

The ordered phase of quantum crystalline membranes

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Université Pierre et Marie Curie

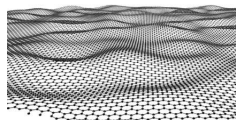
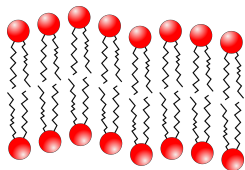
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ERG2016

20th September 2016

Introduction

Membrane: **2D** fluctuating system in an **embedding space**

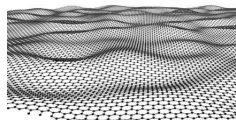
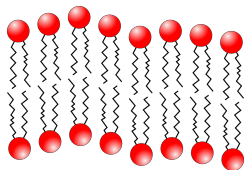


Examples:

- ▶ Phospholipid bilayers (*cell walls, ...*)
- ▶ Interface between immiscible substances
- ▶ Crystalline membranes (*graphene, hBN, MoS₂, spectrin networks...*)
- ▶ Worksheets (*string theory*)

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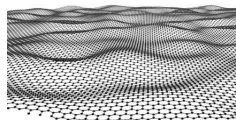
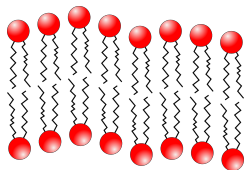


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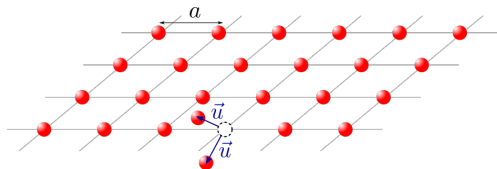
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- ▶ Worksheets (*string theory*)

Is the ordered phase stable against fluctuations?

Elasticity theory:

$$\mathcal{S}_{el} = \int d^2x \left[\frac{\mu}{4} (\partial_i \vec{u} \cdot \partial_j \vec{u} - \zeta^2 \delta_{ij})^2 + \frac{\lambda}{8} (\partial_i \vec{u} \cdot \partial_i \vec{u} - 2\zeta^2)^2 \right]$$



$g_{ij}^0 = \zeta^2 \delta_{ij}$: ground state metric

$\partial_i = \partial/\partial x^i$

μ, λ : Lamé coefficients

ζ : extension parameter

Positional order:

$$\langle \vec{u}(x) \cdot \vec{u}(0) \rangle \sim \log \left(\frac{x}{a} \right)$$

Phonons: 2D Crystal

Elasticity theory:

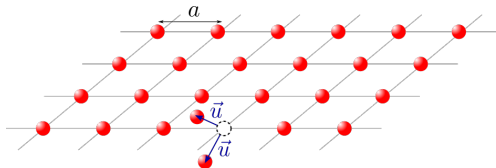
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Mermin Wagner's theorem \Rightarrow **No long range order**

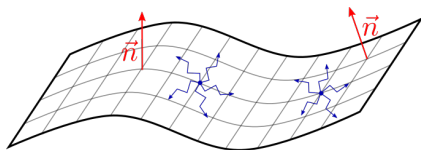
Flexurons: Fluid Membrane

Geometrical theory (No elasticity):

$$\mathcal{S}_{geo} = \int d^2x \sqrt{g} \left[\sigma + \frac{\kappa}{2} (\partial_i \vec{n})^2 \right]$$

σ : surface tension
 κ : bending energy
 g : fluctuating metric
 \vec{n} : normal vector

Free diffusion on the surface



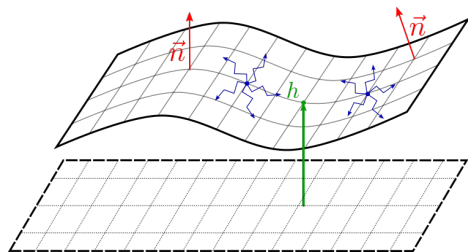
Orientational order:

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$$\mathcal{S}_{geo} = \int d^2x \sqrt{g} \left[\sigma + \frac{\kappa}{2} (\Delta h)^2 \right]$$



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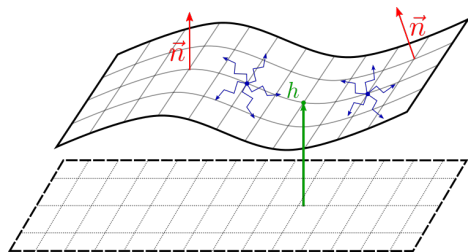
Orientational order:

$$\langle \partial_i h(x) \cdot \partial_i h(0) \rangle \sim \log \left(\frac{x}{a} \right)$$

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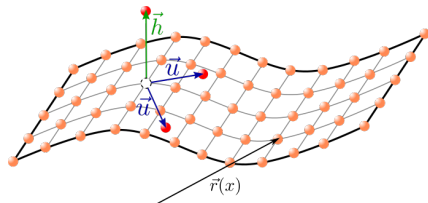
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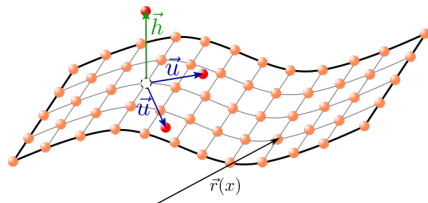
Phonons + Flexurons: Crystalline Membrane

$$\mathcal{S} = \int d^2x \left[\frac{\kappa}{2} (\partial^2 \vec{r})^2 + \frac{\mu}{4} (\partial_i \vec{r} \cdot \partial_j \vec{r} - \zeta^2 \delta_{ij})^2 + \frac{\lambda}{8} (\partial_i \vec{r} \cdot \partial_i \vec{r} - 2\zeta^2)^2 \right]$$



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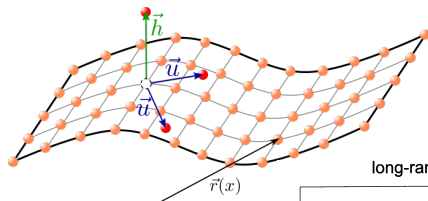
$$\langle \vec{u}(x) \cdot \vec{u}(0) \rangle \sim x^{-2\eta}$$

$$\langle h(x)h(0) \rangle \sim x^{2-\eta}$$

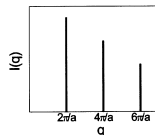
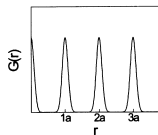
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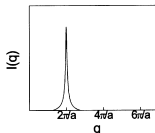
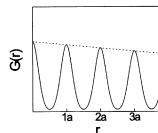
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long-range order (LRO)



quasi-LRO



Positional order:

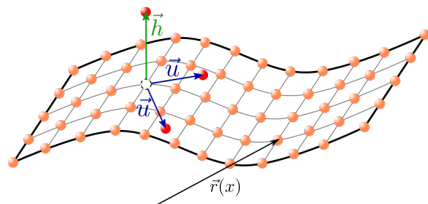
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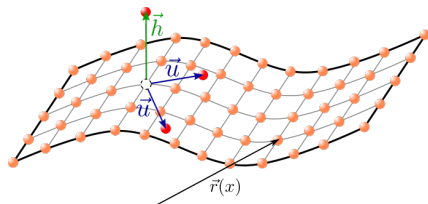
$$\langle \partial_i \vec{u}(x) \cdot \partial_i \vec{u}(0) \rangle \sim x^{-2-2\eta}$$

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Long range order

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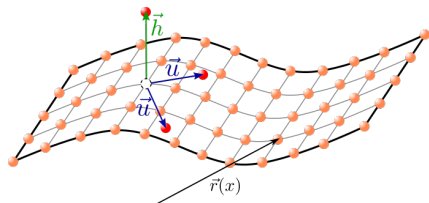
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Long range order

$\eta > 0 \Rightarrow$ Mermin-Wagner

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Long range order

Stable ordered (flat) phase

Breakdown of perturbation theory

$$D_c^{sup} = 4$$

$$D_c^{inf} = 2 - \eta$$

$\Rightarrow D = 4 - \epsilon$ or $d \gg 1$ expansions

Physical membranes: $D = 2$, $d = 3$

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Analytical models

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- ▶ **N**on **P**erturbative **R**enormalization **G**roup
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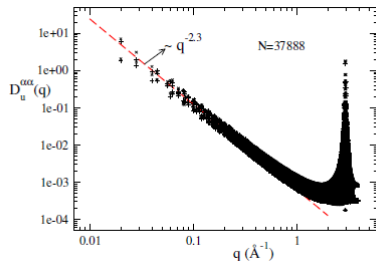
Numerical computations

- ▶ **M**onte **C**arlo
- ▶ **M**olecular **D**ynamics

The anomalous dimension in the flat phase

Computation of η

SCSA	$\eta = 0.821$	Le Doussal,Radzihovsky
SC $d \gg 1$	$\eta = 0.789$	Gazit
NPRG	$\eta = 0.849$	Essafi,Kownacki,Mouhanna
NPRG	$\eta = 0.85$	Braghin,Hasselmann
MD	$\eta = 0.81$	Zhang,Davis,Kroll
MC	$\eta = 0.72$	Bowick,Caterall,Falcioni,Thorleifsson,Anagnostopoulos
MC	$\eta = 0.85$	Los,Katsnelson,Yazyev,Zakharchenko,Fasolino
MC	$\eta = 0.795$	Tröster
RGMC	$\eta = 0.79$	Tröster



What about **quantum** fluctuations?

Previous computations

▶ SCSA, $d \gg 1$:

Gaussian flat phase fixed point $\Rightarrow \eta = 0$

Stable flat phase in IR

P.San-José, J.González and F.Guinea, Phys. Rev. Lett. **106**, 045502 (2011)

F.Guinea, P. Le Doussal, and K.J. Wiese, Phys. Rev. B **89**, 125428 (2014)

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▶ Perturbations, $D = 2, d = 3$:

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Destabilization of the flat phase

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▶ SCBA:

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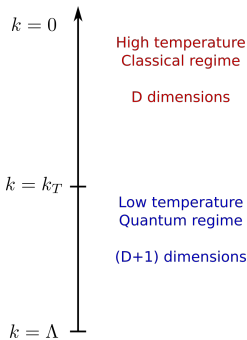
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Quantum to classical crossover

$$\mathcal{S} = \int_0^\beta d\tau \int d^D x \left[\frac{\rho}{2} (\partial_\tau \vec{r})^2 + \frac{\kappa}{2} (\partial^2 \vec{r})^2 + \frac{\mu}{4} (\partial_i \vec{r} \cdot \partial_j \vec{r} - \zeta^2 \delta_{ij})^2 + \frac{\lambda}{8} (\partial_i \vec{r} \cdot \partial_i \vec{r} - \zeta^2 D)^2 \right]$$

Thermal crossover

$$\bar{\beta}_{k_T} \sim 1 \Rightarrow \boxed{k_T = \sqrt{\frac{k_B T}{\hbar}} \sqrt{\frac{\rho}{\kappa}}}$$

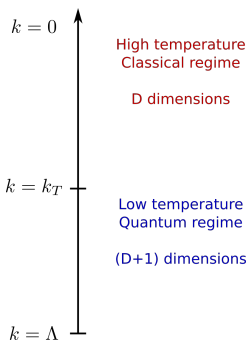


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RG flow at 0 temperature

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Ginzburg length

$$k_G^q \sim \Lambda e^{-\frac{16\pi(\bar{\lambda}_\Lambda + 2\bar{\mu}_\Lambda)}{21(\bar{\mu}_\Lambda + \bar{\lambda}_\Lambda)}}$$

$$D_c^{sup} = 2$$

Flat phase fixed point:

$$\bar{\lambda}^* = \bar{\mu}^* = 0$$

Anomalous dimension:

$$\eta^* = 0$$

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$$\left. \begin{aligned} \langle \partial_i u(x) \partial_i u(0) \rangle &\sim x^{-4} \\ \langle \partial_i h(x) \partial_i h(0) \rangle &\sim x^{-2} \end{aligned} \right\} \Rightarrow \boxed{\text{Stable flat phase}}$$

RG flow at high temperature

$$\mathcal{S} = \int d^D x \left[\frac{\kappa}{2} (\partial^2 \vec{r})^2 + \frac{\mu}{4} (\partial_i \vec{r} \cdot \partial_j \vec{r} - \zeta^2 \delta_{ij})^2 + \frac{\lambda}{8} (\partial_i \vec{r} \cdot \partial_i \vec{r} - \zeta^2 D)^2 \right]$$

Ginzburg length

$$k_G^{cl} \sim \sqrt{\frac{7\mu_\Lambda^{cl}(\lambda_\Lambda^{cl} + \mu_\Lambda^{cl})}{4\pi(\lambda_\Lambda^{cl} + 2\mu_\Lambda^{cl})} \frac{k_B T}{\kappa^2}}$$

$$D_c^{sup} = 4$$

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Flat phase fixed point:

$$\bar{\lambda}^* = -3.10, \bar{\mu}^* = 6.21$$

Anomalous dimension:

$$\eta^* = 0.849$$

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Stable flat phase

Overall picture

Four possible evolution pathways:

- ▶ $k_G^{cl} < k_T < k_G^q$:
Quantum Ultraviolet – Anharmonic
- ▶ $k_G^{cl}, k_G^q < k_T$:
Quantum Ultraviolet – Quantum Gaussian – Anharmonic
- ▶ $k_T < k_G^{cl}, k_G^q$:
Quantum Ultraviolet – Weak Coupling – Anharmonic
- ▶ $k_G^q < k_T < k_G^{cl}$:
Quantum Ultraviolet – Quantum Gaussian – Weak Coupling – Anharmonic

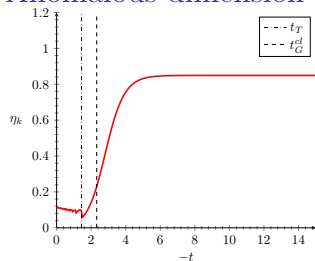
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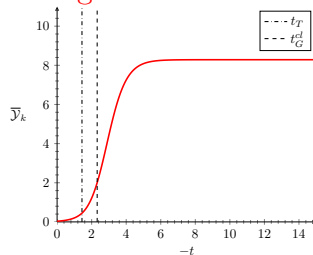
- ▶ $k_G^{cl} < k_T < k_G^q$:
Quantum Ultraviolet – Anharmonic
- ▶ $k_G^{cl}, k_G^q < k_T$:
Quantum Ultraviolet – Quantum Gaussian – Anharmonic
- ▶ $k_T < k_G^{cl}, k_G^q$:
Quantum Ultraviolet – Weak Coupling – Anharmonic
- ▶ $k_G^q < k_T < k_G^{cl}$:
Quantum Ultraviolet – Quantum Gaussian – Weak Coupling – Anharmonic

Application to graphene

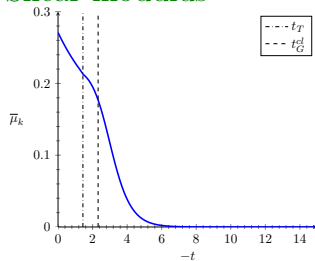
Anomalous dimension



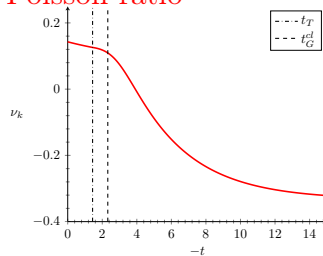
Young modulus



Shear modulus



Poisson ratio



Conclusion

- ▶ Quantum fluctuations **do not** destabilize the flat phase

Quantum fluctuations dominate up to k_T

O. Coquand, D. Mouhanna, cond-mat arxiv 1607.03335

to be published in Phys. Rev. E

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- ▶ Work in progress:

- ▶ Compute other thermodynamic quantities
(*Thermal expansion coefficient, heat capacity,...*)
- ▶ Electron-phonon coupling
- ▶ Effects of disorder

O. Coquand, D. Mouhanna, J.P. Kownacki