Background Independence in a Background Dependent Renormalization Group

Peter Labus

SISSA Trieste

Juergen A. Dietz, Tim R. Morris, JHEP 1504 (2015)

PL, Tim R. Morris and Zoë H. Slade, PRD 94 (2016)

Juergen A. Dietz, Tim R. Morris and Zoë H. Slade, 1605.07636

Background independence

Background field method:

- Split field into background plus fluctuations to be able to evaluate traces in flow equation, e.g.

$$\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$$

- But physical quantities are functions of full field $\tilde{g}_{\mu\nu}$ only, so there should be no artificial dependence of the split

But in gravity additionally:

- Have to define scale via spectrum of Laplacian operator $-\bar{\nabla}^2$ built from background metric $\bar{g}_{\mu\nu}$.
- Then Background Independence lost (at intermediate k) through cutoff function $R_k(-\bar{\nabla}^2)$.
- Modified split Ward identity (msWI) restores background independence in limit $k \to 0$.

Conformally Reduced Gravity

We want to study the functional RG of conformally reduced gravity:

$$\tilde{g}_{\mu\nu} = f(\tilde{\phi})\hat{g}_{\mu\nu} = f(\chi + \tilde{\varphi})\hat{g}_{\mu\nu} \quad \text{and} \quad \bar{g}_{\mu\nu} = f(\chi)\hat{g}_{\mu\nu}\,,$$

where the **conformal factor** $f(\tilde{\phi})$ is kept *arbitrary*.

Set $\hat{g}_{\mu\nu} = \delta_{\mu\nu}$, split conformal factor field $\tilde{\phi} = \chi + \tilde{\varphi}$ into background and fluctuation field and define *classical fluctuation field* $\varphi = \langle\tilde{\varphi}\rangle$.

The parametrisation *does not depend on k*, because it is introduced at the bare level and does not depend on the infrared cutoff.

No gauge fixing required, no ghosts.

Flow Equation and Ward Identity

The effective action satisfies the Functional RG Equation (FRGE)

$$\partial_t \Gamma_k[\varphi, \chi] = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} + R_k[\chi]\right]^{-1} \partial_t R_k[\chi]\,, \quad t = \log(k/\mu)$$

Background independence is obtained by imposing *split symmetry*:

$$\tilde{\varphi}(x) \mapsto \tilde{\varphi}(x) + \varepsilon(x)\,, \qquad \chi(x) \mapsto \chi(x) - \varepsilon(x)\,.$$

The msWI encodes the extent to which the effective action violates this symmetry.

$$\frac{1}{\sqrt{\bar{g}}}\left(\frac{\delta\Gamma_k}{\delta\chi} - \frac{\delta\Gamma_k}{\delta\varphi}\right) = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} + R_k[\chi]\right]^{-1} \frac{1}{\sqrt{\bar{g}}}\left\{\frac{\delta R_k[\chi]}{\delta\chi} + \frac{d}{2}\,\partial_\chi \ln f\, R_k[\chi]\right\}$$

The full functional system

Compatibility: msWI is compatible with the flow if it is satisfied everywhere **along the flow**: $\mathcal{W}_{k_0} = 0 \Rightarrow \mathcal{W}_k = 0, \forall k$

Rewriting Ward Identity as $\mathcal{W} = 0$ the flow of the msWI reads

$$\partial_t \mathcal{W}_\omega = -\frac{1}{2} \operatorname{tr} \left(\Delta \, \dot{r} \, \Delta \, \frac{\delta^2}{\delta\varphi\delta\varphi} \right) \mathcal{W}_\omega$$

- *Trivial* on the functional level, because both identities are derived form the same partition function.
- Not clear however is the *overlap of information* of flow and msWI
- Compatibility also *non-trivial* within truncations

The derivative expansion to $\mathcal{O}(\partial^2)$

Assume slowly varying background field χ.

$$\Gamma_k[\varphi,\chi] = \int d^dx\sqrt{\bar{g}}\left(-\frac{1}{2}K(\varphi,\chi)\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi,\chi)\right)$$

Two equations at every order of expansion.

Flow and msWI, here only potential V:

$$\partial_t V(\varphi,\chi) = f(\chi)^{-\frac{d}{2}}\int dp\, p^{d-1} Q_p \dot{R}_p$$

$$\partial_\chi V - \partial_\varphi V + \frac{d}{2}\partial_\chi \ln f\, V = f(\chi)^{-\frac{d}{2}}\int dp\, p^{d-1} Q_p \left(\partial_\chi R_p + \frac{d}{2}\partial_\chi \ln f\, R_p\right)$$

with the propagator

$$Q_p = \left(\partial_\varphi^2 V - p^2\frac{K}{f} + R_p\right)^{-1}$$

Compatibility in the derivative expansion I

Flow of Ward identity for V:

$$\dot{\mathcal{W}}^{(V)} = -\int_p Q_p^2 \dot{R}_p \left(\partial_\varphi^2 \mathcal{W}^{(V)} - p^2 \mathcal{W}^{(K)}\right) - \int_{p,q} Q_p^2 \left(\partial_\varphi^2 Q_q - 2p^2 P_q\right) [\dot{R}, \partial_\chi R + \gamma R]_{qp} \,,$$

where $\int_p \equiv f(\chi)^{-d/2} \int dpp^{d-1}$ and $\gamma \equiv \frac{d}{2}\partial_\chi \ln f$.

Compatibility may be realised by setting

$$\left[\dot{R}, \partial_\chi R + \gamma R\right]_{qp} \equiv \dot{R}_q(\partial_\chi R_p + \gamma R_p) - \dot{R}_p(\partial_\chi R_q + \gamma R_q) = 0$$

This implies the *compatibility condition*:

$$\boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\, \dot{R}_p}$$

Compatibility in the derivative expansion II

Flow of Ward identity for K:

- more involved
- additional commutator-like terms

$$\left(\partial_\chi R_p + \gamma R_p\right) \partial^k_{p^2} \dot{R}_p - \dot{R}_p \partial^k_{p^2} \left(\partial_\chi R_p + \gamma R_p\right)$$

But again, vanish if

$$\boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\, \dot{R}_p}$$

This provides a necessary and sufficient condition to ensure compatibility in the derivative expansion.

Required form of cutoff R_k

$$\boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\,\dot{R}_p}$$

In dimensionless variables one can show:

- If $\eta = 0$ compatibility condition automatically satisfied
- If $\eta \neq 0$ the compatibility condition implies

$$\hat{p}\frac{d}{d\hat{p}} r(\hat{p}^2) = -2n\, r(\hat{p}^2)$$

for some constant $n = d/2(\eta F/(d_v F - \gamma) - 1)$ and thus

$$r(\hat{p}^2) \propto \hat{p}^{-2n}$$

1. Fixed points are forbidden in general

In dimensionless variables

$$\partial_t \bar{V} + d_V \bar{V} - \frac{\eta}{2}\,\bar{\varphi}\,\frac{\partial \bar{V}}{\partial \bar{\varphi}} - \frac{\eta}{2}\,\bar{\chi}\,\frac{\partial \bar{V}}{\partial \bar{\chi}} = \int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{d_R\, r - \frac{d_V}{d}\,\hat{p}\, r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}} \bar{V}}$$

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\,\bar{V} = \bar{\gamma}\int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{r - \frac{1}{d}\,\hat{p}\, r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}} \bar{V}}$$

where crucially

$$\bar{\gamma} = \frac{d}{2}\frac{\partial}{\partial \bar{\chi}} \ln \bar{f}\left(\mathrm{e}^{\eta t/2}\mu^{\eta/2}\bar{\chi}\right) .$$

Ward identity forces $\bar{V}$ to depend on t through $\bar{\gamma}$, thus **fixed points are forbidden in general** unless again

1. $\eta = 0$ or
2. set f to be power law: $f \propto \chi^\rho \implies \bar{\gamma} = \frac{d}{2}\frac{\rho}{\bar{\chi}}$, ρ const.

2. Incompatibility implies no solutions

Combine (incompatible) equations to ($d = 4$, opt. cutoff):

$$2\partial_t \bar{V} + \eta \bar{V} - (\eta \bar{\varphi} - \alpha \bar{\chi})\, \partial_{\bar{\varphi}} \bar{V} - (\eta + \alpha) \bar{\chi} \partial_{\bar{\chi}} \bar{V} = 0$$

Separate scale dependence by method of characteristics:

$$\bar{V} = \mathrm{e}^{-\eta t/2}\, \hat{V}(\hat{\phi}, \hat{\chi})\,,$$

where hatted variables $\hat{V}, \hat{\phi}, \hat{\chi}$ are initial data. Substitute $\bar{V}$ back into flow equation:

$$\hat{\chi} \partial_{\hat{\chi}} \hat{V} + 2\rho \hat{V} = \frac{\rho}{3} \frac{1}{\mathrm{e}^{-\frac{\eta}{2} t} - \partial^2_{\hat{\phi}} \hat{V}}$$

No solutions unless $\eta = 0$.

3. Background independent flow for $\eta = 0$

Flow and msWI can be combined and after redefinition of scale

$$\hat{t} = t + \frac{\ln \bar{f}}{2 - d_f}$$

one finds background independent flow equation

$$\partial_{\hat{t}} \hat{V} + d_V \hat{V} = \frac{d_V}{6} \frac{1}{1 - \partial_\phi^2 \hat{V}} \, .$$

- independent of χ and parametrisation f.
- Fixed points in t coincide with fixed points in $\hat{t}$:

$$\partial_t \bar{V} = \partial_{\hat{t}} \hat{V}$$

- When flow and Ward identity compatible can uncover a background independent (and f independent) description.

Summary and conclusions

- Investigated the potential conflict between fixed points and background independence.
- Compatibility guaranteed at exact level. But in the derivative expansion, compatibility only guaranteed if $\eta = 0$ or cutoff R_k is power law.
- If incompatible then no solutions - confirmed with LPA example.
- If compatible, fixed points can still be forbidden.
- If compatible, can combine flow and Ward identity to uncover a background independent description.