

Smooth potentials, continuum limit

$$\boxed{-3\partial_x^2\alpha + (\partial_x V)^2\alpha = \partial_x(\partial_\lambda V)}$$

This gauge potential defines the best local co-moving frame. Does not require diagonalization of the Hamiltonian. Maps quantum to classical problem

$$H_{\rm CD} = H_0 + \dot\lambda \mathcal{A}^*_\lambda = -J\sum_j \left[c^\dagger_{j+1}c_j\left(1 - i\frac{\alpha_j\dot\lambda}{J}\right) + c^\dagger_j c_{j+1}\left(1 + i\frac{\alpha_j\dot\lambda}{J}\right)\right] + \sum_j V_j(\lambda)c^\dagger_j c_j$$

Perform a phase (Pierls) transformation: $c_j \to c_j \mathrm{e}^{-i\phi_j}$

$$H_{CA} = -\sum_i J_{eff}(i)\left(c^\dagger_{i+1}c_i + c^\dagger_i c_{i+1}\right) + \sum_i V_{eff}(i)c^\dagger_i c_i,$$

$$V_{eff}(x,t) = V(\lambda,x) - \ddot\lambda\int_a^x \frac{\alpha(x')}{1+\dot\lambda^2(\alpha(x'))^2}\mathrm{d}x', \quad J_{eff}(x,t) = \sqrt{1+\dot\lambda^2(\alpha(x))^2},$$

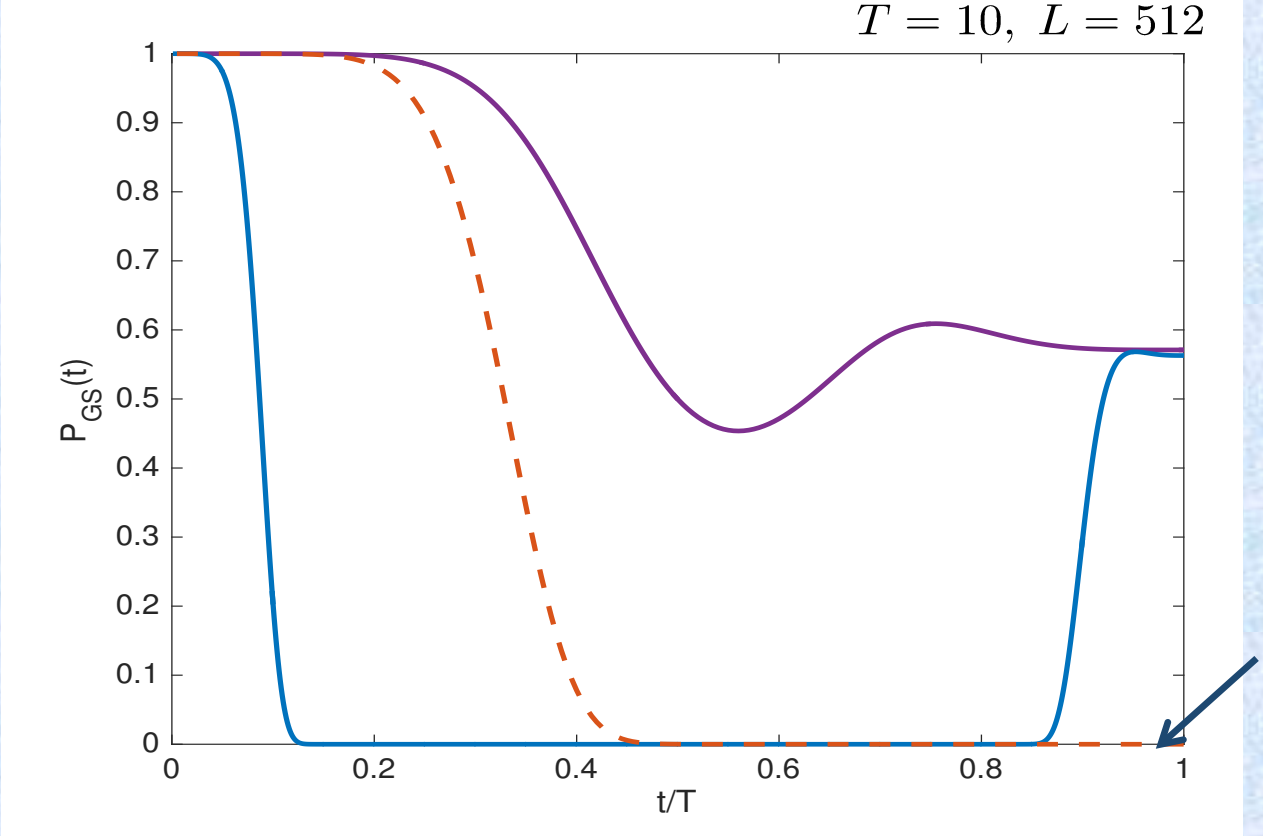
Imaginary CD protocol is only sensitive to velocity. Real CD protocol also knows about acceleration

Small velocity: potential renormalization (slowing particles in front)

Large velocity: need to locally renormalize hopping = local time rescaling or introducing a local refraction index

Example: inserting Eckart's potential (fighting Anderson orthogonality catastrophe). Half filling, 512 sites

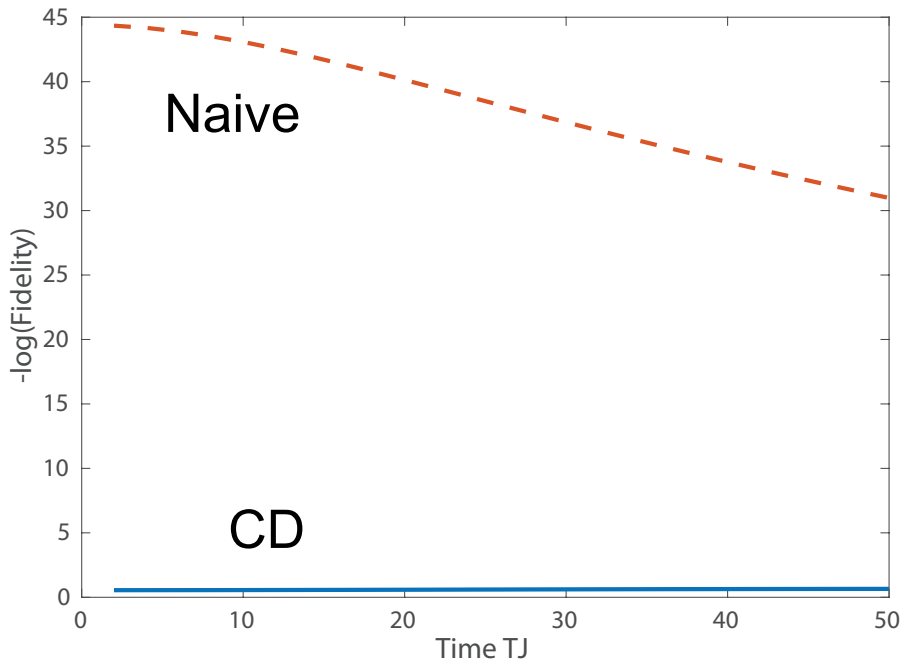
$$V(j,\lambda) = \frac{\lambda(t)}{\cosh^2(j/\xi)} \qquad \lambda(t) = 2\sin^2\left(\frac{\pi}{2}\sin^2(\pi t/2T)\right)$$



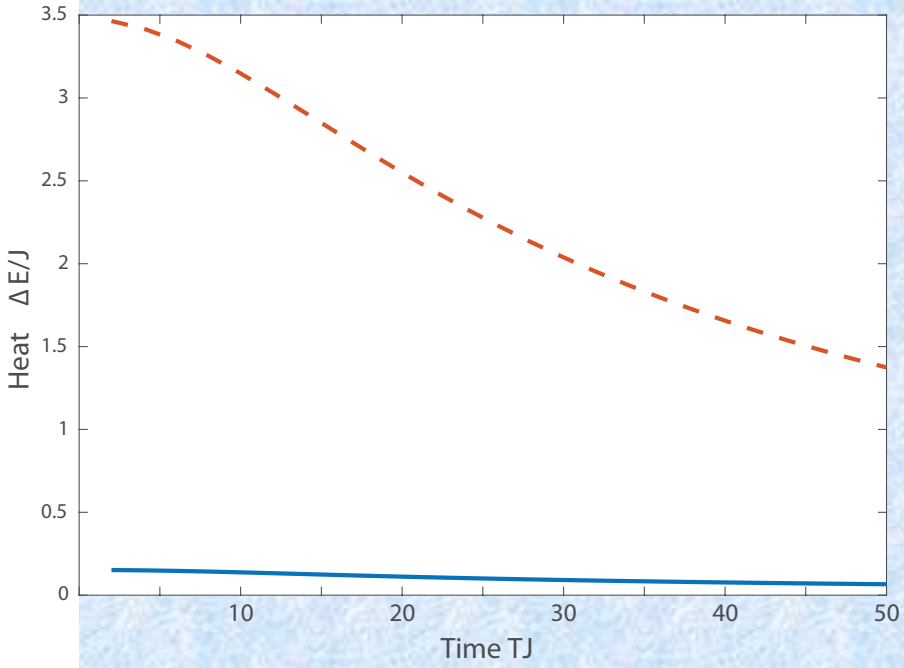
Like throwing a stone into quantum water (gas) without generating ripples.

Inserting the Eckart potential

Log Fidelity



Excess energy

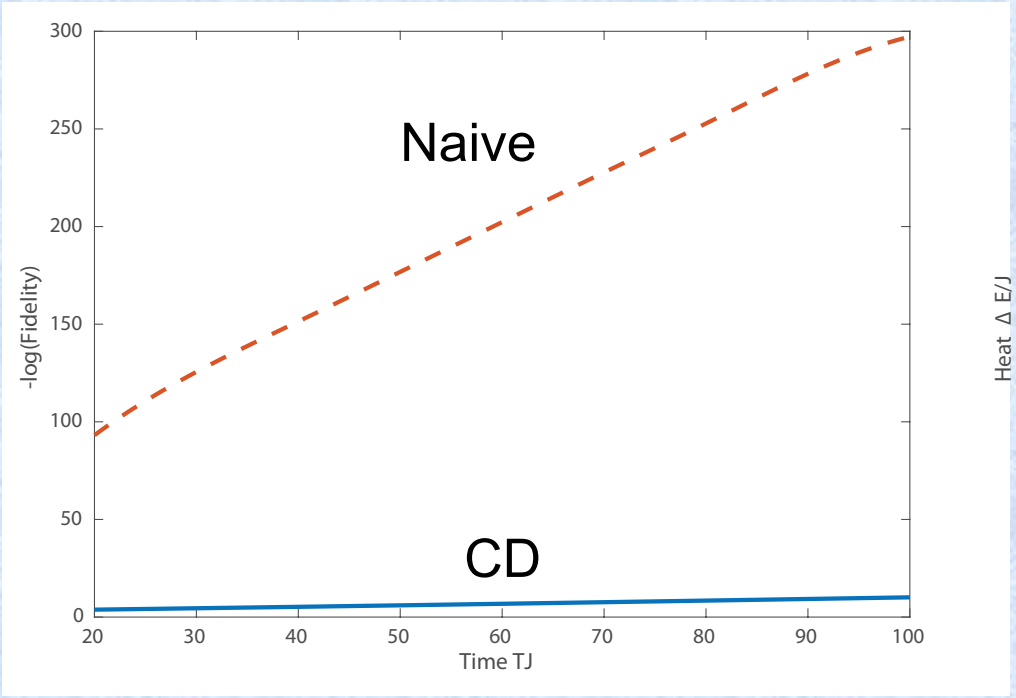


CD driving outperforms naïve protocol by many orders of magnitude.

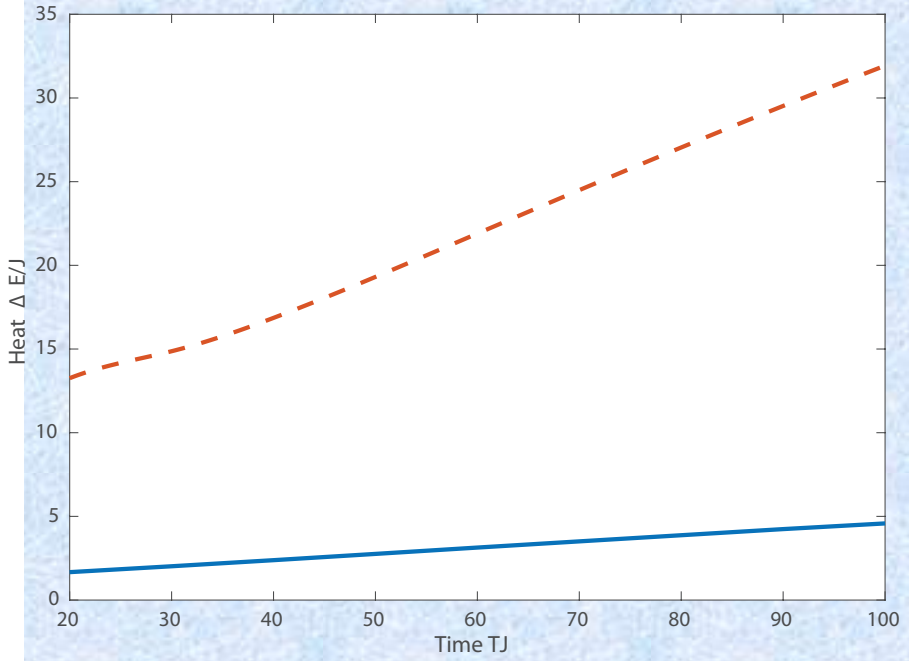
Moving local potential. Fighting friction.

$$V(j,\lambda) = \frac{2}{\cosh^2((j-\lambda(t))/\xi)}$$

Log Fidelity

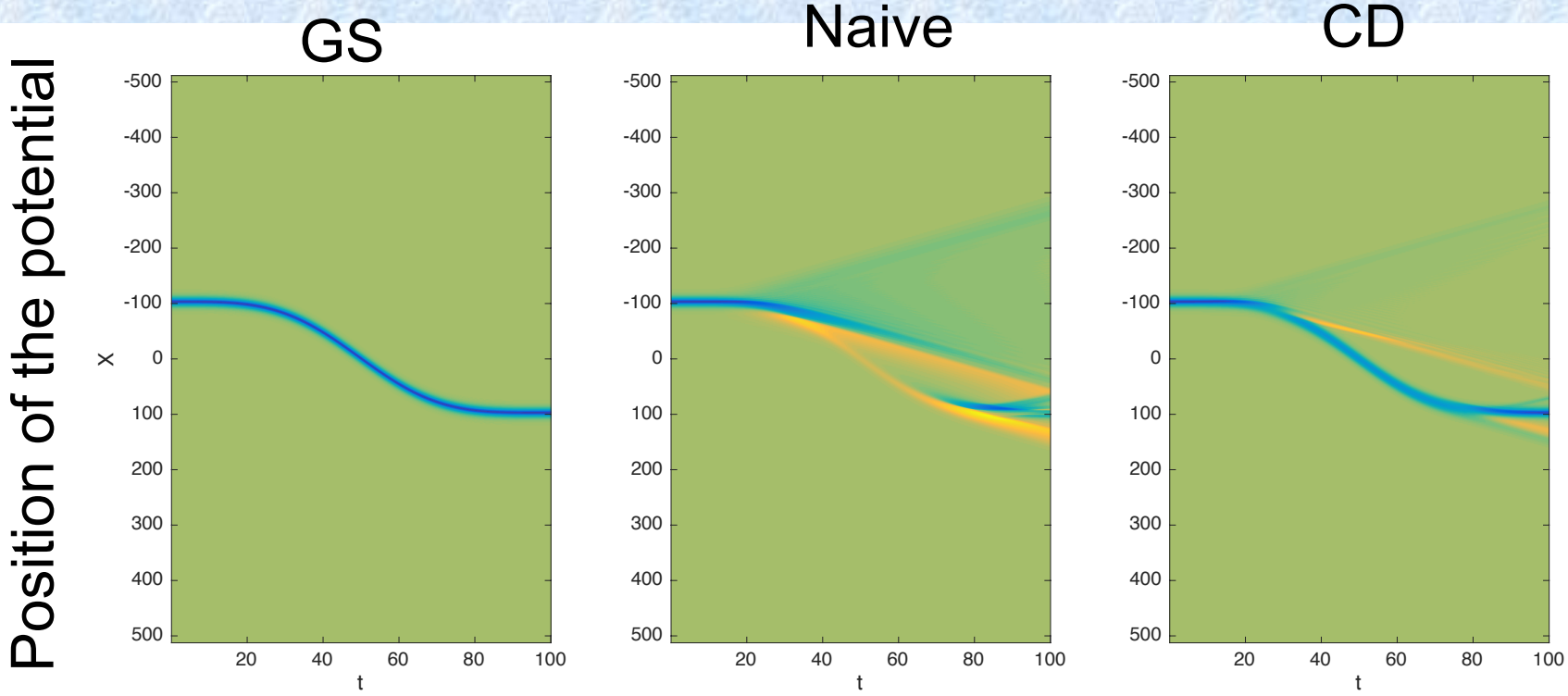


Dissipation (heating)



Suppress dissipation, increase fidelity.

Fermion density profiles



Time

Emit much fewer particles with CD driving.

Large gain in energy and fidelity for loading to a quasi-periodic smooth potential

Large gain in energy (but not fidelity) for loading into a random potential.

Ergodic spin system (going around QCP)

$$H_0 = -J\sum_j \sigma_j^z\sigma_{j+1}^z - h_z\sum_j \sigma_j^z - h_x\sum_j \sigma_j^x, \quad \mathcal{A}_\lambda^* = \alpha\sum_j \sigma_j^y$$

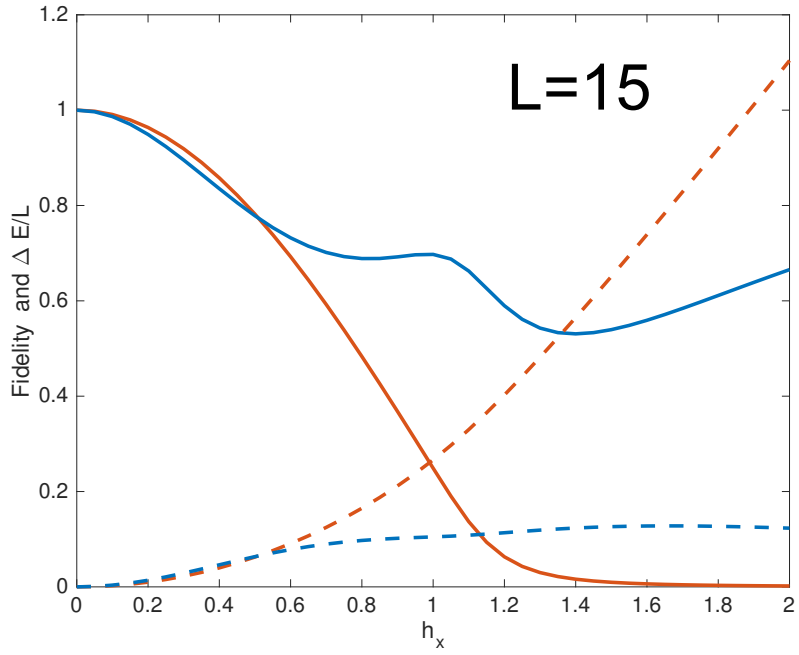
Variational solution:
(exact for J=0).

$$\alpha = \frac{1}{2}\frac{h_x h_z' - h_z h_x'}{h_z^2 + h_x^2 + 2J^2},$$

Gauge equivalent real solution:

$$h_x^{\mathrm{CD}} = \sqrt{h_x^2 + \dot\lambda^2\alpha^2}, \quad h_y^{\mathrm{CD}} = 0$$

$$h_z^{\mathrm{CD}} = h_z - \frac{1}{2}\frac{\ddot\lambda h_x\alpha + \dot\lambda^2(h_x\alpha' - \alpha h_x')}{h_x^2 + \dot\lambda^2\alpha^2}$$



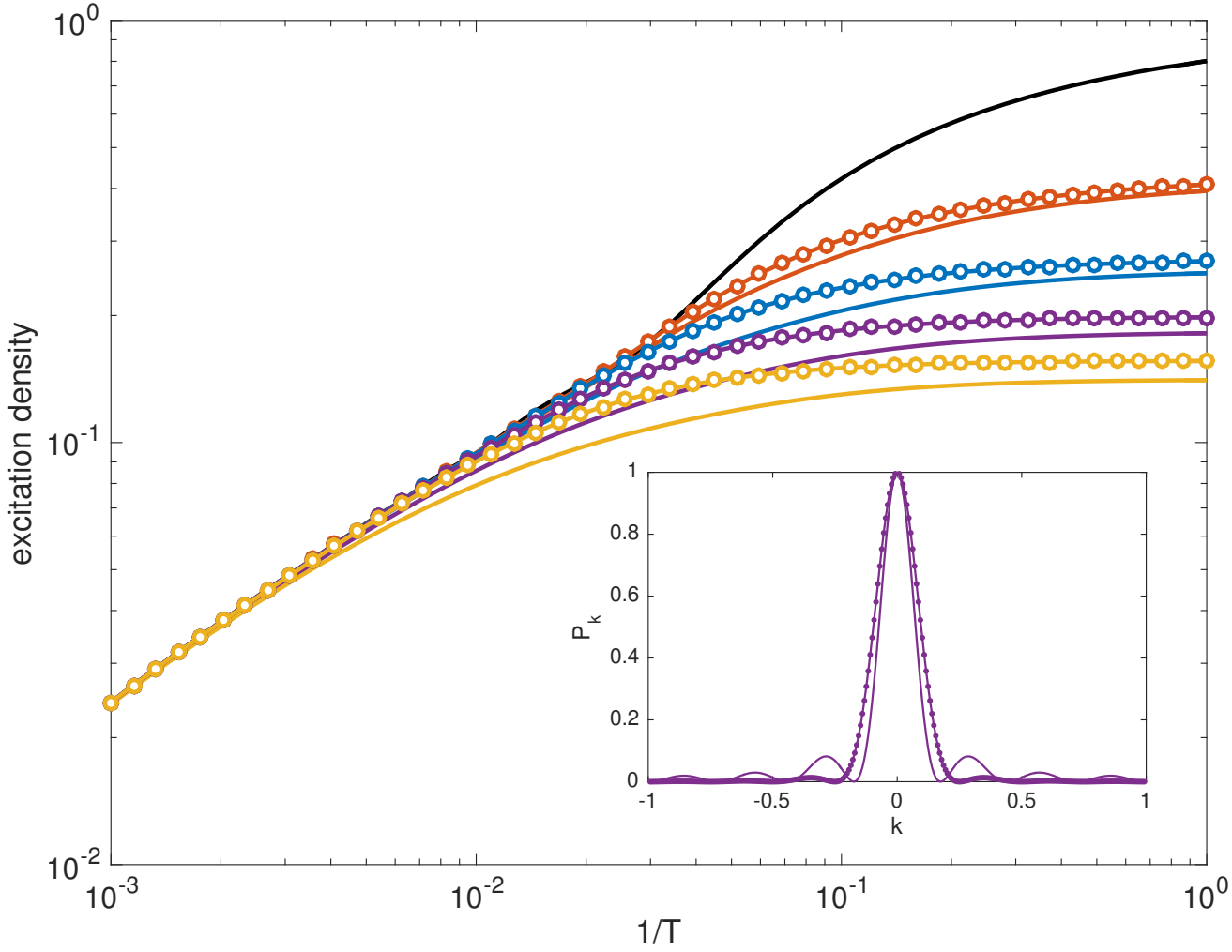
Loading across critical point.

$$h_z = 0.02,\ h_x = h_0\lambda(t),\ J = \lambda(t)$$

CD protocol strongly outperforms naïve protocol if driving across QCP.

No information about GS, location of QCP etc. is needed!

CD driving through QCP in the TFI model

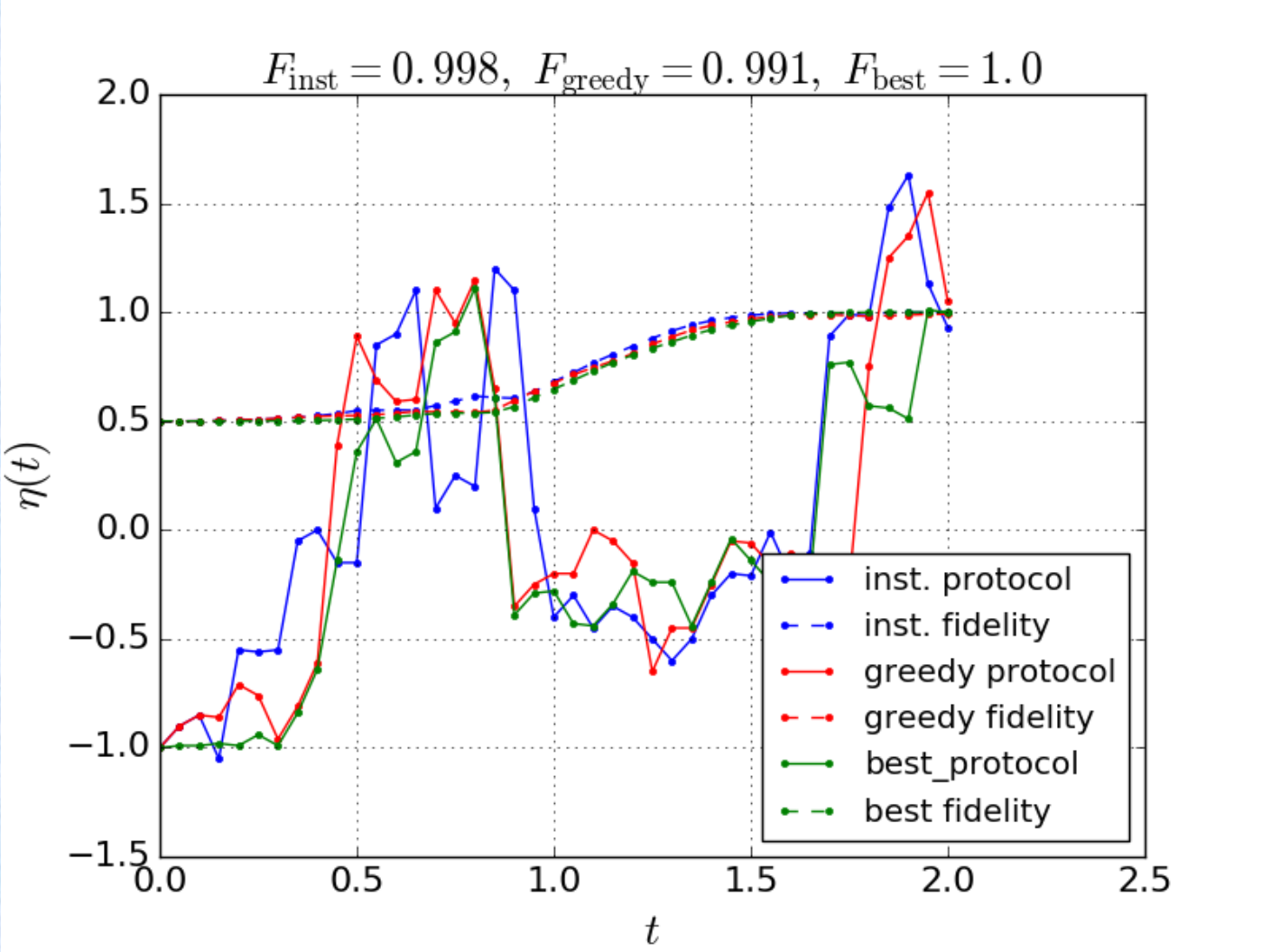


Dots: variational solutions with string lengths 1-4; lines: exact truncated solutions. Variational solution can be generalized to a nonintegrable chain.

Machine learning optimization (final fidelity as a reinforcer)

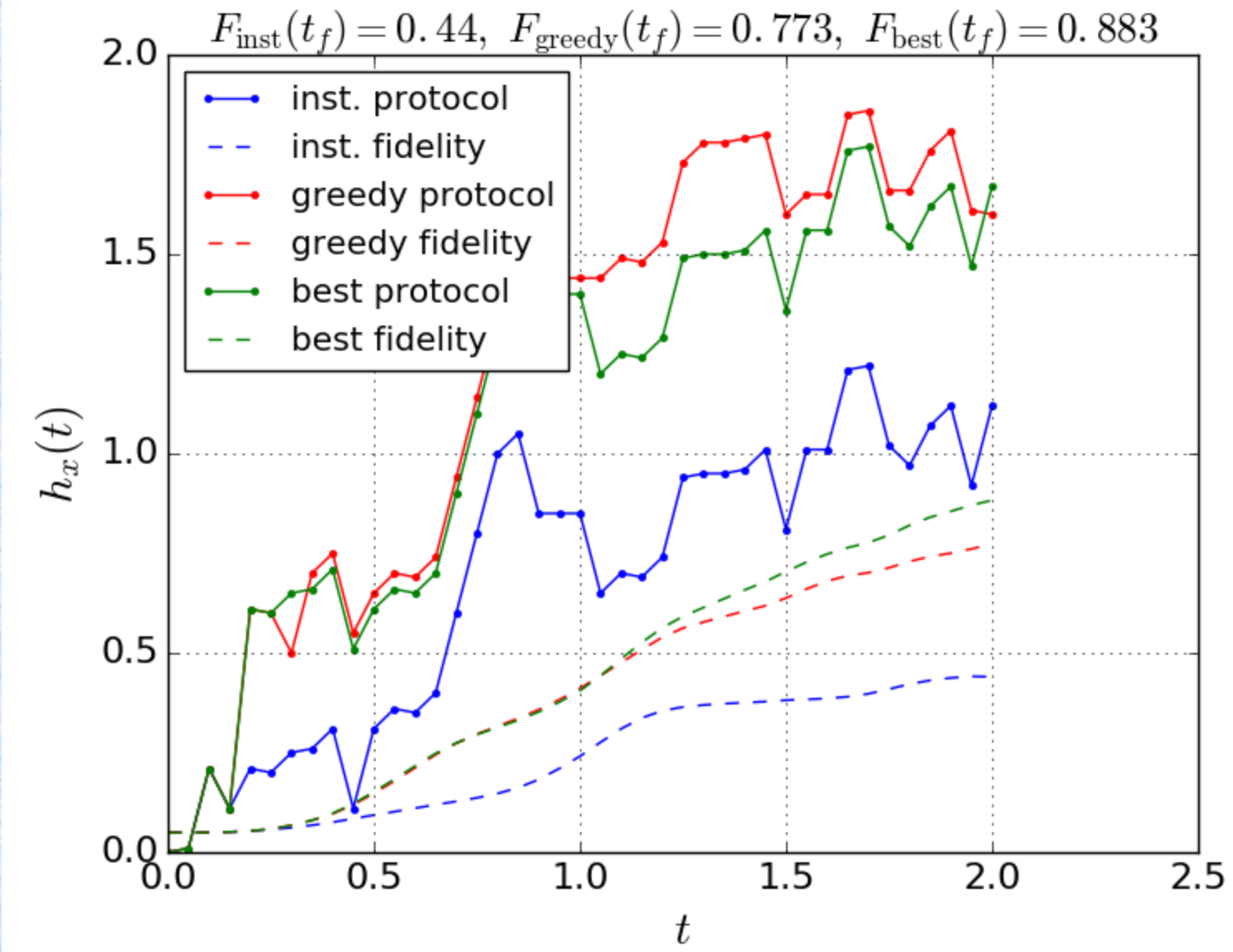
(in progress, lead by M. Bukov and P. Mehta)

Landau-Zener pr$h_z = 1, \quad h_x \in [-1, 1], \quad \sim 1000$ runs



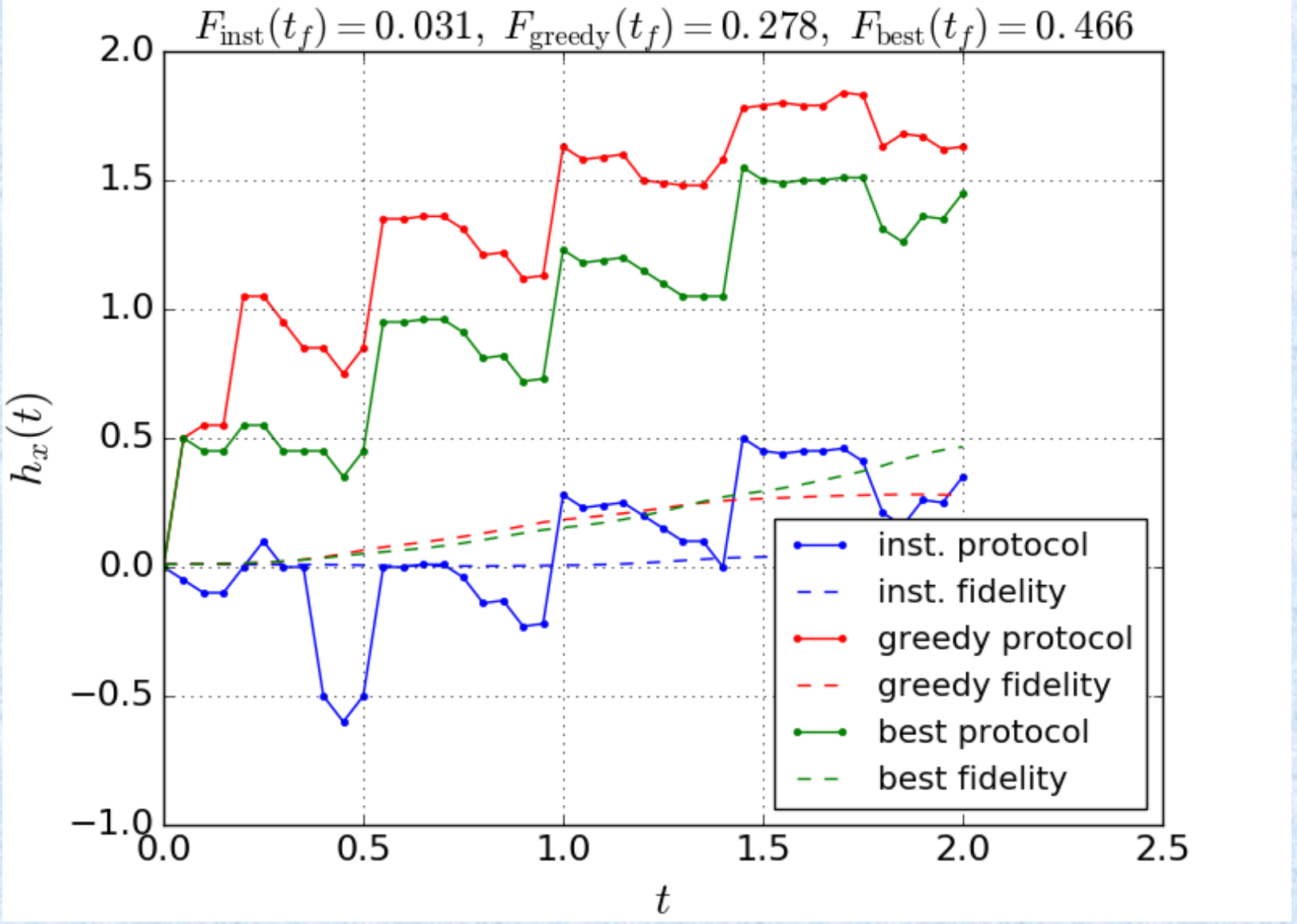
14 cite chain, no QCP, 1000 runs

$$H_0 = -1.23 \sum_j \sigma_j^z \sigma_{j+1}^z + 0.02 \sum_j \sigma_j^z - h_x(t) \sum_j \sigma_j^x, \quad h_x \in [0,2],\ t \in [0,2]$$



14 cite chain, crossing QCP, 1000 runs

$$H_0 = 1.23 \sum \sigma_j^z \sigma_{j+1}^z + 0.02 \sum \sigma_j^z - h_x(t) \sum \sigma_j^x, \quad h_x \in [0,2],\ t \in [0,2]$$



Summary

- Deep connections between non-adiabatic response and geometry.
- Imaginary time dynamics can be used to simulate real time quantum annealing in nontrivial regimes
- Geometric approach to optimum annealing protocols
- Local counter-adiabatic driving: robust, easy to implement
- Reinforcement learning can be used to solve non-trivial optimization problems (crushed my intuition and cost me a bottle of a good cognac).

$$M_b \approx M_b^{(0)} + \hbar F_{ba} \dot{\lambda}_a$$

Imagine motion in momentum space (equivalently gauge potential space)

$$\lambda_x = \frac{1}{c} A_x, \ \lambda_y = \frac{1}{c} A_y$$

$$\dot{\lambda}_x = \frac{1}{c} \frac{\partial A_x}{\partial t} = E_x \qquad M_y = -c \langle \frac{\partial \mathcal{H}}{\partial A_y} \rangle = J_y$$

Recover the standard Hall effect

$$J_y = \hbar F_{xy} E_x$$

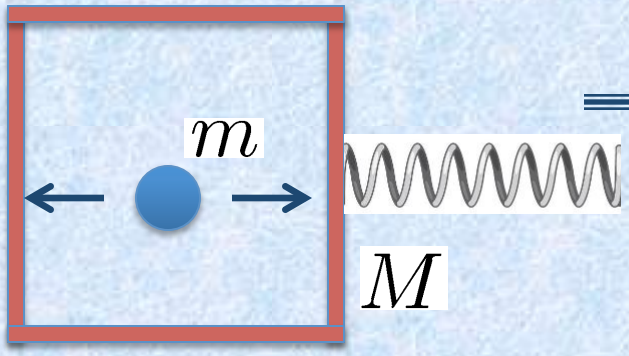
Quantization of the Chern number (when we integrate F over a closed manifold) implies the quantum Hall effect

$$\phi = \frac{e}{c\hbar} \int \vec{A} d\vec{l} \Rightarrow \left(\frac{A_x}{c}, \frac{A_y}{c} \right) \in [0, 2\pi\hbar/L_x e] \times [0, 2\pi\hbar/L_y e]$$

$$F_{xy} \frac{4\pi^2\hbar^2}{e^2 S} = 2\pi n \ \Rightarrow \ \sigma_{xy} = \frac{\hbar F_{xy}}{S} = \frac{e^2}{2\pi\hbar} n$$

QHE can be interpreted as measurement of the quantized Coriolis force.

How can we understand the mass in a simple setup?

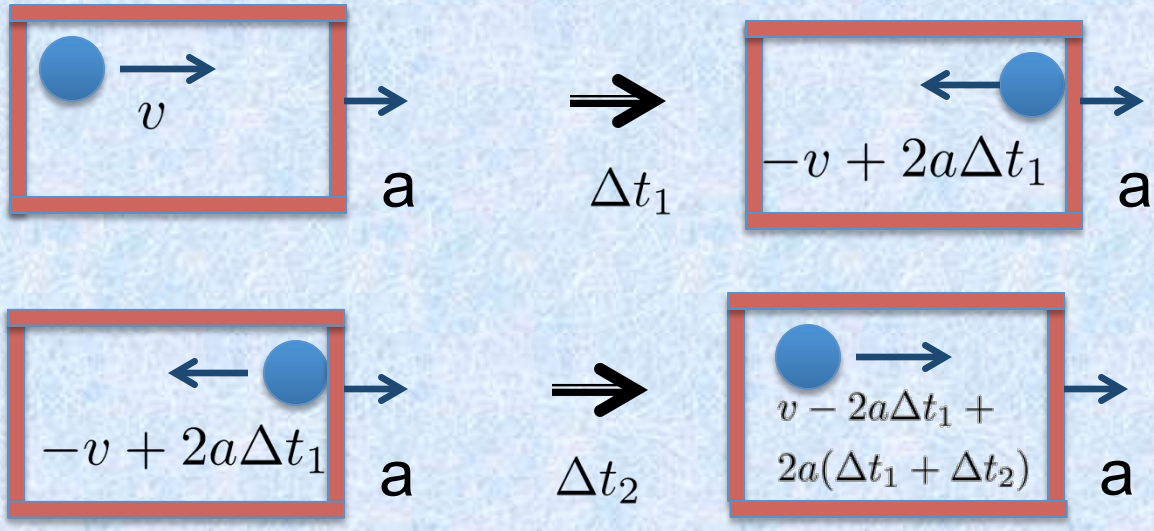


F

Take container and start slowly accelerating it to velocity *v*.

Compute the force (or work).

Assume particle is fast compared to the container



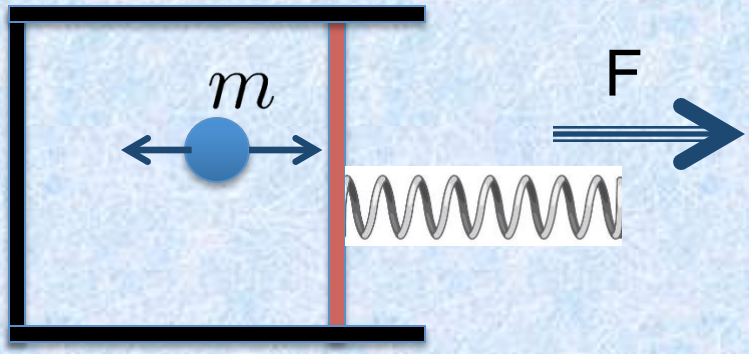
$$\Delta p_1 = -2mv + 2a\Delta t_1$$

$$\Delta p_2 = 2mv + 2a(\Delta t_2 - \Delta t_1)$$

$$F = \frac{\Delta p_1 + \Delta p_2}{\Delta t_1 + \Delta t_2} = \frac{2ma\Delta t_2}{\Delta t_1 + \Delta t_2} \approx ma$$

Only valid near the adiabatic limit, where $\Delta t_1 \approx \Delta t_2$

Two ways of measuring generalized force



1. Measure force as a pressure using some calibrated device like a spring and third Newton's law.

2. Measure as the generalized force

$$F = \int dx \rho(x) \partial_x V(x) = \langle \psi | \partial_x V | \psi \rangle, \quad V(x)$$ is the wall potential

Y. Kafri, M. Kardar, … non-existence of pressure as a function of state in active (non-equilibrium) matter (2014)