

Spontaneous spin textures in Hubbard systems

Jan Kuneš

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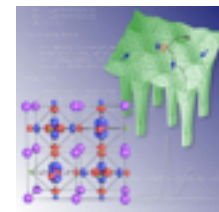
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(Masaryk University, Brno)



European Research Council

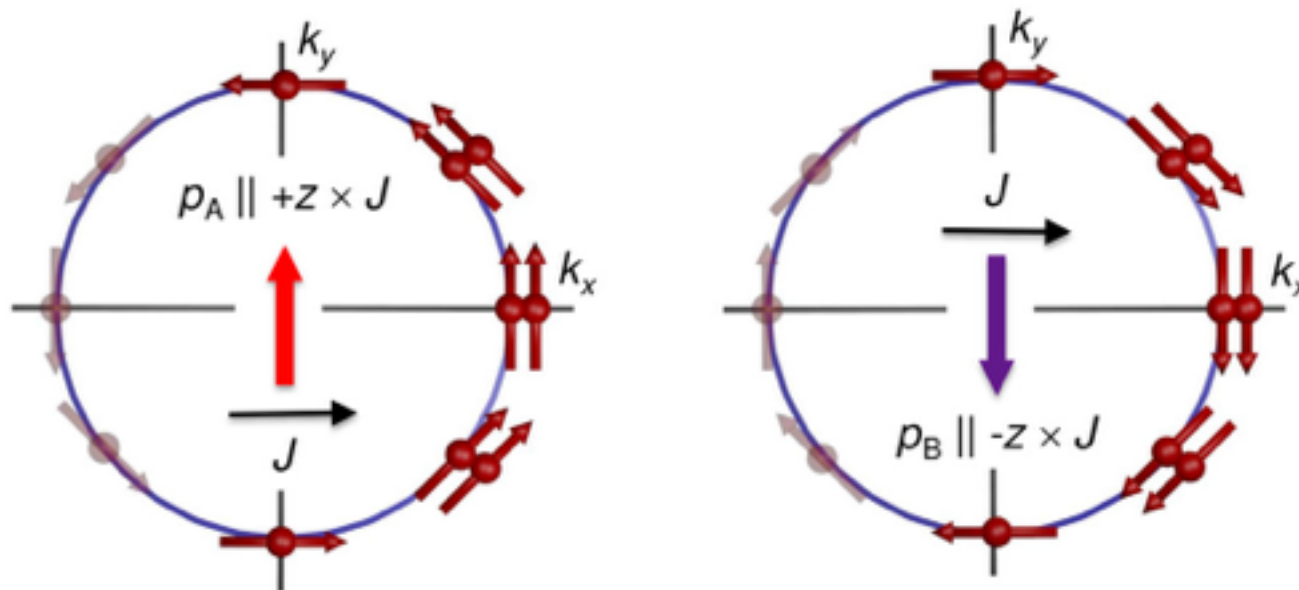


DFG Research Unit FOR1346

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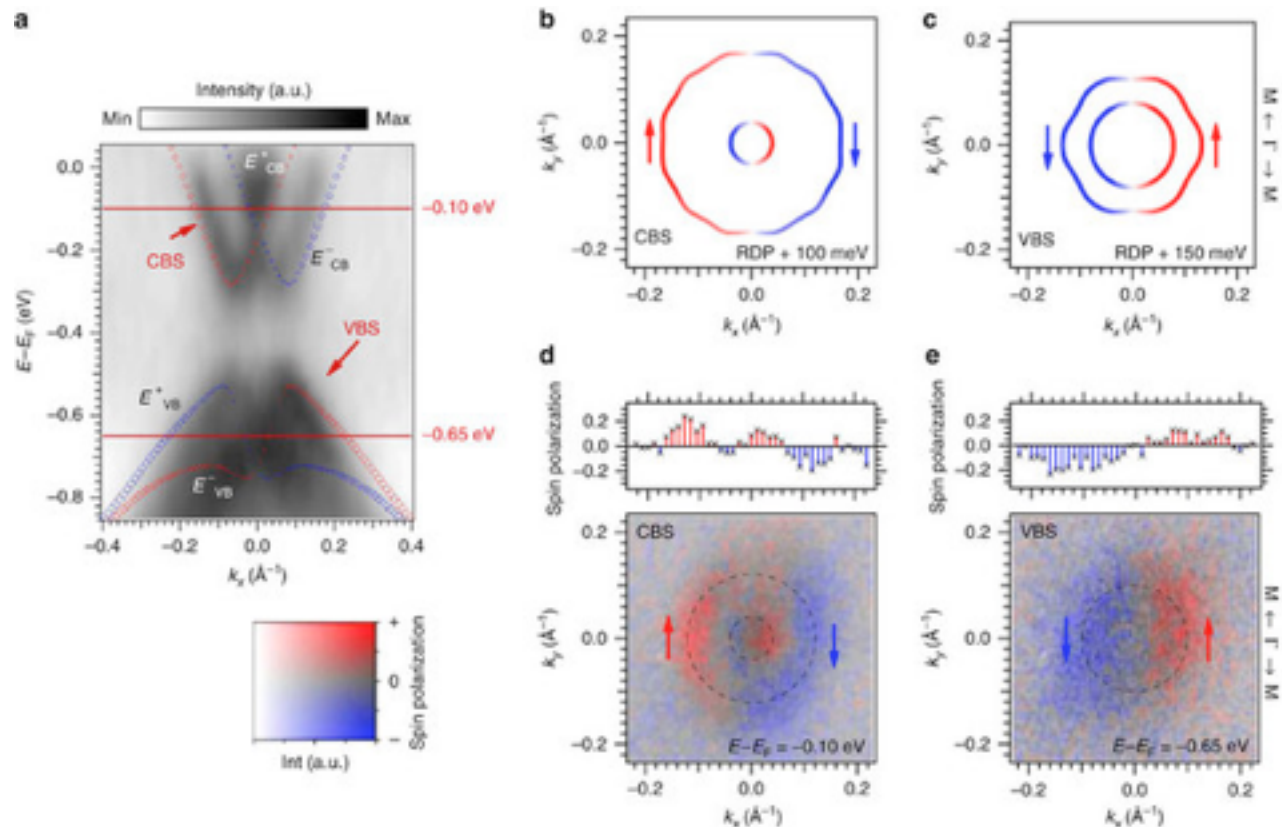
Electrical switching of an antiferromagnet

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Spin-texture inversion in the giant Rashba semiconductor BiTeI

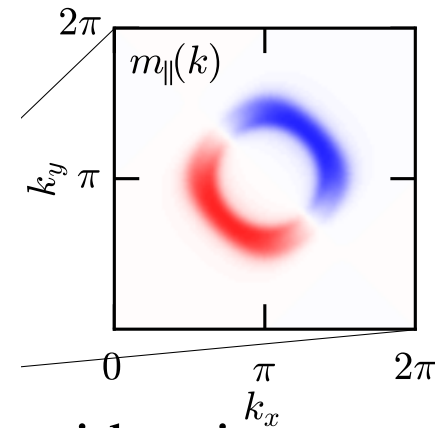
Henriette Maaß¹, Hendrik Bentmann¹, Christoph Seibel¹, Christian Tusche², Sergey V. Eremeev^{3,4,5}, Thiago R.F. Peixoto¹, Oleg E. Tereshchenko^{5,6,7}, Konstantin A. Kokh^{5,7,8}, Evgueni V. Chulkov^{4,5,9,10}, Jürgen Kirschner² & Friedrich Reinert¹



Spin texture

- no spin polarization in direct space $\mathbf{m}(\mathbf{r}) = 0$
- finite polarization in the k -space $\mathbf{m}_{-\mathbf{k}} = -\mathbf{m}_{\mathbf{k}}$

~ can be realized in non-centrosymmetric systems with spin-orbit coupling (Rashba/Dresselhaus SOC).



Here I will present how spin textures can be generated by spontaneous symmetry breaking in multi-band Hubbard models.

The key ingredients are:

condensation of spinful excitons
generalized double-exchange

The model

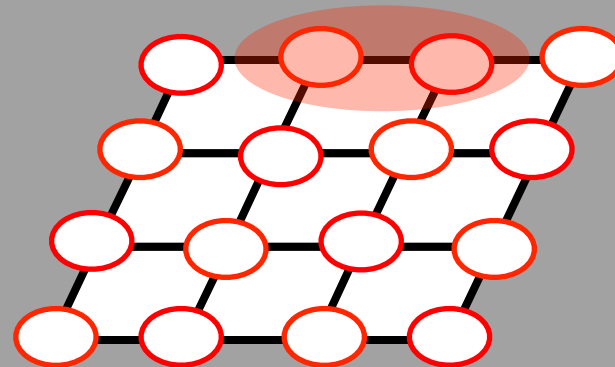
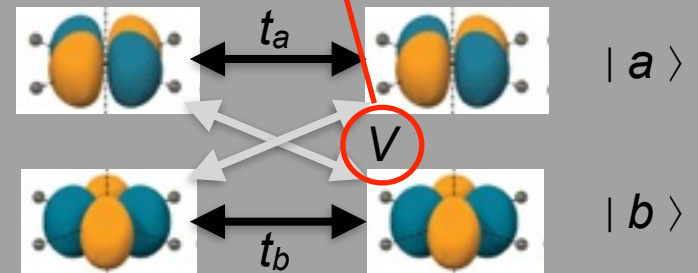
Two-band Hubbard model at $n=2$ (half filling)

$$\begin{aligned}
 H_t &= \frac{\Delta}{2} \sum_{i,\sigma} (n_{i\sigma}^a - n_{i\sigma}^b) + \sum_{i,j,\sigma} (t_a a_{i\sigma}^\dagger a_{j\sigma} + t_b b_{i\sigma}^\dagger b_{j\sigma}) \\
 &\quad + \sum_{\langle ij \rangle, \sigma} (V_1 a_{i\sigma}^\dagger b_{j\sigma} + V_2 b_{i\sigma}^\dagger a_{j\sigma} + c.c.) \\
 H_{\text{int}}^{\text{dd}} &= U \sum_i (n_{i\uparrow}^a n_{i\downarrow}^a + n_{i\uparrow}^b n_{i\downarrow}^b) + (U - 2J) \sum_{i,\sigma} n_{i\sigma}^a n_{i-\sigma}^b \\
 &\quad + (U - 3J) \sum_{i\sigma} n_{i\sigma}^a n_{i\sigma}^b \\
 H'_{\text{int}} &= J \sum_{i\sigma} a_{i\sigma}^\dagger b_{i-\sigma}^\dagger a_{i-\sigma} b_{i\sigma} + J' \sum_i (a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger b_{i\downarrow} b_{i\uparrow} + c.c.).
 \end{aligned}$$



John Hubbard

doping
cross-hopping



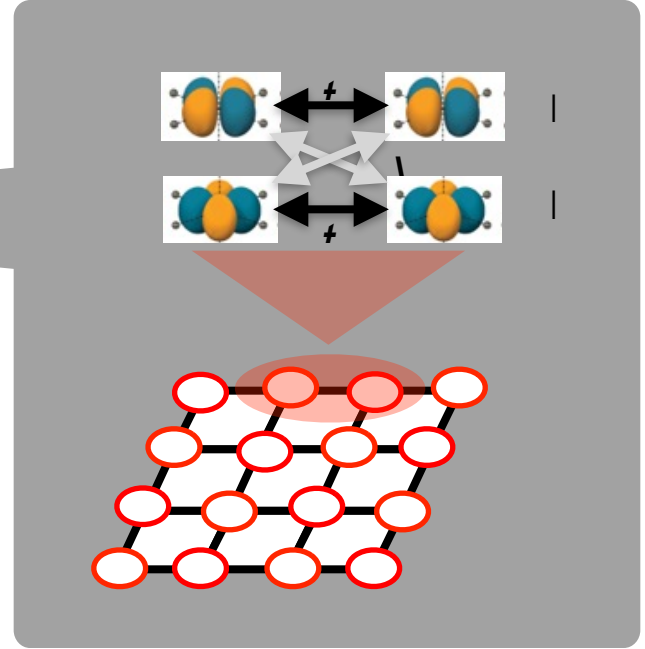
Proximity to spin-state crossover

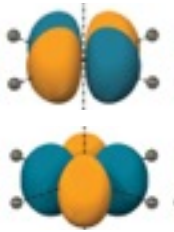
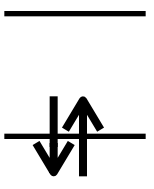
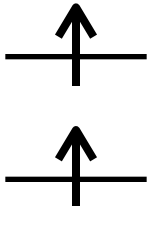

Two-band Hubbard model at $n=2$ (half filling)

$$H_t = \frac{\Delta}{2} \sum_{i,\sigma} (n_{i\sigma}^a - n_{i\sigma}^b) + \sum_{i,j,\sigma} (t_a a_{i\sigma}^\dagger a_{j\sigma} + t_b b_{i\sigma}^\dagger b_{j\sigma})$$

$$+ \sum_{\langle ij \rangle, \sigma} (V_1 a_{i\sigma}^\dagger b_{j\sigma} + V_2 b_{i\sigma}^\dagger a_{j\sigma} + c.c.)$$

$$H_{\text{int}}^{\text{dd}} = U \sum (n_{i\uparrow}^a n_{i\downarrow}^a + n_{i\uparrow}^b n_{i\downarrow}^b) + (U - 2J) \sum n_{i\uparrow}^a n_{i-\sigma}^b$$



	low spin $S=0$	high spin (triplet) $S=1$	
			

Competition of Hund's coupling J and crystal-field Δ

We are interested in $E_{LS} \approx E_{HS}$



Friedrich H. Hund

Strong-coupling limit (hard-core bosons)



What is exciton condensate?

Strong coupling theory

Balents 2000

Rademaker et al. 2012-2014

bilayer Heisenberg model

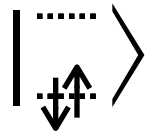
- Define restricted low-energy Hilbert space

Fermions

Bosons (hard-core)

low-spin state

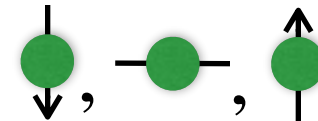
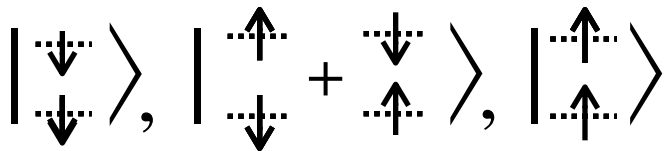
vacuum



.....

high-spin state

S=1 boson



LS → HS transition

creation of a boson

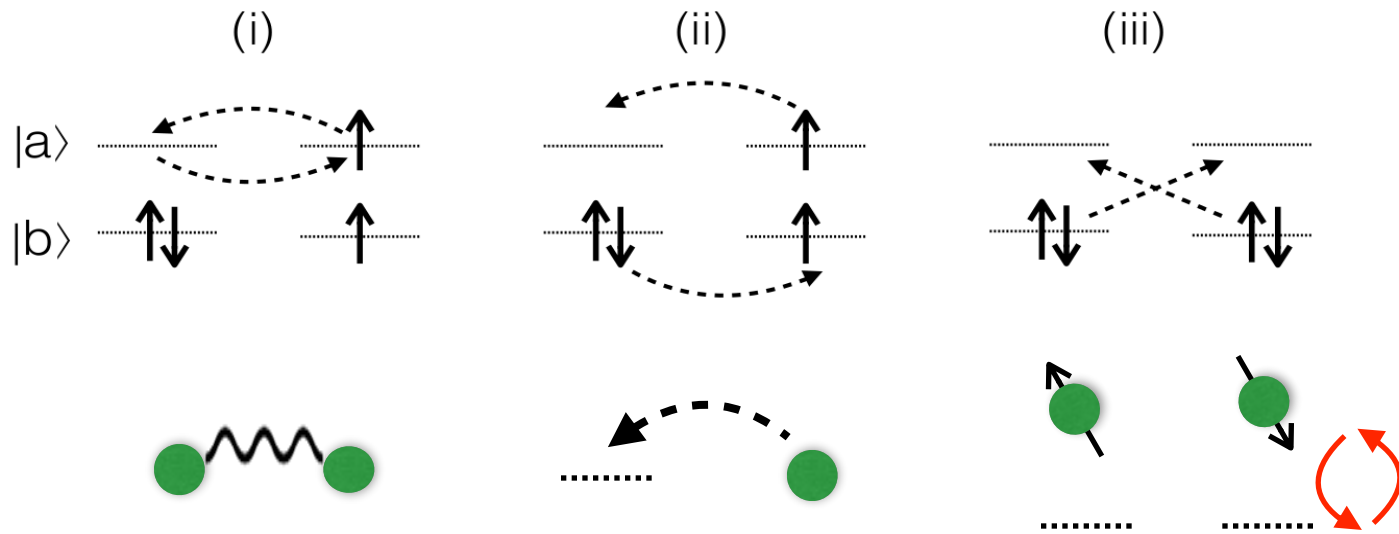
$$a_{\downarrow}^{\dagger} b_{\uparrow}, \dots$$

$$d_{-1}^{\dagger}, \dots$$

Strong coupling theory

- Decouple it from the high-energy states (Schrieffer-Wolff transformation)

Typical 2nd order processes:



Effective Hamiltonian:

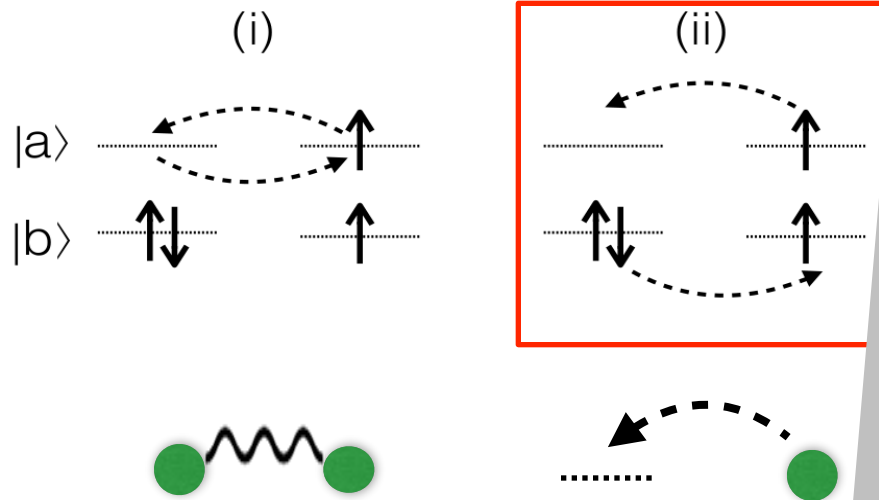
$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} \left(\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c. \right) +$$

$$K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + K_1 \sum_{\langle ij \rangle} \left(\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j^{\dagger} + H.c. \right) + \dots$$

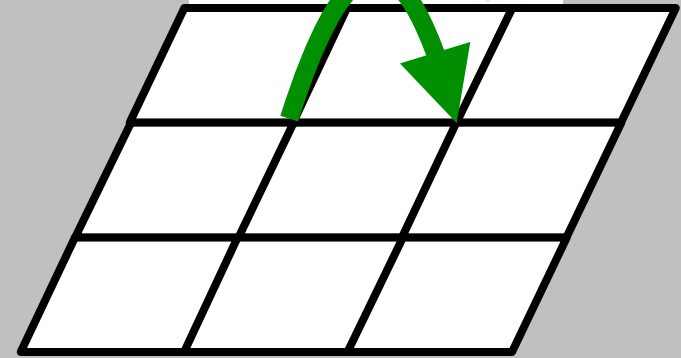
Strong coupling

- Decouple it from the high-energy states (Schrieffer)

Typical 2nd order processes:



d-bosons are mobile !



Effective Hamiltonian:

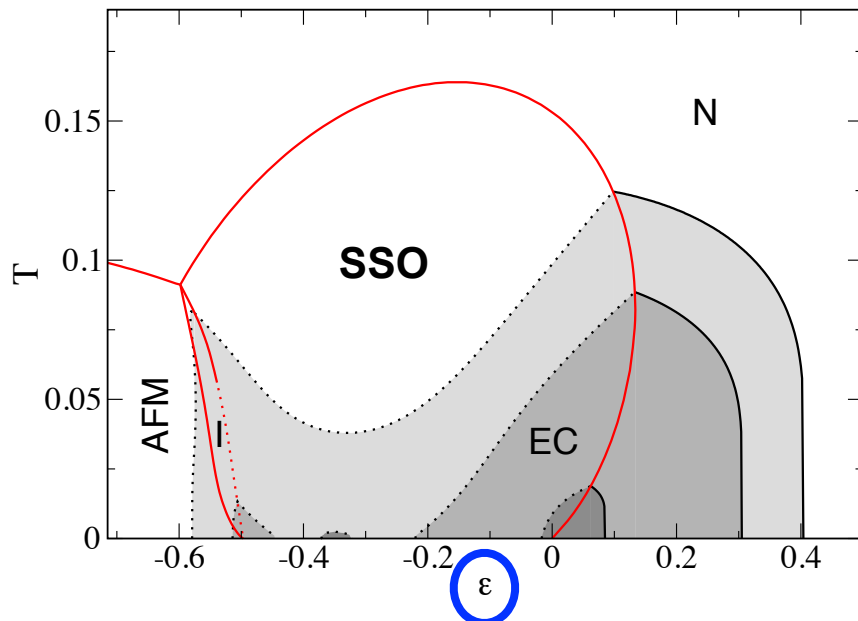
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Mean-field theory

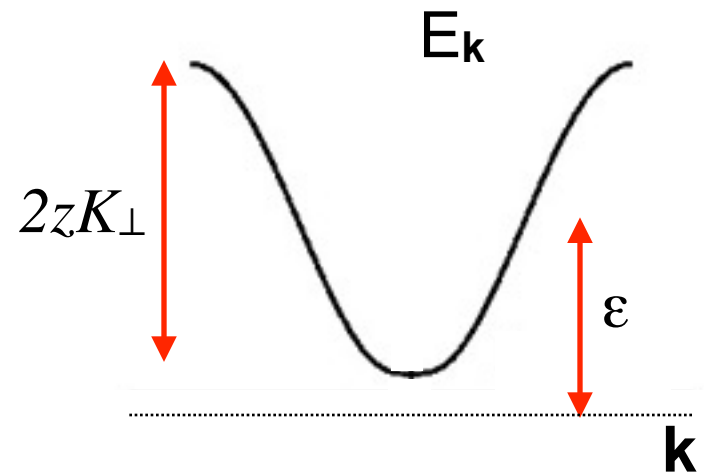
$$H_{\text{eff}} = \epsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} (\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c.) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

MF phase diagram:



Augustinský and Kuneš, 2014

excitations in the normal state

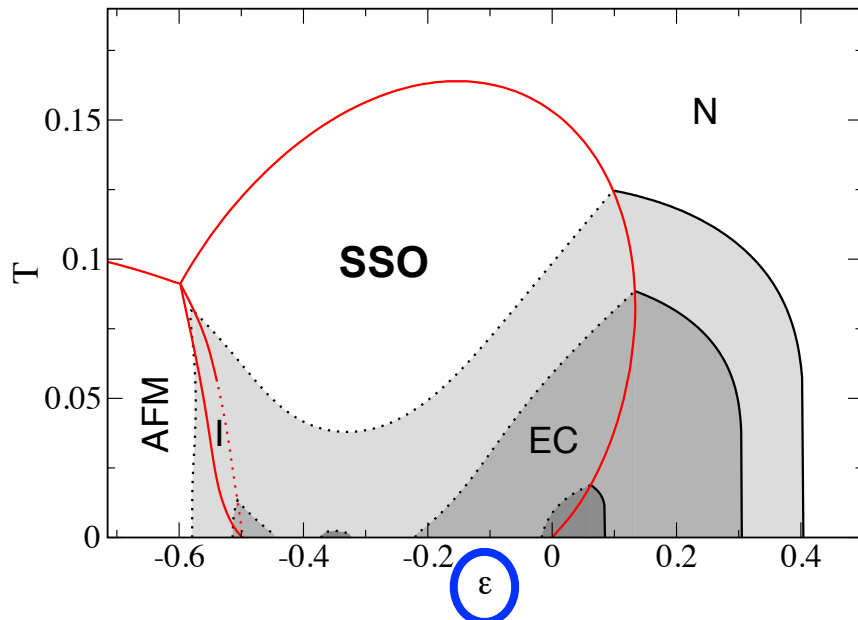


d-occupancy: $\langle n \rangle = 0$

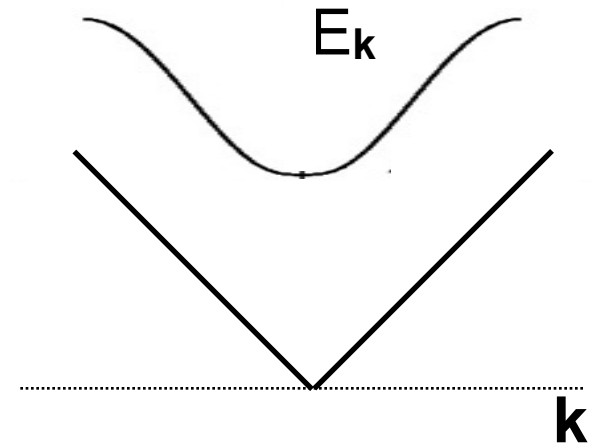
Mean-field theory

$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} (\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c.) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

MF phase diagram:



excitations of the condensate

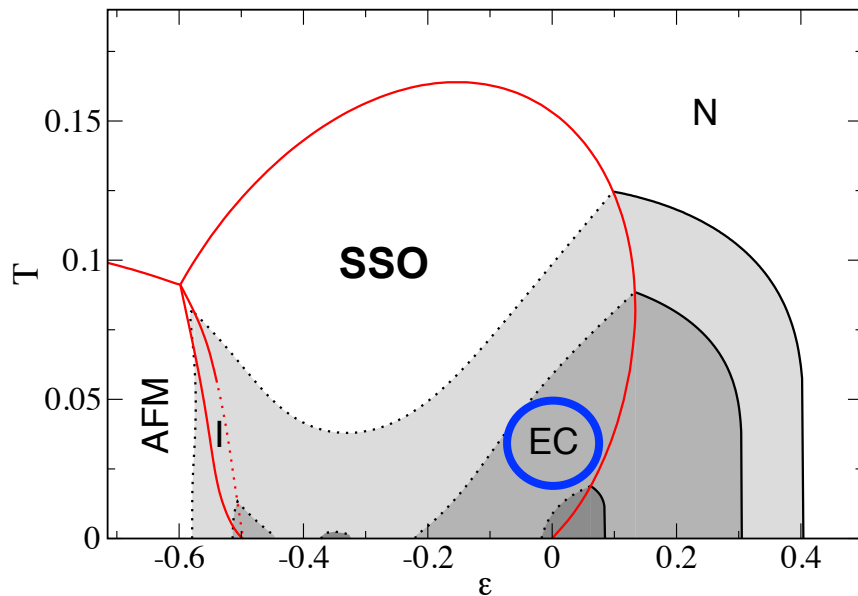


Augustinský and Kuneš, 2014

Mean-field theory

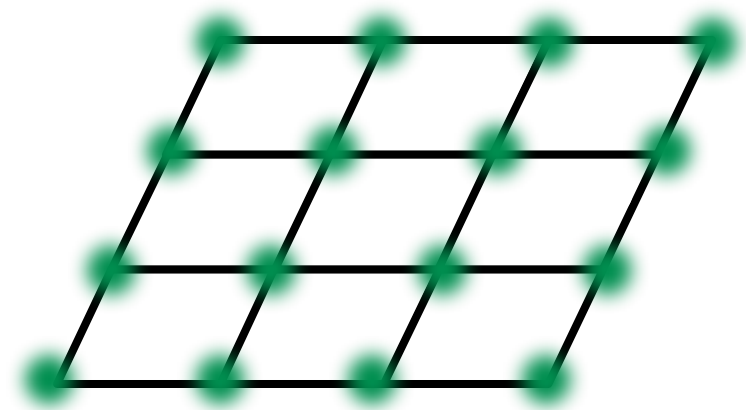
$$H_{\text{eff}} = \varepsilon \sum_i n_i + K_{\perp} \sum_{\langle ij \rangle} \left(\mathbf{d}_i^{\dagger} \cdot \mathbf{d}_j + H.c. \right) + K_{\parallel} \sum_{\langle ij \rangle} n_i n_j + K_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

MF phase diagram:



Augustinský and Kuneš, 2014

Bose-Einstein condensate
of d-bosons



Order parameter: $\langle d_s \rangle$

d-occupancy: $0 < \langle n \rangle < 1$

Exciton condensate

Degenerate excitations \rightarrow distinct condensates possible

$$|C_i\rangle \begin{cases} \alpha |\cdots\rangle + \beta |\downarrow\rangle & \textit{ferromagnetic condensate} \\ \alpha |\cdots\rangle + \beta' |\downarrow\rangle + \beta'' |\uparrow\rangle & \textit{polar condensate} \end{cases}$$

$$|C\rangle = \prod_i |C_i\rangle \quad \text{approx. condensate wavefunction}$$

order parameter

$$\vec{\phi} = \sum_{\sigma, \sigma'} \langle a_{\sigma}^{\dagger} b_{\sigma'} \rangle \vec{T}_{\sigma\sigma'} \sim \alpha^* \beta$$

Back to fermions (DMFT)



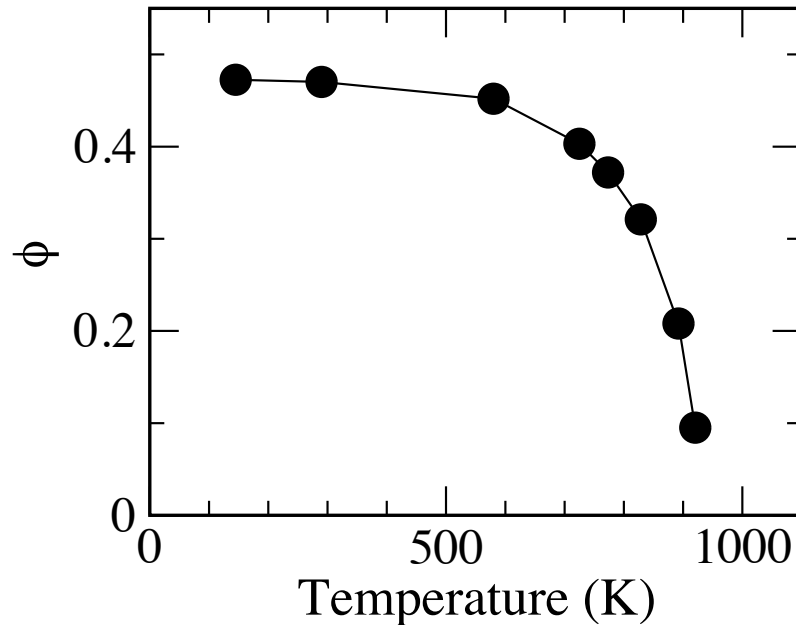
“ fermion = boson^{1/2} ” adds a lot of extra structure

Undoped system - polar condensate

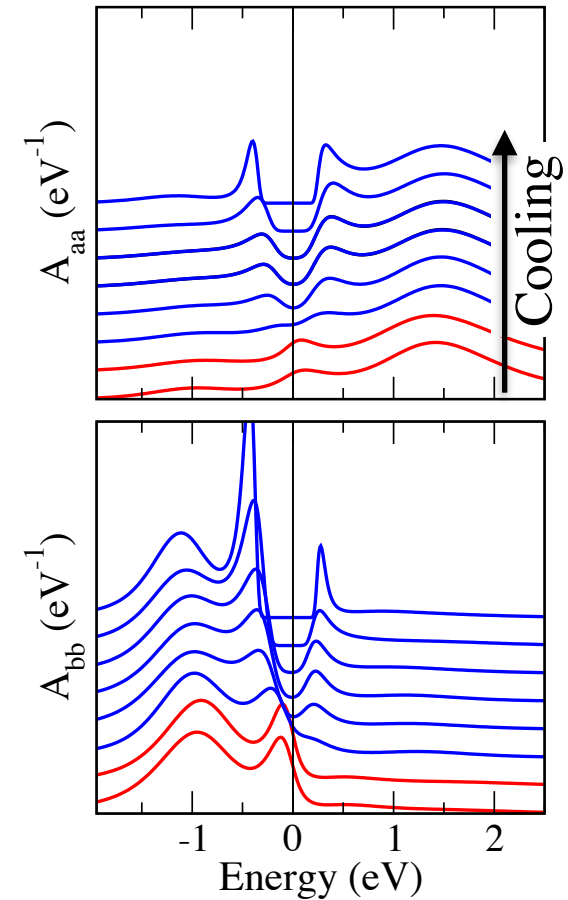
order parameter

$$\vec{\phi} = \sum_{\sigma, \sigma'} \langle a_{\sigma}^{\dagger} b_{\sigma'} \rangle \vec{\tau}_{\sigma\sigma'}$$

$$\vec{\phi} = (\phi, 0, 0)$$



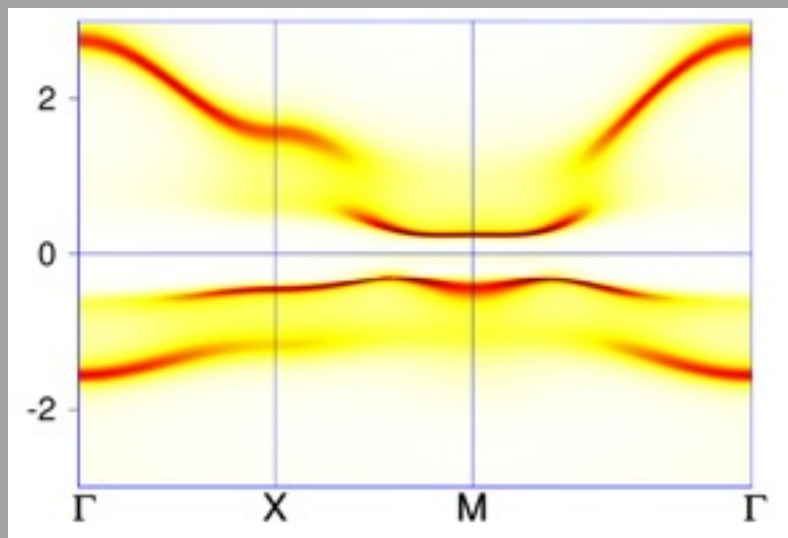
Spectral density (diagonal elements)



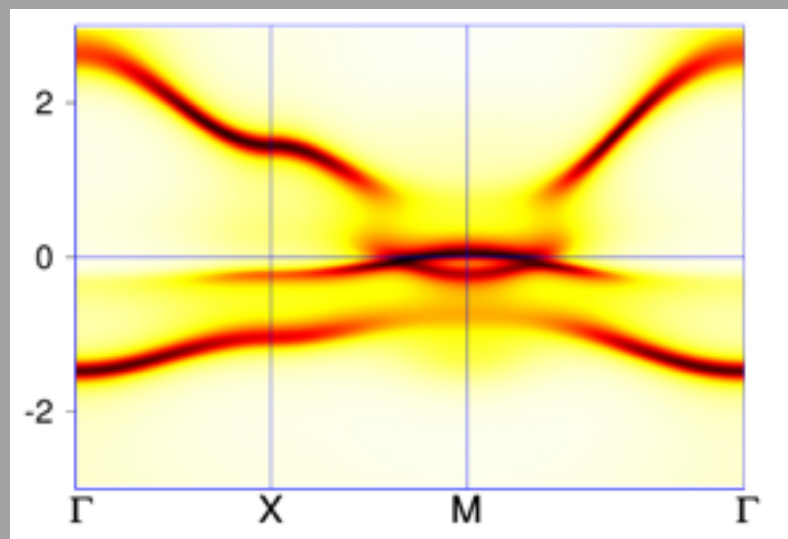
Undoped system - polar condensate

Opening of charge gap

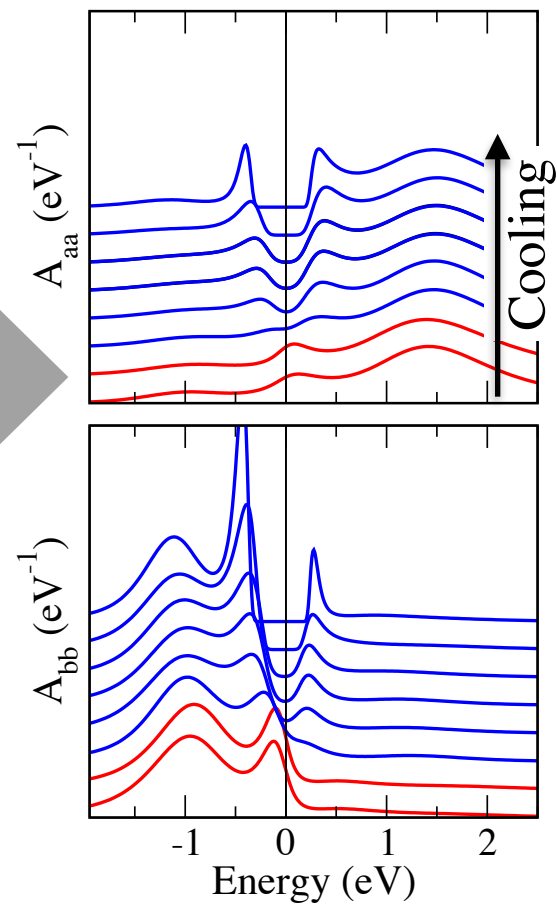
EC phase
(low T)



normal phase
(high T)

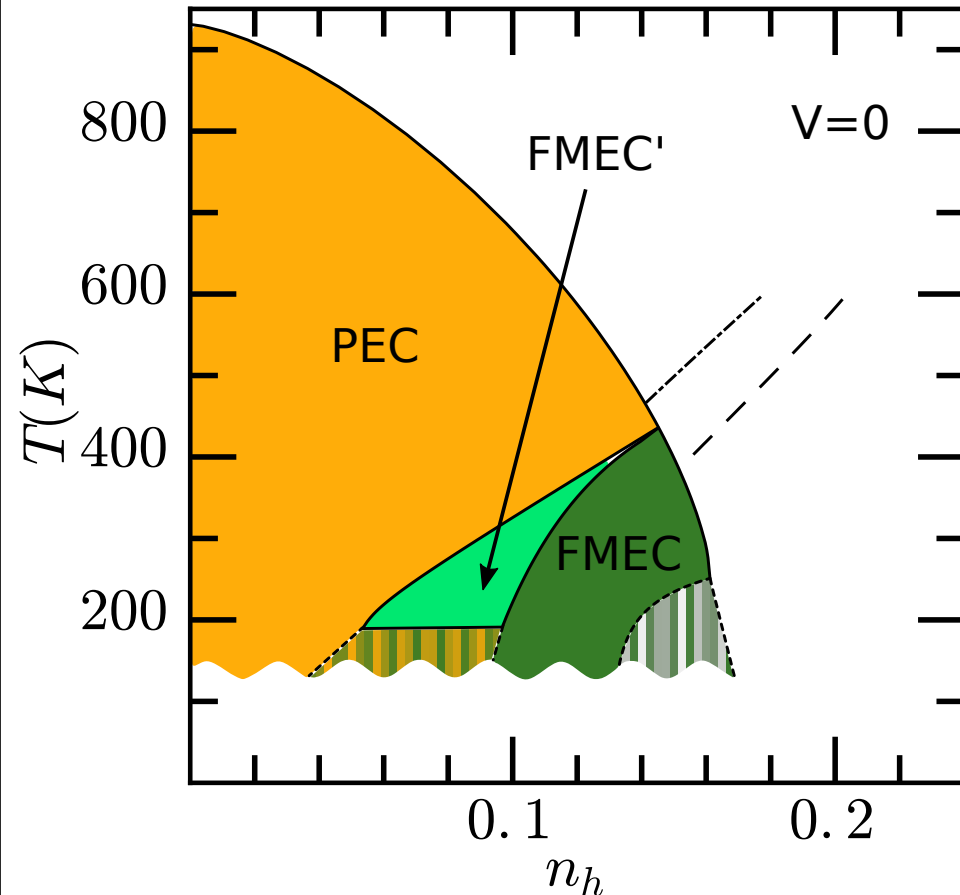


density (diagonal elements)



Doping ($V=0$)

n-T phase diagram



PEC - polar condensate

FMEC - ferromagnetic condensate

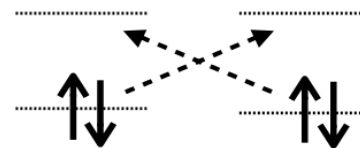
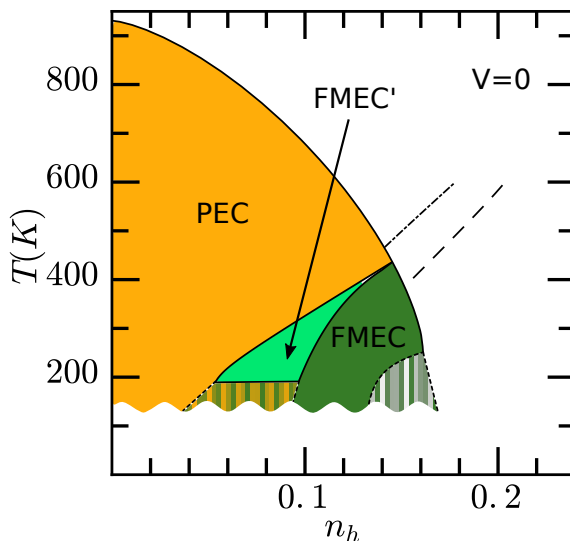
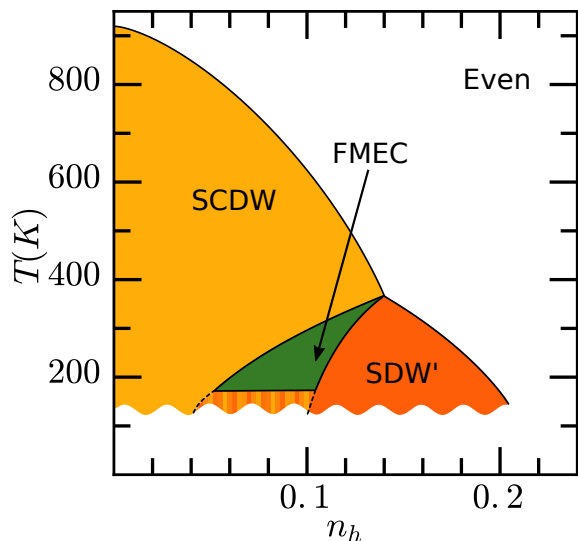
Competition between
AFM super-exchange (**PEC**) and
double-exchange (FMEC)

$$\vec{\phi}_i = \sum_{\sigma, \sigma'} \langle a_{\sigma}^{\dagger} b_{\sigma'} \rangle \vec{\tau}_{\sigma\sigma'}$$

$$\vec{\phi} = \exp(i\varphi) \vec{x}$$

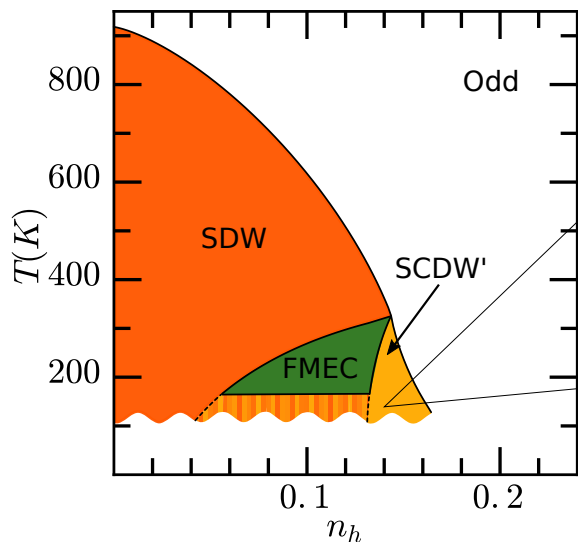
$$\vec{M} = i \vec{\phi}^* \wedge \vec{\phi}$$

Finite cross-hopping



$V_1=V_2$ even cross-hopping
 $V_1=-V_2$ odd cross-hopping

PEC { SDW $\vec{\phi} = \vec{x}$
 SCDW $\vec{\phi} = i\vec{x}$

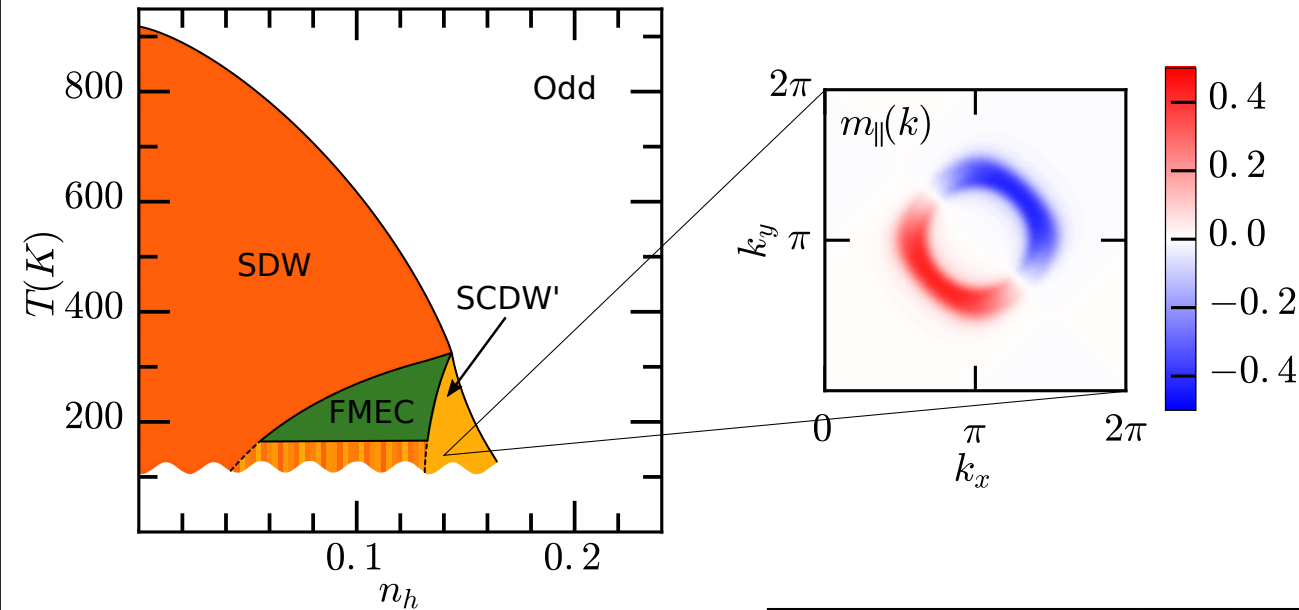


Condensate state	M_{\perp}	M_{\parallel}	$\mathbf{m}(\mathbf{r})$	$\mathbf{m}_{\mathbf{k}}$	$\text{Re } \phi$	$\text{Im } \phi$
FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓, 0	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

$$\mathbf{m}_{\mathbf{k}} = \sum_{\alpha\beta} \sigma_{\alpha\beta} \langle a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} + b_{\mathbf{k}\alpha}^{\dagger} b_{\mathbf{k}\beta} \rangle$$

$$\phi = \sum_{\alpha,\beta} \sigma_{\alpha\beta} \langle a_{i\alpha}^{\dagger} b_{i\beta} \rangle$$

Spin texture



Condensate state	M_{\perp}	M_{\parallel}	$\mathbf{m}(\mathbf{r})$	$\mathbf{m}_{\mathbf{k}}$	$\text{Re } \phi$	$\text{Im } \phi$
FMEC	✓	✓, 0	✓	✓	✓	✓
SDW	0	0	✓	0	✓	0
SCDW	0	0	0	0	0	✓
SDW'	0	✓	✓	✓	✓	0
SCDW'	0	0	0	✓	0	✓

Dynamically generated Dresselhaus-Rashba spin-orbit coupling
 centrosymmetric Hamiltonian, no spin-orbit coupling

Propagation of doped carriers in the condensate

$$H_{\text{eff}} = \sum_{\langle ij \rangle} \bar{h}_{\alpha\beta} c_{i\alpha}^\dagger c_{j\beta} + \text{h.c.}$$

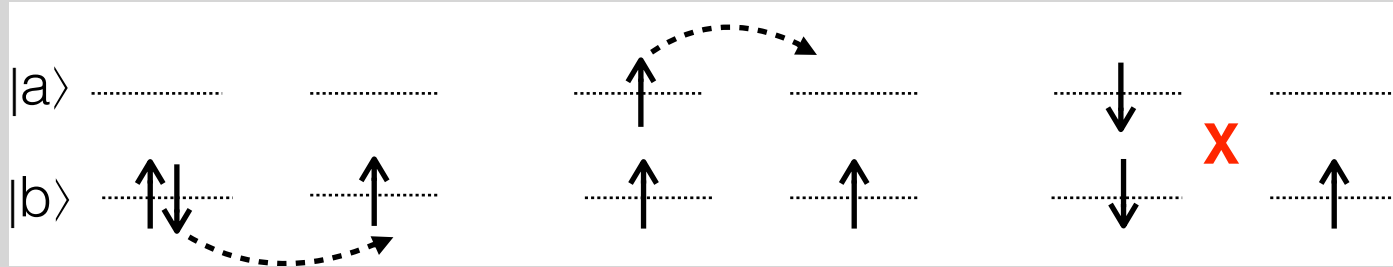
$$h_{\alpha\beta}^{(ij)} = \langle \alpha_i C_j | H | C_i \beta_j \rangle$$
$$\alpha, \beta \in \{\uparrow, \downarrow\}$$

$$\bar{h} = t_s \bar{I} - \frac{it_a}{4s^2} (\phi^* \wedge \phi) \cdot \bar{\sigma} + \frac{1}{2} (V_1 \phi + V_2 \phi^*) \cdot \bar{\sigma}$$

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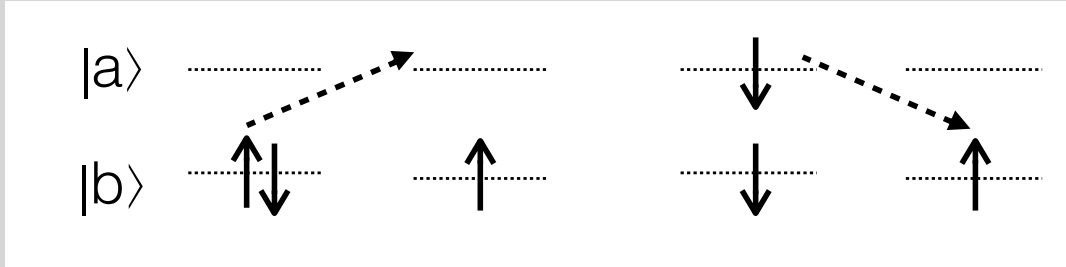


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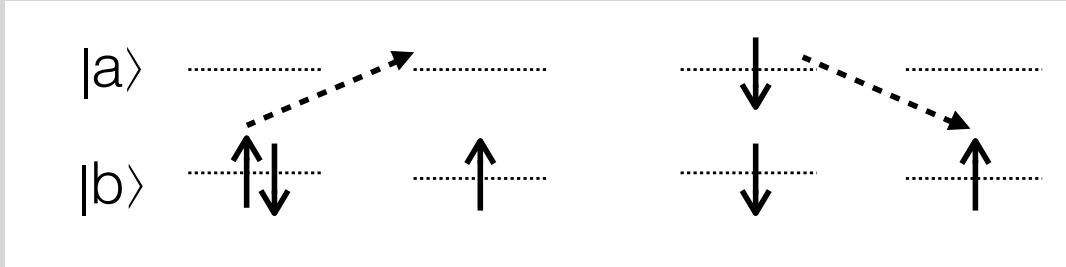


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Field acting on bonds is local in k-space:

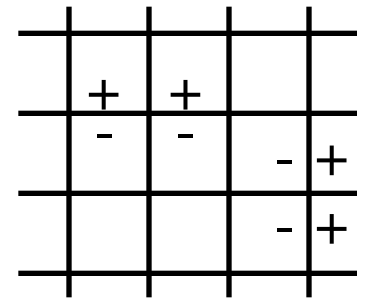
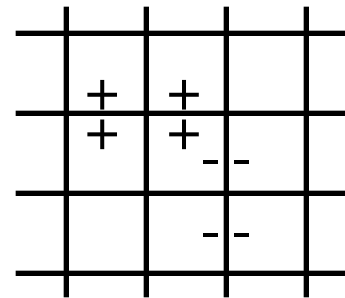
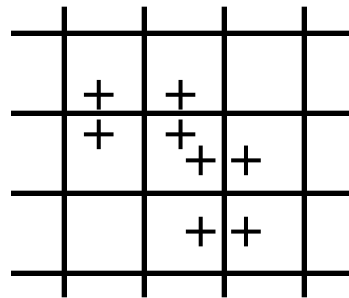
$$\bar{h}_{\mathbf{k}} = 2t_s \bar{I} C_{\mathbf{k}} + 2V_1 \phi \cdot \bar{\sigma} \begin{cases} C_{\mathbf{k}} & \text{SDW} \\ iS_{\mathbf{k}} & \text{CSDW} \end{cases}$$

with $C_{\mathbf{k}} = \cos k_x + \cos k_y$, $S_{\mathbf{k}} = \sin k_x + \sin k_y$.

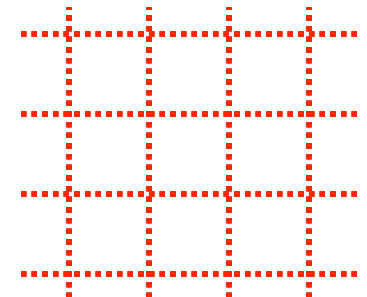
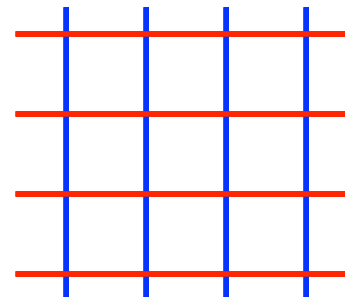
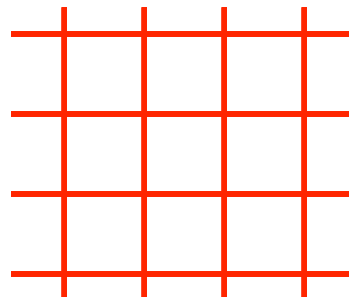
How to realize spin textures

- PEC state (intra atomic level)
- effective exchange on the bonds (inter atomic level)
- global spin-currents or FM polarization (inter cell level)

Cross-hopping:



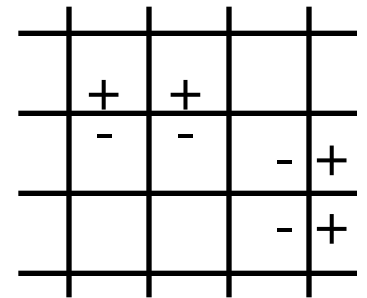
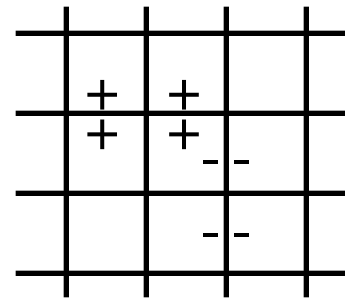
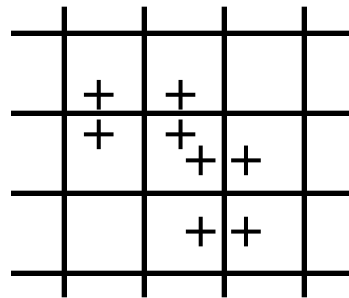
Bond exchange field:



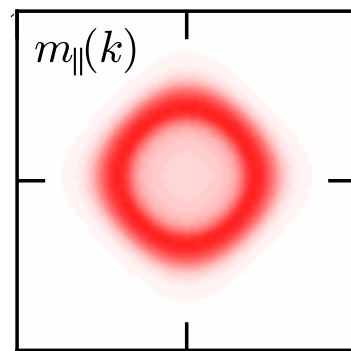
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- global spin-currents or FM polarization (inter cell level)

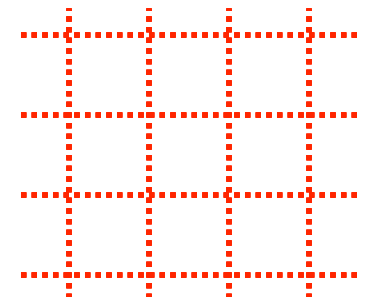
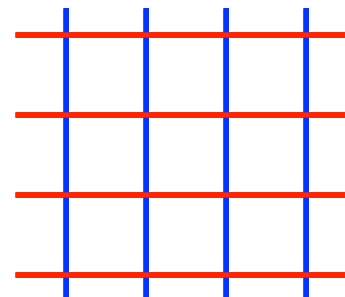
Cross-hopping:



Bond exchange field:



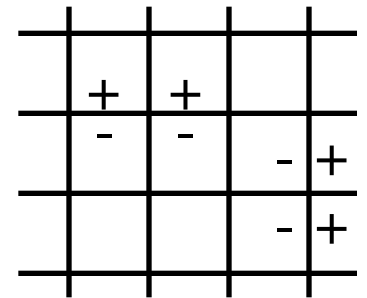
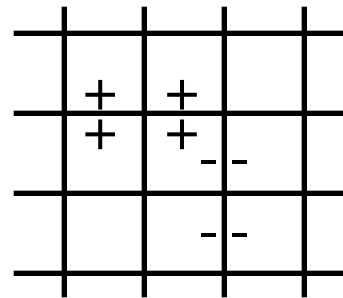
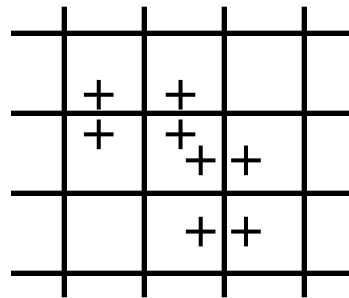
$$\cos k_x + \cos k_y$$



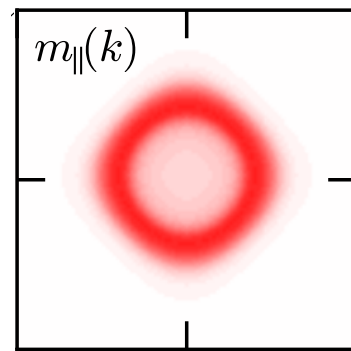
How to realize spin textures

- PEC state (intra atomic level)
- effective exchange on bonds (inter atomic level)
- global spin-currents or FM polarization (inter cell level)

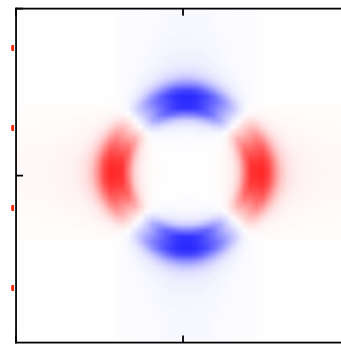
Cross-hopping:



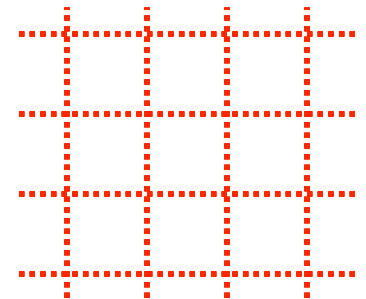
Bond exchange field:



$$\cos k_x + \cos k_y$$



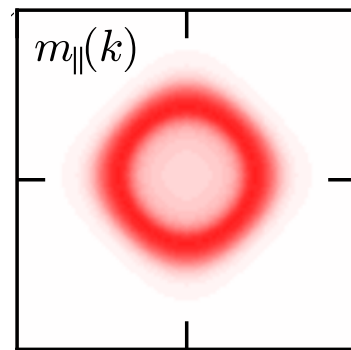
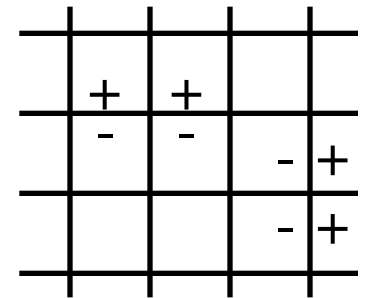
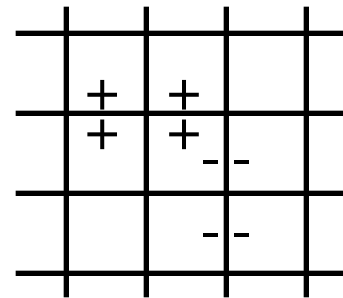
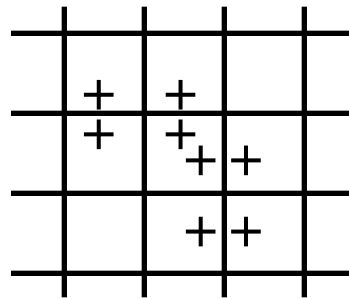
$$\cos k_x - \cos k_y$$



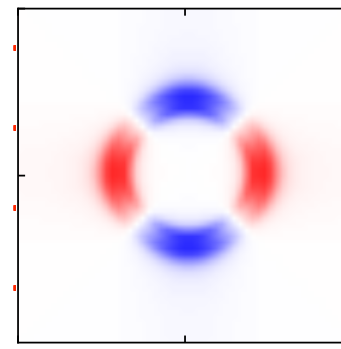
How to realize spin textures

- PEC state (intra atomic level)
- effective exchange on bonds (inter atomic level)
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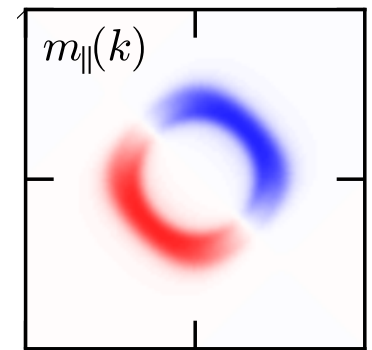
Cross-hopping:



$$\cos k_x + \cos k_y$$



$$\cos k_x - \cos k_y$$



$$\sin k_x + \sin k_y$$

Bond exchange field:

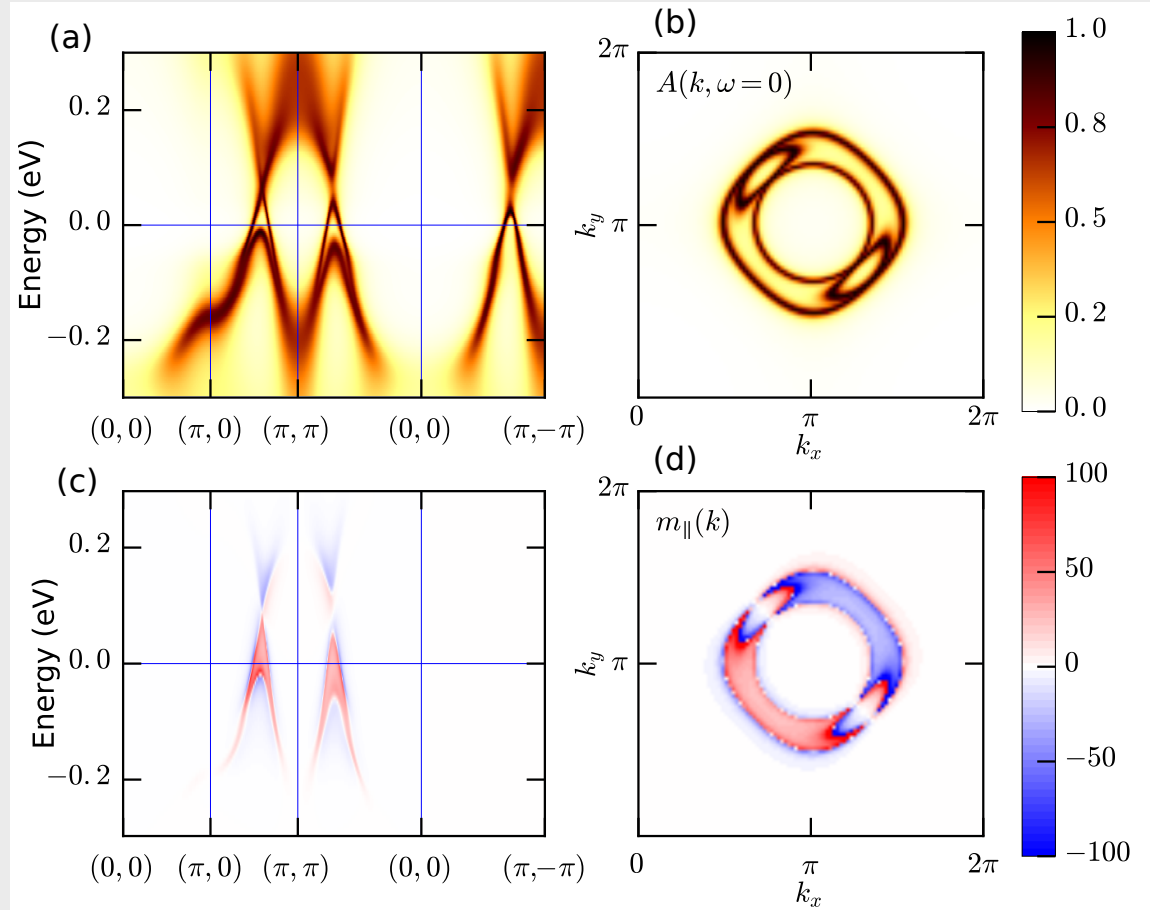
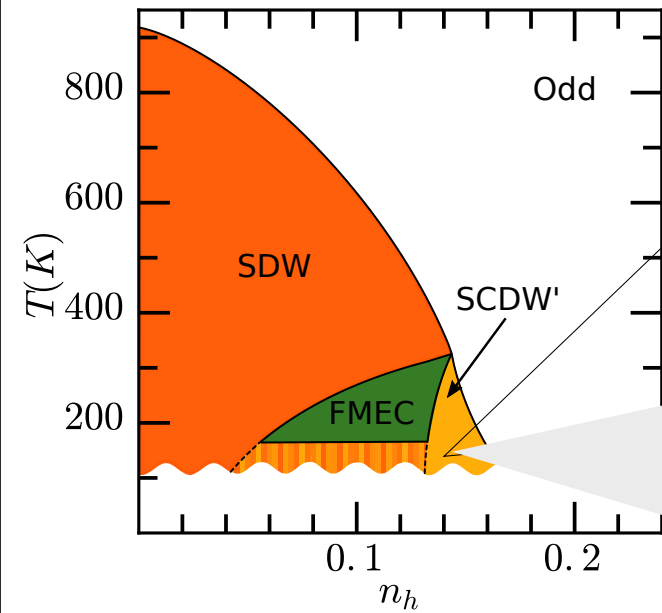
Conclusions

- Solids close to **spin-state transition** are unstable towards **condensation of spinful excitons**.
- Excitonic condensation can give rise to a **number of phases** with rather diverse properties.
- **Doping** activates generalised double-exchange mechanism with interesting consequences (e.g. spontaneous spin texture)

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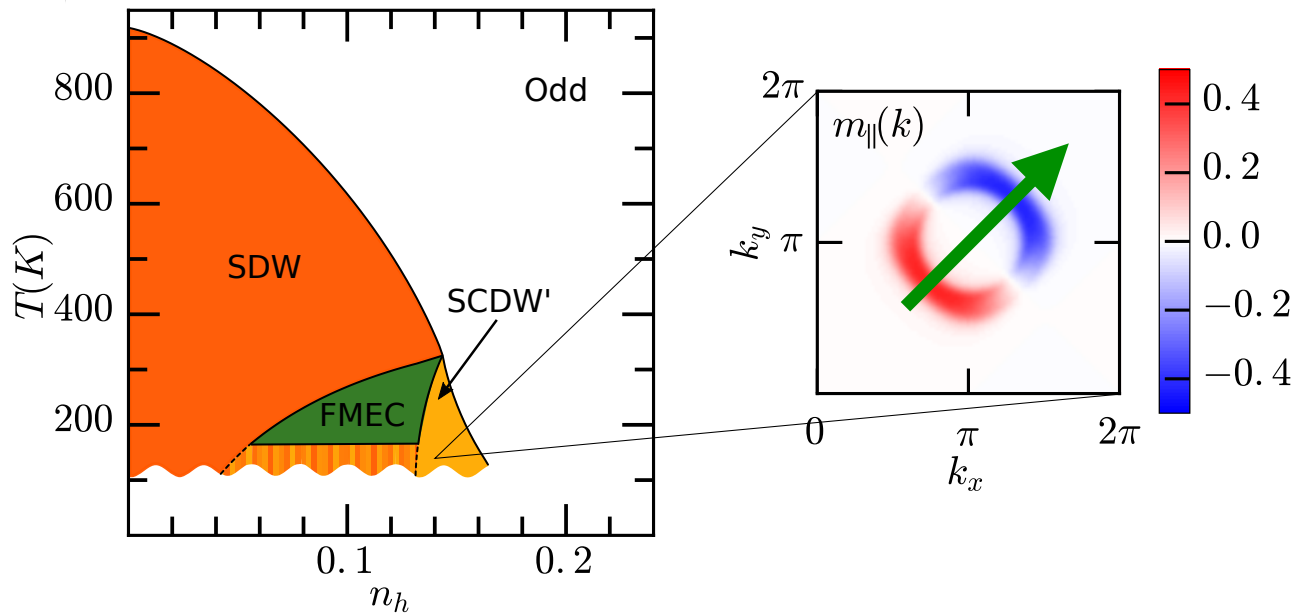
Phys. Rev. Lett. **116**, 256403 (2016)

Spin texture



Spin-galvanic effect

Magnetic field generates charge current

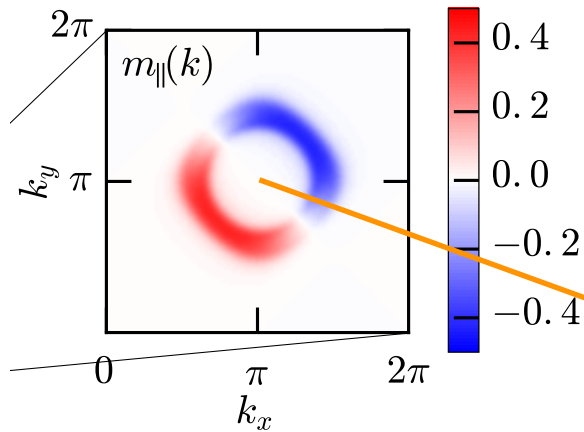


Spin-galvanic effect

Magnetic field generates charge current

poor man's treatment (using $\Sigma_{B=0}$)

spin distribution $m_{\mathbf{k}}$ @ $B=0$



$$\delta n_{\mathbf{k}} = n_{\mathbf{k}}(B) - n_{\mathbf{k}}(0)$$

charge distribution $\delta n_{\mathbf{k}}$ @ $B=0.001 \text{ eV}/\mu_B$

