Suppression of inhomogeneous broadening in qubit ensemble under optimized driving

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motivation

NV centers in diamond:

- large inhomogeneous broadening
- large coherence time

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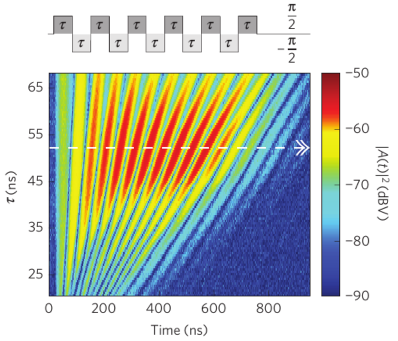
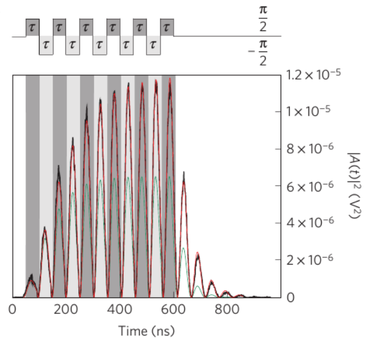
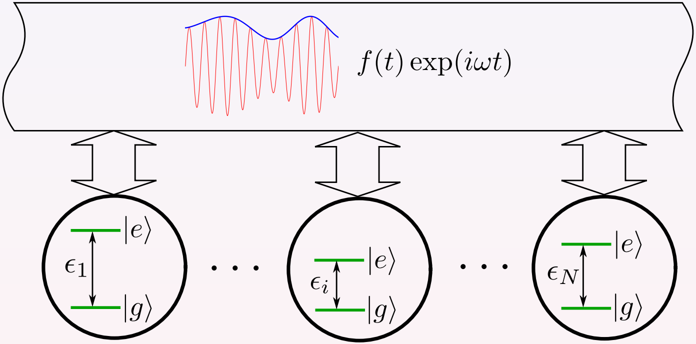
Increase of a response for optimally choosen pulse series





S. Putz et al, Nature Physics (2014); arXiv:1404.4169

problem formulation



Hamiltonian for N qubits in classical field in rotating frame basis:

$$\mathcal{H} = \sum_{i=1}^{N} \left[\frac{\Delta_i}{2} \sigma_z^i + f(t) \sigma_x^i \right], \qquad \Delta_i = \epsilon_i - \omega$$

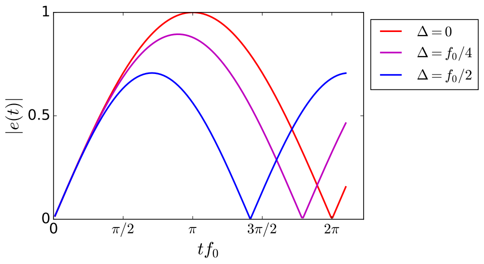
no decoherence

Schrödinger equation

$$\mathrm{i}\hbar \frac{d}{dt}|\psi(t)\rangle = \frac{1}{2}\begin{pmatrix} \Delta & f(t) \\ f^*(t) & -\Delta \end{pmatrix} |\psi(t)\rangle, \qquad |\psi(t)\rangle = \begin{pmatrix} e(t) \\ g(t) \end{pmatrix}$$

exact solution 1: Rabi oscillations

$$f(t) = f_0 = \mathrm{const}, \qquad \Omega_R = \sqrt{|f_0|^2 + \Delta^2}$$



no decoherence

exact solution 2

$$\Delta = 0, \quad f(t) \text{ real}$$

$$\varphi(t) = \int_0^t f(t_1) dt_1$$

$$e(t) = A \cos \frac{\varphi(t)}{2} - \mathrm{i} B \sin \frac{\varphi(t)}{2}$$

$$g(t) = B \cos \frac{\varphi(t)}{2} - \mathrm{i} A \sin \frac{\varphi(t)}{2}$$

arbitrary Δ, $f(t)$

perturbation theory for A, B on Δ

perturbation theory on Δ

solution for ground state amplitude $g(t) = B(t)\cos\frac{\varphi(t)}{2} - \mathrm{i}A(t)\sin\frac{\varphi(t)}{2}$:

$$B(t) = 1 + \frac{\Delta \mathrm{i}}{2}\int_0^t dt_1 \cos\varphi(t_1) - \frac{\Delta^2}{4}\int_0^t dt_1 \int_0^{t_1} dt_2\, \cos\left[\varphi(t_2) - \varphi(t_1)\right] + \dots$$

$$A(t) = -\frac{\Delta}{2}\int_0^t dt_1 \sin\varphi(t_1) + \frac{\Delta^2 \mathrm{i}}{4}\int_0^t dt_1 \int_0^{t_1} dt_2\, \sin\left[\varphi(t_2) - \varphi(t_1)\right] + \dots$$

initial conditions:

$$|\psi(0)\rangle \equiv \begin{pmatrix} e(0) \\ g(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \varphi(0) = 0$$

π-pulse:

$$\varphi(T) = \pi$$

qubit excitation condition

after π-pulse ($\varphi(T) = \pi$) ground state is not occupied ($g(T) = 0$):

$$\frac{d}{d\Delta}A(t)\Big|_{\substack{t=T\\ \Delta=0}} = 0: \quad \int\limits_0^T \sin\varphi(t)dt = 0$$

$$\frac{d^2}{d\Delta^2}A(t)\Big|_{\substack{t=T\\ \Delta=0}} = 0: \quad \int\limits_0^T dt_1 \int\limits_0^{t_1} dt_2 \sin\left[\varphi(t_1) - \varphi(t_2)\right] = 0$$

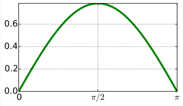
$$\frac{d^3}{d\Delta^3}A(t)\Big|_{\substack{t=T\\ \Delta=0}} = 0: \quad \int\limits_0^T dt_1 \int\limits_0^{t_1} dt_2 \int\limits_0^{t_2} dt_3 \sin\left[\varphi(t_1) - \varphi(t_2) + \varphi(t_3)\right] = 0$$

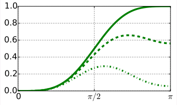
$$\dots$$

$$f(t) = \Omega \left[k_1 \sin \Omega t + k_3 \sin 3\Omega t + k_5 \sin 5\Omega t + \ldots \right], \qquad T = \frac{\pi}{\Omega}$$

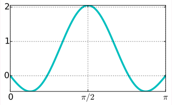
$f(t)$ $\qquad$ $|e(t)|^2;\ \Delta = 0, \Omega, 2\Omega$

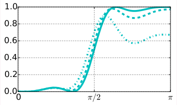
$N = 1$



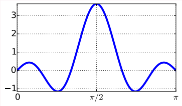


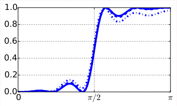
$N = 2$



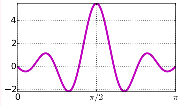


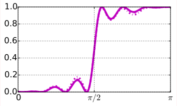
$N = 3$





$N = 4$

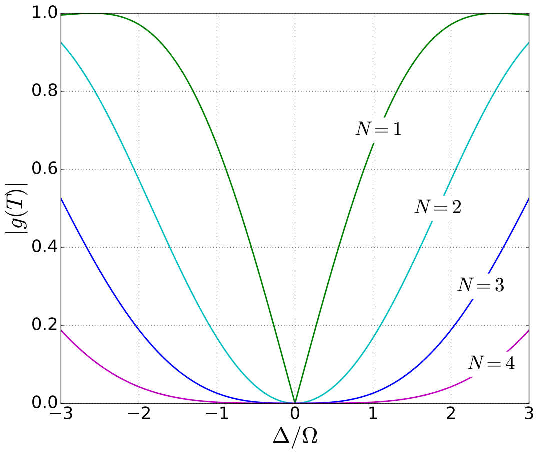




results

$$f(t) = \Omega\left[k_1 \sin\Omega t + k_3 \sin 3\Omega t + k_5 \sin 5\Omega t + \dots\right], \qquad T = \frac{\pi}{\Omega}$$

ground state amplitude after π-pulse



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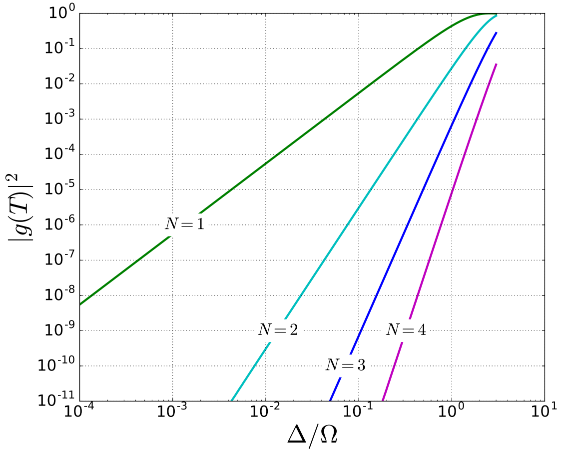
residual population of the ground state after π-pulse



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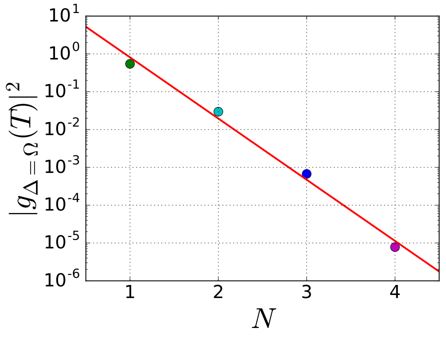


$$|g_\Delta(T)|^2 \approx 34\left(0.16\frac{\Delta}{\Omega}\right)^{2N}$$

Effect of dissipation

N qubits. Hamiltonian:

$$\mathcal{H} = \sum_{i=1}^{N} \left[\frac{\epsilon_i}{2}\sigma_z^i + f(t)\sigma_x^i\right]$$

Lindblad equation

$$\frac{\partial}{\partial t}\hat{\rho} = -i[\mathcal{H}, \hat{\rho}] + \Gamma\rho, \qquad \langle x(t)\rangle = \mathrm{Tr}\rho(t)x$$

Lindblad operator

$$\Gamma\rho = \frac{\Gamma_1}{2}\sum_i \left(2\sigma_i^-\rho\sigma_i^+ - \rho\sigma_i^+\sigma_i^- - \sigma_i^+\sigma_i^-\rho\right) + \frac{\Gamma_\varphi^*}{2}\left(\sigma_i^z\rho\sigma_i^z - \rho\right)$$

effect of decoherence

in rotating frame basis

$$\Delta_i = \epsilon_i - \omega$$

Bloch equations for a qubit

$$\begin{aligned}
\partial_t \left\langle \sigma^+ \right\rangle &= (\,\mathrm{i}\,\Delta - \Gamma_\varphi)\left\langle \sigma^+ \right\rangle - \frac{\mathrm{i}\,f(t)}{2}\left\langle \sigma^z \right\rangle \\
\partial_t \left\langle \sigma^- \right\rangle &= (-\,\mathrm{i}\,\Delta - \Gamma_\varphi)\left\langle \sigma^- \right\rangle + \frac{\mathrm{i}\,f(t)}{2}\left\langle \sigma^z \right\rangle \\
\partial_t \left\langle \sigma^z \right\rangle &= -\,\Gamma_1(\left\langle \sigma^z \right\rangle + 1) - \,\mathrm{i}\,f(t)\big(\left\langle \sigma^+ \right\rangle - \left\langle \sigma^- \right\rangle\big)
\end{aligned}$$

no decoherence:

$$\begin{aligned}
\left\langle \sigma^+ \right\rangle &= e(t)^* g(t) \\
\left\langle \sigma^z \right\rangle &= |e(t)|^2 - |g(t)|^2
\end{aligned}$$

effect of decoherence

after π-pulse

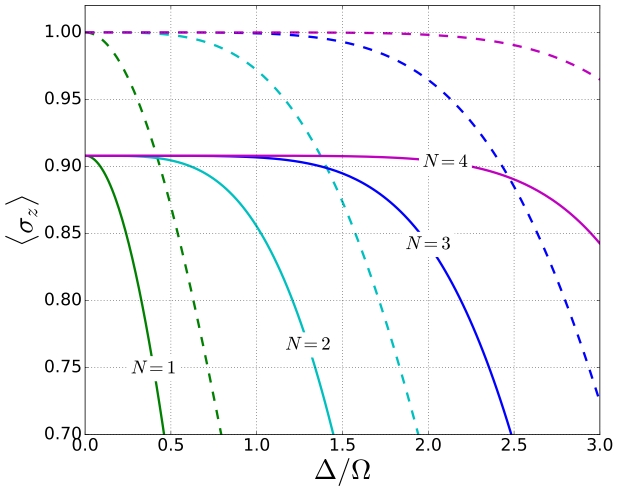
$$\langle \sigma^z \rangle = 1 - \frac{\Delta^2}{2} \left[\int_0^T \sin \frac{\varphi(t)}{2}\, dt \right]^2 + \ldots$$

$$- \left[2\Gamma_1 \int_0^T \sin^4 \frac{\varphi(t)}{2}\, dt + \left(\Gamma_\varphi - \frac{\Gamma_1}{2} \right) \int_0^T \sin^2 \varphi(t)\, dt \right] + \ldots$$

detuning

dissipation

effect of decoherence



$\Gamma_1 = 0.03\Omega$

$\Gamma_\varphi = 0.03\Omega$

decoherence correction: $\delta \langle \sigma_z \rangle \sim \Gamma \tau$

conclusions

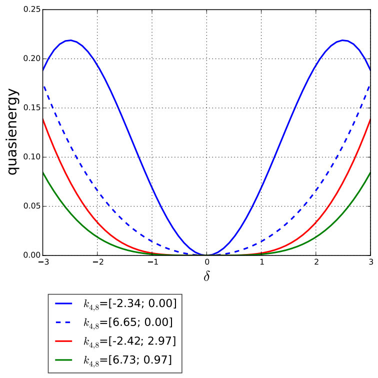
- we propose the model of smooth shaped single π-pulse which can be applied to realistic disordered qubit ensembles coupled to a transmission line
- such a π-pulse provides an effective suppression of the inhomogeneous broadening and can be used as qubit synchronization technique
- similar technique can be effectively applied to create, say, $\pi/2$-pulse to prepare entangled states of the inhomogeneously broadened qubits

Phys. Rev. A 92, 053814 (2015); arXiv:1508.00821

Thank you for attention

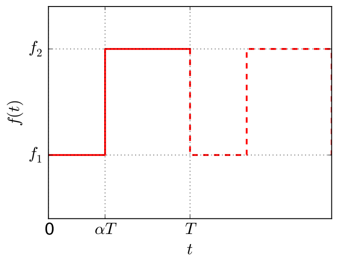
$$f(t) = \Omega\left[k_1 \sin \Omega t + k_3 \sin 3\Omega t + k_5 \sin 5\Omega t + \ldots\right]$$

band flattering



results. driving envelope in the form of steps

form of envelope

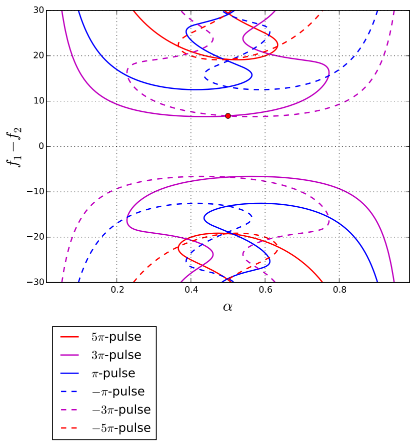


The problem is solvable exactly. Condition for band flattering:

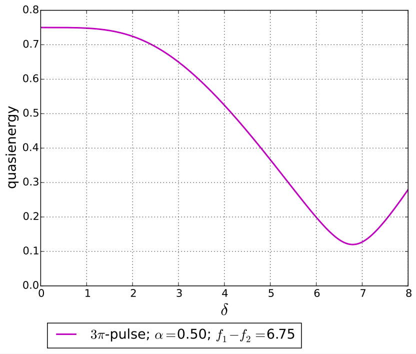
$$\frac{2\left(\frac{1}{f_2}-\frac{1}{f_1}\right)^2 \sin\frac{f_1\alpha T}{2}\sin\frac{f_2(1-\alpha)T}{2}}{T\sin\left(\frac{f_1\alpha T}{2}+\frac{f_2(1-\alpha)T}{2}\right)} = \frac{1-\alpha}{f_2}+\frac{\alpha}{f_1}$$

results. driving envelope in the form of steps

map of band flattering condition after $(2n+1)\pi$-pulse



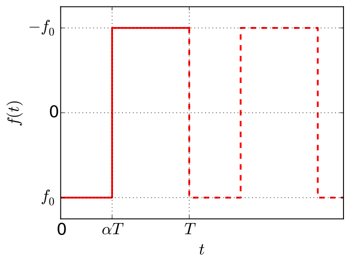
band flattering after 3π-pulse



$$f_1 = 4.37$$
$$f_2 = -2.37$$

results. driving envelope in the form of steps

form of envelope

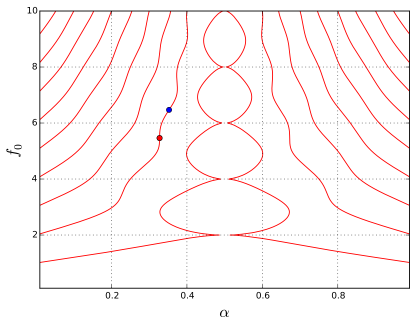


Condition for band flattering:

$$8 \sin \frac{\alpha f_0 T}{2} \sin \frac{(\alpha - 1) f_0 T}{2} = (2\alpha - 1) f_0 T \sin \frac{(2\alpha - 1) f_0 T}{2}$$

results. driving envelope in the form of steps

map of band flattering condition



band flattering