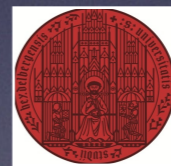


Towards solving hierarchy problem with asymptotically safe gravity

Masatoshi Yamada

(Kanazawa Univ. → Kyoto Univ. → Heidelberg Univ.)



with Kin-ya Oda (Osaka Univ.)

and

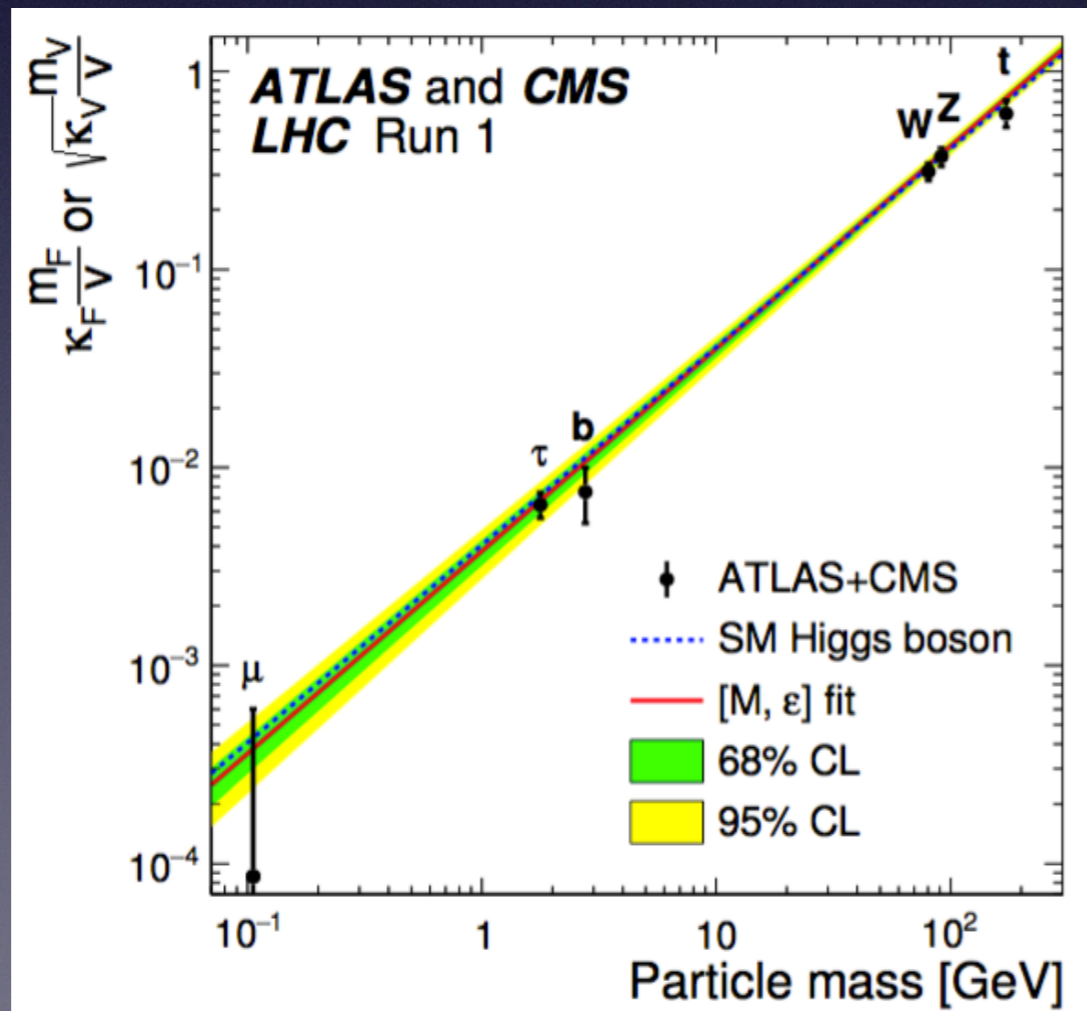
Yuta Hamada (KEK & Wisconsin Univ.)

ERG2016@Trieste

LHC

- Discovery of Higgs boson with 125 GeV
- The SM well describes the physics up to TeV.

coupling to Higgs



$$g_i = \frac{1}{\langle h \rangle} m_i$$

Particle mass

Nothing else...

- No new particles appear yet.
- Diphoton (750 GeV) Olympic was held and finished.
 - More than 400 papers entry.



- 17 MeV?? (not LHC)

taken from resonances

A. Krasznahorkay et al, Phys. Rev. Lett. 116 (2016) no. 4, 042501
cf. J. L. Feng, arXiv: 1604.07411, 1608.03591

There are mysteries.

- Neutrino mass
- Dark matter
- Baryon number
- Quantum gravity
- Hierarchy problem
- Origin of electroweak scale
- Quantization of charge
- Flavor structure etc...

What can we do at present?
How to approach to them?

Purpose of study

- Quantum gravity must exist.
 - Asymptotically safe gravity is one of possible candidates.
- Attack to the hierarchy problem.
- Can we establish a new paradigm?
 - Symmetry? Principle?

Plan

- Revisit Hierarchy problem
- Asymptotically safe gravity
- Higgs-Yukawa model non-minimally coupled to gravity

Hierarchy problem

- Renormalized Higgs mass

$$\int^{\Lambda} d^4p \frac{1}{p^2} \sim \Lambda^2$$

$$\overline{\text{---} \times \text{---}}^{m_R^2} = \overline{\text{---} \times \text{---}}^{m_{\Lambda}^2} + \frac{\text{---} \bigcirc \text{---}}{\lambda} + \dots$$

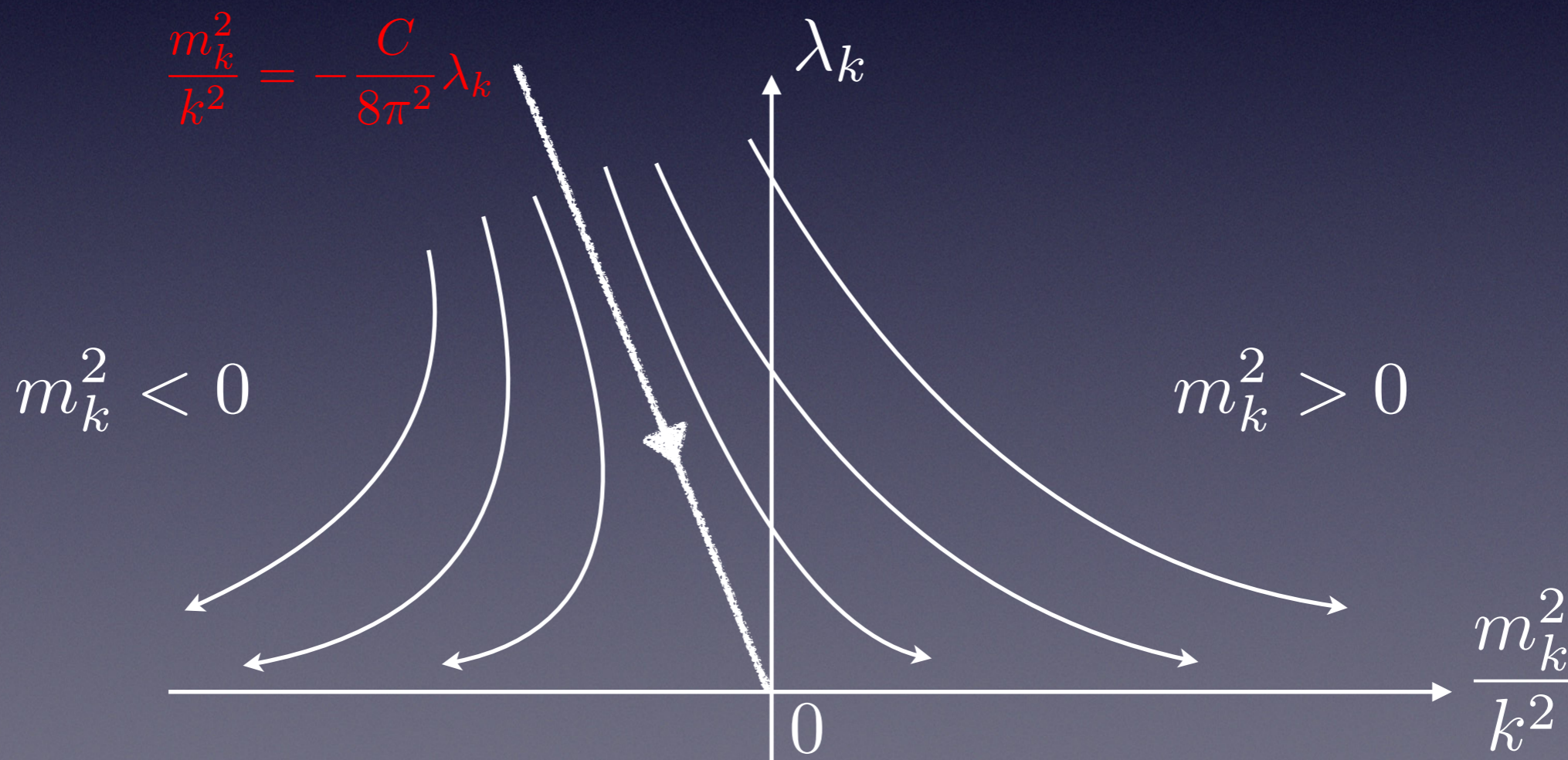
$$m_R^2 = m_{\Lambda}^2 + \frac{\Lambda^2}{16\pi^2} (\lambda + \dots)$$

- $O(10^2 \text{ GeV})^2 = O(10^{19} \text{ GeV})^2 - O(10^{19} \text{ GeV})^2$

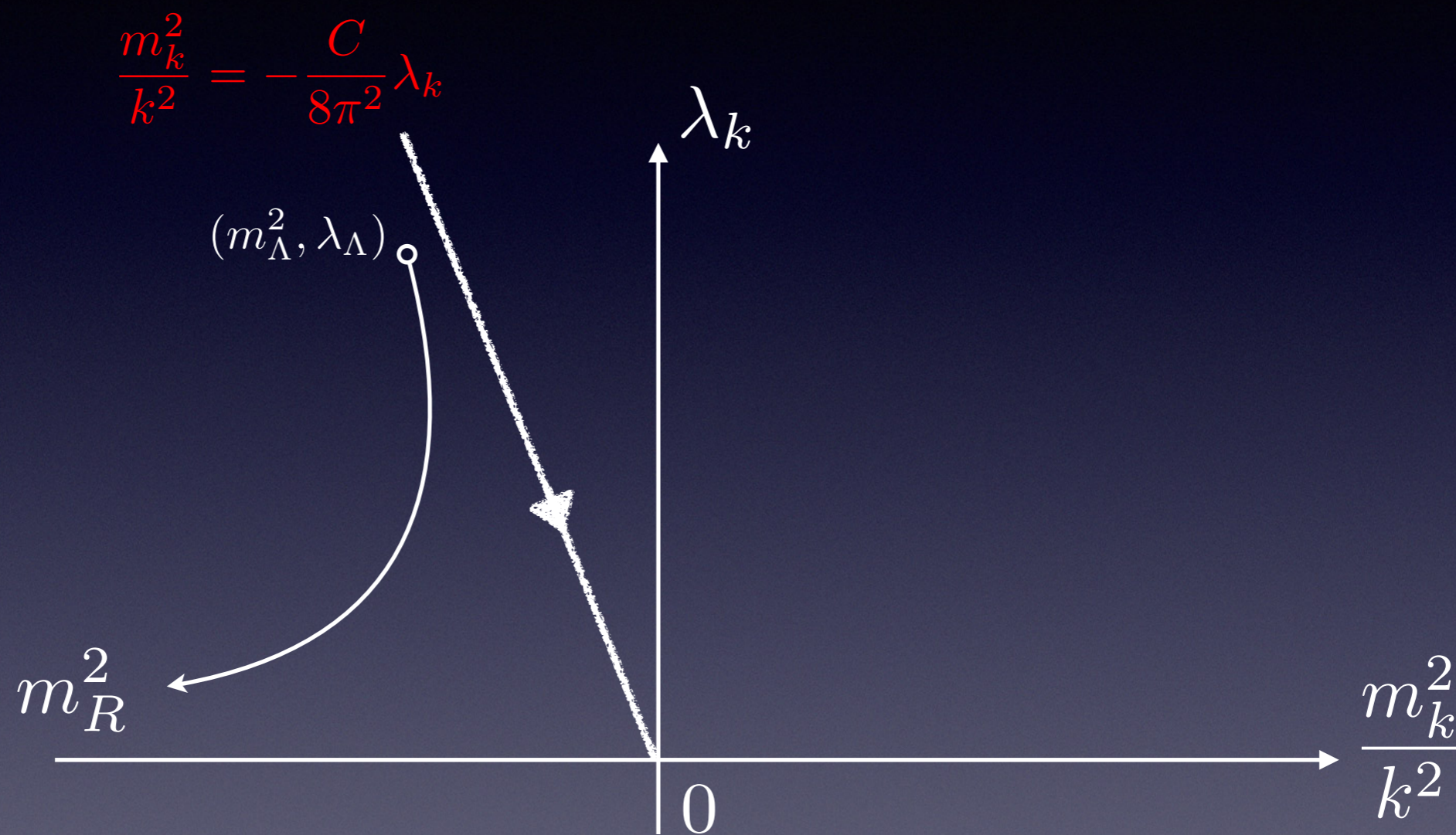
$$m_R^2 \ll m_{\Lambda}^2$$

In viewpoint of Wilson RG

$$\Gamma_k = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 - \frac{m_k^2}{2} \phi^2 - \frac{\lambda_k}{4} \phi^4 \right]$$



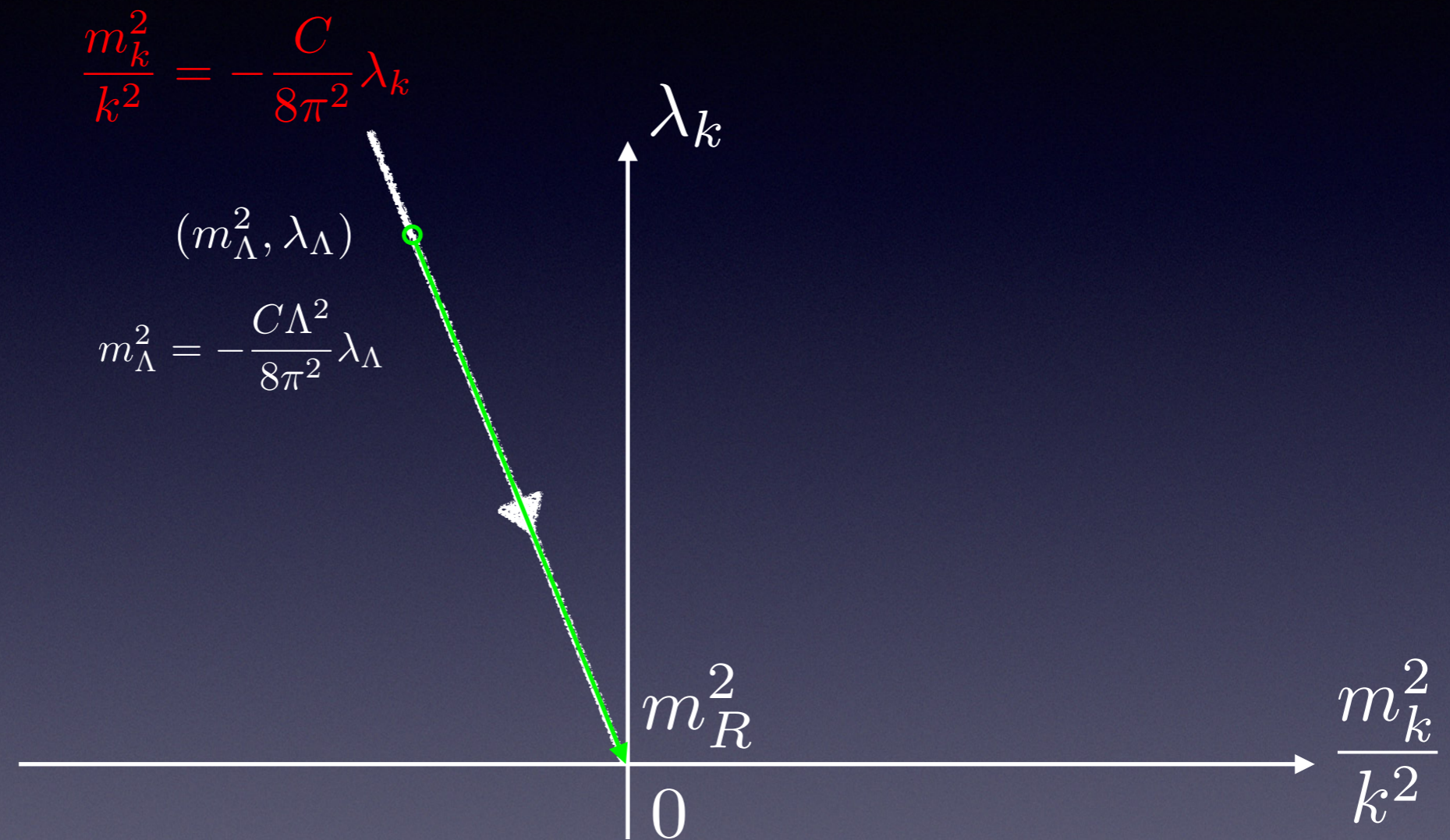
In viewpoint of Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2} \lambda_{k=0}$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

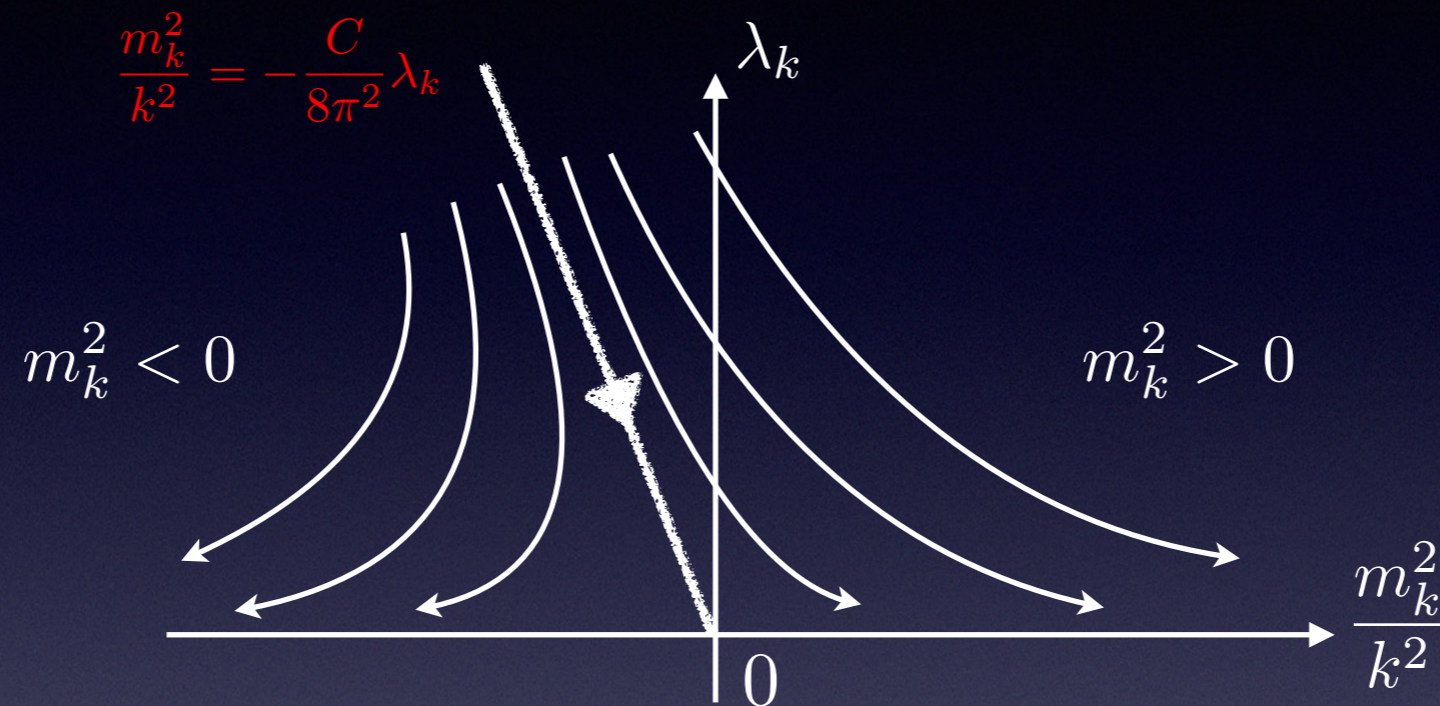
In viewpoint of Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2} \lambda_{k=0} = 0$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

In viewpoint of Wilson RG



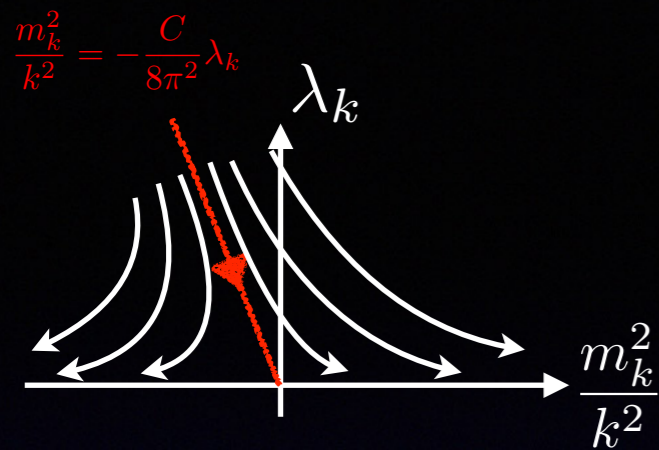
- Λ^2 determines the position of phase boundary (critical line).
- The phase boundary corresponds to the massless (critical) theory.
- To obtain small m_R , put the bare parameters close to the phase boundary.

In viewpoint of Wilson RG

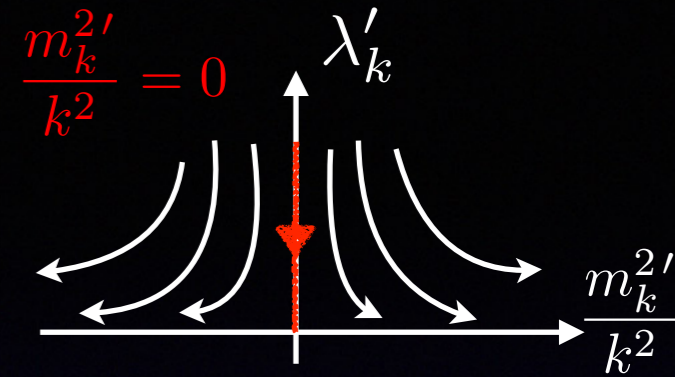
Hierarchy problem = Criticality problem

Why is the Higgs close to critical?





Comment



W. A. Bardeen, FERMILAB-CONF-95-391-T
H. Aoki, S. Iso, Phys. Rev. D86, 013001

- Λ^2 is spurious?
 - The **position** of phase boundary is physically meaning**less**.
 - The distance between the flow and the boundary is physically meaning**ful**.
 - In perturbation theory, Λ^2 is always subtracted by the counter term or dimensional regularization.

• Rotation of coordinate. $\rightarrow C = 0$

$$\frac{m_k^2}{k^2} = -\frac{C}{8\pi^2}\lambda_k \quad \rightarrow \quad \frac{m_k^{2'}}{k^2} = 0$$

- Hierarchy problem \Leftrightarrow The bare theory of Higgs is almost **scale (conformally) invariant**.
- If $m_\Lambda=0$, $m_R=0$ is realized.

cf. RG eq. of m in perturbation

$$\mu \frac{dm^2}{d\mu} = \frac{m^2}{16\pi^2} (12\lambda + \dots)$$

• Idea of **classical scale (conformal) invariance**

- How to generate the scale related to v_{EW} ?
- Dimensional transmutation or Dynamical symmetry breaking with TeV scale.

Summary so far

- Hierarchy problem is criticality problem.
 - Higgs have to be close to the phase boundary.
- Λ^2 is physically meaningful or not.
 - Classical scale (conformal) invariance?
 - Gravitational effect?

Plan

- Revisit Hierarchy problem
- Asymptotically safe gravity
- Higgs-Yukawa model non-minimally coupled to gravity

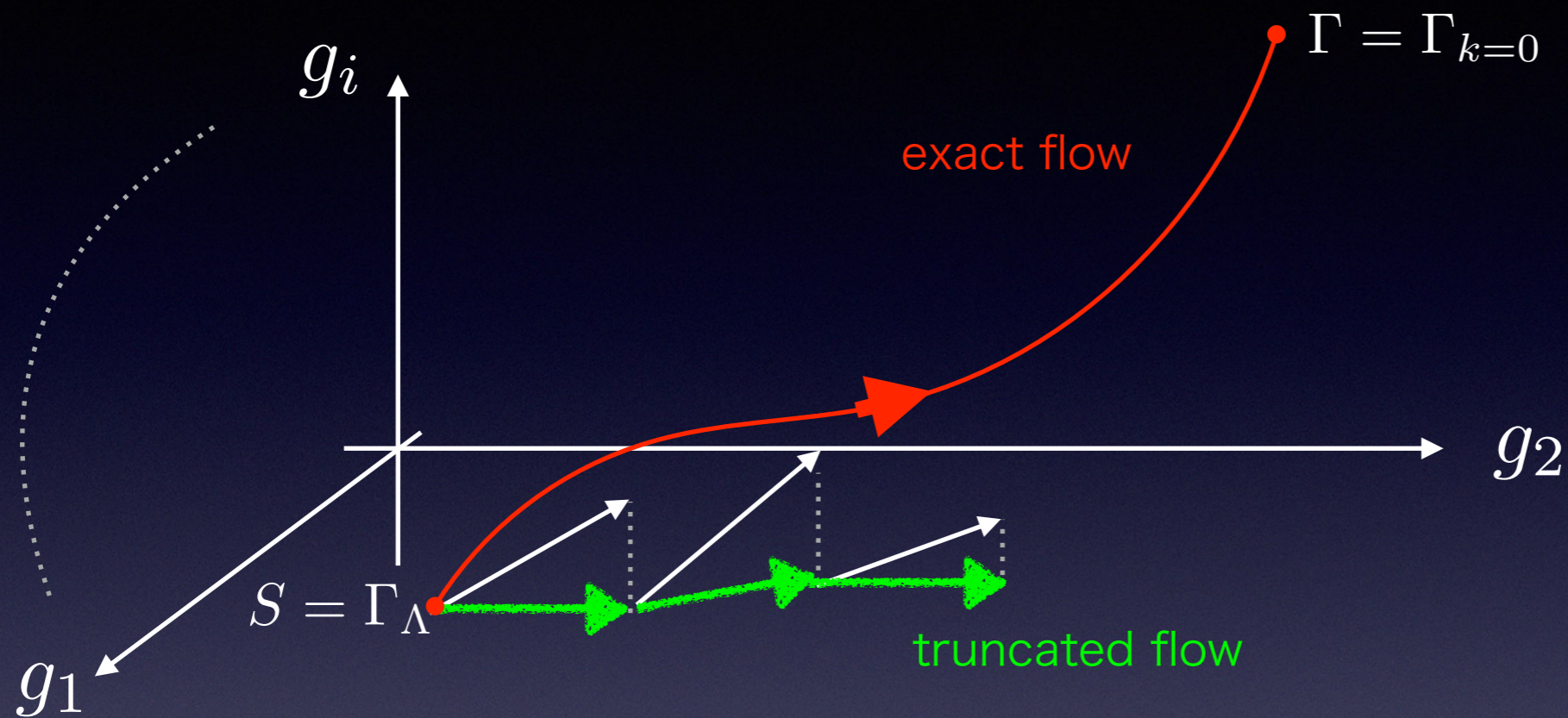
Asymptotically safe gravity

- Suggested by S. Weinberg S. Weinberg, Chap 16 in General Relativity
- Existence of UV fixed point
 - Continuum limit $k \rightarrow \infty$.
 - UV critical surface (UV complete theory) is defined by relevant operators.
 - Its dimension = number of free parameters.
- Generalization of asymptotically free

skippable!

釈迦に説法
(preaching to the experts)

Functional renormalization group



$$k\partial_k\Gamma_k = \frac{1}{2}\text{Str}[(\Gamma_k^{(2)} + R_k)^{-1}k\partial_k R_k]$$

$$\Gamma_k = \int d^4x [g_1\mathcal{O}_1 + g_2\mathcal{O}_2 + \cdots + g_i\mathcal{O}_i + \cdots]$$



$$\Gamma_k \simeq \int d^4x [g_1\mathcal{O}_1 + g_2\mathcal{O}_2]$$

Critical exponent

- Classification of flow around FP

- RG eq. around FP g^*

$$\partial_t g_i = \cancel{\beta_i(g^*)} + \left. \frac{\partial \beta_i}{\partial g_j} \right|_{g=g^*} (g_j - g_j^*) + \dots$$

$\partial_t = -k \partial_k$

- Solution of RG eq.

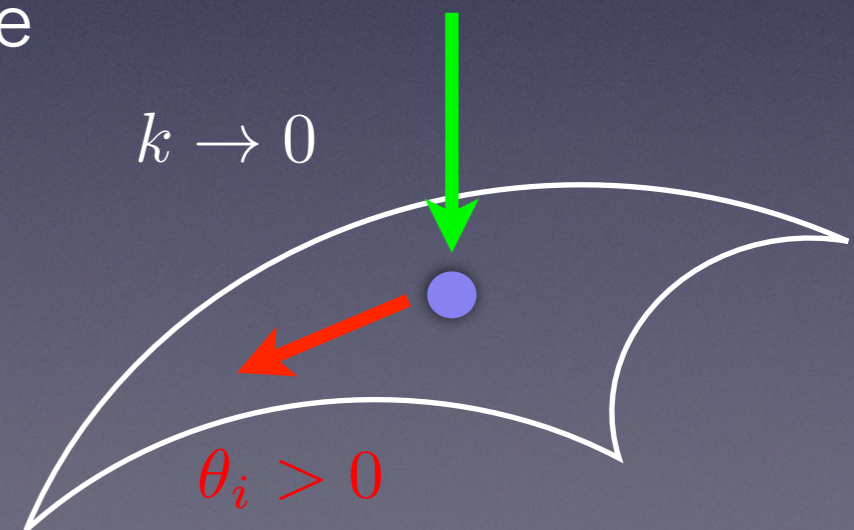
$$g_i(k) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{\Lambda}{k} \right)^{\theta_j}$$

eigenvalue

irrelevant

$\theta_i < 0$

$k \rightarrow 0$



relevant

Earlier studies

- Pure gravity

- Truncation: $f(R), \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu}, \text{ etc.}$
- Number of relevant operators: 3

- With matters

- stability of FP
- scalar-gravity system
- Higgs-Yukawa system
- gauge field system
- Fermionic system

- Prediction of Higgs mass

Cf.

O. Lauscher, M. Reuter, Phys. Rev. D66, 025016

K. Falls, et. al., arXiv: 1301.4191

D. Benedetti, et. al., Mod. Phys. Lett. A24, 2233

Cf.

R. Percacci, D. Perini, Phys. Rev. D67, 081503

P. Dona, et. al., Phys. Rev. D89, 084035

J. Meibohm, et. al., Phys.Rev. D93, 084035

R. Percacci, D. Perini, Phys. Rev. D68, 044018

G. Narain, R. Percacci, CQG 27, 075001

R. Percalli et. al, Phys. Lett. B689, 90

G. P. Vacca, O. Zanusso, Phys. Rev. Lett. 105, 231601

J. Daum et. al., JHEP 01, 084

U. Harst, M. Reuter, JHEP 05, 119

A. Eichhorn, H. Gies, New Phys. J., 113, 125012

A. Eichhorn, Phys. Rev. D86, 105021

etc.

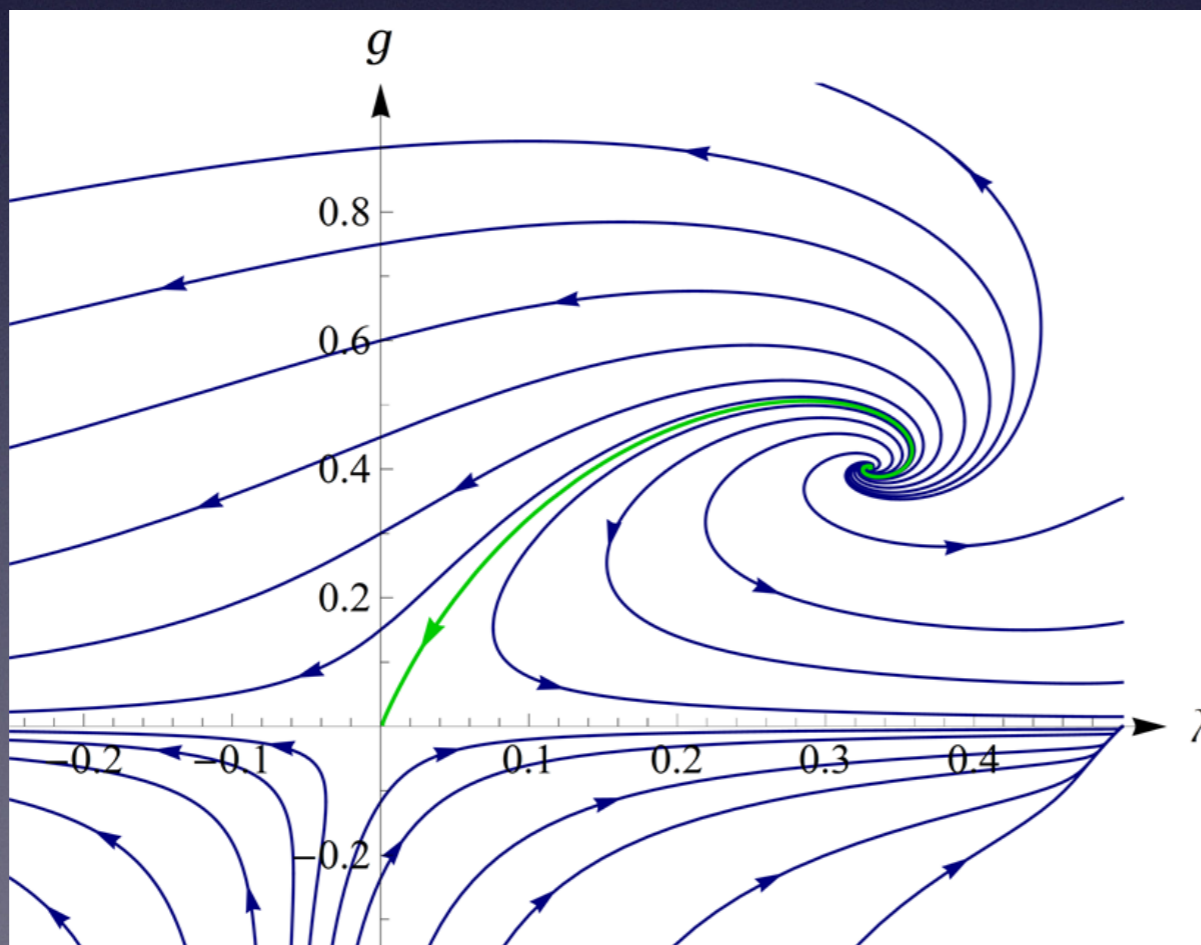
M. Shaposhnikov, C. Wetterich, Phys. Lett. B683, 196

Hierarchy problem for Λ_{cc}

- Why is Λ_{cc} small?



Why is the universe critical?



$$\Lambda_{cc} \ll 10^{-120}$$

Taken from Wiki
M, Reuter, F. Saueressing, Phys. Rev. D65, 065016

Plan

- Revisit Hierarchy problem
- Asymptotically safe gravity
- Higgs-Yukawa model non-minimally coupled to gravity

Higgs-Yukawa model

- Effective action

$$\Gamma_k = \int d^4x \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi^2) - F(\phi^2) R + \bar{\psi} \not{\nabla} \psi + y \phi \bar{\psi} \psi \right] + S_{\text{gf}} + S_{\text{gh}}$$

- Potentials

$$V(\phi^2) = \Lambda_{\text{cc}} + m^2 \phi^2 + \lambda \phi^4 + \dots$$

$$F(\phi^2) = M_{\text{pl}}^2 + \xi \phi^2 + \dots$$

- Toy model of Higgs-inflation (mentioned in latter)

Set-up

- Background field method $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 - de-Sitter metric is used.

- de-Donder (Landau) gauge

- Cutoff function: Litim cutoff;

$$R_k(z) = (k^2 - z)\theta(k^2 - z) \quad \text{scalar and gravity}$$

$$R_k(z - R/4) = (k^2 - (R/4))\theta(k^2 - (z - R/4)) \quad \text{fermion}$$

Without fermion

- Scalar-gravity system

R. Percacci, D. Perini, Phys. Rev. D68, 044018
G. Narain, R. Percacci, CQG 27, 075001

- 5 dimensional theory space

$$\{M_{\text{pl}}^2, \Lambda_{\text{cc}}, m^2, \xi, \lambda\}$$

- Gaussian-matter FP:

$$\bar{M}_{\text{pl}}^{2*} = 2.38 \times 10^{-2}$$

$$\bar{\Lambda}_{\text{cc}}^* = 8.82 \times 10^{-3}$$

$$\bar{m}^{2*} = \bar{\xi}^* = \bar{\lambda}^* = 0$$

- Critical exponents:

| | $M_{\text{pl}}^2, \Lambda_{\text{cc}}$ | m^2, ξ | λ |
|--------------|--|--------------------|-----------|
| $\theta_i =$ | $2.143 \pm 2.879i$ | $0.143 \pm 2.879i$ | -2.627 |

With a fermion

- Higgs-Yukawa system

K. Oda, M. Y., CQG 33, 125011

Without non-minimal coupling:

R. Percacci et. al, Phys. Lett. B689, 90

G. P. Vacca, O. Zanusso, Phys. Rev. Lett. 105, 231601

- 6 dimensional theory space

$$\{M_{\text{pl}}^2, \Lambda_{\text{cc}}, m^2, \xi, \lambda, y\}$$

- Gaussian-matter FP:

$$\bar{M}_{\text{pl}}^{2*} = 1.63 \times 10^{-2}$$

$$\bar{\Lambda}_{\text{cc}}^* = 3.72 \times 10^{-3}$$

$$\bar{m}^{2*} = \bar{\xi}^* = \bar{\lambda}^* = \bar{y}^* = 0$$

- Critical exponents:

| | M_{pl}^2, Λ | m^2, ξ | λ | y |
|--------------|----------------------------|----------------------|-----------|----------|
| $\theta_i =$ | $1.509 \pm 2.4615i$ | $-0.4909 \pm 2.461i$ | -2.6069 | -1.464 |

Result

- Fermionic effect makes m^2 and ξ **irrelevant**.
- m^2 , ξ are **not** free parameters.
- M_{pl}^2 , Λ_{cc} determine low energy physics.
- m^2 , ξ , λ are generated.

Is criticality of m^2 solved?

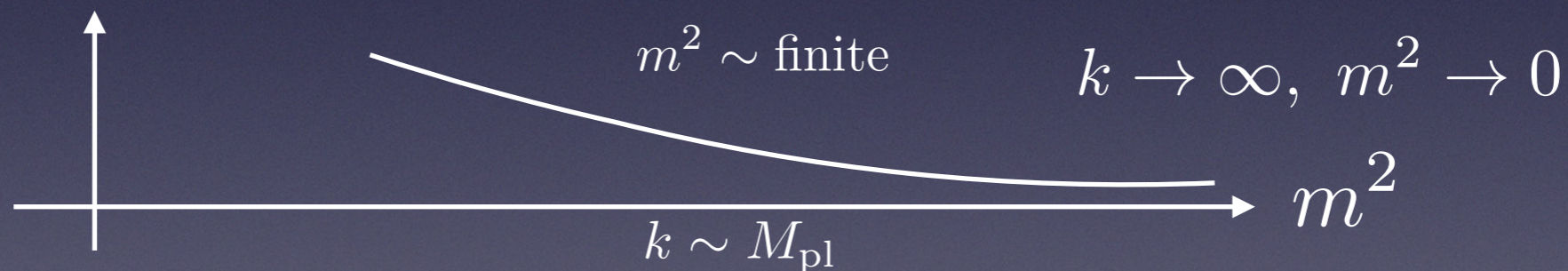
Is criticality of m^2 solved?

No

Flow of scalar mass

- RG eq.

$$\begin{aligned} \partial_t \bar{m}^2 = 2\bar{m}^2 - \frac{1}{48\pi^2} & \left[\frac{9\bar{\Lambda}_{\text{cc}}(1+2\bar{\xi})}{2(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})^2} - \frac{9(2\bar{\Lambda}_{\text{cc}} - \bar{M}_{\text{pl}})(1+2\bar{\xi})^2}{2(1+2\bar{m}^2)(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})^2} - \frac{9(1+2\bar{\xi})^2}{2(1+2\bar{m}^2)^2(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})} - \frac{18\lambda}{(1+2\bar{m}^2)^2} \right] \\ & + \frac{\partial_t \bar{M}_{\text{pl}} - 2\bar{M}_{\text{pl}}}{96\pi^2 \bar{M}_{\text{pl}}} \left[-\frac{2\bar{\xi}}{\bar{M}_{\text{pl}}} + \frac{3\bar{M}_{\text{pl}}(1+2\bar{\xi})}{2(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})^2} - \frac{3\bar{M}_{\text{pl}}(1+2\bar{\xi})^2}{2(1+2\bar{m}^2)(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})^2} \right] \\ & + \frac{1}{96\pi^2} \frac{\partial_t \bar{\xi}}{\bar{M}_{\text{pl}}} \left[2 - \frac{3\bar{M}_{\text{pl}}}{\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}}} + \frac{6\bar{M}_{\text{pl}}(1+2\bar{\xi})}{(1+2\bar{m}^2)(\bar{M}_{\text{pl}} - \bar{\Lambda}_{\text{cc}})} \right] - \frac{y^2}{8\pi^2}, \quad \partial_t = -k\partial_k \end{aligned}$$



- Once m^2 is generated, m^2 grows up due to the canonical scaling ($2m^2$).
- Fine-tuning of M_{pl}^2 , Λ_{cc} is still needed.

Higgs mass
 m^2

Gravitational
couplings
 $\{M_{pl}, \Lambda_{cc}\}$

- Higgs is controlled by the gravitational effect.
- But, why are they located at the critical place?

Criticality of Higgs mass \Leftrightarrow Criticality of the universe

Comment on irrelevant ξ

- $\xi \phi^2 R$ becomes also irrelevant.
- ξ plays crucial role in **Higgs-inflation**.

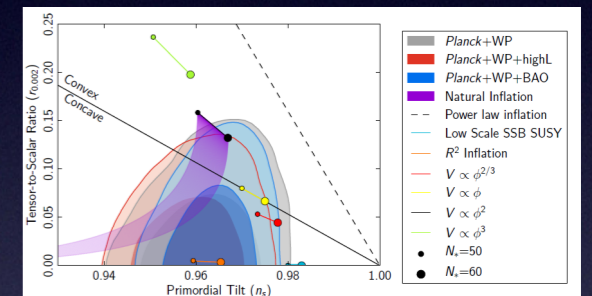
Brief review on Higgs-inflation

- Higgs-inflation can explain the Planck obs.

F. Bezrukov, M. Shaposhnikov, Phys. Lett. B659, 703

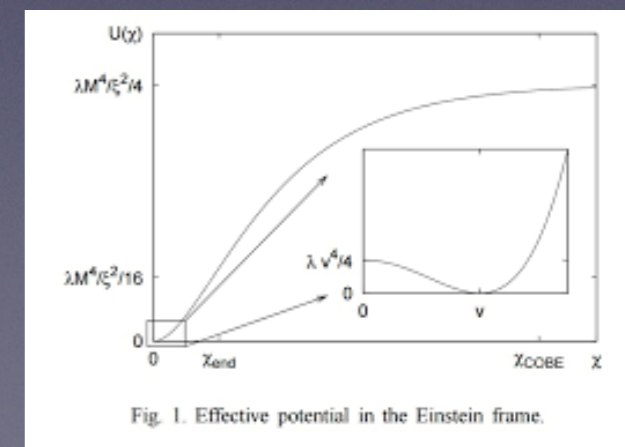
- Action (Jordan frame)

$$S_J = \int d^4x \sqrt{-g} \left[\left(1 + \xi \frac{h^2}{M_{\text{pl}}^2} + \dots \right) \frac{M_{\text{pl}}^2}{2} R + \frac{1}{2} (\partial_\mu h)^2 - V(h^2) \right]$$



- Conformal transformation (Jordan \rightarrow Einstein)

$$\left(1 + \xi \frac{h^2}{M_{\text{pl}}^2} + \dots \right) g_{\mu\nu} \rightarrow g_{\mu\nu}^E \rightarrow V(h^2) \rightarrow \frac{V(h^2)}{\left(1 + \xi \frac{h^2}{M_{\text{pl}}^2} + \dots \right)^2}$$



Brief review on Higgs-inflation

- To explain the experimental data, need large ξ

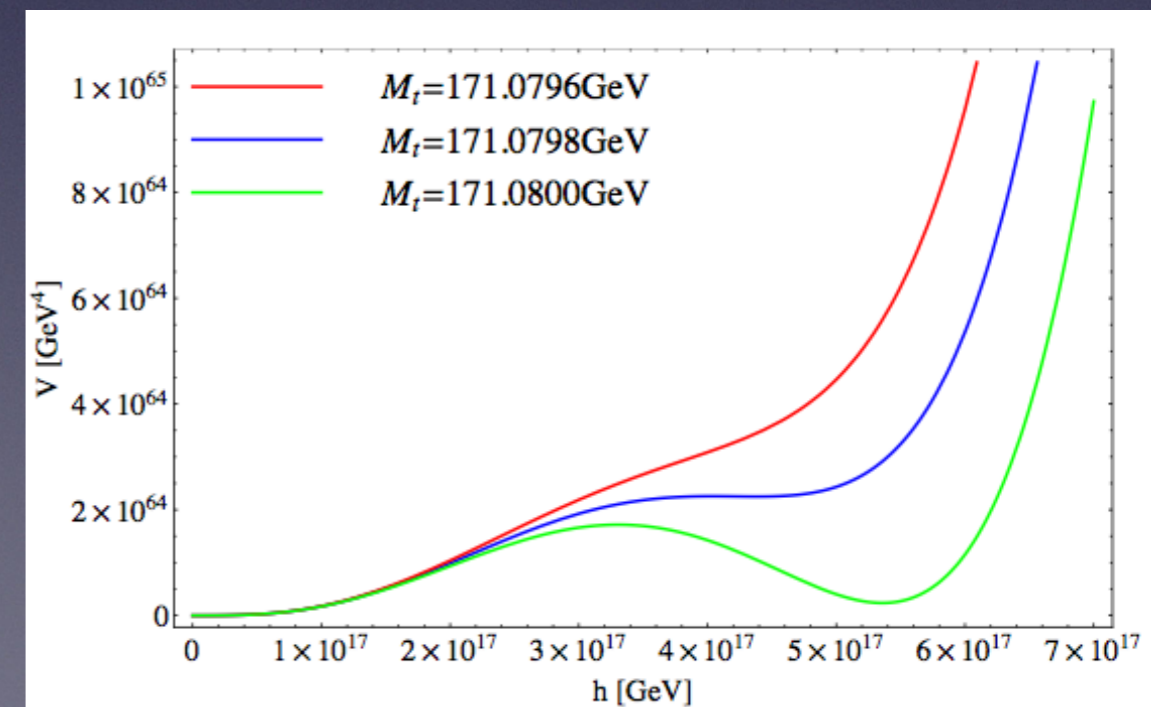
- $\xi \sim 10^5$ for $\lambda \sim 0.1$

F. Bezrukov, M. Shaposhnikov, Phys. Lett. B659, 703

- $\xi \sim 10$ for $\lambda \sim \lambda^*$

Y. Hamada, et. al., Phys. Rev. Lett., 112, 241301

J. L. Cook, et. al., Phys. Rev. D89, 103525



Is large ξ possible?

- RG eq. of ξ (canonical dim. =0)
- Large suppression factor $\rightarrow \xi$ basically is not large.

$$\begin{aligned}
 \partial_t \xi = & -\frac{1}{576\pi^2} \left[\frac{1+2\bar{m}^2}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} \left(9 + \frac{39\bar{M}_{\text{pl}}^2}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} + \frac{60\bar{M}_{\text{pl}}^2}{(\xi_0 - \bar{\Lambda}_{\text{cc}})^2} \right) + \frac{3(3+32\xi)}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} - \frac{6\bar{M}_{\text{pl}}^2(11+2\xi)}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} \right. \\
 & - \frac{60\bar{M}_{\text{pl}}^2(1+2\xi)}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^3} + \frac{216\xi(1+2\xi)^2}{(1+2\bar{m}^2)^3(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})} + \frac{9[\bar{\Lambda}_{\text{cc}}(5-2\xi) - 2\bar{M}_{\text{pl}}^2(1+2\xi)](1+2\xi)}{(1+2\bar{m}^2)(\xi_0 - \bar{\Lambda}_{\text{cc}})^2} \\
 & \left. + \frac{27(1+2\xi)(1-10\xi-16\xi^2)}{(1+2\bar{m}^2)^2(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})} + \frac{108\bar{M}_{\text{pl}}^2\xi(1+2\xi)^2}{(1+2\bar{m}^2)^2(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} + \frac{72\lambda_4}{(1+2\bar{m}^2)^2} \frac{1+12\xi+2\bar{m}^2}{1+2\bar{m}^2} \right] \\
 & + \frac{\partial_t \bar{M}_{\text{pl}}^2 - 2\bar{M}_{\text{pl}}^2}{1152\pi^2 \bar{M}_{\text{pl}}^2} \left[\frac{1+2\bar{m}^2}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} \left(3 + \frac{18\bar{M}_{\text{pl}}^2}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} + \frac{20\bar{M}_{\text{pl}}^2}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} \right) + \frac{15\xi}{\bar{M}_{\text{pl}}^2} - \frac{6(1+\xi)}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} - \frac{10\bar{M}_{\text{pl}}^2(3+4\xi)}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} \right. \\
 & \left. - \frac{20\bar{M}_{\text{pl}}^2(1+2\xi)}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^3} - \frac{3[\bar{\Lambda}_{\text{cc}} - \bar{M}_{\text{pl}}^2(5-4\xi)](1+2\xi)}{(1+2\bar{m}^2)(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} + \frac{36\bar{M}_{\text{pl}}^2\xi(1+2\xi)^2}{(1+2\bar{m}^2)^2(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} \right] \\
 & + \frac{\partial_t \xi}{1152\pi^2 \bar{M}_{\text{pl}}^2} \left[-15 + \frac{54\bar{M}_{\text{pl}}^2}{\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}}} + \frac{20\bar{M}_{\text{pl}}^2}{(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})^2} - \frac{6\bar{M}_{\text{pl}}^2(7+2\xi)}{(1+2\bar{m}^2)(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})} - \frac{144\bar{M}_{\text{pl}}^2\xi(1+2\xi)}{(1+2\bar{m}^2)(\bar{M}_{\text{pl}}^2 - \bar{\Lambda}_{\text{cc}})} \right] \\
 & - \frac{y^2}{48\pi^2},
 \end{aligned}$$

Summary

- Hierarchy problem = Criticality problem
- Asymptotically safe gravity is candidate of quantum gravity.
- Higgs-Yukawa model
 - m^2 , ξ becomes irrelevant.
 - Unification of Hierarchy problems
 - How to fine-tune the relevant parameters?
 - Higgs-inflation

Future works

- Extension of theory space
 - More fermions, gauge fields
- Yukawa coupling...
 - It should not be irrelevant because ~~$\phi \bar{\Psi} \Psi$~~ by chiral symmetry: $\beta_y \propto y$
 - Cf. A. Eichhorn's talk and A. Eichhorn, A. Held, J. M. Pawłowski: arXiv:1604.02041
 - Higgs-Yukawa with higher derivative gravity (in progress with Y. Hamada)

Future aims

- Why do m^2 and Λ_{cc} prefer the critical?
 - Do we have a relationship that both m^2 and Λ_{cc} become small?
 - Can we guarantee it?
 - What is the relationship with physical values in low energy.
- If we believe the classical scale invariance scenario,
 - can we guarantee it within asymptotically safe gravity?

ご清聴ありがとうございました!

Thank you for your attention!

Appendix

17 MeV

PRL 116, 042501 (2016)

PHYSICAL REVIEW LETTERS

week ending
29 JANUARY 2016

Observation of Anomalous Internal Pair Creation in ^8Be : A Possible Indication of a Light, Neutral Boson

A. J. Krasznahorkay,^{*} M. Csatlós, L. Csige, Z. Gácsi, J. Gulyás, M. Hunyadi, I. Kuti, B. M. Nyakó, L. Stuhl, J. Timár, T. G. Tornyai, and Zs. Vajta

Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), P.O. Box 51, H-4001 Debrecen, Hungary

T. J. Ketel

Nikhef National Institute for Subatomic Physics, Science Park 105, 1098 XG Amsterdam, Netherlands

A. Krasznahorkay

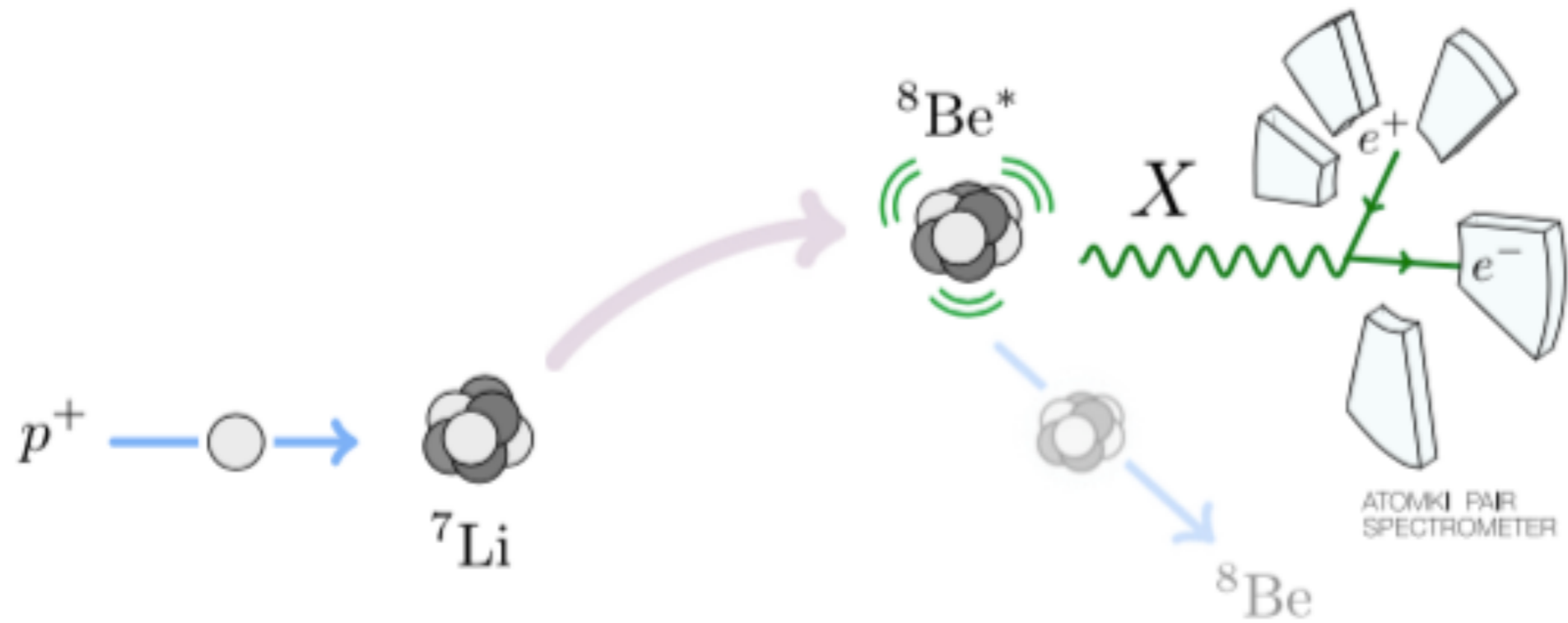
CERN, CH-1211 Geneva 23, Switzerland and Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), P.O. Box 51, H-4001 Debrecen, Hungary

(Received 7 April 2015; published 26 January 2016)

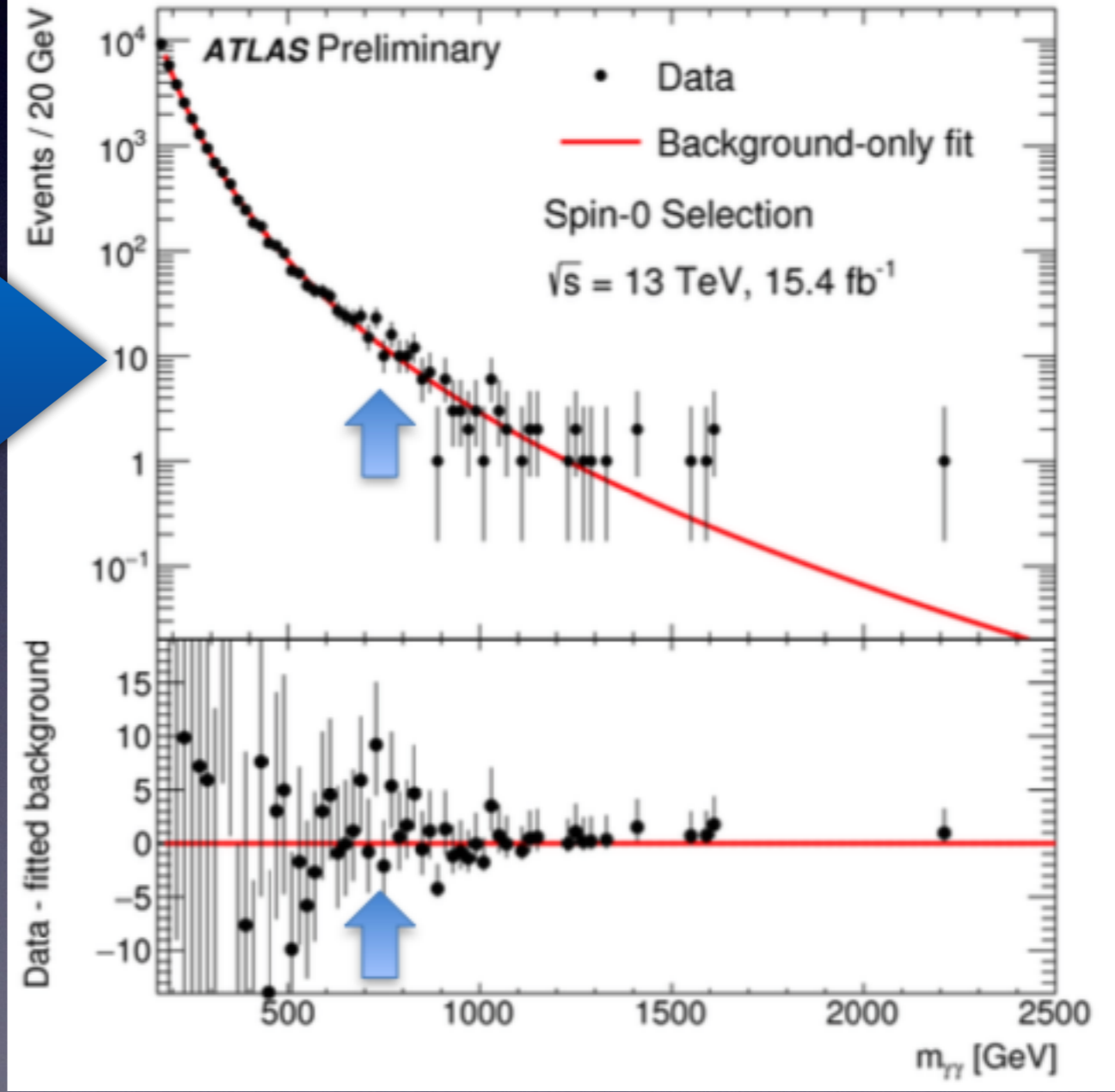
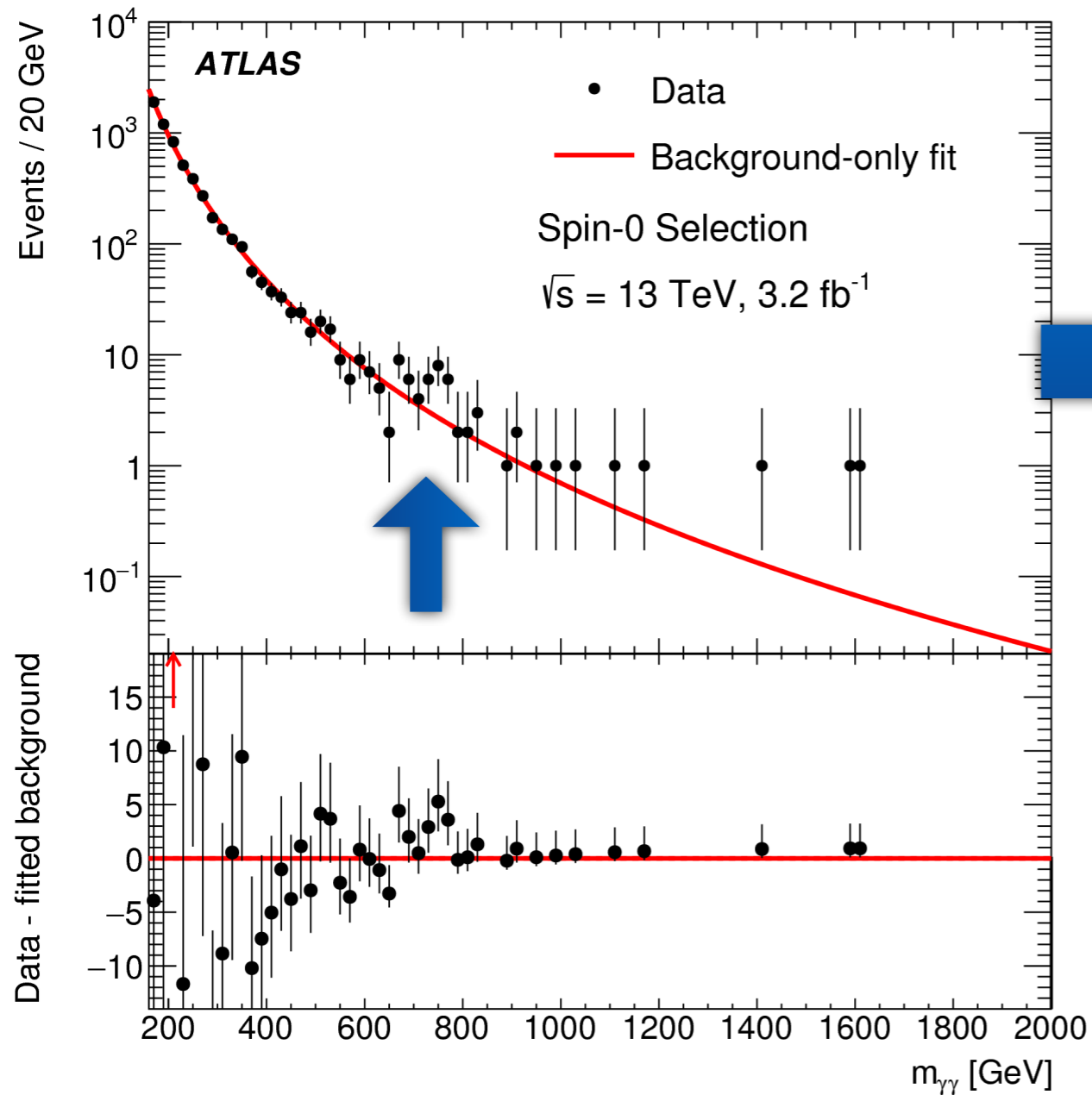
Electron-positron angular correlations were measured for the isovector magnetic dipole 17.6 MeV ($J^\pi = 1^+$, $T = 1$) state \rightarrow ground state ($J^\pi = 0^+$, $T = 0$) and the isoscalar magnetic dipole 18.15 MeV ($J^\pi = 1^+$, $T = 0$) state \rightarrow ground state transitions in ^8Be . Significant enhancement relative to the internal pair creation was observed at large angles in the angular correlation for the isoscalar transition with a confidence level of $> 5\sigma$. This observation could possibly be due to nuclear reaction interference effects or might indicate that, in an intermediate step, a neutral isoscalar particle with a mass of $16.70 \pm 0.35(\text{stat}) \pm 0.5(\text{syst}) \text{ MeV}/c^2$ and $J^\pi = 1^+$ was created.

DOI: 10.1103/PhysRevLett.116.042501

17 MeV



Diphoton



Gauge-fixing and ghost actions

$$S_{\text{gf}} = \frac{1}{\alpha} \int d^4x \sqrt{\bar{g}} F(\phi^2) \bar{g}^{\mu\nu} \Sigma_\mu \Sigma_\nu$$

$$\Sigma^\mu = \bar{D}_\nu h^{\nu\mu} - \frac{\beta + 1}{4} \bar{D}^\mu h$$

$$S_{\text{gh}} = \int d^4x \sqrt{\bar{g}} \bar{C}_\mu \left[-\delta_\mu^\rho \bar{D}^2 - \left(1 - \frac{1 + \beta}{2}\right) \bar{D}_\mu \bar{D}^\rho + \bar{R}_\mu^\rho \right] C_\rho$$

Why m and ξ become irrelevant?

- Effect of fermionic fluctuation

$$\bar{M}_{\text{pl}}^{2*} = 2.38 \times 10^{-2}$$

$$\bar{\Lambda}_{\text{cc}}^* = 8.82 \times 10^{-3}$$



$$\bar{M}_{\text{pl}}^{2*} = 1.63 \times 10^{-2}$$

$$\bar{\Lambda}_{\text{cc}}^* = 3.72 \times 10^{-3}$$

- Matrix

$$\frac{\partial \beta_i}{\partial g_j} \simeq \begin{pmatrix} \frac{\partial \beta_\xi}{\partial \xi} & \frac{\partial \beta_\xi}{\partial m^2} \\ \frac{\partial \beta_{m^2}}{\partial \xi} & \frac{\partial \beta_{m^2}}{\partial m^2} \end{pmatrix}$$



$$\theta_i \simeq \frac{1}{2} \left(\frac{\partial \beta_\xi}{\partial \xi} + \frac{\partial \beta_{m^2}}{\partial m^2} \right)$$

$$\begin{pmatrix} 2.85544 & -6.51993 \\ 2.40051 & -2.57031 \end{pmatrix}$$

$$\begin{pmatrix} 1.6814 & -5.39674 \\ 1.99718 & -2.66334 \end{pmatrix}$$

$$\theta_i \simeq \frac{2.85544 - 2.57031}{2} > 0$$

$$\theta_i \simeq \frac{1.6814 - 2.66334}{2} < 0$$