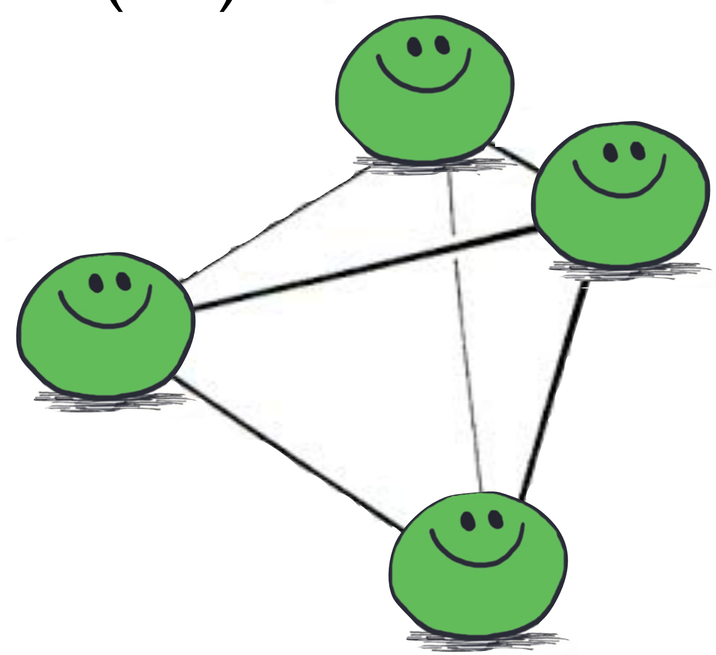
(De)-localization in mean-field quantum glasses

Chris R. Laumann (BU)



Chris Baldwin (UW/BU)
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ICTP Workshop on Theory and Practice of Adiabatic Quantum Computers and Quantum Simulation

August 26, 2016

Plan

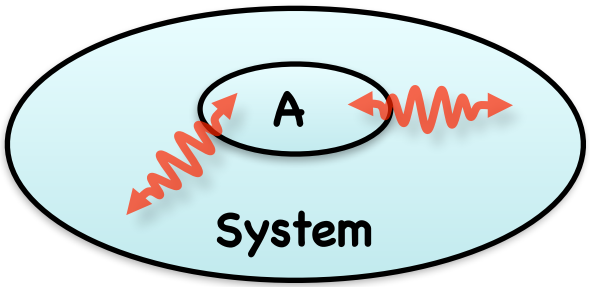
- Why? Behavior of eigenstates
- Quantum Random Energy Model (QREM)
  - Thermodynamics and eigenstates
- Annealing the QREM
  - Lower bounds and estimates
- Quantum p-Spin Model
  - Thermodynamics and eigenstates
- Summary

Highly excited eigenstates

Isolated Quantum Many-Body Systems

Eigenstate Thermalization Hypothesis (ETH)	Many-Body Localization (MBL)
Eigenstates thermal	Eigenstates frozen
Energies repel (GOE)	No repulsion (Poisson)
No \`Ψ-Chaos’	\`Ψ-Chaos’

Thermal Eigenstates



ETH systems behave as their own bath.

Many-body eigenstates agree with equilibrium on subsystems.

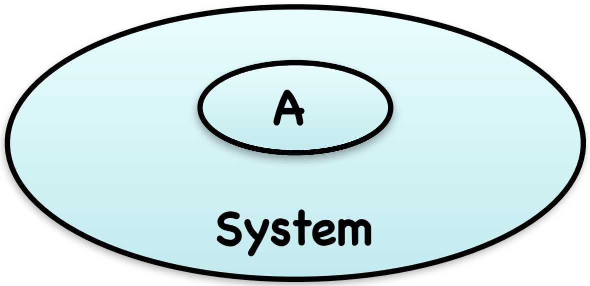
$$H\,|E_i\rangle = E_i\,|E_i\rangle$$

$$\rho_A = \mathrm{Tr}_{S\setminus A}\,|E_i\rangle\langle E_i| = \mathrm{Tr}_{S\setminus A}\,e^{-\beta H}$$

(At correct temperature for E)

Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii (2008), Pal and Huse (2010), ...

Frozen Eigenstates



Localized systems can't exchange energy…

Many-body eigenstates locally frozen in particular patterns

- Eigenstate Edwards-Anderson order parameter

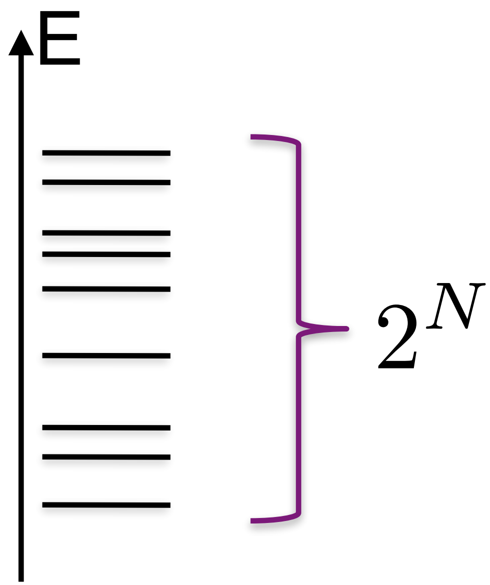
$$q_{ES} = \frac{1}{N}\sum_i \langle\psi|\sigma_i^z|\psi\rangle^2$$

- Note: possibly distinct from q_{EA} in Gibbs state

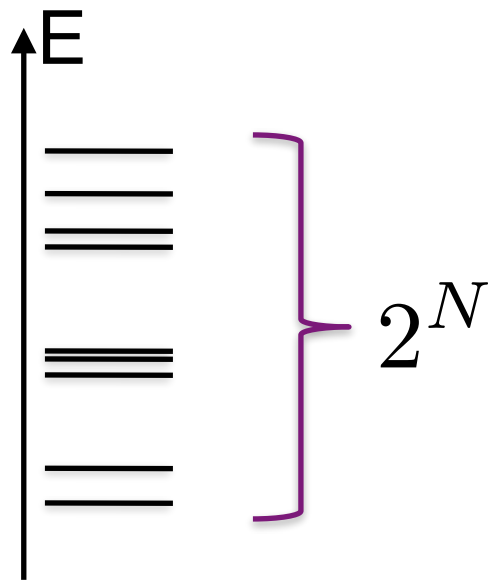
Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii (2008), Pal and Huse (2010), …

Many-body Level Statistics

ETH



MBL

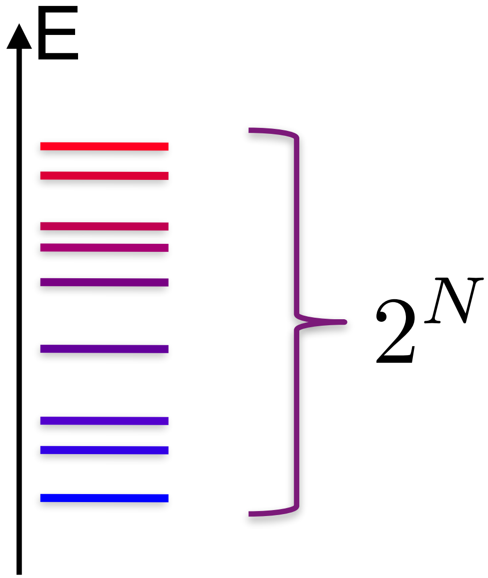


Spectrum exhibits level repulsion (GOE, etc)

Spectrum is Poisson

Eigenstate Chaos ('Ψ-Chaos'?)

ETH

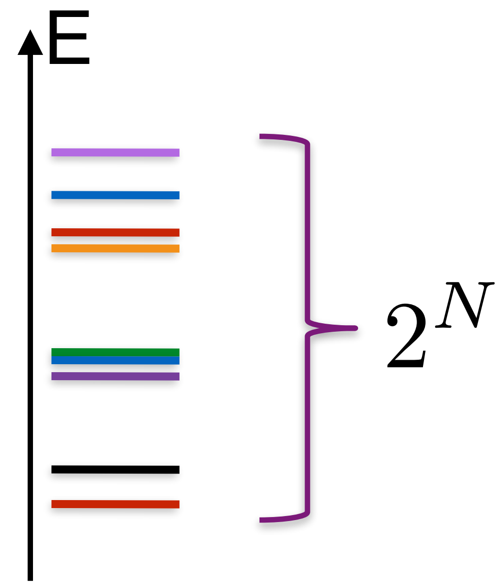


Local observables smooth

$$M(n) = \langle n|S_0^z|n\rangle = M(\epsilon_n)$$

$$\frac{\delta M(n)}{\delta n} \approx \frac{dM(\epsilon)}{d\epsilon}\frac{\delta \epsilon}{\delta n} \approx M'(\epsilon)e^{-Ns(\epsilon)}$$

MBL



Local observables frozen per eigenstate

$$\frac{\delta M}{\delta n} = \langle n+1|S_0^z|n+1\rangle - \langle n|S_0^z|n\rangle = O(1)$$

Looking for tractable models

- Analytically tractable ETH-MBL transitions/phases?
- Finite energy density mobility edges?
- Localization is ultimate glass — any connection?

Quantum Random Energy Model

$$H = E(\{\sigma_i^z\}) - \Gamma \sum_i \sigma_i^x$$

Classical Random Energy Model

"The simplest spin glass"

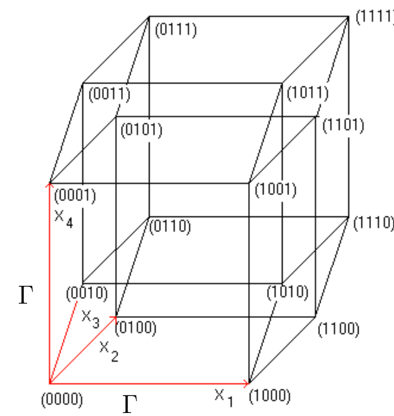
Random energy for each z-state

$$P(E) = \frac{1}{\sqrt{\pi N}} e^{-\frac{E^2}{N}}$$

N-body generalization of SK model

Transverse field

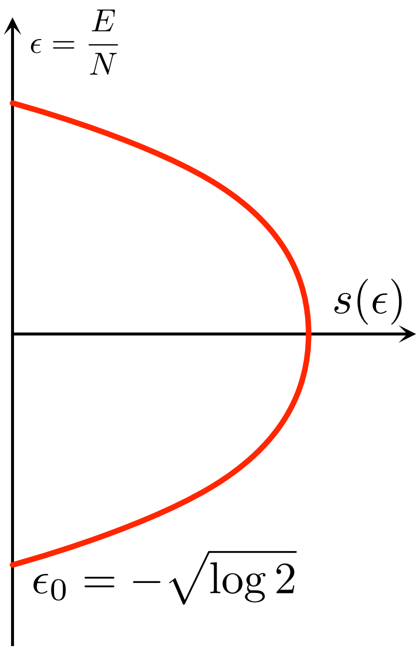
Provides dynamics



B. Derrida (1980,1981), Y. Goldschmidt (1990), T. Jorg, et al (2008), CRL, Pal, Scardicchio (2014), Baldwin, CRL, Pal, Scardicchio (2016)

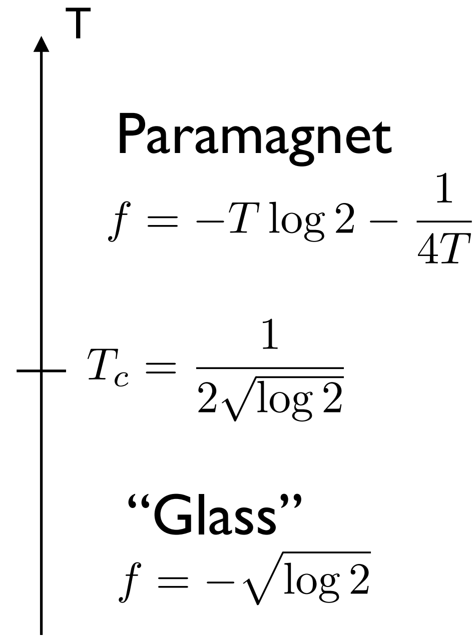
Classical Limit: Statistical Mechanics

Microcanonical



$$s(\epsilon) = \log 2 - \epsilon^2$$

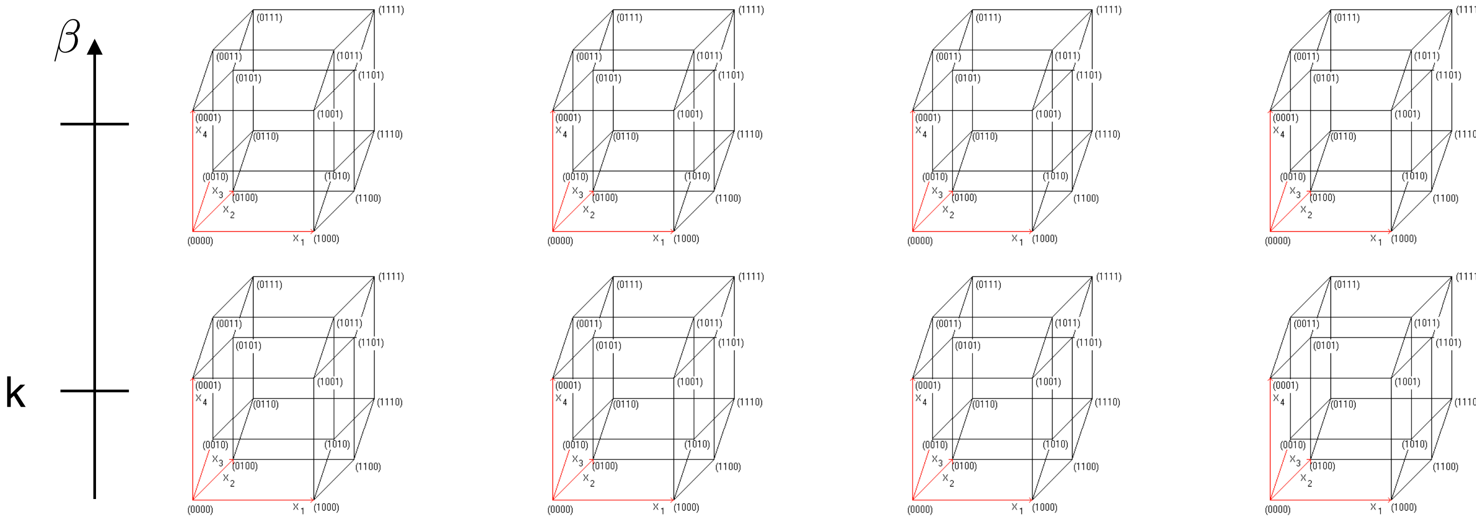
Canonical



$$\overline{n(E)} = 2^N P(E) \sim e^{N s(E/N)}$$

B. Derrida (1980)

Replica Solution of QREM

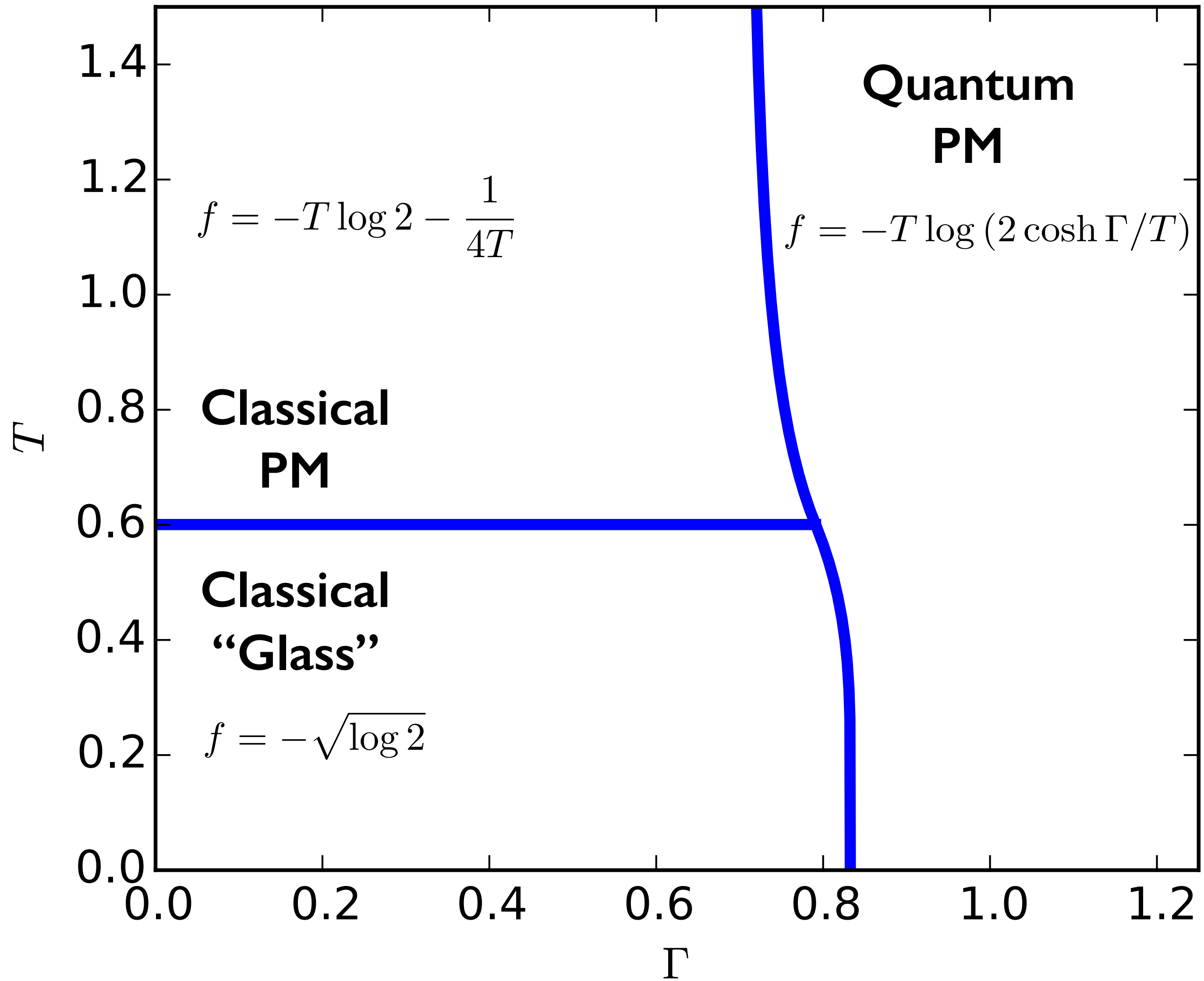


- Replica trick in imaginary time representation
- Time and replica dependent order parameters
- *Static* RS and 1RSB ansatzes give three phases…

$$Q_{kk'}^{\alpha\alpha'}(\sigma) = \frac{1}{N}\sum_i \sigma_i^\alpha(k)\sigma_i^\beta(k')$$

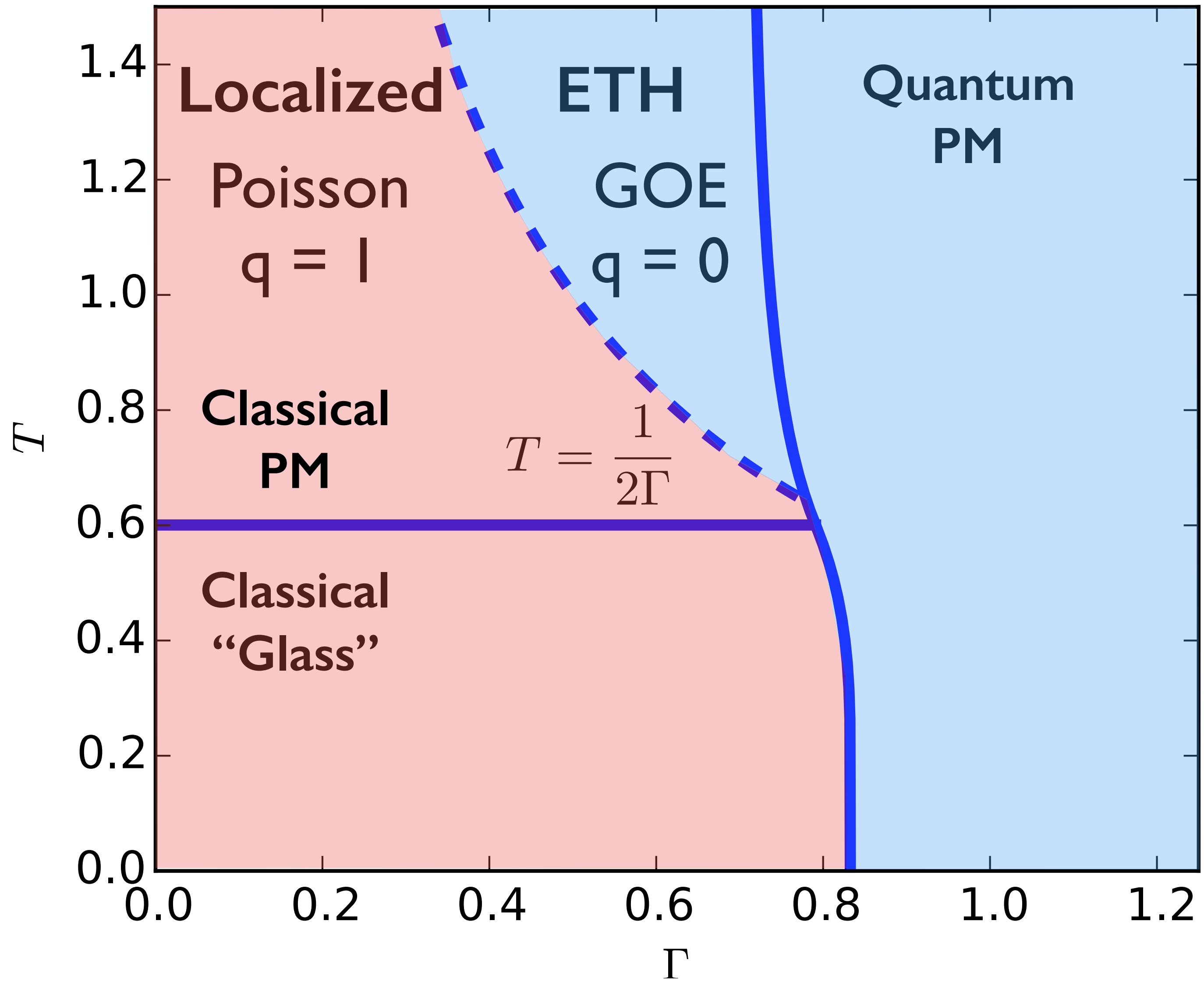
Y. Goldschmidt (1990)

Canonical Phase Diagram



Y. Goldschmidt (1990)

Dynamical Phase Diagram



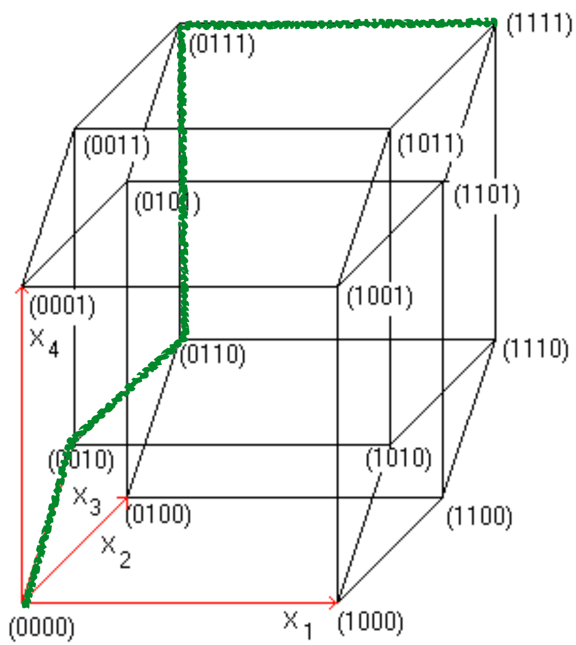
CRL, Pal, Scardicchio PRL 2014; Baldwin, CRL, Pal, Scardicchio PRB 2016

Forward Scattering

- Leading perturbative wavefunction `forward scattering'

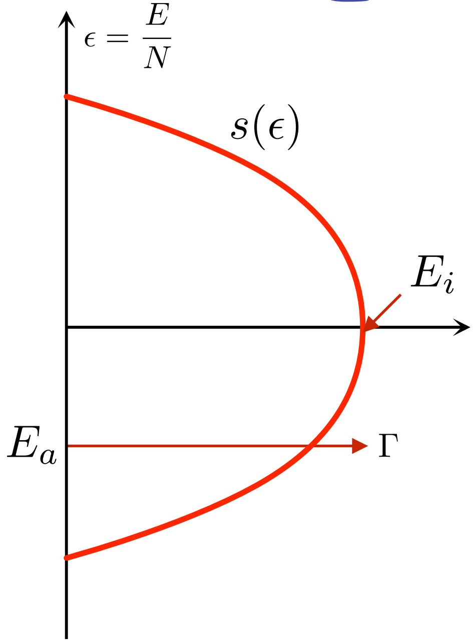
$$\psi_b = \sum_{\text{paths } p:a\to b} \prod_{i\in p} \frac{\Gamma}{E_a - E_i}$$

- Localization: amplitudes decay to system size n=N



- Directed random polymer on hypercube
- Amenable to numerical transfer matrix treatment
- Replica treatment of polymer problem identifies transitions as well

Extensive Energies



- Gap typically O(N) so expand

$$\psi_b = \sum_{\text{paths } p:a\to b} \prod_{i\in p} \frac{\Gamma}{E_a - E_i}$$

$$\psi_b \approx \left(\frac{\Gamma}{E_a}\right)^n \sum_p \left(1 + \sum_{i\in p} \frac{E_i}{E_a} + \cdots\right)$$

- Fluctuations small in N

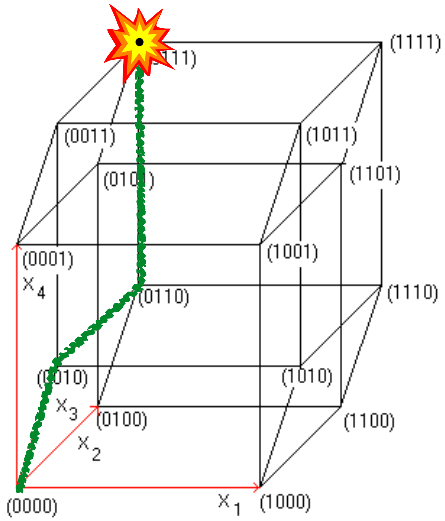
$$\overline{\psi_b} \approx n! \left(\frac{\Gamma}{E_a}\right)^n \qquad \frac{\delta\psi_b}{\psi_b} \sim \frac{1}{\sqrt{N}}$$

- Demand typical amplitude decreasing uniformly to n = N

$$\Gamma_c = \epsilon_a$$

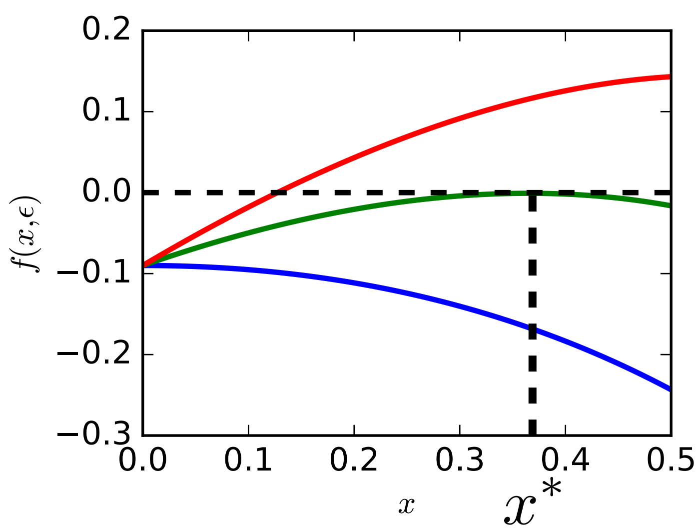
- Paths eventually resonate: can we do better?

Proliferation of Resonances



- SRA: Paths typical till resonance at x
- Expected number of resonances $e^{Nf(x,\epsilon)}$
- Entropy function

$$f(x,\epsilon) \equiv x \ln \frac{x\Gamma}{e\epsilon} - \epsilon^2 - x \ln x - (1-x)\ln(1-x)$$



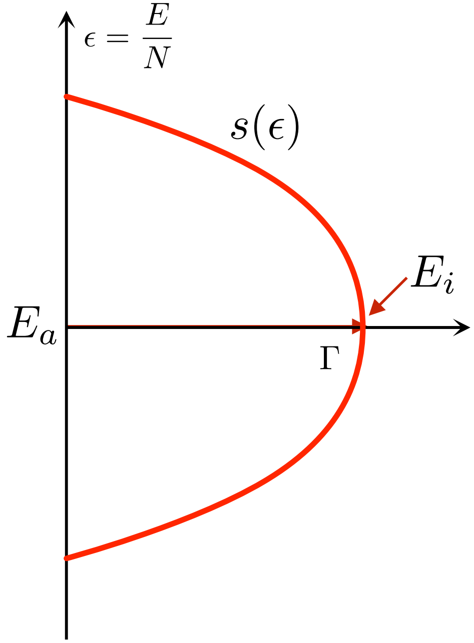
- Resonances proliferate beyond

$$\epsilon_c = -\left(\Gamma - \sqrt{2}\Gamma^2 + O(\Gamma^3)\right)$$

- At distance

$$x^* = \sqrt{2}\Gamma + O(\Gamma^2)$$

Central Energies



- Gap typically $\sqrt{N}$, fluctuations large

$$\psi_b = \sum_{\text{paths } p: a \to b} \prod_{i \in p} \frac{\Gamma}{E_a - E_i}$$

- Bound by greedy path at distance n

$$\psi_g \sim \left(\frac{\Gamma N}{\sqrt{N}}\right)\left(\frac{\Gamma(N-1)}{\sqrt{N}}\right)\cdots\left(\frac{\Gamma \cdot 1}{\sqrt{N}}\right) \sim (\Gamma\sqrt{N})^n$$

- Demand amplitudes small gives upper bound on delocalization

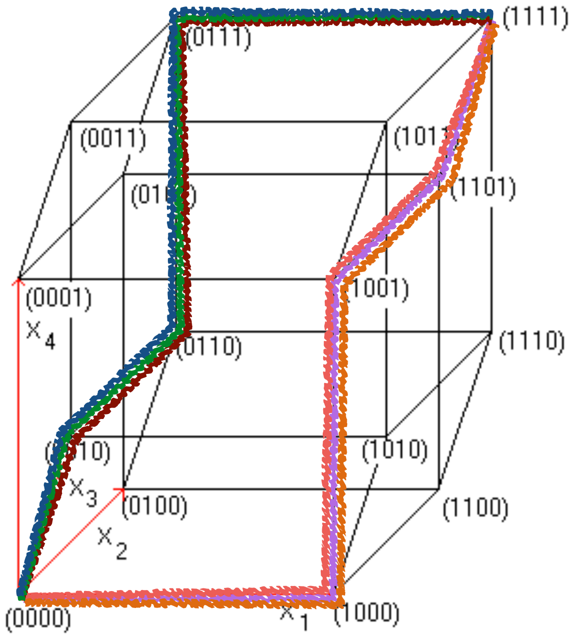
$$\Gamma_c < \frac{1}{\sqrt{N}}$$

- Counting resonances more carefully gives log correction

Replicated Polymers

- Typical amplitudes given by quenched average

$$\overline{\ln \psi} = \lim_{n \to 0} \frac{\overline{\psi^n} - 1}{n}$$



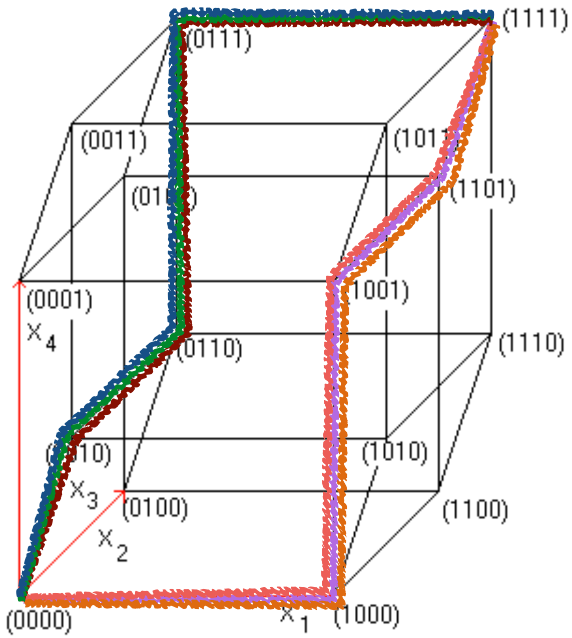
- n interacting paths on hypercube

$$\overline{\psi^n} = \sum_{p_1 \cdots p_n} \prod_i \overline{w_i^{r_i(p_1 \cdots p_n)}}$$

$$r_i = \sum_{a=1}^{n} \mathbf{1}\left[i \in p_a\right]$$

Replicated Polymers

- 1RSB Ansatz: polymers clump in n/x groups of x paths



$$\overline{Z^n} \approx \left(\sum_p \prod_{i \in p} \overline{w_i^x} \right)^{n/x} = \exp\left[n f(x) \right]$$

$$f(x) = \frac{L}{x} (\log L - 1 + \log \overline{w_i^x})$$

$$\overline{w^x} = \int \frac{dE}{\sqrt{\pi N}} e^{-E^2/N} \left(\frac{\Gamma}{|E_a - E|} \right)^x$$

Finite-size form at zero energy

- Replica recipe: $f_{1RSB} = \min_{x \in [0,1]} f(x)$

$$f(x) = \frac{L}{x}(\log L - 1 + \log \overline{w_i^x})$$

$$\overline{w^x} = \int \frac{dE}{\sqrt{\pi N}} e^{-E^2/N} \left(\frac{\Gamma}{|E_a - E|} \right)^x$$

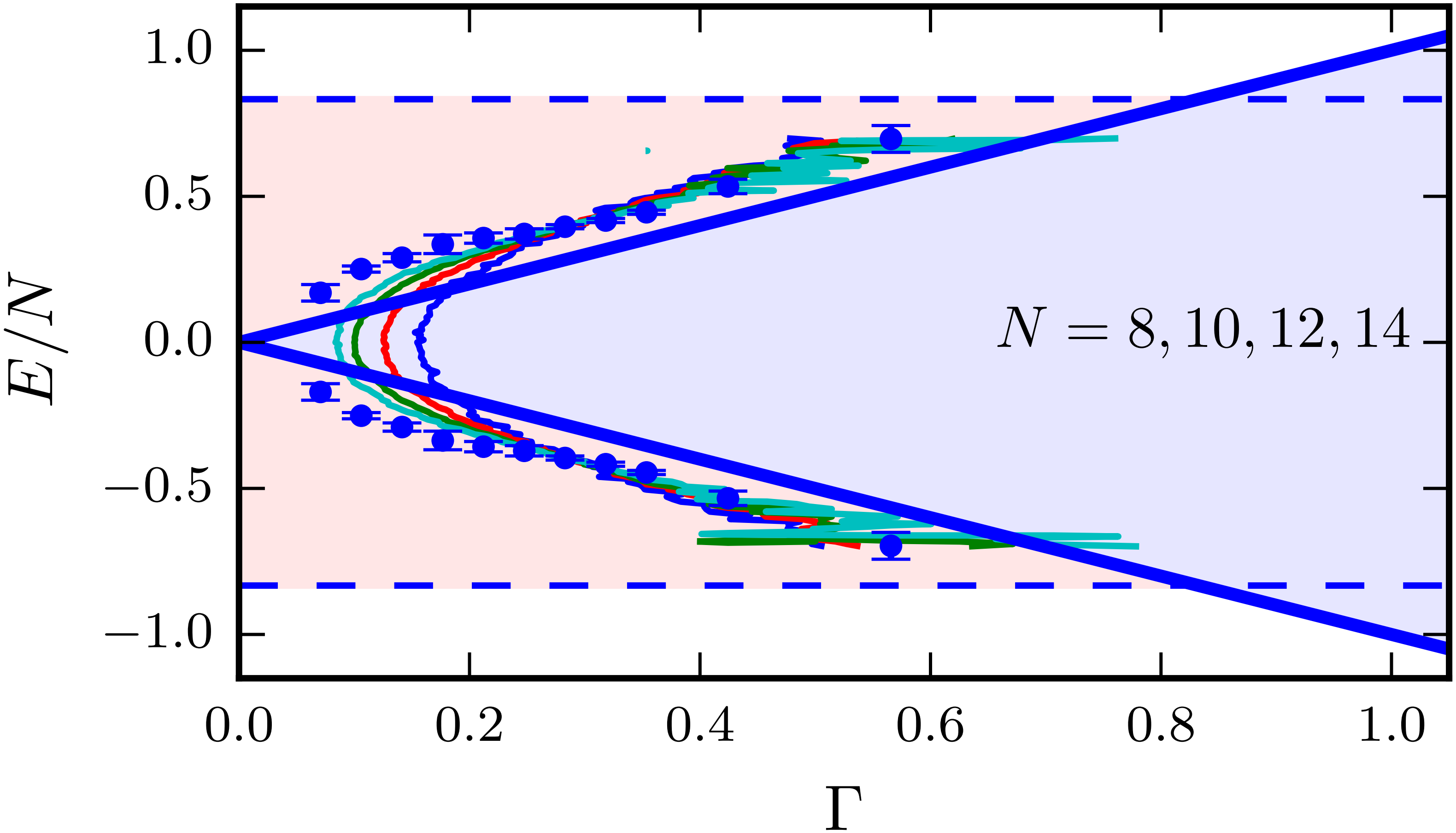
- 1RSB holds at finite L but is swamped slowly by L! paths

$$x^* = 1 - \frac{1}{\log \sqrt{\frac{2}{\pi} L}} + O(\log \log L / \log^2 L)$$

- Demanding amplitude decays at size L = N

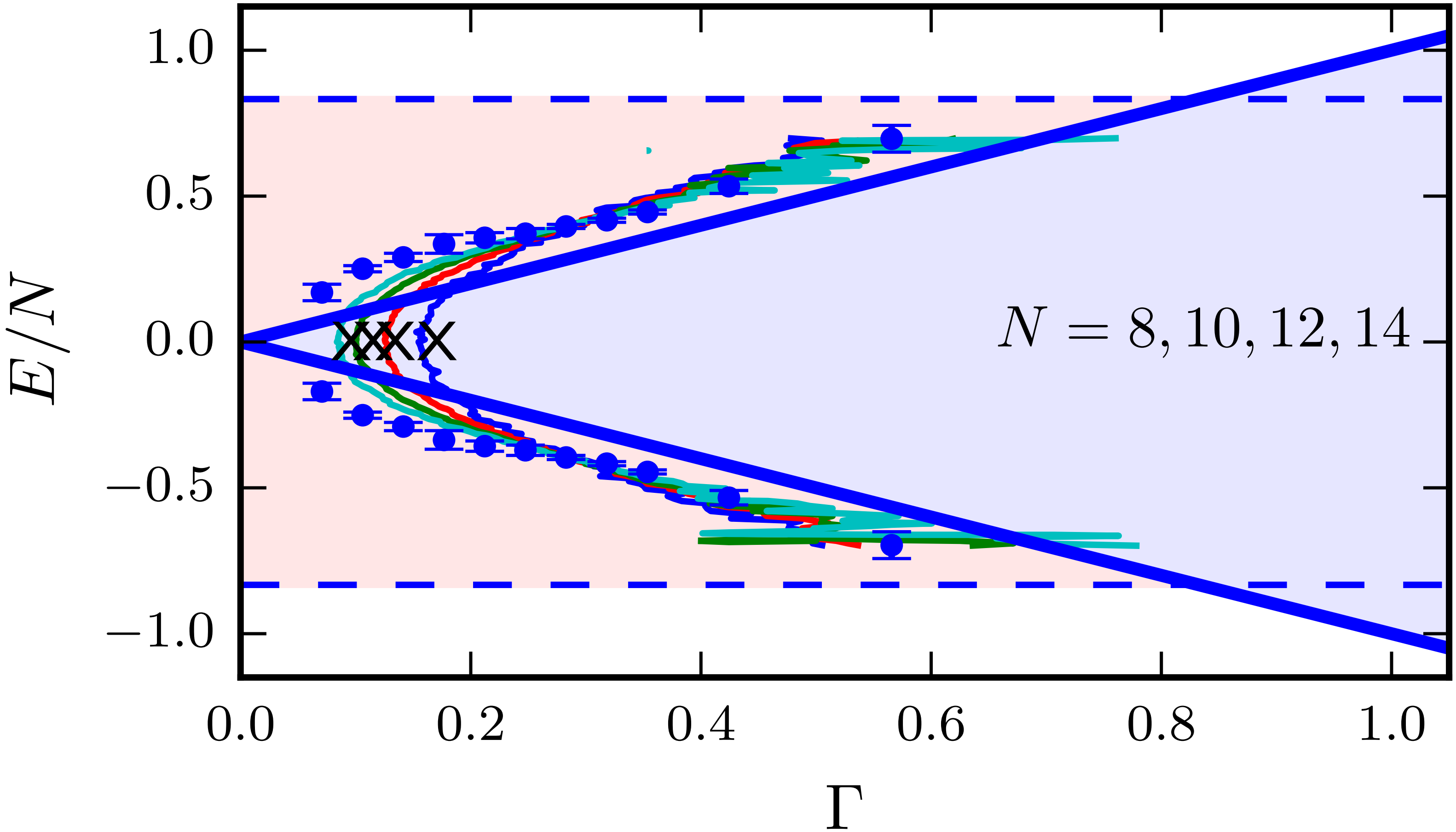
$$\Gamma_c = \frac{\sqrt{\pi}}{2\sqrt{N} \log \sqrt{2/\pi N}} + \cdots$$

Numerical Phase Diagram



Contours of level statistics ratio $[r] = 0.48$

Numerical Phase Diagram



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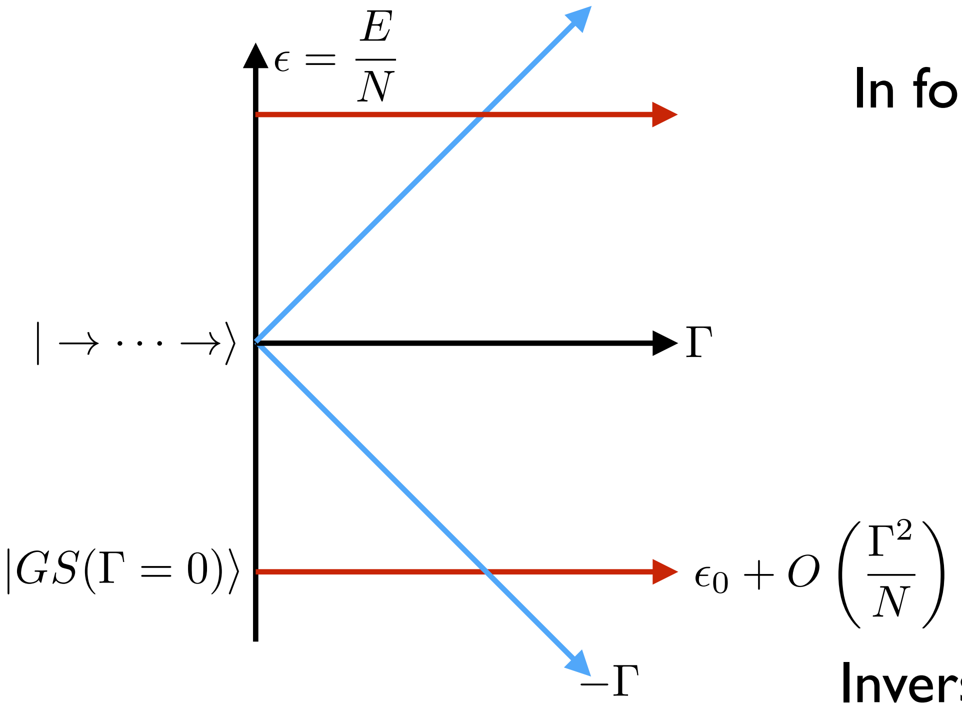
X marks the replica formula estimate

Perturbative Rigidity



In forward scattering:

$$E_0(\Gamma) = E_0 - \Gamma^2 \sum_{i=1}^{N} \frac{1}{E_i - E_0} + \cdots$$

$$\approx E_0 - \Gamma^2 \frac{1}{\sqrt{2\log(2)}}$$

O(N) O(1)

Inverse Participation Ratio:

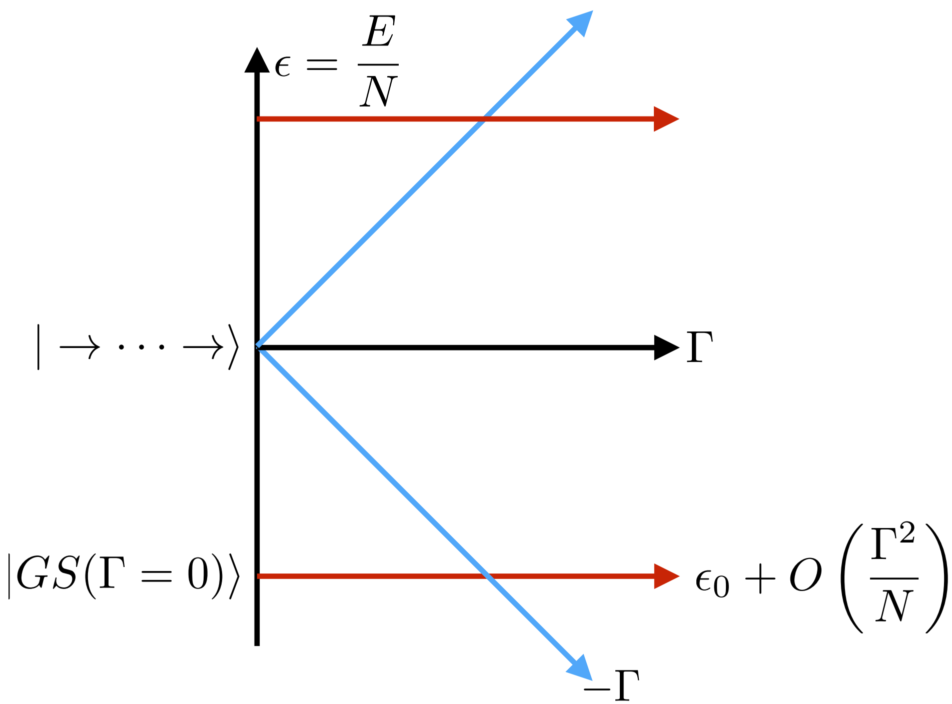
$$\text{IPR} = 1 - O(1/N)$$

- All orders give O(1) corrections to extensive energies
- States localized strongly to corners of hypercube
- Cross with paramagnetic state at boundary

T. Jorg et al (2008); Baldwin, CRL, Pal, Scardicchio (2016)

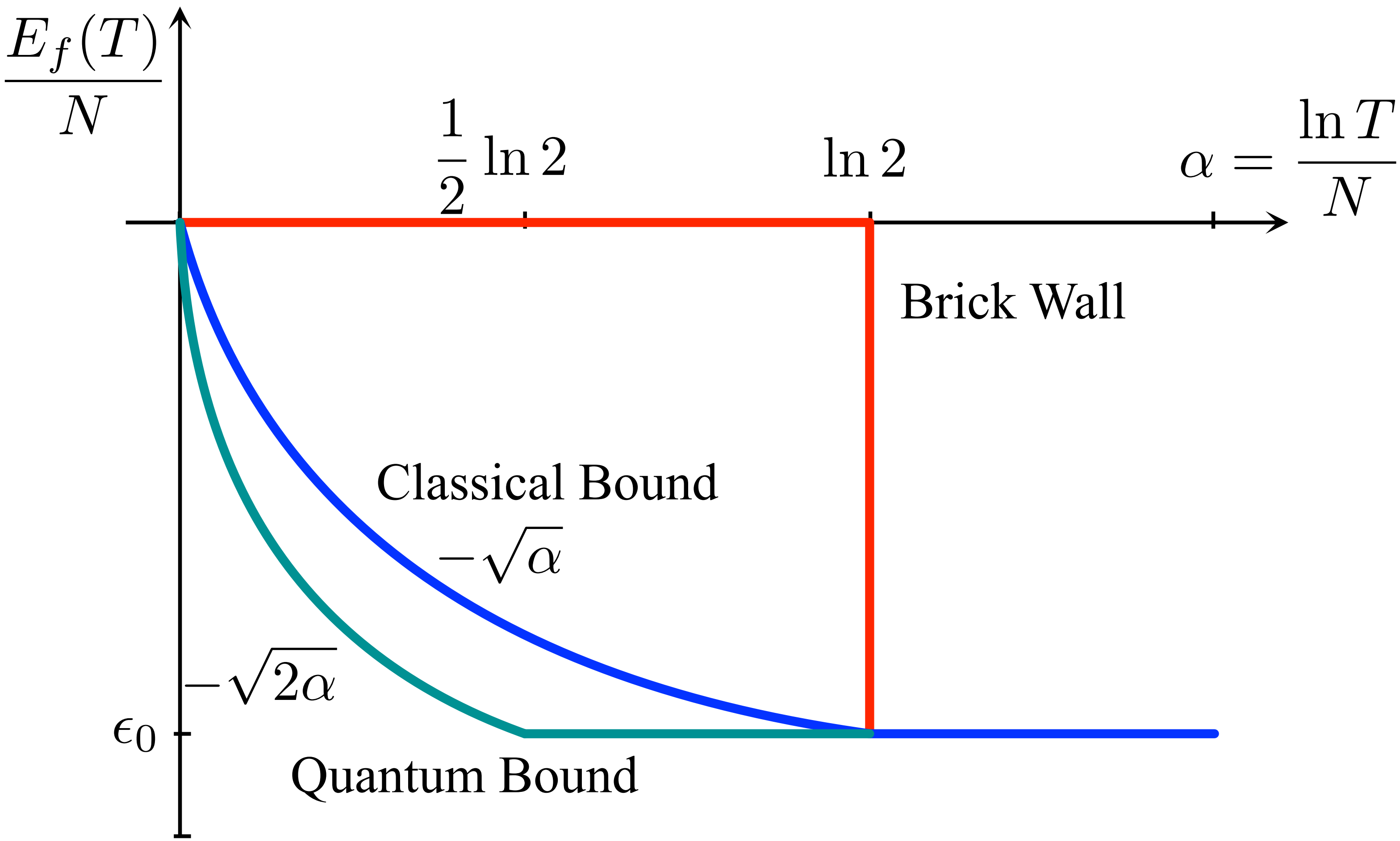
Quantum Annealing

- Annealing transverse field — ground state search
- Final finite energy density — ‘approximate’ search
- Scaling of final energy density with time — how hard is approximation?



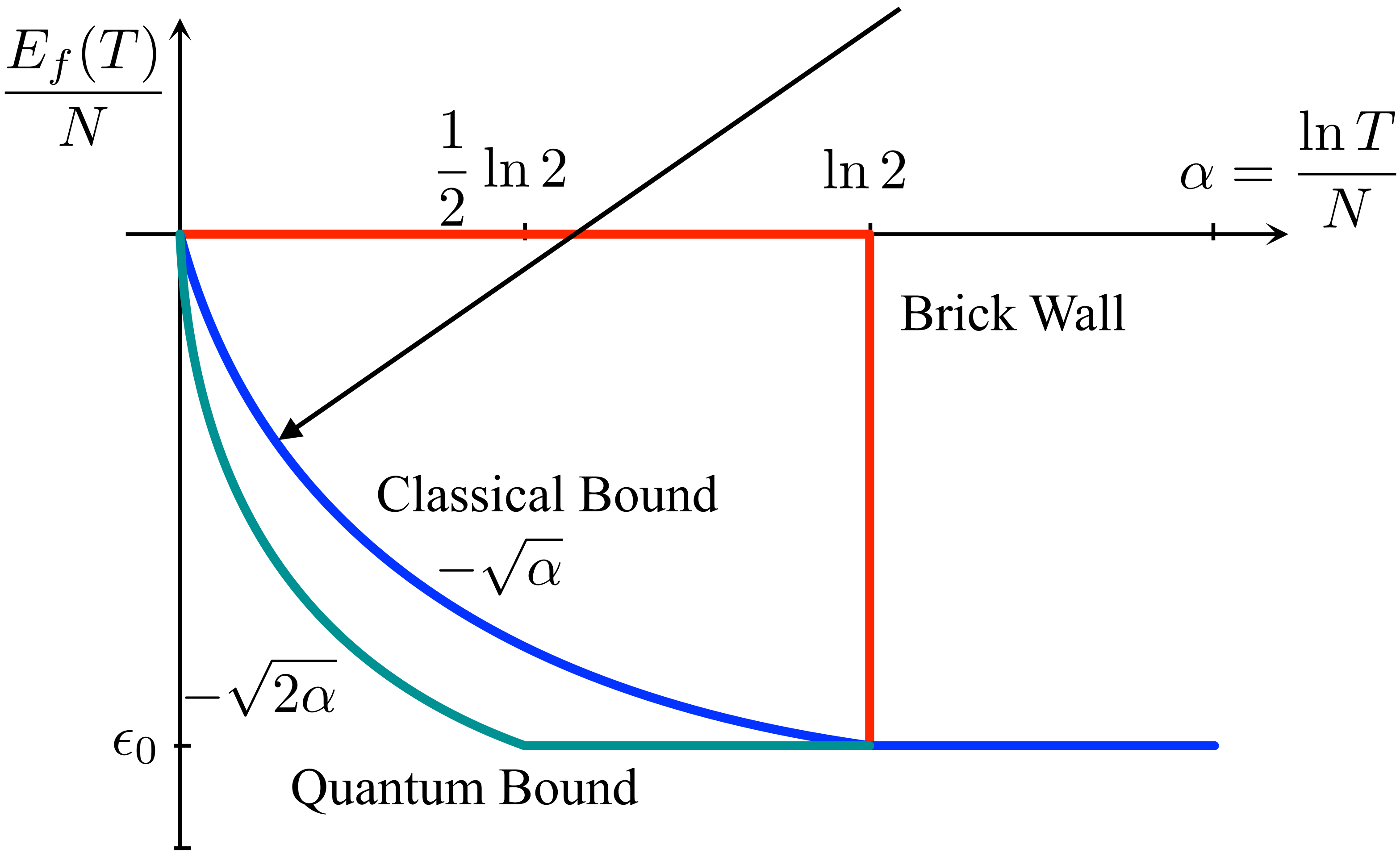
Annealing the QREM

- Unstructured cost function — exponential lower bounds



Annealing the QREM

- Linear ramping — iterated Landau-Zener problem



A bit more structure?

- Random energy model is completely non-local
- More local (/ less tractable?) model?

Quantum p-Spin Glass

$$H_p = -\sum_{(i_1\cdots i_p)} J_{i_1\ldots i_p}\hat{\sigma}^z_{i_1}\cdots\hat{\sigma}^z_{i_p} - \Gamma\sum_i \hat{\sigma}^x_i .$$

$$P(J_{i_1\ldots i_p}) = \sqrt{\frac{N^{p-1}}{\pi p!}}\, e^{-\frac{N^{p-1}}{p!}J^2_{i_1\cdots i_p}} .$$

Classical p-Spin Glass

"The (next) simplest spin glass"

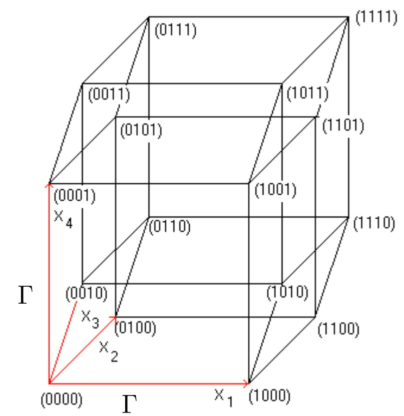
Recovers REM as $p \to \infty$

p-body generalization of SK model

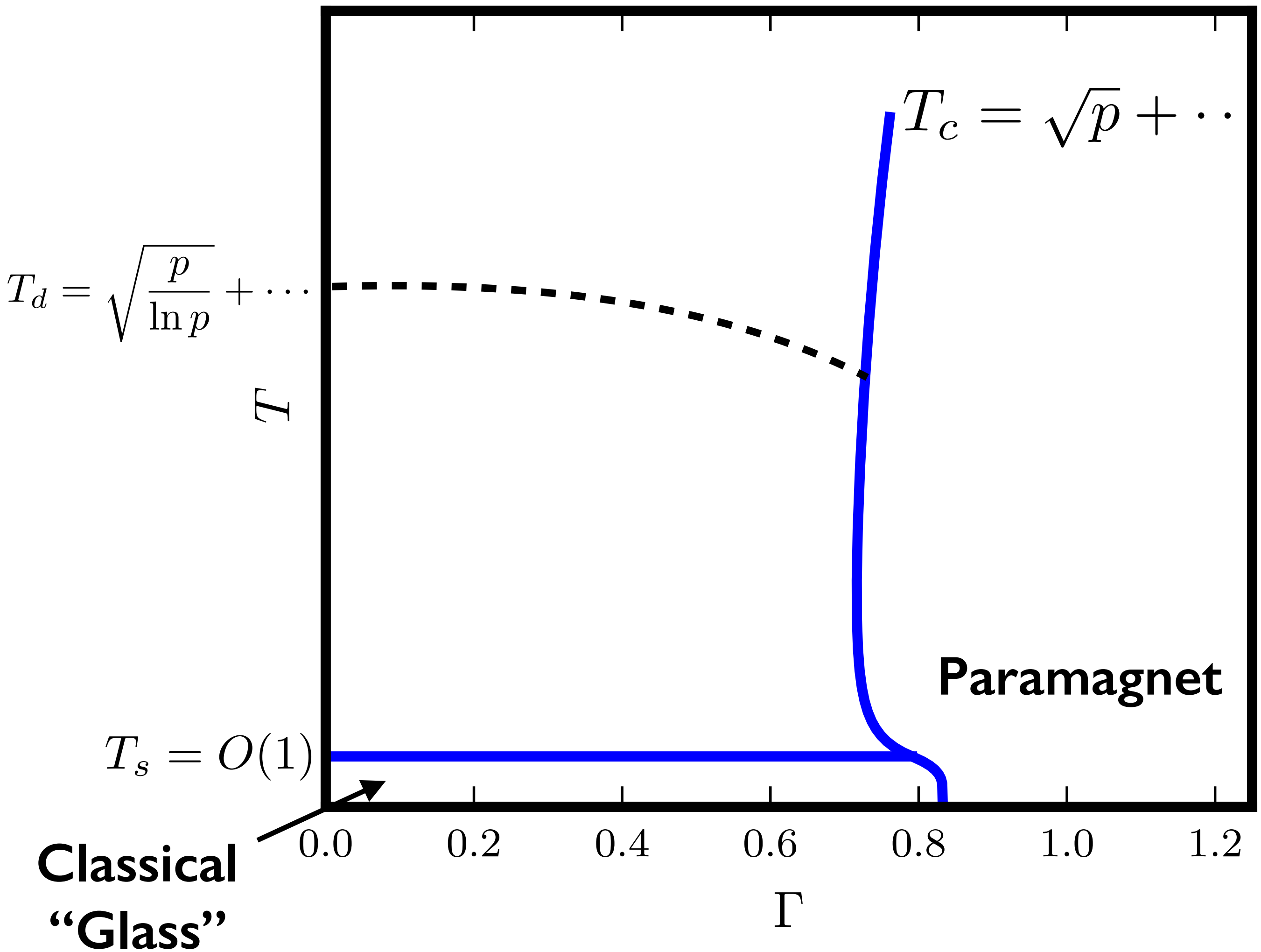
Typical fields O(p)

Transverse field

Provides dynamics



Canonical Phase Diagram

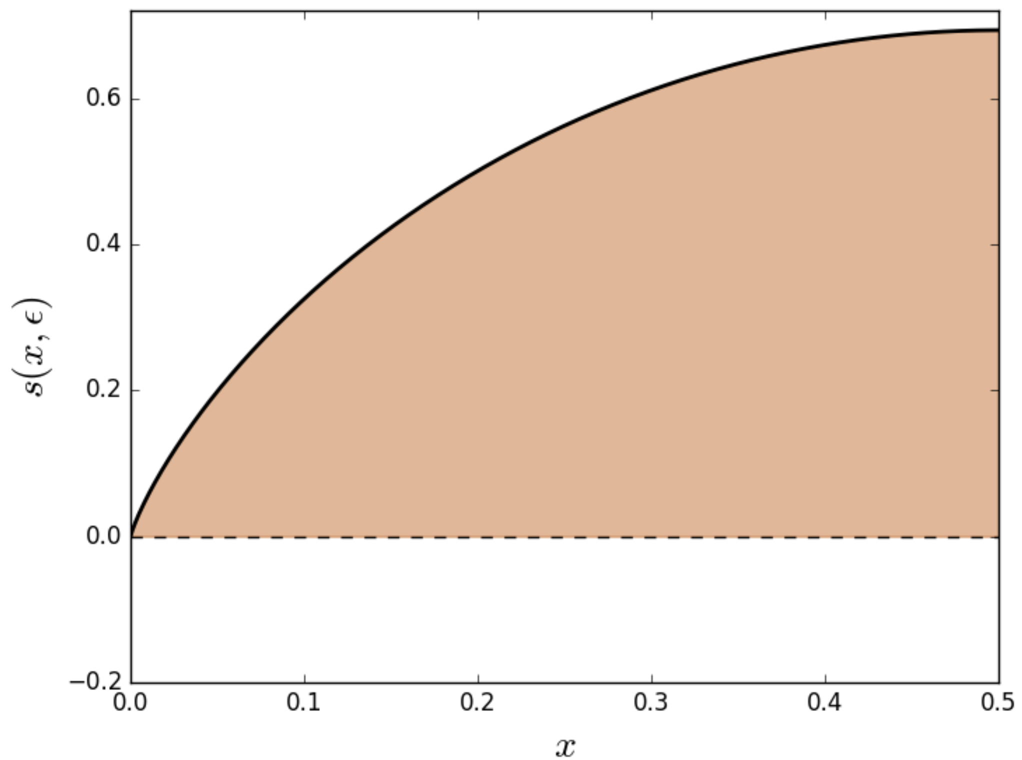


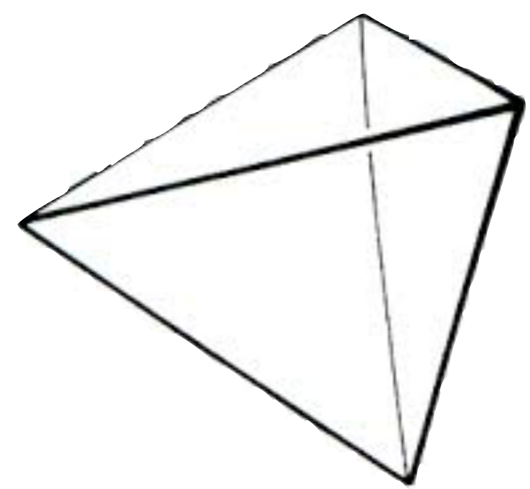
Y. Goldschmidt (1990); Th. M. Nieuwenhuizen, F. Ritort (1998)

Classical Clustering

- Entropy of states at distance x with energy matching origin

$$s(x,\epsilon) = -x\ln x - (1-x)\ln(1-x) - \frac{1-(1-2x)^p}{1+(1-2x)^p}\epsilon^2$$





Baldwin, CRL, Pal, Scardicchio (soon)

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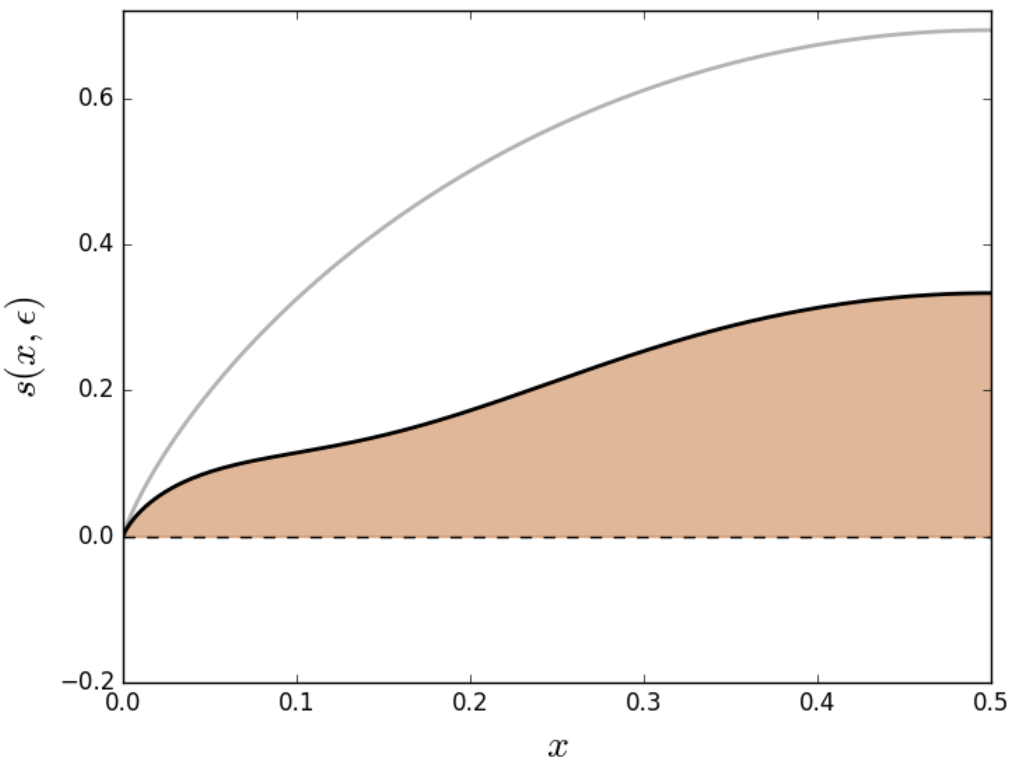

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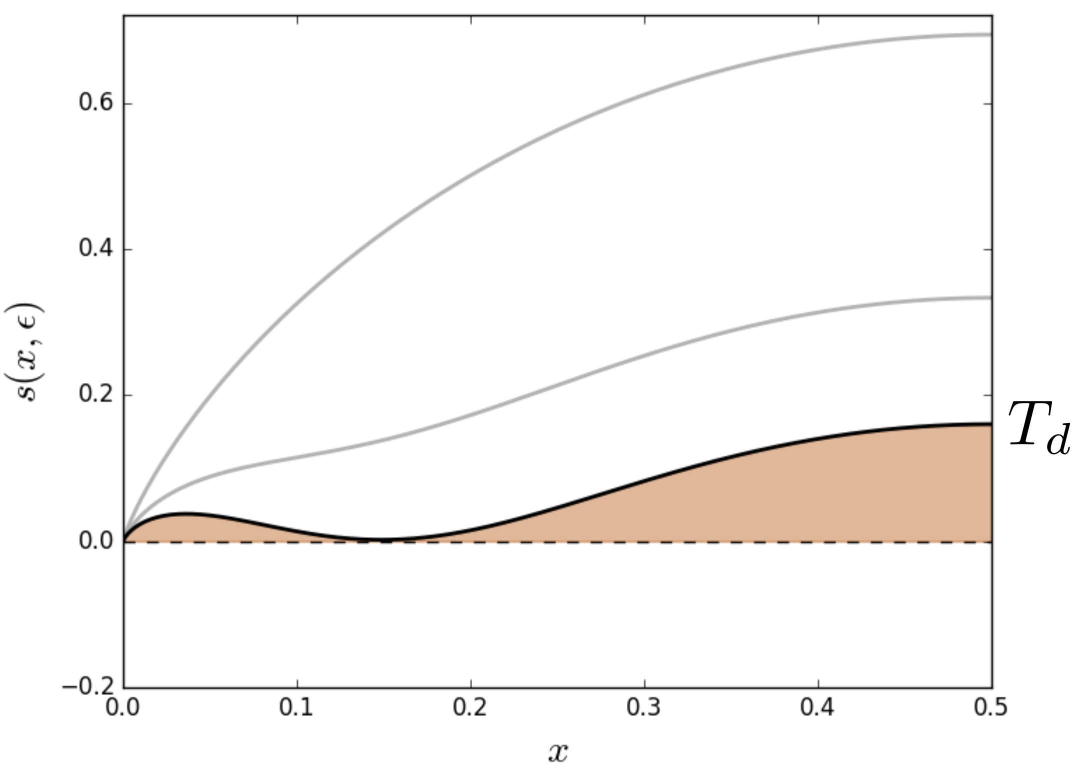

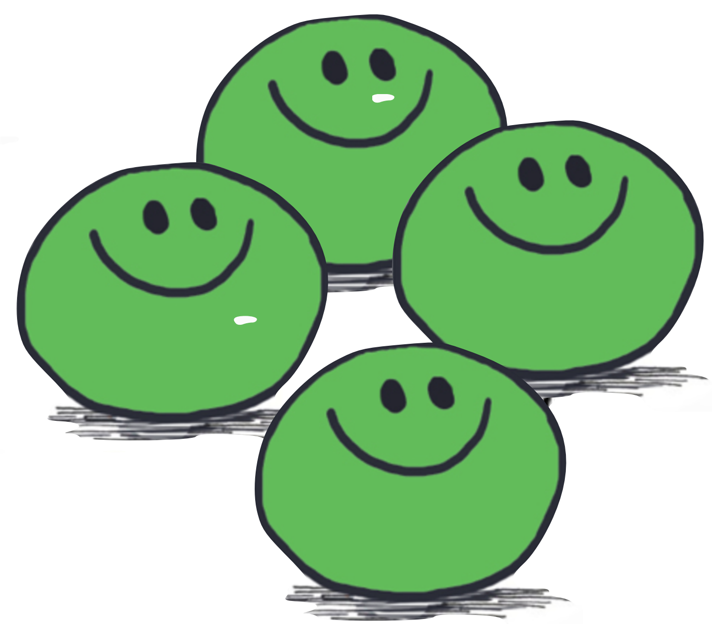

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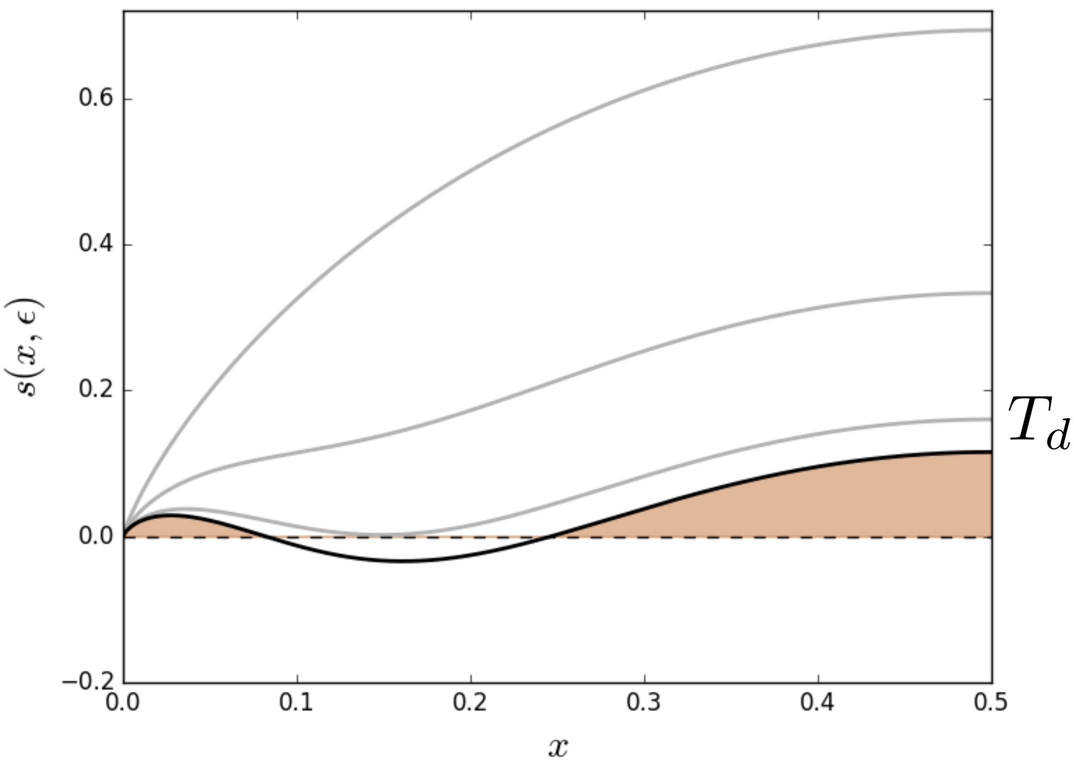

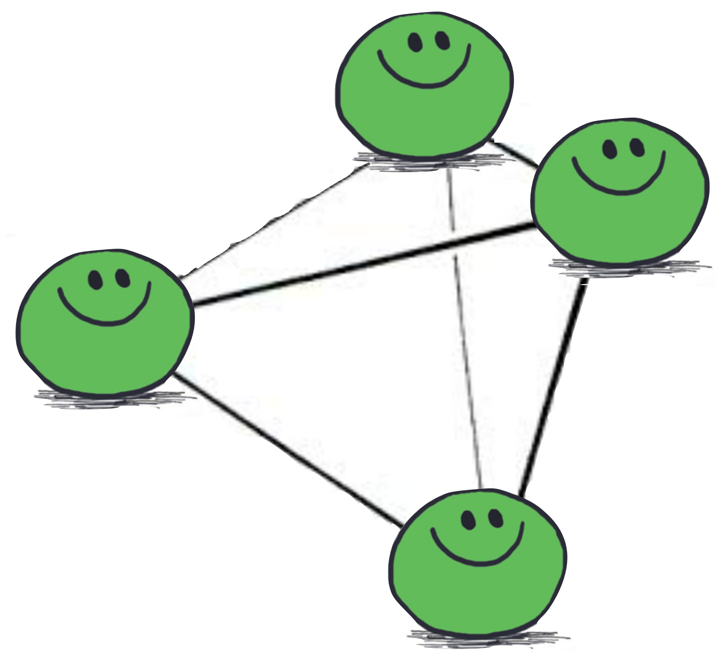

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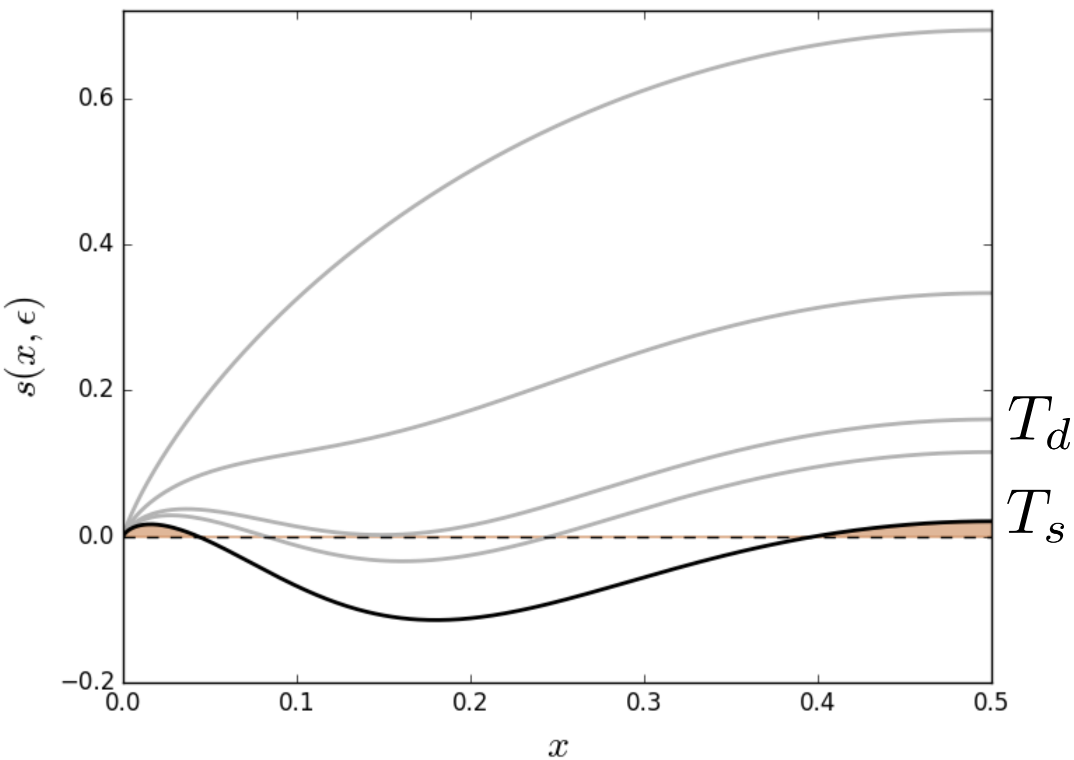
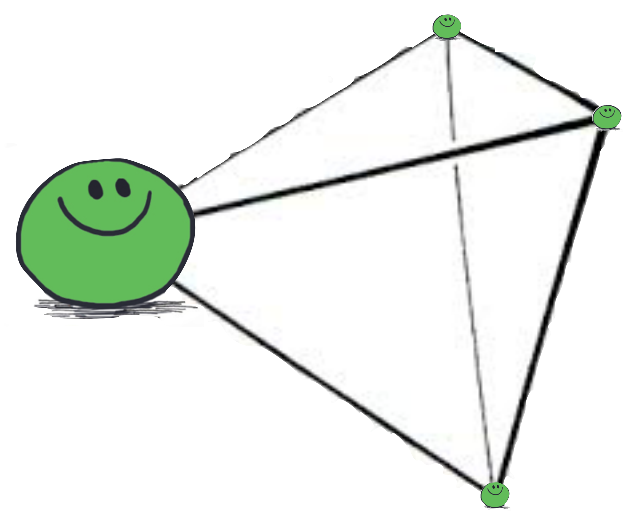
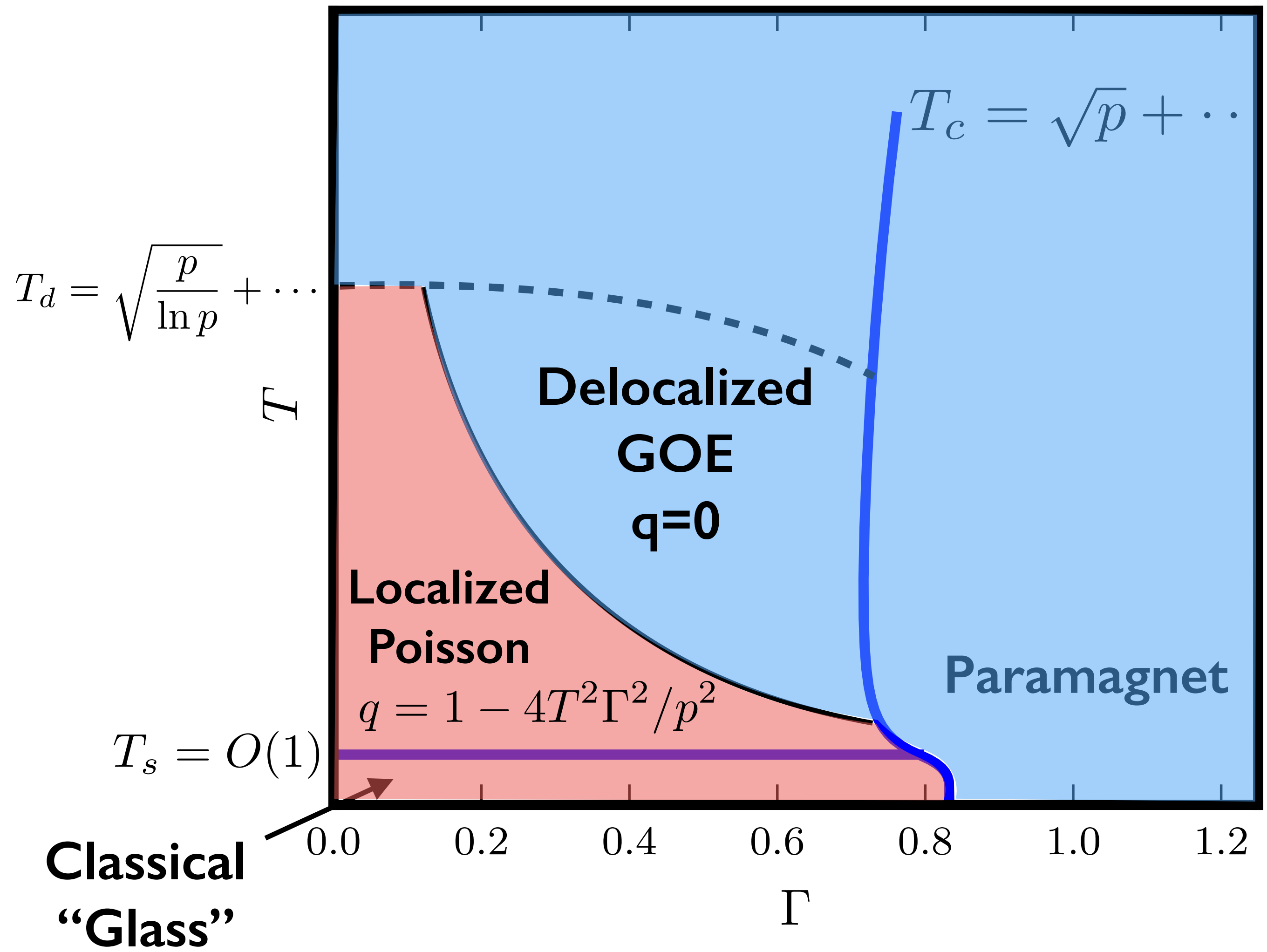




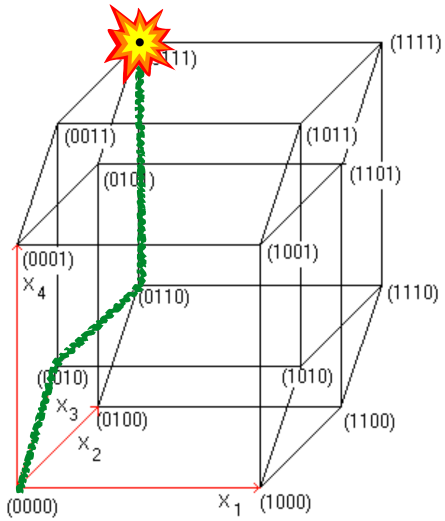
Baldwin, CRL, Pal, Scardicchio (soon)

Dynamical Phase Diagram

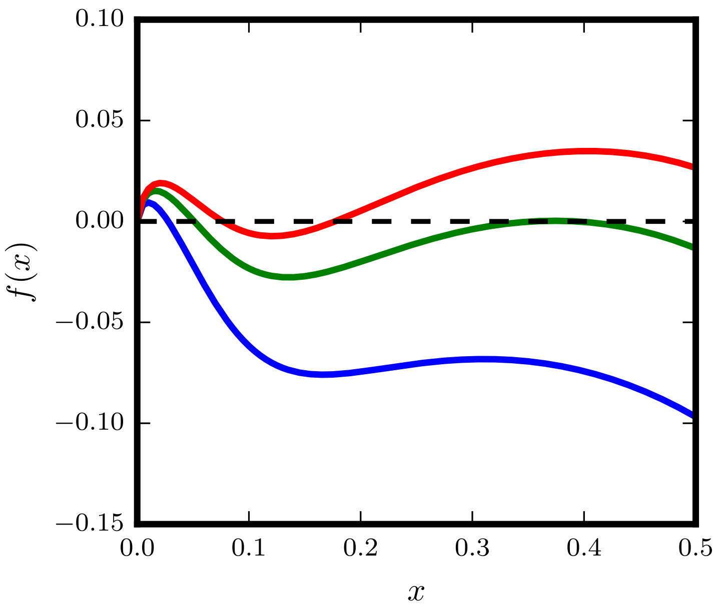


Baldwin, CRL, Pal, Scardicchio (soon)

Proliferation of Resonances

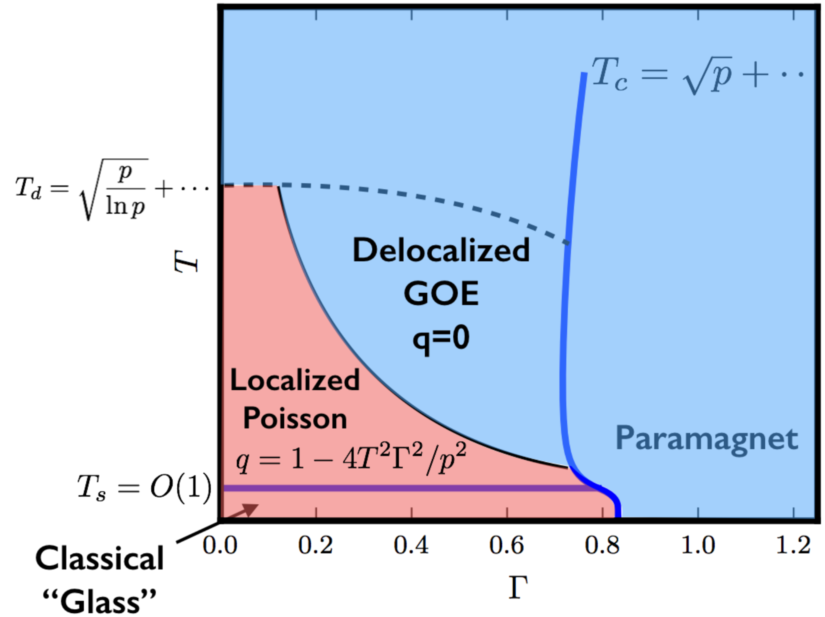


- SRA: Paths typical till resonance at x
- Neglect initial cluster; ok at low temperature
- Underestimate resonance proliferation
- Gives O(1/p) correction to phase boundary



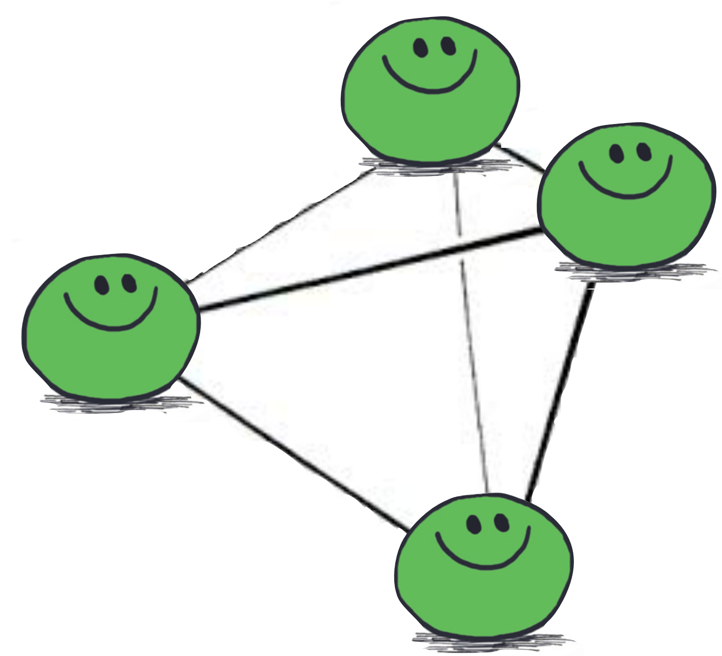
Summary

The p-Spin and QREM provides a 'mean-field' model of MBL, and the MBL-ETH transition at finite energy density mobility edge. First order eigenstate transition.



Perturbative treatment in forward approximation — directed random polymer on hypercube.

De-localization transition inside canonical 'paramagnetic' phase, but below T_d.



Annealing the QREM is hard. What about p-Spin?

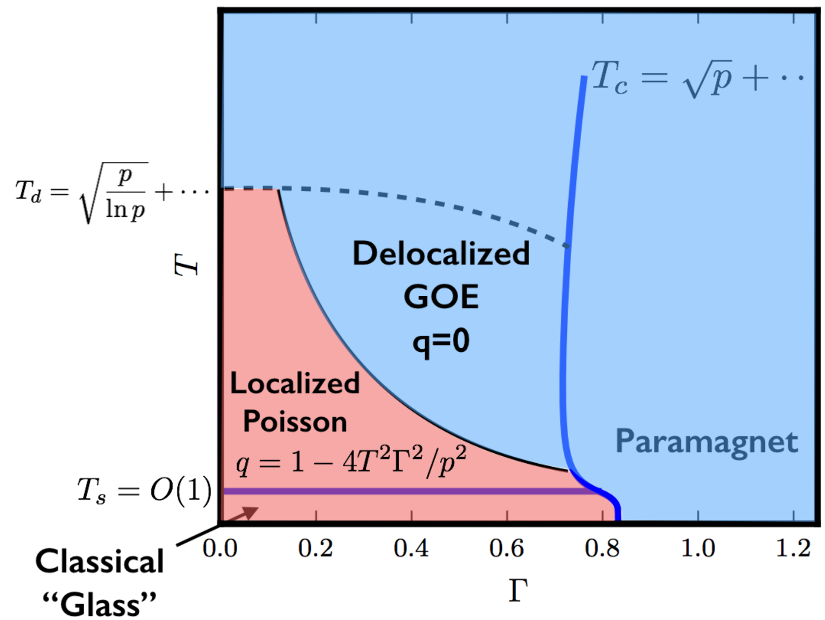
Open Questions

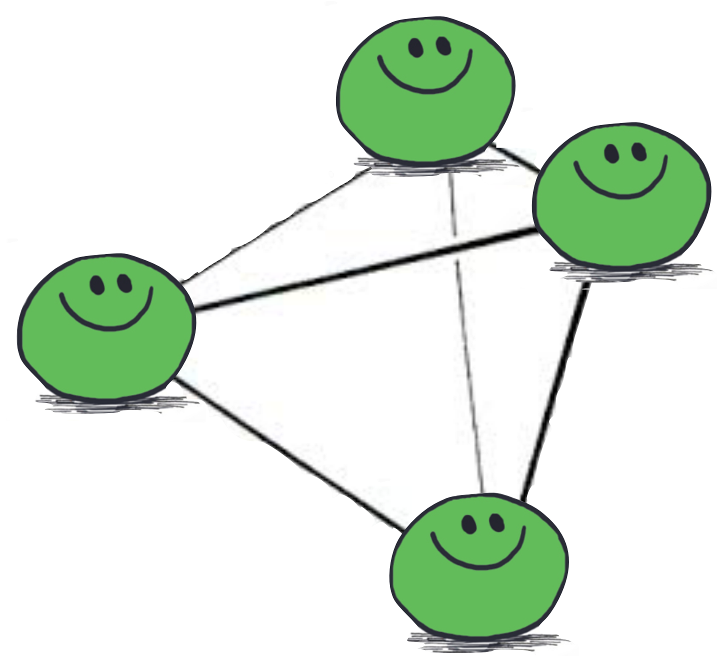
Complete analytic solution of p-Spin/ QREM?

Do thermodynamics reflect dynamical transition? (Existing canonical phase diagrams perhaps not complete.)

No infinite temperature MBL — feature of long-range interactions?

Expected outcome for approximate quantum annealing in p-Spin? Grover speed up somehow?





Many-body Level Statistics

- Level statistics diagnose dynamical phase transition
- Ratio diagnostic cancels DOS fluctuations

$$r_\alpha^{(n)} = \min\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\} / \max\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\}$$

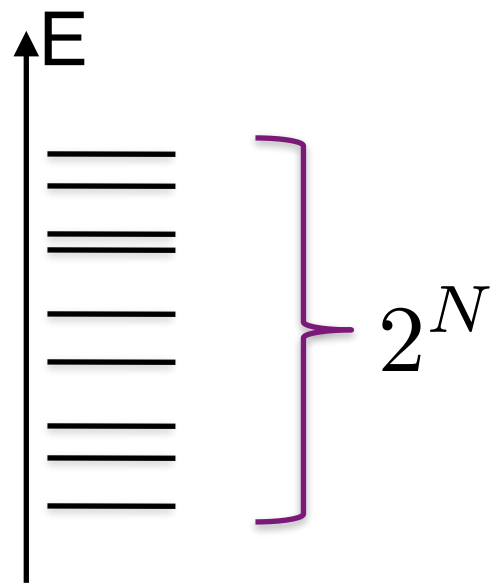
$$\delta_\alpha^{(n)} = |E_\alpha^{(n)} - E_\alpha^{(n-1)}|$$

MBL: Poisson level statistics

$$[r] \approx 0.39$$

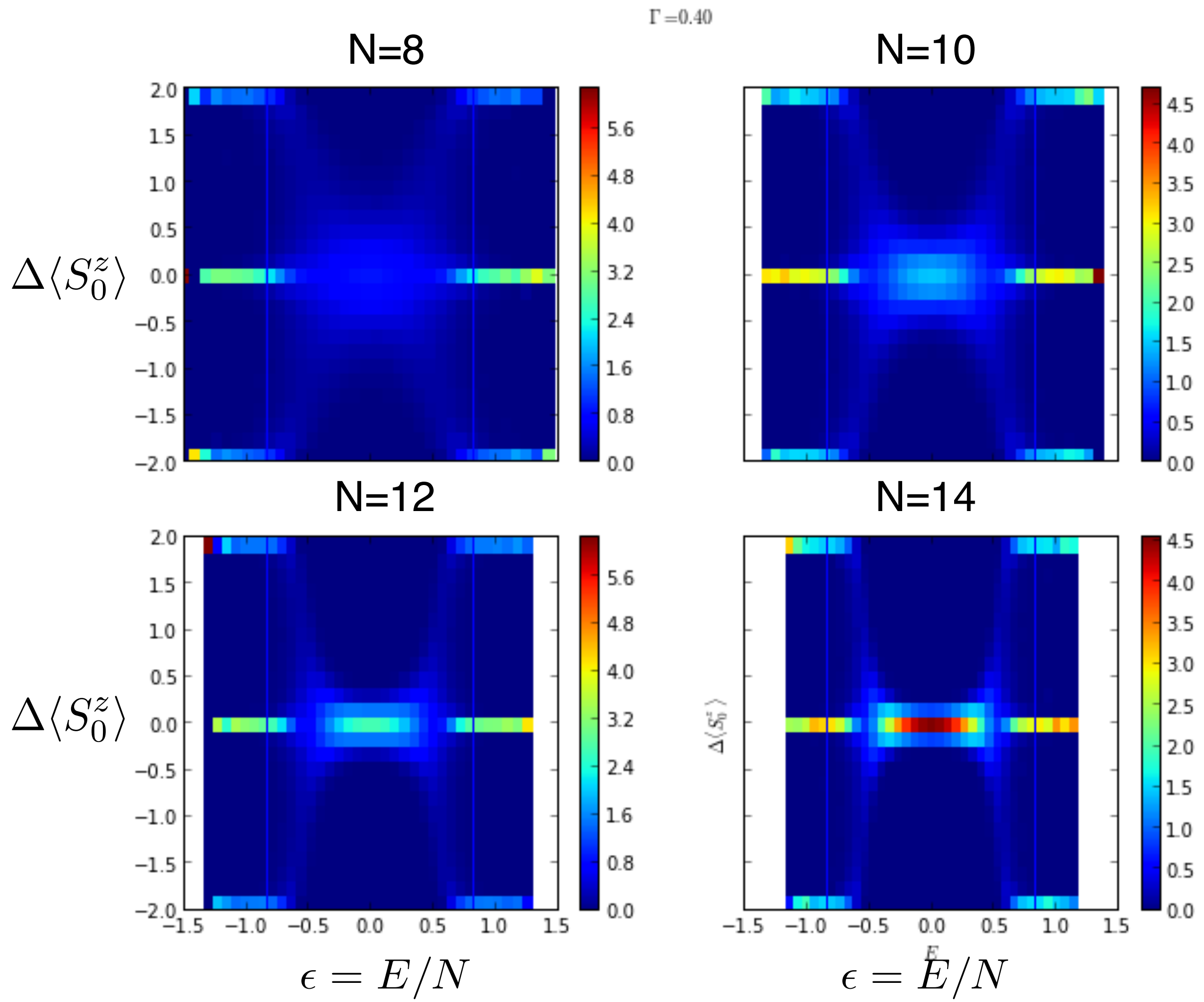
ETH: GOE level statistics

$$[r] \approx 0.53$$

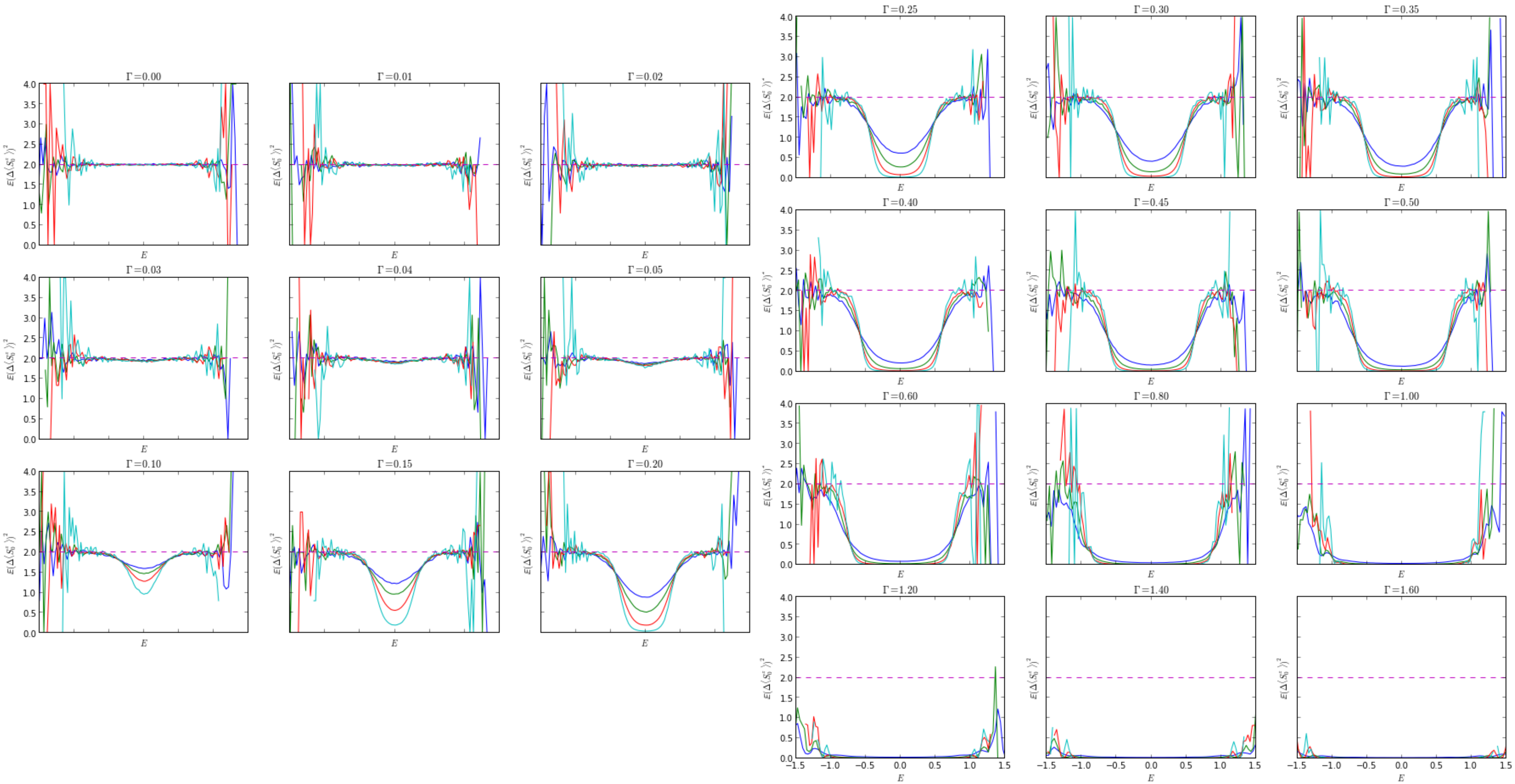


Spider diagrams

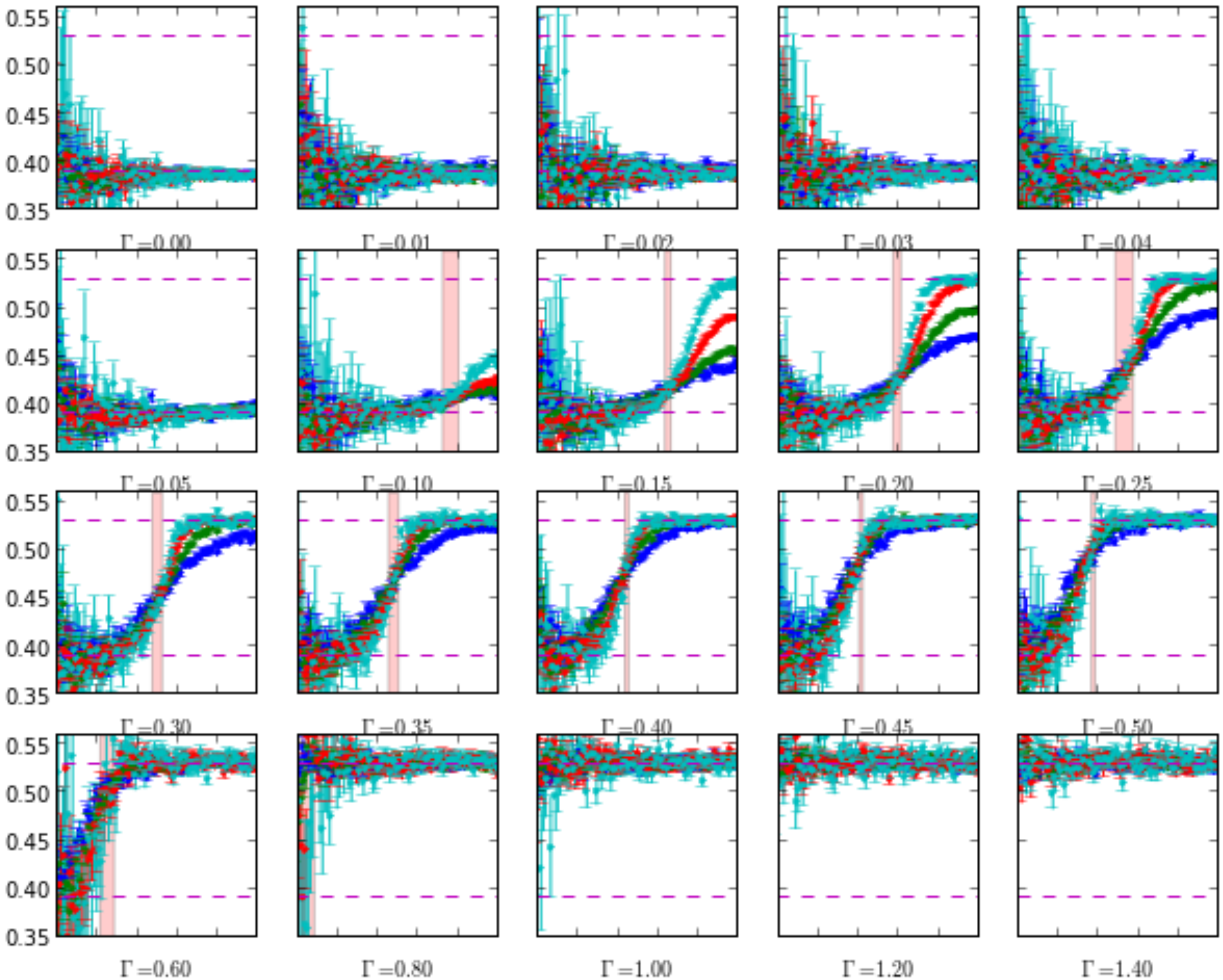
Histogram of delta Z-Magnetization across eigenstates



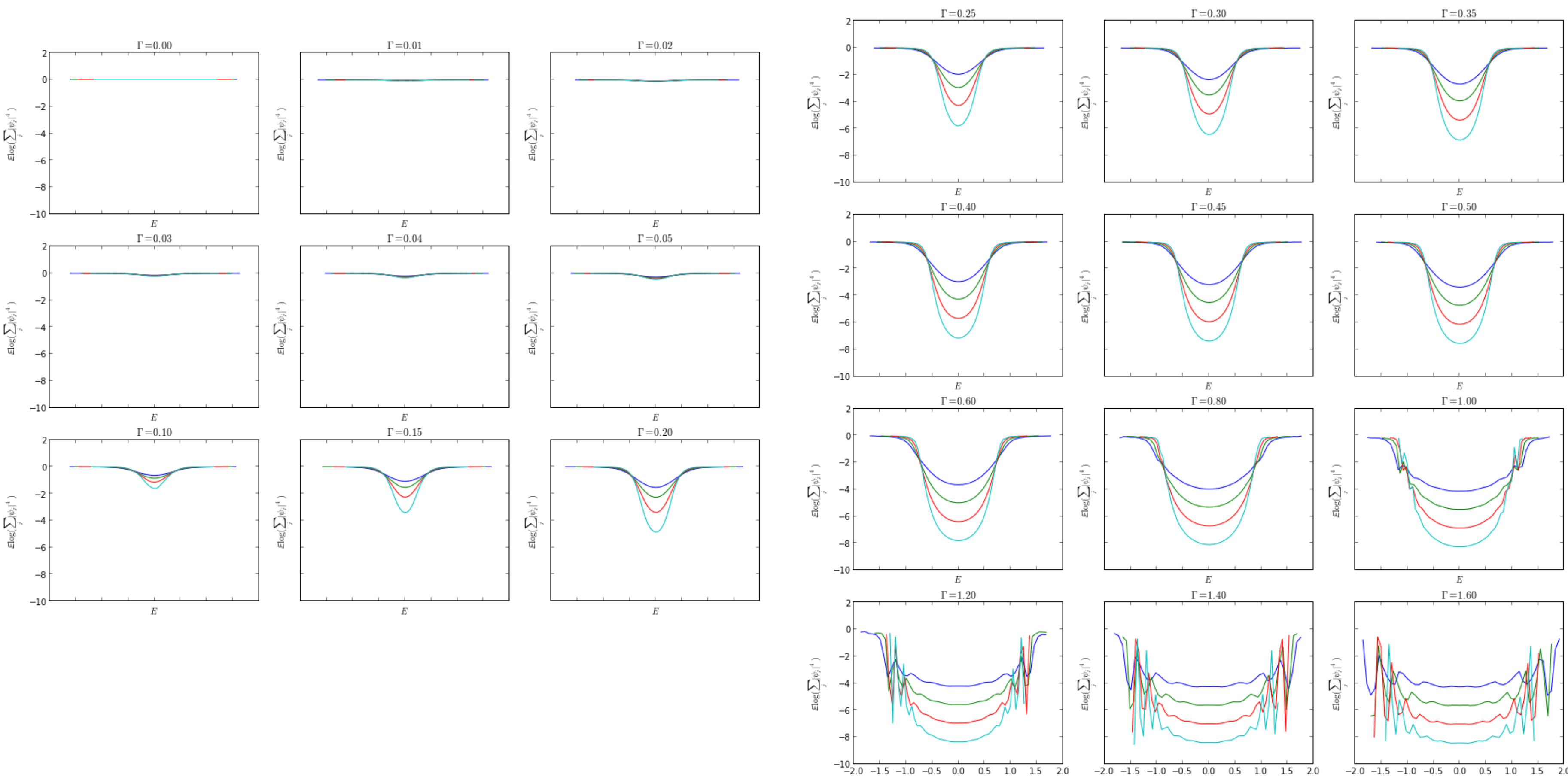
Spider cuts (flow)

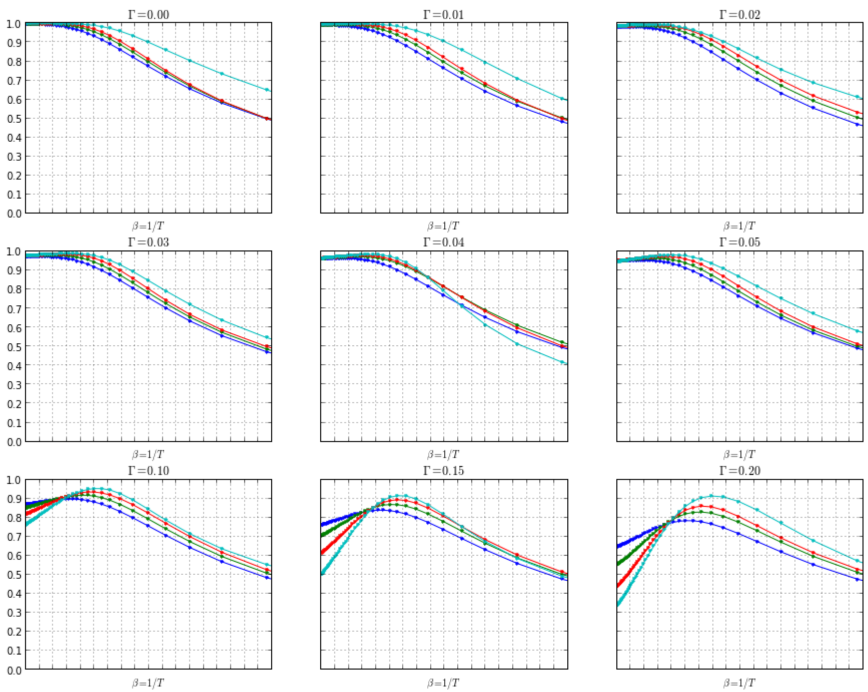


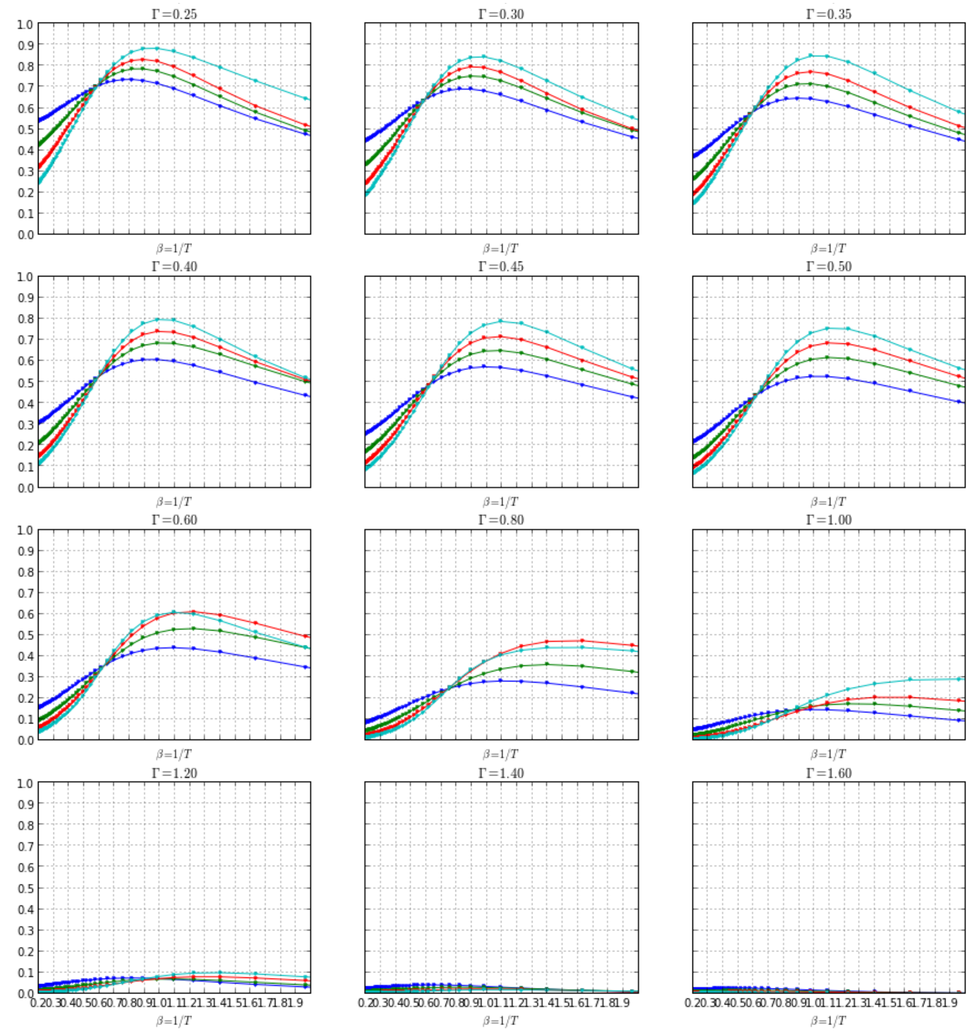
[r] flow (fixed field cuts)



QREM IPRs


Γ=0.00
Γ=0.01
Γ=0.02
Γ=0.03
Γ=0.04
Γ=0.05
Γ=0.10
Γ=0.15
Γ=0.20
Γ=0.25
Γ=0.30
Γ=0.35
Γ=0.40
Γ=0.45
Γ=0.50
Γ=0.60
Γ=0.80
Γ=1.00
Γ=1.20
Γ=1.40
Γ=1.60
E
2
0
−2
−4
−6
−8
−10
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0





$$\lim_{T\to\infty} 1/T \int dt < S^z(t) S^z(0) >^c_\beta$$