Infrared QCD: perturbative or non perturbative?

M. Peláez[1], U. Reinosa[2], J. Serreau[3],
M. Tissier[4] and N. Wschebor[1]

[1]Instituto de Física, Facultad de Ingeniería, Udelar,
Montevideo, Uruguay,
[2]Centre de Physique Théorique, Ecole Polytechnique, Palaiseau, France,
[3]APC, Université Paris Diderot, France,
[4]LPTMC, Université de Paris VI, France.

Trieste, September 2016

Introduction

Standard lore:

- There is a well defined procedure to fix the gauge in gauge-theories (Faddeev-Popop).
- This gives a well grounded lagrangian for gauge-fixed QCD.
- QCD is perturbative for $p \gg 1$ GeV (asymptotic freedom).
- For $p \lesssim 1$ GeV the coupling become too big to apply perturbation theory: QCD becomes non-perturbative.
- All these considerations apply as well to pure gauge theory (Yang-Mills theory).

We will discuss below that many of these ideas are not correct:

- Faddeev-Popop lagrangian is not under control in the infrared.
- For $p \lesssim 1$ GeV the coupling in Yang-Mills theory is not large.
- If a simple and renormalizable lagrangian is chosen:
    - Perturbation theory reproduces Landau-gauge correlation functions with good precision.
    - With appropriate modifications works also well at $T > 0$ (see Serreau's talk).

QCD and non-abelian gauge symmetry

- QCD presents the non-abelian gauge symmetry $SU(3)$:

$$\begin{cases} \delta\Psi(x) = igt^a\epsilon^a(x)\Psi(x) \\ \delta A^a_\mu(x) = \partial_\mu\epsilon^a(x) + gf^{abc}A^b_\mu(x)\epsilon^c(x) \end{cases}$$

- The corresponding Lagrangian (in an euclidean space) is:

$$\mathcal{L}_{QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \sum_{i\in flavors} \bar{\Psi}_i(i\gamma_\mu(\partial_\mu + gA^a_\mu t^a) + m_i)\Psi_i$$

  with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$

- Physical quantities are gauge-invariant. Examples:
  $\langle\bar{\Psi}(x)\Psi(x)\bar{\Psi}(y)\Psi(y)\rangle$, or $\langle F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)F^b_{\rho\sigma}(y)F^b_{\rho\sigma}(y)\rangle$
- The QCD Lagrangian is only used in Monte-Carlo simulations in a periodic lattice givin physical spectrum, reaction amplitudes, etc.
  However: We do not know how to use gauge-invariant Lagrangians in the continuum without fixing the gauge.

Gauge-fixing (I)

- Gauge-fixing in non-abelian case is very cumbersome.
- Let us consider here the quenched case (pure Yang-Mills).
- Perturbatively, the covariant gauge condition

$$\partial_\mu A^a_\mu = 0$$

gives the famous Faddeev-Popov Lagrangian:

$$\mathcal{L}_{FP} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2$$

where: c and $\bar{c}$ $\rightarrow$ ghost and anti-ghost fields,
$D_\mu c^a = \partial_\mu c^a + g f^{abc} A^b_\mu c^c \rightarrow$ covariant derivative of c

- ξ is the gauge-fixing parameter. Important cases:
  $\xi = 1 \rightarrow$ Feynman gauge,
  $\xi \rightarrow 0 \rightarrow$ Landau gauge
- After gauge fixing, no more gauge symmetry! However: residual BRST symmetry.

Gauge-fixing (II)

- BRST symmetry expresses itself at a quantum level via Slavnov-Taylor identities.
- In particular, it implies that in usual perturbation theory gluons have no mass.
- Beyond perturbation theory, things are much more involved:
  1. Faddeev-Popov Lagrangian is not justified because there are many solutions for the covariant gauge condition $\partial_\mu A_\mu^a = 0$ (Gribov copies).
  2. BRST symmetry is not well defined (Neuberger zero problem).
  3. Slavnov-Taylor identities and the Faddeev-Popov Lagrangian must be reconsidered.
- In particular, it not clear at all to which gauge-fixed Lagrangian corresponds Landau-gauge lattice simulations.

Validity of usual perturbation theory

- Let us consider the predictions of standard perturbative Faddeev-Popov action.
- At one-loop the running of the coupling is:

$$\frac{g^2(\mu)}{16\pi^2} = \frac{1}{22\log(\mu/\Lambda_{QCD})}$$

- In the ultraviolet $(\mu \gg \Lambda_{QCD}) \to$ asymptotic freedom.
- In the infrared $(\mu \sim \Lambda_{QCD}) \to$ the coupling explodes (Landau pole).
- No indication from lattice simulation of such infrared singularity.
- Most interesting properties of QCD (confinement, χSB, mass gap, etc.) take place in the infrared regime $\to$ beyond standard perturbation theory.
- Very involved approximation schemes not based on perturbation theory have been proposed to study such regime.

Correlation functions

- Correlation functions are not gauge invariant (typically).
- Require gauge-fixing $\rightarrow$ haunted by Gribov problem.
- However: easiest object for (semi-) analytical methods.
- Can be obtained (non-perturbatively) in lattice calculations.
- Good testing ground for various approximation schemes/models.
- Have been used for various confinement or spontaneous symmetry braking scenarios.
- Most studies (both analytical and from simulations) are done in Landau gauge ($\xi \rightarrow 0$).
- Studies have been performed:
  - Mainly for 2-point functions, but also for higher correlators
  - Not only for $SU(3)$ but also for $SU(2)$
  - Not only for $d = 4$ but also for $d = 3$ and $d = 2$
  - In quenched or unquenched cases have been considered.
  - Both at $T = 0$ and at $T \neq 0$.

Main previous results in Landau gauge

- Most results coming from lattice and Schwinger-Dyson (SD), Non-Perturbative Renormalization Group (NPRG) approaches. [R. Alkofer and L. von Smekal, Phys.Rept.**353** (2001) 281; C. S. Fischer and H. Gies, JHEP **0410** (2004) 048.]
- Main difference with perturbation theory: No IR Landau pole.
- Gluon propagator shows violation of positivity→ No Källén-Lehmann representation.
- SD/NPRG solutions of two types:
  →Scaling solution: Vanishing Gluon propagator at zero momentum, Ghost propagator more singular than bare in IR.
  →Massive solution: Non-zero finite Gluon propagator at zero momentum, Ghost propagator as singular as bare in IR.
  [A.C.Aguilar *et al.*, Phys.Rev.**D78**(2008)025010;
  P.Boucaud *et al.*, JHEP0806(2008)099.]
- Lattice results tend to favor massive solution for $d = 4$ and $d = 3$ and scaling solution in $d = 2$.

A model for Yang-Mills correlators (Landau gauge)

- Gribov problem → correct Lagrangian for YM unknown (beyond perturbation theory).
- At perturbative level, Faddeev-Popov Lagrangian (quenched case):

$$\mathcal{L}_{FP} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2$$

- Beyond perturbation theory: Gribov and Zwanzinger propose a change of Lagrangian (but not fully first-principles). [V. N. Gribov, Nucl. Phys. B **139** (1978) 1; D. Zwanziger, Nucl. Phys. B **323**, 513 (1989) and Nucl. Phys. B **399**, 477 (1993)]
- GZ approach may give (depending on approximations) scaling or massive solutions. Also reproduces violation of positivity.
- Given the lattice results, we propose the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{FP} + \frac{m^2}{2} A^a_\mu A^a_\mu$$

One-loop results for propagators (I)

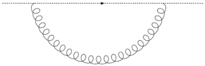
- We define:

$$G^{ab}(p) = \delta_{ab} F(p)/p^2, \quad G^{ab}_{\mu\nu}(p) = \Big(\delta_{\mu\nu} - p_\mu p_\nu / p^2\Big)\delta_{ab} G(p).$$

- We adopt the following renormalization conditions:

$$G(p=0) = 1/m^2, \quad G(p=\mu) = 1/(m^2+\mu^2), \quad F(p=\mu) = 1.$$

- At 1-loop the ghost self-energy is obtained from the diagram:

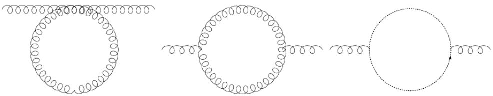


- We obtain : $(s = p^2/m^2)$
  [Phys.Rev. D82 (2010) 101701,Phys.Rev. D84 (2011) 045018.]

$$F^{-1}(p) = 1 + \frac{g^2 N}{64\pi^2}\Big\{ - s\log s + (s+1)^3 s^{-2}\log(s+1) \\ - s^{-1} - (s \to \mu^2/m^2)\Big\}$$

One-loop results for propagators (II)

- At 1-loop the gluon self-energy is obtained from the diagrams:



- We obtain: $(s = p^2/m^2)$
  [Phys.Rev. D82 (2010) 101701,Phys.Rev. D84 (2011) 045018.]

$$G^{-1}(p)/m^2 = s + 1 + \frac{g^2 N s}{384\pi^2}\Big\{111 s^{-1} - 2s^{-2} + (2 - s^2)\log s$$

$$+ (4s^{-1}+1)^{3/2}\left(s^2 - 20s + 12\right)\log\left(\frac{\sqrt{4+s}-\sqrt{s}}{\sqrt{4+s}+\sqrt{s}}\right)$$

$$+ 2(s^{-1}+1)^3\left(s^2 - 10s + 1\right)\log(1+s) - (s \to \mu^2/m^2)\Big\}$$

Renormalization-Group effects

- Previous expressions are one-loop results obtained at a fixed renormalization scale.
- When considering momenta $p \gg m$, it is necessary to take into account RG effects.
- That is, one must take into account that for $\mu \gg m$ the coupling runs.
- If an appropriate renormalization scheme is chosen, the coupling saturates for $\mu \simeq m \to$ No infrared Landau pole!!!

Comparison with lattice results: $SU(2)$, $d = 4$

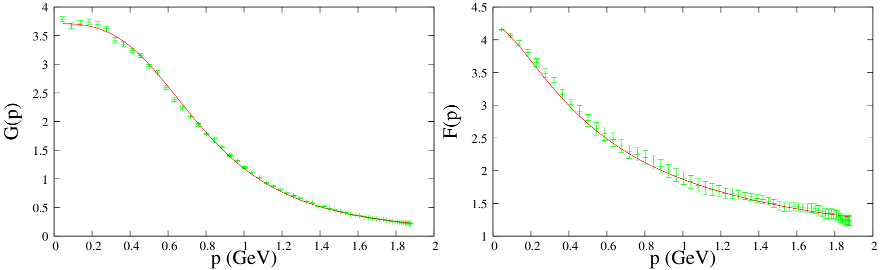


Figure : Left: Gluon propagator. Right: Ghost dressing function.

Red curves: present work with $g = 7.5$ and $m = 0.68$ GeV for $\mu = 1$ GeV.

Green points: A. Cucchieri and T. Mendes, Phys. Rev. Lett. (2008) 100.

Comparison with lattice results: $SU(3)$, $d = 4$

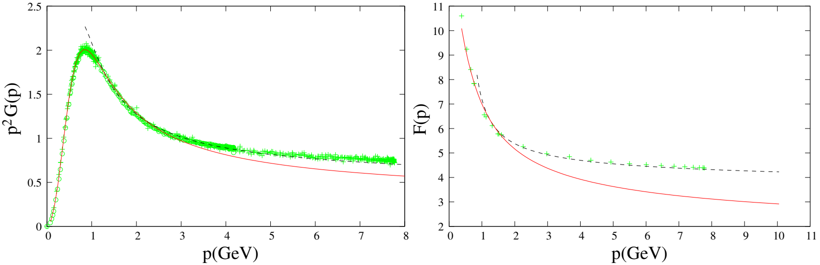


Figure : Left: Gluon dressing function. Right: Ghost dressing function.

Red curves: present work with $g = 4.9$ and $m = 0.54$ GeV for $\mu = 1$ GeV.
Green points: I. L. Bogolubsky *et al.*, Phys. Lett. B **676**, 69 (2009) and
D. Dudal, O. Oliveira and N. Vandersickel, Phys. Rev. D (2010) 074505.

Comparison with lattice results: $SU(2)$, $d = 3$

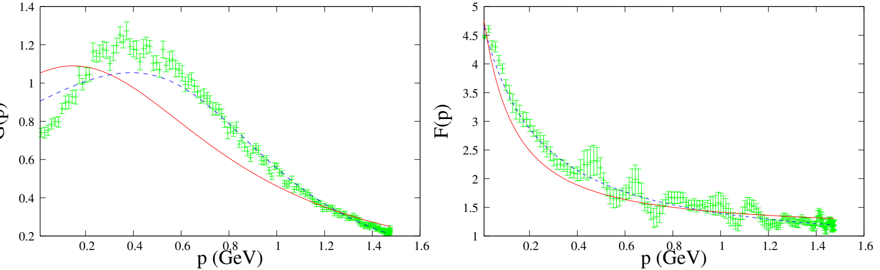


Figure : Left: Gluon propagator. Right: Ghost dressing function.

Red curves: present work with $g = 3.7\ \sqrt{\text{GeV}}$ and $m = 0.89$ GeV for $\mu = 1$ GeV. Blue curves: Idem with $g = 1.6\ \sqrt{\text{GeV}}$ and $m = 0.35$ GeV for $\mu = 11$ GeV.

Green points: A. Cucchieri and T. Mendes, Phys. Rev. Lett. (2008) 100.

Positivity violations (I): $SU(3)$, $d = 4$

- Consider the spectral decomposition of gluon propagator:

$$G(p) = \int_0^\infty \frac{d\mu}{2\pi} \frac{\rho(\mu)}{p^2 + \mu^2} \tag{1}$$

- If all states with positive norm, the Källén-Lehmann spectral density $\rho(\mu)$ must be positive.
- $\rho(\mu)$ difficult to measure on the lattice.
- Consider instead

$$C(t) = \int_{-\infty}^\infty \frac{dp}{2\pi} e^{ipt} G(p) = \int_0^\infty \frac{d\mu}{2\pi} \rho(\mu) \frac{e^{-\mu|t|}}{2\mu} \tag{2}$$

- If $\rho(\mu)$ is positive, so is $C(t)$.

Positivity violations (II): $SU(3)$, $d = 4$

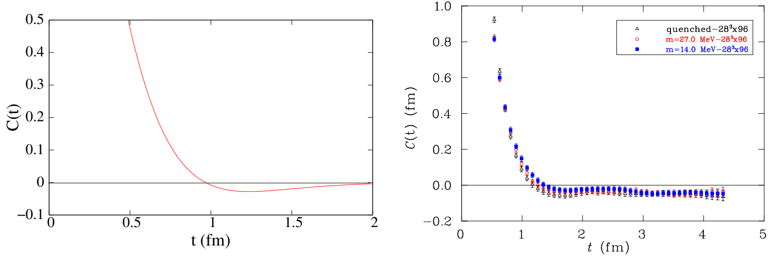


Figure : Left: Present work. Right: Lattice results.

Present work with $g = 4.9$ and $m = 0.54$ GeV for $\mu = 1$ GeV.
Lattice results: P. O. Bowman *et al.*, Phys. Rev. D **76**, 094505 (2007).

A moderate coupling in the infrared (gluon-ghost sector)

- One may be worried by the values of the couplings in the infrared $g \sim 5 - 7$. Why perturbation theory should work?
- The naive expansion parameter for $SU(N)$ in $d = 4$ is

$$\frac{g^2 N}{16\pi^2} \sim 0.4 - 0.6 \qquad \textit{for } p \simeq m \tag{3}$$

- Moreover, given the massive behaviour of gluons, there are further kinematical suppresions for typical momenta $p \ll m$.
- In the worst case, the expansion parameter for $p \ll m$ is

$$\frac{g^2 N}{16\pi^2} \frac{p^2}{m^2} \qquad \text{for } p \ll m \tag{4}$$

- An naive interpolation for arbitrary momenta is

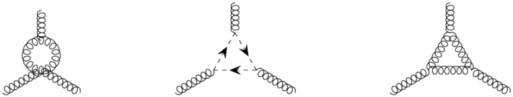
$$\frac{g^2 N}{16\pi^2} \frac{p^2}{m^2 + p^2} \lesssim 0.2 - 0.3 \tag{5}$$

- This may also explain why SD or NPRG truncations does work!

Three-point functions (I)

- The same procedure has been used in order to calculate ghost-gluon and 3-gluon vertices [M. Peláez, M. Tissier, N. Wschebor, Phys.Rev. D88 (2013) 125003].
- For the ghost-gluon vertex the 1-loop diagrams are

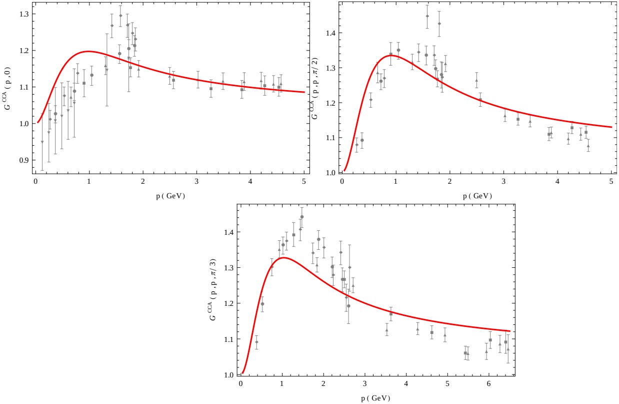
- For the 3-gluon vertex the 1-loop diagrams are



- The complete one-loop calculation has been performed. No further approximations. All momenta configurations. All tensorial structures. No new parameters (already fixed with propagators). Calculations done in $d = 4$ and $d = 3$.

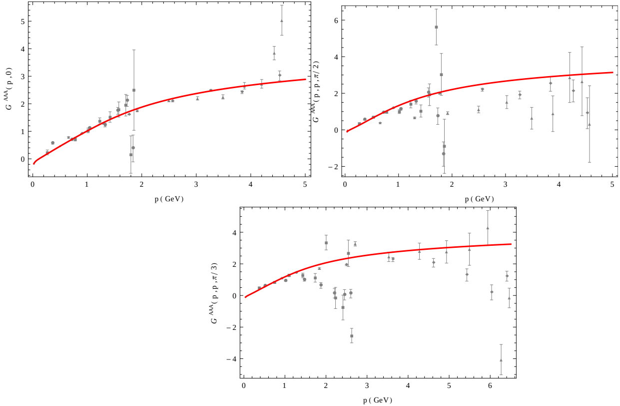
Three-point functions (II): Ghost-gluon vertex

- Let us compare with lattice simulations [A. Cucchieri, A. Maas and T. Mendes, Phys. Rev. D 77, 094510 (2008)] for the ghost-gluon vertex ($d = 4$).
- A single tensorial structure is tested but many momenta configurations.



Three-point functions (III): 3-gluon vertex

- Let us compare with lattice simulations [A. Cucchieri, A. Maas and T. Mendes, Phys. Rev. D 77, 094510 (2008)] for the 3-gluon vertex ($d = 4$).
- A single tensorial structure is tested but many momenta configurations.



Yang-Mills: perturbative RG or NPRG?

- Can-we reproduce NPRG or SD results with simple one loop calculations?
- The answer is YES:

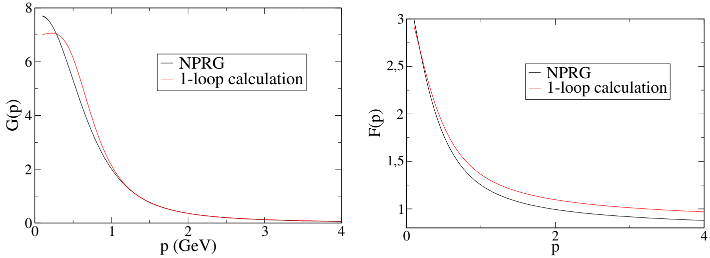


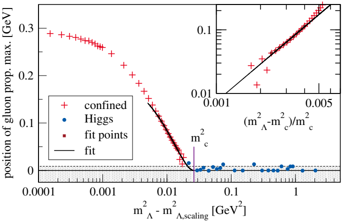
Figure : Left: Gluon propagator. Right: Ghost dressing function.

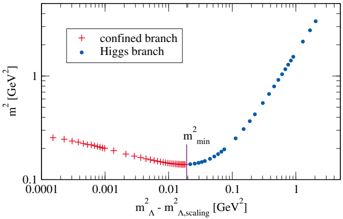
Red curves: present work with $g = 5.2$ and $m = 0.5$ GeV for $\mu = 1$ GeV and $N = 3$.

Black curves: [Cyrol et al., Phys.Rev. D94 (2016) no.5, 054005].

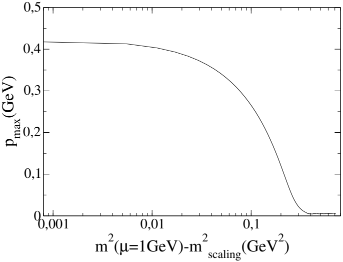
Yang-Mills: perturbative RG or NPRG? (II)

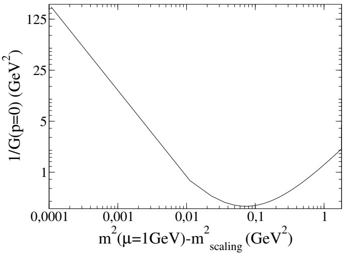
Results from [Cyrol et al., Phys.Rev. D94 (2016) no.5, 054005]:





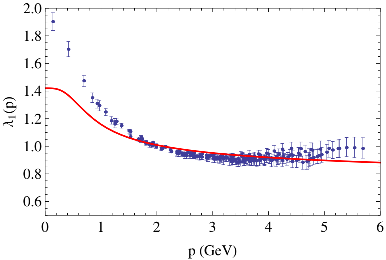
Results from one-loop calculation:





Full QCD: really non-perturbative

- The succes in the ghost-gluon sector has been explained for a moderate expansion parameter ($0.2 \to 0.3$).
- However, α_S is not universal in the infrared.
- $\lambda_1(p^2)$ is a mesure of the quotient of the quark-gluon coupling compared to the Taylor (ghost-gluon) coupling.
- It is measured in the lattice.
- In the infrared it introduces a factor 2 in the coupling ($\Rightarrow$ factor 4 in α_S.
- Accordigly in the quark sector the expansion parameter is ~ 1.



- The small expansion parameter is lost in the quark sector!
- One-loop calculations give qualitative agreement with lattice.
- This is needed for sponeaneous chiral symmetry breaking.

Conclusion

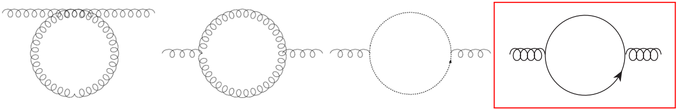
- We are able to reproduce quantitatively propagators and 3-point vertices in Landau gauge Yang-Mills theory in all momenta regimes with 1-loop calculations.
- We introduce of a gluon mass term.
- Reason: no infrared Landau pole and moderate coupling.
- In the quark sector the agreement is only qualitative.
- Reason for that: the infrared coupling is larger.
- Finite temperature: see Serreau's talk.
- Work in progress:
  - Resumations in the quark sector (large coupling).
  - Spontaneous chiral symmetry breaking.
- For the future:
  - confinement?
  - origin of the mass?
  - higher orders of calculations?
  - unitarity? physical space?

Including quarks: ghost and gluons (I)

- The same procedure has been used in the unquenched case.
- Consider firt 2-point functions. [M. Peláez, M. Tissier, N. Wschebor, Phys.Rev. D90 (2014) 065031]
- One add the quark term in the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{FP} + \mathcal{L}_m + \sum_{i \in flavors} \bar{\Psi}_i (i\gamma_\mu(\partial_\mu + gA_\mu^a t^a) + m_i)\Psi_i \quad (6)$$

- At one loop, diagrams for the gluon 2-point function are now



- The result agree well with lattice simulations.

Including quarks: ghost and gluons (II)

- Comparison for various values of number of flavors Nf.

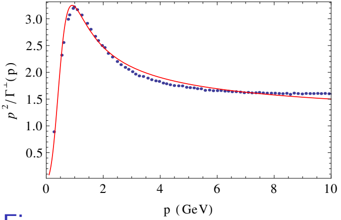


Figure : Gluon propagator for $Nf = 2$. Lattice data from [A. Sternbeck et al., PoS LATTICE2012, 243 (2012)]

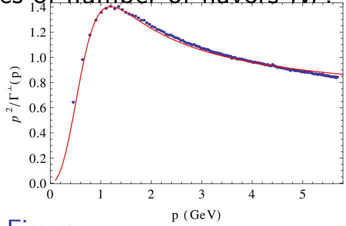


Figure : Gluon propagator for $Nf = 2+1+1$. Lattice data from [P. O. Bowman et al. Phys.Rev.D 70 (2004) 034509]

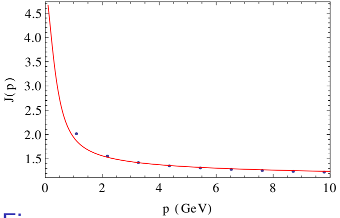


Figure : Ghost propagator for $Nf = 2$. Lattice data from [A. Ayala et al., Phys.Rev.D 86 (2012) 74512.]

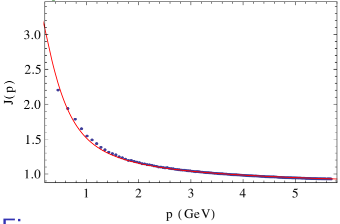


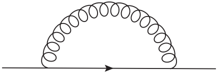
Figure : Ghost propagator for $Nf = 2+1+1$. Lattice data from [A. Ayala et al., Phys.Rev.D 86 (2012) 74512.]

Quark propagator (I)

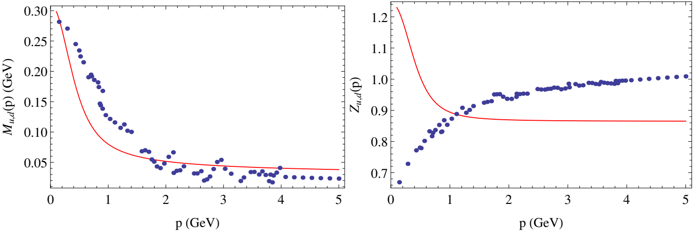
- One can also analyze the quark propagator parametrized as

$$\Gamma^{(2)}_{\psi\bar{\psi}}(p) = Z^{-1}(p)\,(i\not{p} + M(p)) \qquad (7)$$

- At one loop it requires to calculate the diagram

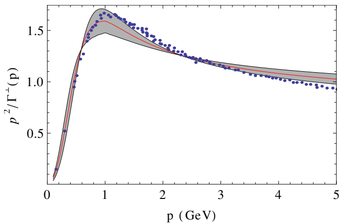


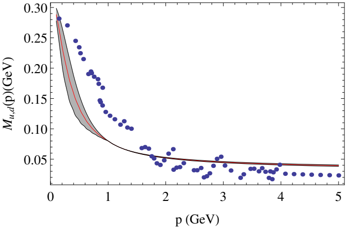
- The agreement is not as good as in the gluon-ghost sector:

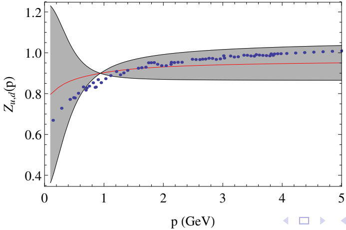


Quark propagator (II)

- Discrepancy on $Z(p^2) \leftrightarrow$ very small one-loop contributions.
- More preciselly, one-loop corrections vanishes for $m \to 0$.
- For $Z(p^2)$ two-loop contributions dominates (two loops estimated):

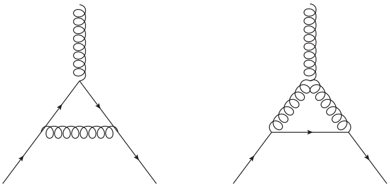






Quark-gluon vertex (I)

- One can also calculate the quark-gluon vertex. [M. Peláez, M. Tissier, N. Wschebor, Phys.Rev. D92 (2015) 4, 045012].
- One must calculate the diagrams



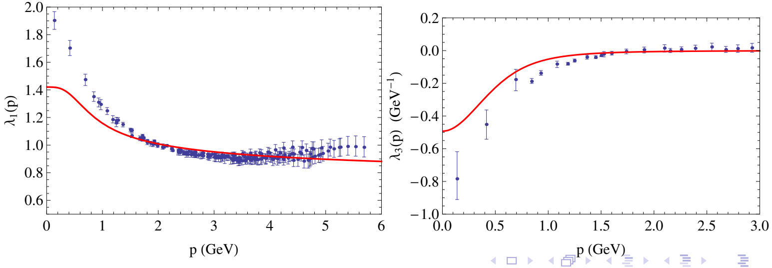
- As for previous 3-point vertices:
  Complete one-loop calculation has been performed. No further approximations. All momenta configurations. All tensorial structures (12!). No new parameters (already fixed with propagators).

Quark-gluon vertex (II)

- Many tensorial components have been simulated.
- We compared to all the available lattice data.
- For brevity, we present here only results gluon momentum=0.
- In that case, the tensorial decomposition is simpler:

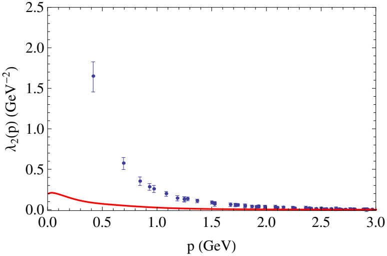
$$\Gamma_\mu(p,-p,0) = -ig\left[\lambda_1(p^2)\gamma_\mu - 4\lambda_2(p^2)\not{p}p_\mu - 2i\lambda_3 p_\mu\right]. \quad (8)$$

- We compare to the lattice data of [J. I. Skullerud et al., JHEP 0304, 047 (2003).].
- $\lambda_1(p^2)$ and $\lambda_3(p^2)$ work quite well:



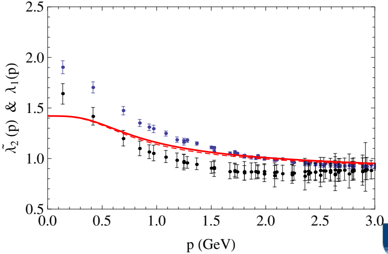
Quark-gluon vertex (III)

- $\lambda_2(p^2)$ does not seem to work.
- The same difficulty for these quantity observed in other approaches. [A. C. Aguilar et al. Phys.Rev.D 90 (2014) 065027.].
- We think that $\lambda_2(p^2)$ is not properly extracted from lattice.



- It is extracted by substracting $\lambda_1(p^2)$ from the quantity

$$\widetilde{\lambda}_2 = -\frac{1}{16 g_B} \sum_\mu \text{Im Tr}\left[\gamma_\mu \Gamma_\mu(p,-p,0)\right]$$



which is directly observed. But: