Critical Casimir forces from the equation of state of quantum critical systems

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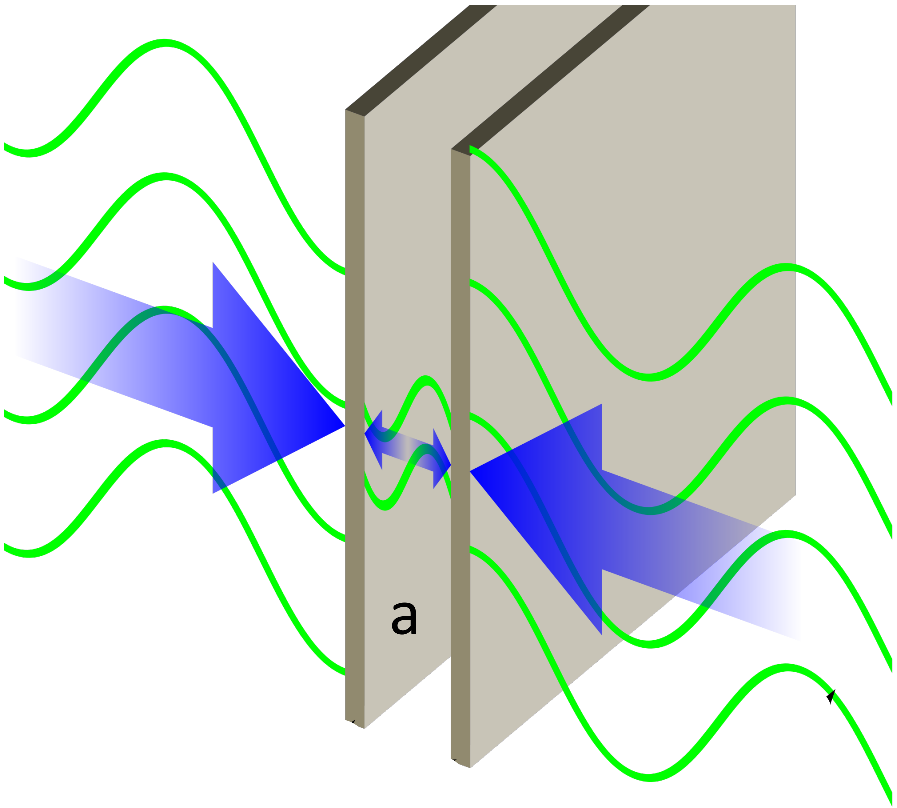
Introduction

Casimir 1948 : Casimir effect



Critical Casimir force

Fisher and de Gennes 1978 :
fluctuating (classical) medium between two plates creates a force
(if correlation length of the order of distance between the plates)

Large correlation length : second order phase transition $\xi \propto |T - T_c|^{-\nu}$
Also implies universality of the force close to criticality :

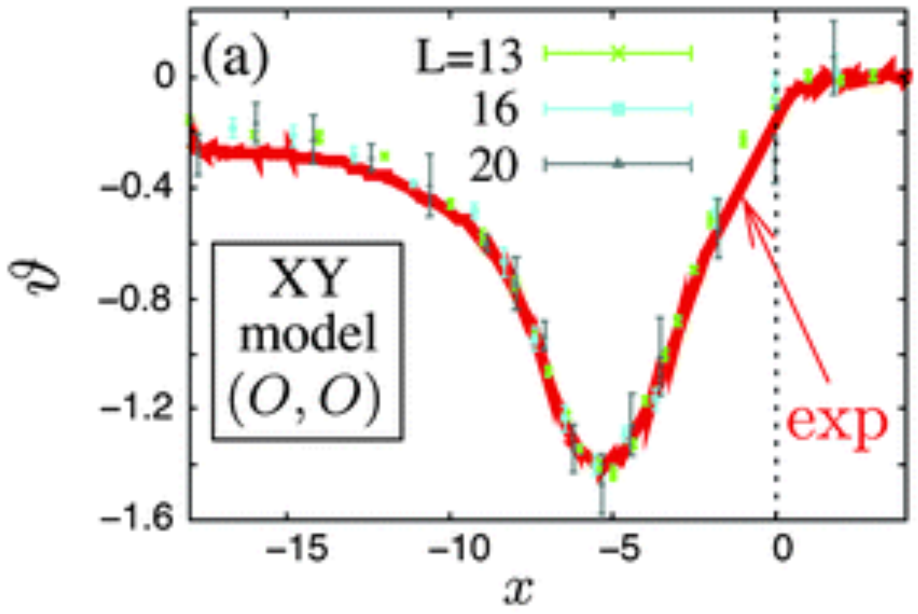
Pressure of fluid on the plates : $P = -\frac{1}{A}\frac{\partial F}{\partial a}$

$$F = aA(f_{\text{bulk}} + f_{\text{ex}}) \qquad f_{\text{ex}} = a^{-3}\tilde{f}_s(a/\xi)$$

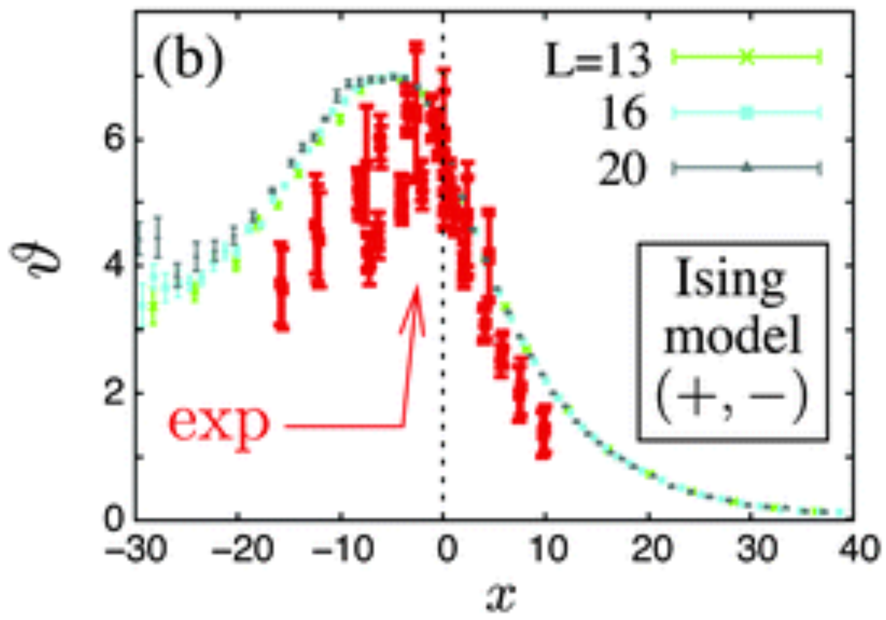
$$P_c = -\partial_a(a\tilde{f}_s) = a^{-3}\vartheta(a/\xi)$$

Experimental realizations

Helium films



Binary mixtures



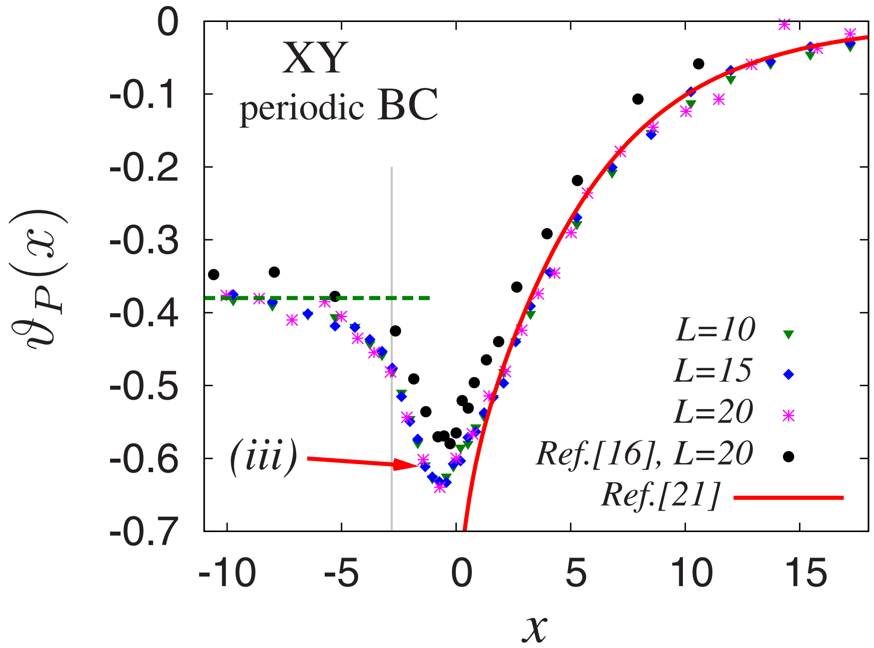
MC simulations : Vasilyev et al. 2007
He experiments : Garcia et al. 1994
Ganshin et al. 2006
Binary mixture : Fukuto et al. 2005

Strict boundary conditions

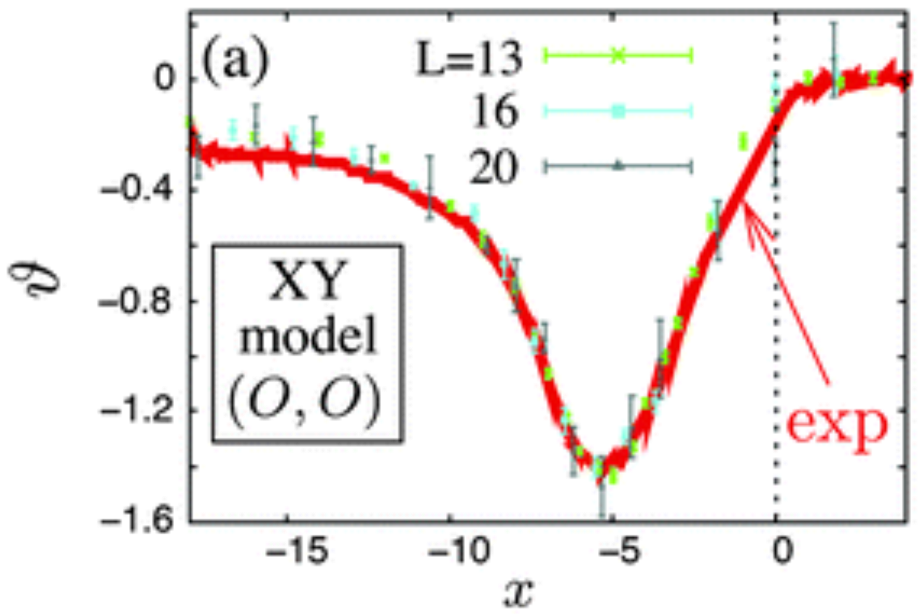
Boundary conditions

Casimir scaling function universal, depends on :

- dimension
- symmetry of order parameter
- geometry / boundary conditions

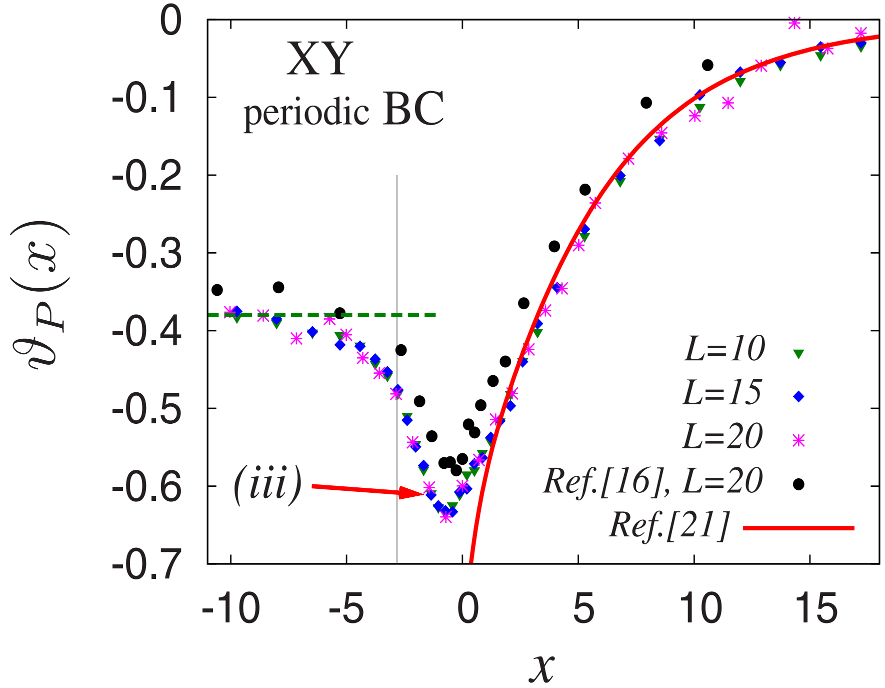


Vasilyev et al. 2009



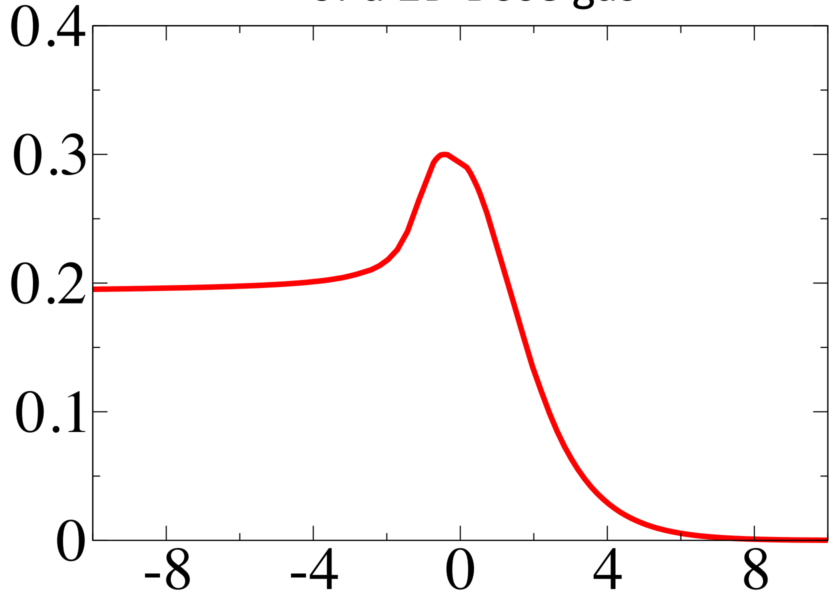
From critical Casimir to quantum critical systems…

Critical classical Casimir force



VS

"Thermal part" of pressure
of a 2D Bose gas



Functional renormalization group study :
AR, Kodio, Dupuis, Lecheminant (2013)

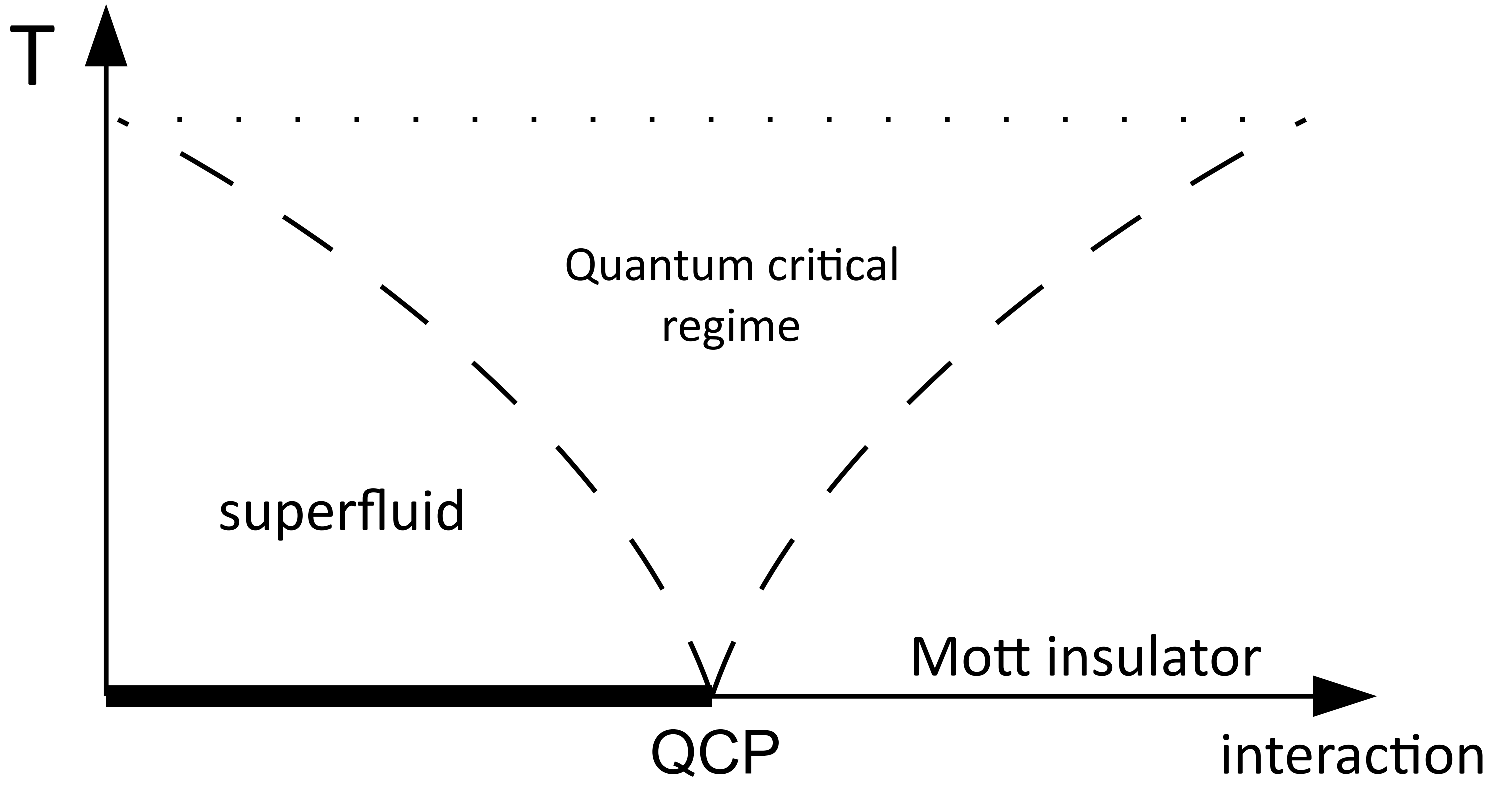
Quantum phase transitions

QPT : transition at zero temperature (change of ground-state) when changing non-thermal parameter δ

Examples : Bosonic Mott transition at constant density (XY universality class)

Ferro-paramagnetic transition in quantum Ising model in transverse field

Phase diagram and critical scaling



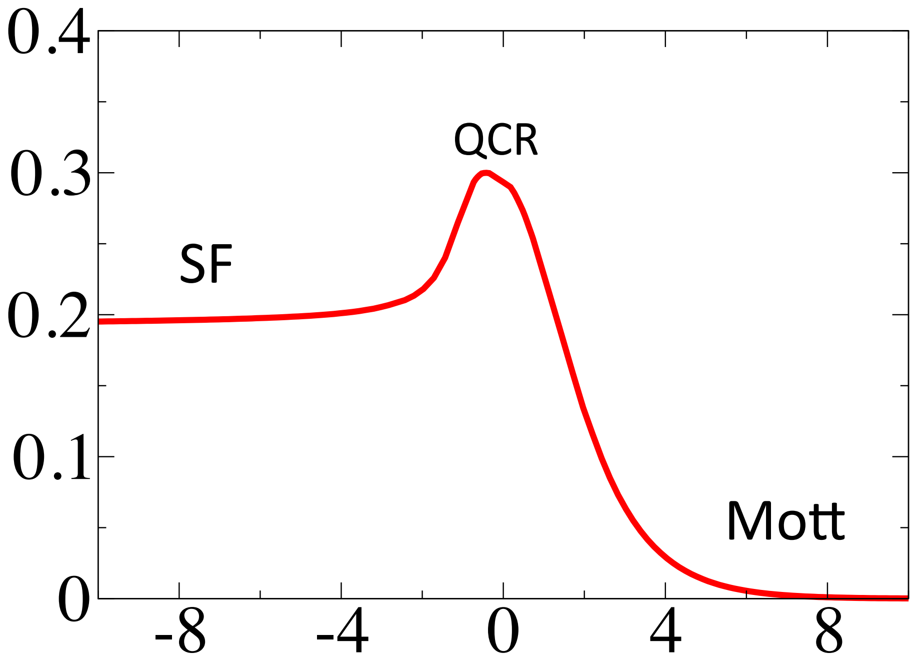
Close to QCP : $f(\delta, T) = \epsilon_0(\delta) + T^3/c^2 \tilde{f}_s(\beta c/\xi)$

Ground-state energy density

$\xi \propto |\delta - \delta_c|^{-\nu}$

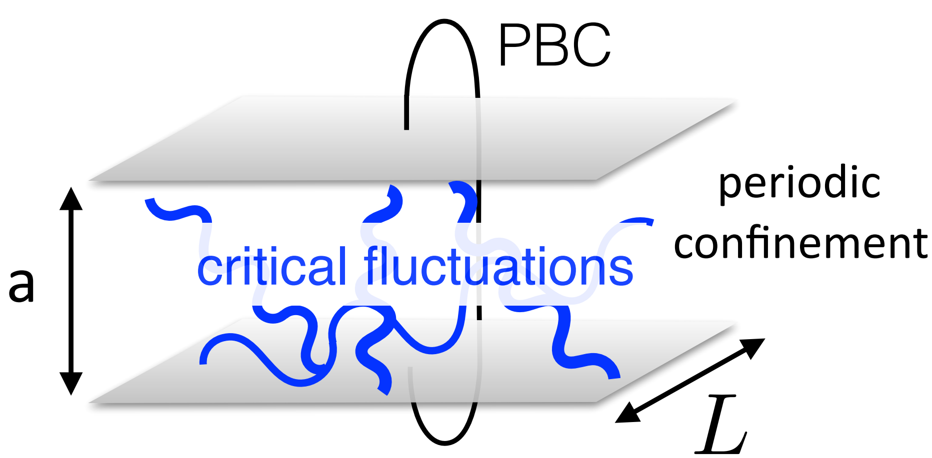
Phase diagram and critical scaling

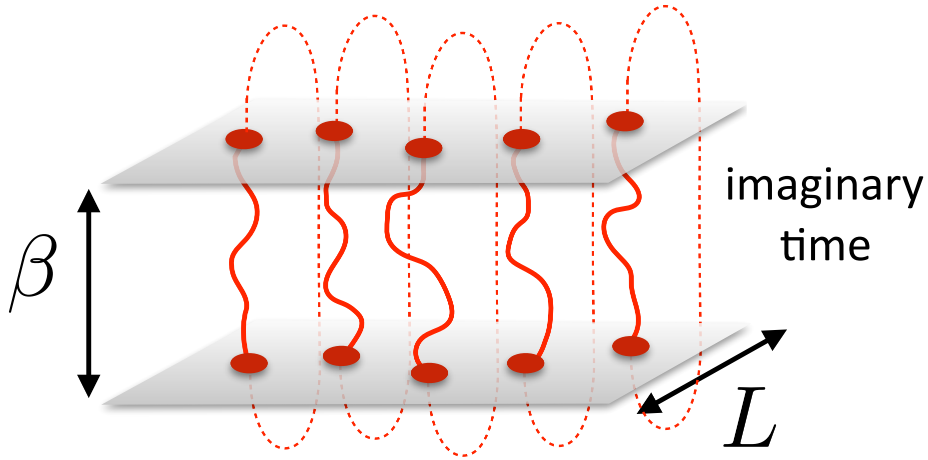
Thermal part of pressure/free energy



Close to QCP : $f(\delta, T) = \epsilon_0(\delta) + T^3/c^2 \tilde{f}_s(\beta c/\xi)$

Quantum classical correspondence





confined
3D classical system

a=c/T

2D quantum system
at finite T

Close to a critical point, universality implies that scaling functions are the same !

See e.g. Sachdev

From critical Casimir to quantum critical systems... and back

$$f(\delta, T) = \epsilon_0(\delta) + T^3/c^2 \tilde{f}_s(\beta c/\xi)$$

Universality : same as confined 3D classical system

$$\epsilon = L^{-2}\langle \hat{H} \rangle = \partial_\beta(\beta f)$$

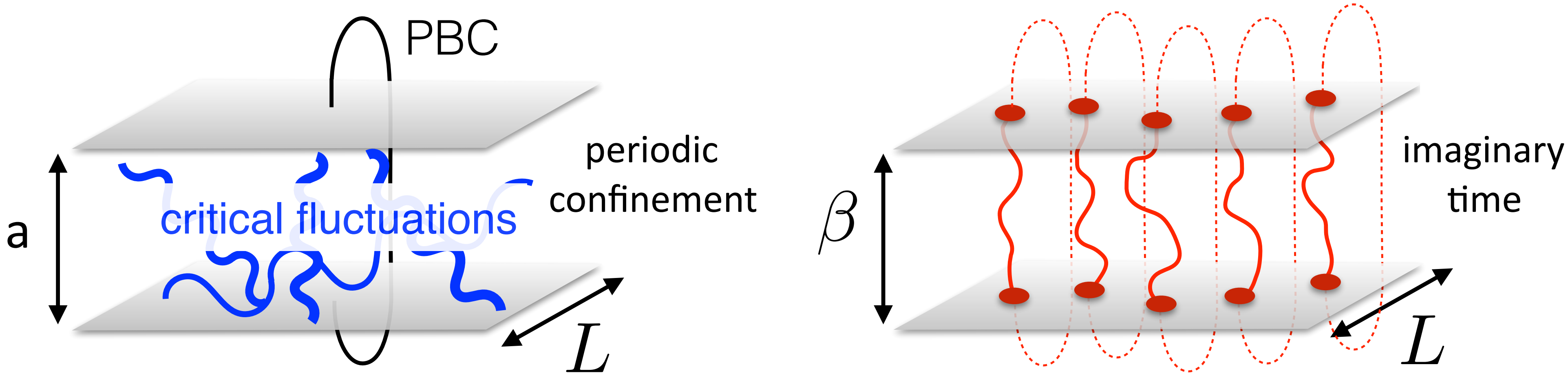
Average energy of a quantum critical system : $\epsilon = \epsilon_0 - \frac{T^3}{c^2}\vartheta_P(\beta c/\xi)$

Thermodynamic stability of quantum systems implies
attractive Casimir force for periodic BC.

$$P_c = -\partial_a(a\tilde{f}_s) = a^{-3}\vartheta(a/\xi)$$

For free bosons : Coleman et al. Am. J. Phys. 2009

Critical Casimir vs Equation of State



Quantum to classical correspondence :

average energy interpreted as (universal) entropic "Casimir" force

Classical to quantum correspondence :

critical Casimir force with PBC can be quantum simulated experimentally!

FRG calculation

Theoretical approaches are scarce to compute scaling functions
(large N or epsilon expansion fail here).

AR, Kodio, Lecheminant and Dupuis (2014) : LPA'+ expansion up to ϕ^4
Not good for Ising.

Improved calculation : 2nd order of Derivative Expansion

$$\Gamma_k[\phi] = \int_0^{\hbar\beta} d\tau \int d^d r \Big\{ \frac{Z_k^x(\rho)}{2} (\boldsymbol{\nabla}\phi)^2 + \frac{Z_k^\tau(\rho)}{2} (\partial_\tau \phi)^2 + \frac{Y_k^x(\rho)}{4} (\boldsymbol{\nabla}\rho)^2 + \frac{Y_k^\tau(\rho)}{4} (\partial_\tau \rho)^2 + U_k(\rho) \Big\},$$

$$f = \lim_{k\to 0} U_k(\rho_{0,k})$$

$\rho = \phi^2/2$

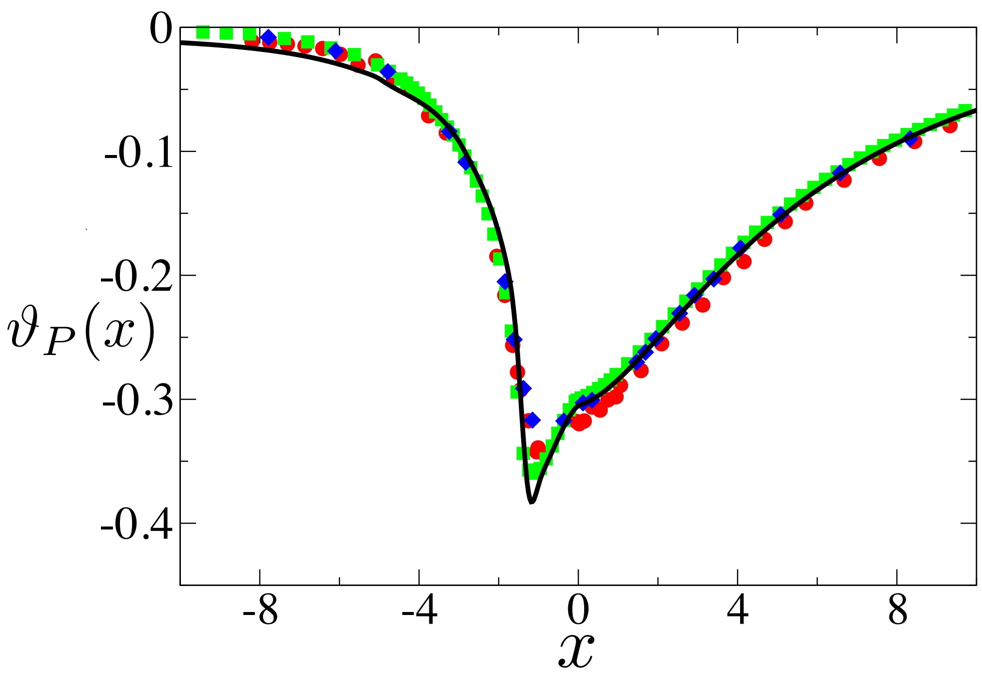
See also : Jakubczyk and Napiorkowski 2013

Casimir critical force from the FRG

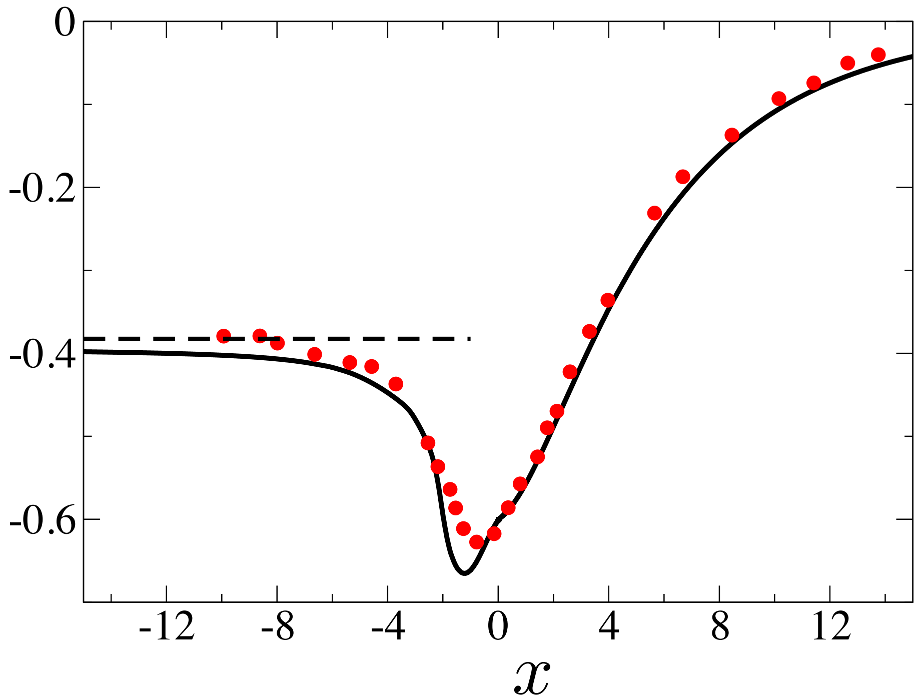
TABLE II. Universal Casimir amplitude $\vartheta(0)/2$

N	1	2	3
NPRG	-0.1527	-0.3006	-0.4472
Monte Carlo [5]	$-0.1520(2)$	$-0.2993(7)$	

Ising universality class



XY universality class

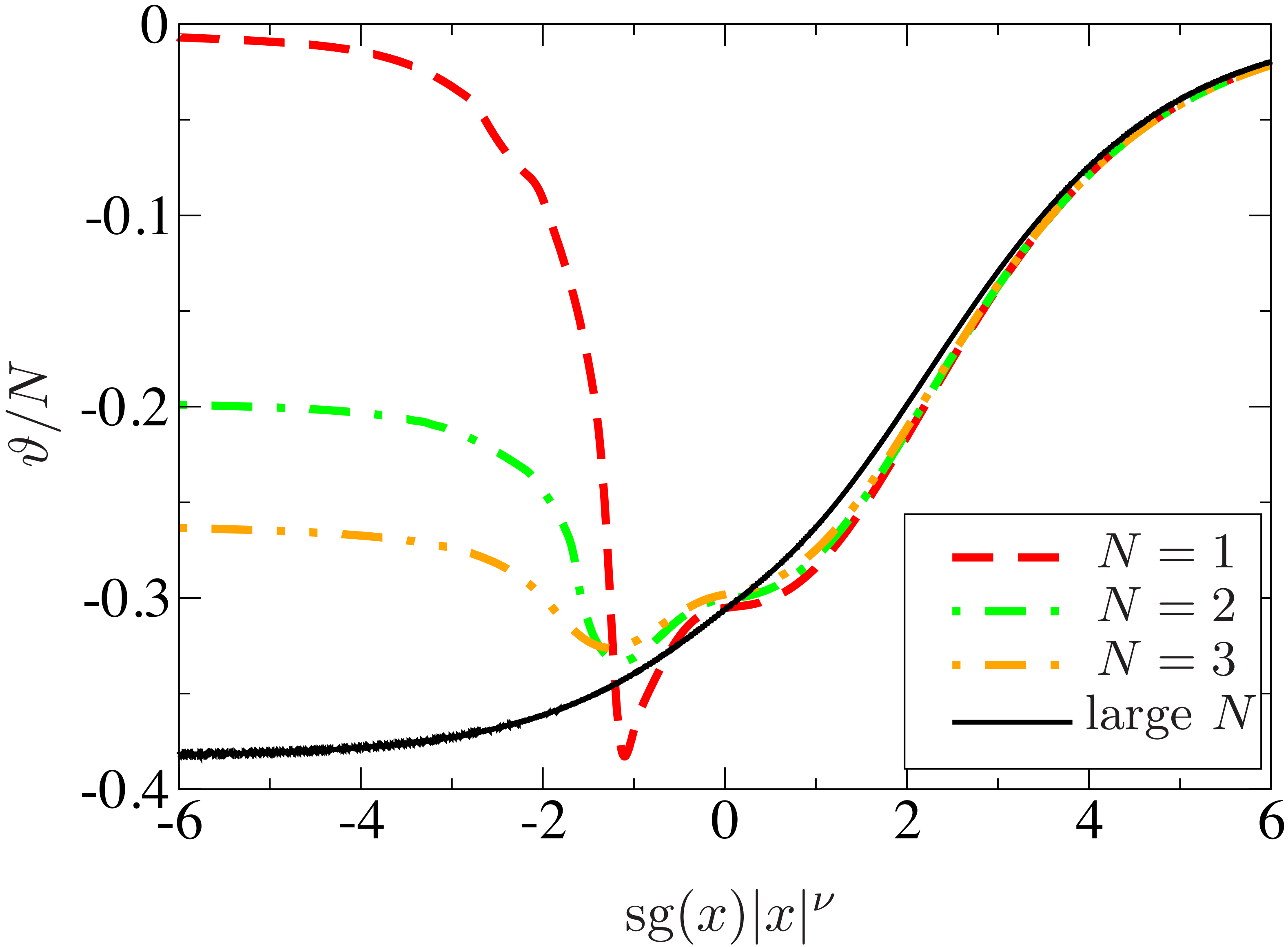


MC : Vasilyev et al. 2009
Hucht et al. 2011
D. Lopes Cardoso (PhD thesis 2015)

$$x = (a/\xi_0)^{1/\nu}(T - T_c)$$

MC : Vasilyev et al. 2009

Scaling functions for different N



Conclusion and perspectives

- Critical Casimir forces with periodic BC is the equation of state of a quantum critical system.

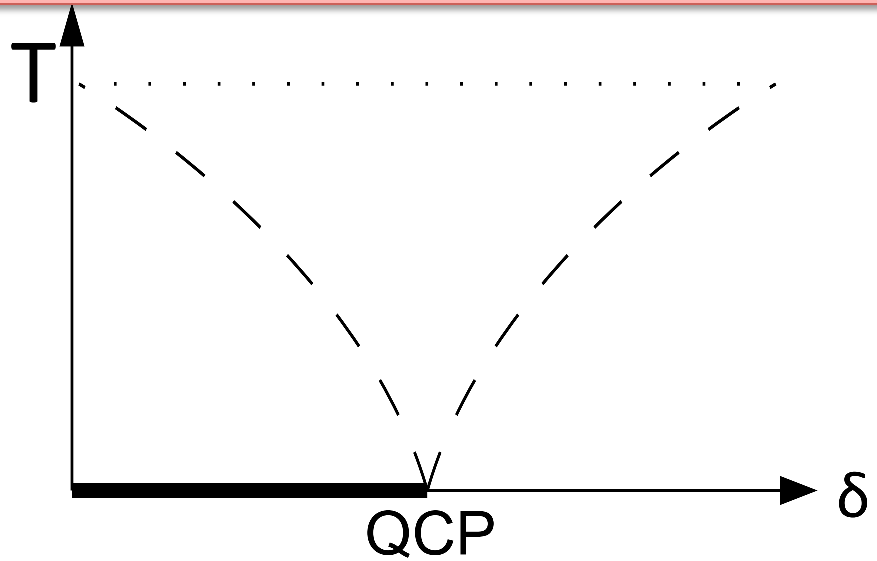
- Corresponding scaling functions could be measured in state of the art experiments on quantum systems.

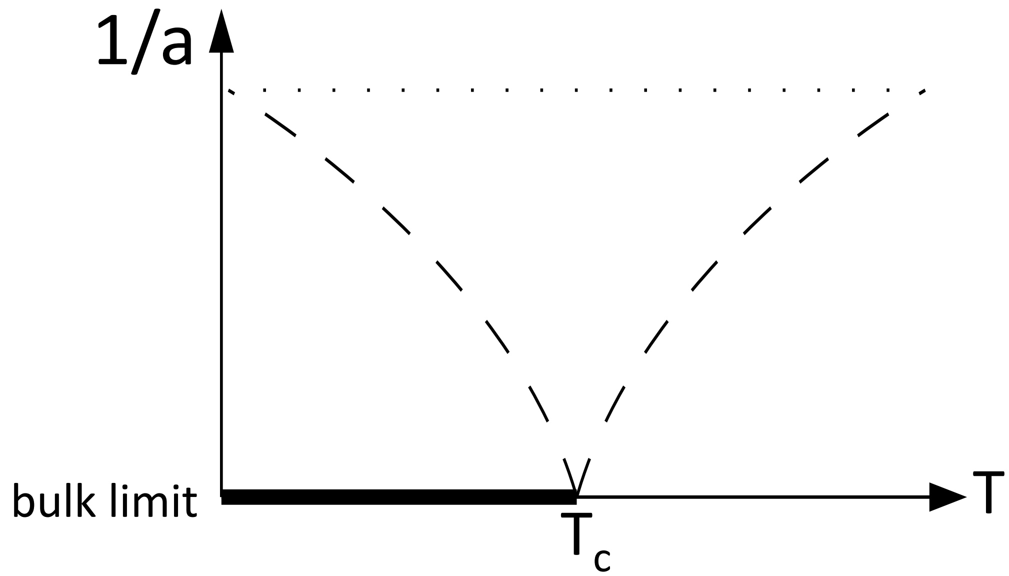
- Tools of quantum many-body problem can be used to study critical Casimir forces (Quantum Monte Carlo).

- Open question : how to tackle other boundary conditions ?
Ex : Free BC, order parameter depends on position, flow equation much harder to solve.

arXiv:1606.03205

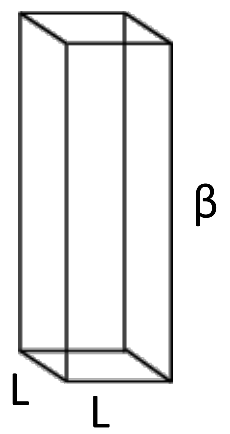
Phase diagram : quantum vs classical





Finite size scaling for quantum systems

QMC: β>>L



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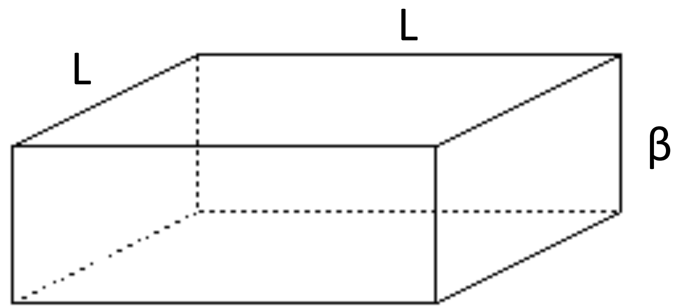
Critical Casimir : β<<L



We can thus expect that the universal coefficient of FSS depends on the ratio ρ=βc/L .

At the critical point δ=0: $$u = E_0 - \frac{T^3}{c^2}\Phi(\rho)$$

$$\Phi(0)$$ Casimir amplitude (=-0.32 for Ising)

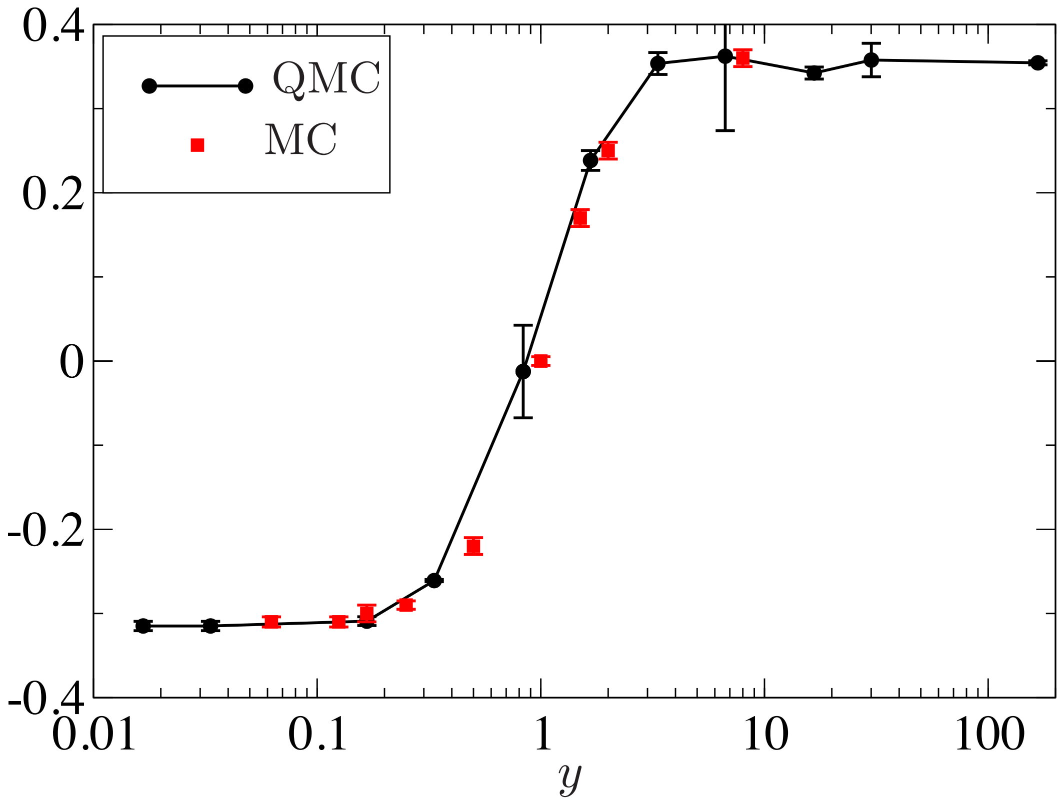
$$\lim_{\rho\to\infty} \tilde{\Phi}(\rho) = \alpha\rho^3$$ with α a universal (non-standard Casimir) amplitude (= 0.37 for Ising)

Aspect ratio and Finite Size Scaling

Dependence on aspect ratio known in context of Casimir forces.

Continuous imaginary time QMC for quantum Ising in transverse field

y=β/L=a/L



MC: Hucht et al. 2011

Non-Pertubative Renormalization Group

Subtle calculation in 4-ε and Large N for periodic BC.

Here : Non-Perturbative Renormalization Group (Wetterich 1993)

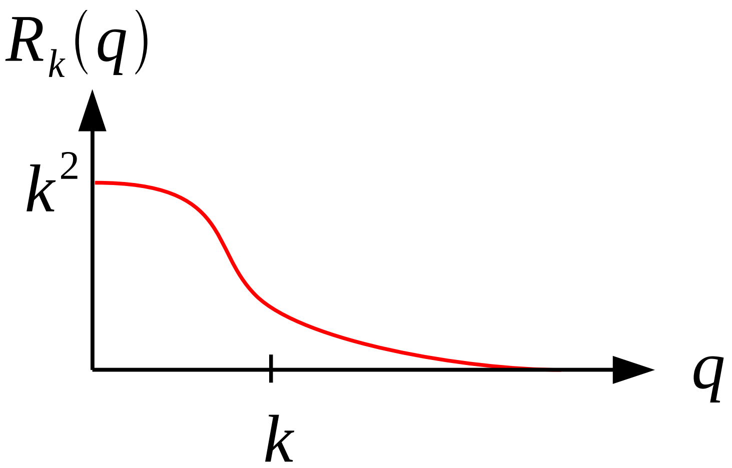
Family of actions indexed by momentum scale k

$S+\Delta S_\Lambda$ — $S+\Delta S_k$ → $S+\Delta S_{k=0}=S$

Reference system — System of interest

RG equation: $\partial_k \Gamma_k[\phi]$

$$\Delta S_k[\psi] = \sum_i \int_q R_k(q)\psi_i(q)\psi_i(-q)$$



Effective Action
("Gibbs" free energy)

Quantum O(N) model

$$S = \int_0^{\hbar\beta} d\tau \int d^2r \left\{ \frac{(\nabla\varphi)^2}{2} + \frac{(\partial_\tau\varphi)^2}{2c^2} + \frac{r\varphi^2}{2} + \frac{u(\varphi^2)^2}{4!} \right\}$$

Effective action : Legendre transform of Free energy with respect to magnetic field.

Depends on the order parameter : $\phi = \langle\varphi\rangle$

Ansatz : Derivative expansion (low energy fluctuations most important close to QCP)

$$\Gamma_k[\phi] = \int_0^{\hbar\beta} d\tau \int d^dr \left\{ \frac{Z_k^x(\rho)}{2}(\nabla\phi)^2 + \frac{Z_k^\tau(\rho)}{2}(\partial_\tau\phi)^2 + \frac{Y_k^x(\rho)}{4}(\nabla\rho)^2 + \frac{Y_k^\tau(\rho)}{4}(\partial_\tau\rho)^2 + U_k(\rho) \right\}$$

$$\rho = \phi^2/2$$