

# Anisotropic magnetic interactions in Iridium oxides from LDA+U calculations

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What about U?

Effects of Hubbard Interactions and Hunds Coupling in Solids  
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- 1 motivations:  $j_{\text{eff}}=1/2$  beauty
- 2 anisotropic exchange in honeycomb  $\text{Na}_2\text{IrO}_3$
- 3 noncollinear ground state in  $\text{Sr}_2\text{IrO}_4$  and  $\text{Sr}_3\text{Ir}_2\text{O}_7$

# anisotropic exchange interaction

for general bi-linear pair interaction between spins:  $H = \sum_{i \neq j} \mathbf{S}_i^T \mathfrak{J}_{ij} \mathbf{S}_j$

where  $\mathfrak{J}_{\alpha\beta}$  is a real  $3 \times 3$  matrix:  $\mathfrak{J} = \mathfrak{J}^+ + \mathfrak{J}^-$

with **symmetric**  $\mathfrak{J}^+ = (\mathfrak{J} + \mathfrak{J}^T)/2$  and **antisymmetric**  $\mathfrak{J}^- = (\mathfrak{J} - \mathfrak{J}^T)/2$

- no spin-orbit coupling (SOC) ( $\mathfrak{J}_{\alpha\alpha}^+ = J$ ,  $\mathfrak{J}_{\alpha\beta}^+ = 0$ ,  $\mathfrak{J}_{\alpha\beta}^- = 0$ )

$$\mathfrak{J} = \begin{pmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J \end{pmatrix}; \quad H = J \mathbf{S}_i \cdot \mathbf{S}_j$$

isotropic Heisenberg

- weak SOC, no inversion ( $\mathfrak{J}_{xy}^- = D \neq 0$ )

$$\mathfrak{J} = \begin{pmatrix} J & D & 0 \\ -D & J & 0 \\ 0 & 0 & J \end{pmatrix}; \quad H = J \mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{D}_{ij} \cdot [\mathbf{S}_i \times \mathbf{S}_j]$$

antisymmetric Dzyaloshinsky-Moriya

- strong SOC (e.g.,  $\mathfrak{J}_{xx}^+ = \mathfrak{J}_{yy}^+ = J$ ,  $\mathfrak{J}_{zz}^+ = J + K$ ,  $\mathfrak{J}_{\alpha\beta}^+ \neq 0, \dots$ )

$$\mathfrak{J} = \begin{pmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J + K \end{pmatrix}; \quad H = J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^z S_j^z$$

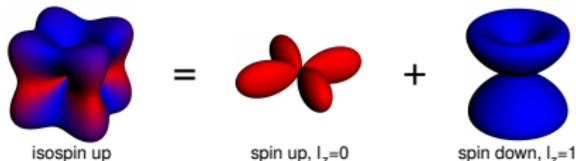
# cubic crystal field + spin-orbit coupling (SOC)

split  $t_{2g}$  states into a  $\Gamma_8$  ( $j_{\text{eff}} = 3/2$ ;  $d_{3/2} + d_{5/2}$ ) quartet:

$$\chi_{\Gamma_8} = \begin{cases} \sqrt{\frac{1}{2}} \left( d_{yz} \chi_{\pm\frac{1}{2}} \pm i d_{zx} \chi_{\pm\frac{1}{2}} \right) \\ \sqrt{\frac{1}{6}} \left( 2d_{xy} \chi_{\mp\frac{1}{2}} \pm d_{yz} \chi_{\pm\frac{1}{2}} + i d_{zx} \chi_{\pm\frac{1}{2}} \right) \end{cases}$$

and a  $\Gamma_6$  ( $j_{\text{eff}} = 1/2$ ; pure  $d_{5/2}$ ) doublet:

$$\chi_{\Gamma_6} = \sqrt{\frac{1}{3}} \left( d_{xy} \chi_{\mp\frac{1}{2}} \mp d_{yz} \chi_{\pm\frac{1}{2}} + i d_{zx} \chi_{\pm\frac{1}{2}} \right)$$

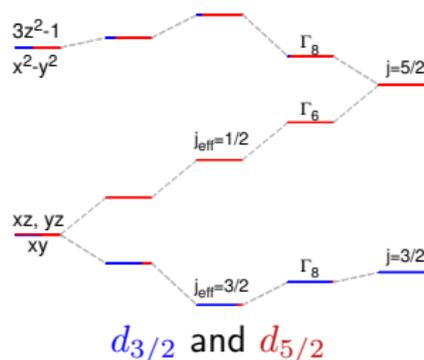


$\text{Ir}^{4+} 5d^5$  ion in octahedral environment:

$j_{\text{eff}}=1/2$  half-filled

$j_{\text{eff}}=3/2$  completely filled

- Mott insulator already for moderate  $U$
- $j_{\text{eff}} = \pm 1/2$  splitting is caused by  $U$  instead of  $J_H$



## Mott Insulators in the Strong Spin-Orbit Coupling Limit: From Heisenberg to a Quantum Compass and Kitaev Models

 G. Jackeli<sup>1,\*</sup> and G. Khaliullin<sup>1</sup>
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(Received 21 August 2008; published 6 January 2009)

We study the magnetic interactions in Mott-Hubbard systems with partially filled  $t_{2g}$  levels and with strong spin-orbit coupling. The latter entangles the spin and orbital spaces, and leads to a rich variety of the low energy Hamiltonians that extrapolate from the Heisenberg to a quantum compass model depending on the lattice geometry. This gives way to “engineer” in such Mott insulators an exactly solvable spin model by Kitaev relevant for quantum computation. We, finally, explain “weak” ferromagnetism, with an anomalously large ferromagnetic moment, in  $\text{Sr}_2\text{IrO}_4$ .

corner-sharing octahedra (a):

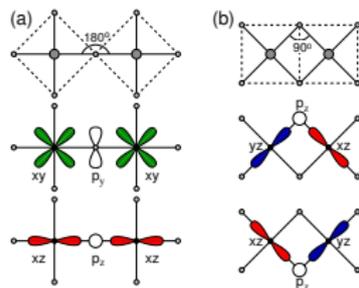
- Heisenberg + weak pseudo-dipolar interaction

$$H = J_1 \mathbf{S}_i \cdot \mathbf{S}_j + J_2 (\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{r}_{ij} \cdot \mathbf{S}_j)$$

- additional anisotropic terms if  $\phi \neq 180^\circ$

$$H = J_1 \mathbf{S}_i \cdot \mathbf{S}_j + J_z S_i^z S_j^z + \mathbf{D} \cdot [\mathbf{S}_i \times \mathbf{S}_j]$$

are responsible for weak FM in  $\text{Sr}_2\text{IrO}_4$



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edge-sharing octahedra (b):

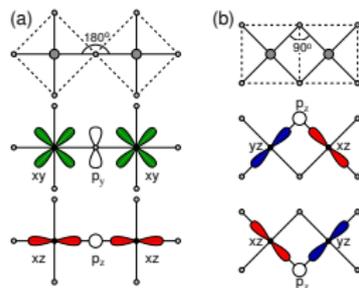
$j_{\text{eff}}=1/2$  hoppings via  $O_1 p_z$  and  $O_2 p_z$  cancel out

- isotropic superexchange  $J$  is suppressed
- strongly anisotropic interaction  $K^{\alpha\beta}$

Kitaev-Heisenberg model:

$$H_{\text{HK}} = K^{\alpha\beta} S_i^\alpha S_j^\beta + JS_i \cdot S_j$$

with exotic spin-liquid ground state



Can one get correct ground state and estimate  $\tilde{J}$  from LDA+ $U$  calculations?

In  $d$  occupation matrix  $n_{mm'}^{\sigma\sigma'}$  has off-diagonal in spin terms ( $n_{mm'}^{\sigma-\sigma} \neq 0$ ):

Coulomb energy:

$$E^U = \frac{1}{2} \sum_{\sigma, \{m\}} [n_{m_1 m_2}^{\sigma\sigma} (\langle m_1 m_3 | V_{ee} | m_2 m_4 \rangle - \langle m_1 m_3 | V_{ee} | m_4 m_2 \rangle) n_{m_3 m_4}^{\sigma\sigma} + n_{m_1 m_2}^{\sigma\sigma} \langle m_1 m_3 | V_{ee} | m_2 m_4 \rangle n_{m_3 m_4}^{-\sigma-\sigma} - n_{m_1 m_2}^{\sigma-\sigma} \langle m_1 m_3 | V_{ee} | m_4 m_2 \rangle n_{m_3 m_4}^{-\sigma\sigma}]$$

$\sigma, m$ -dependent potential:

$$V_{mm'}^{\sigma\sigma'} = \frac{\partial(E^U - E^{dc})}{\partial n_{mm'}^{\sigma\sigma'}}, \quad E^{dc} = \frac{1}{2}UN(N-1) - \frac{1}{2}J \sum_{\sigma} N_{\sigma\sigma}(N_{\sigma\sigma} - 1)$$

A. Liechtenstein, *et al* PRB **52**, R5467 (1995), AY, *et al* PRB **67**, 155103 (2003), ...

rotationally invariant LDA+ $U$ +SOC

- split  $j_{\text{eff}} = 1/2$  states
- but does not change their wavefunction

# how to estimate $\mathfrak{J}$ ?

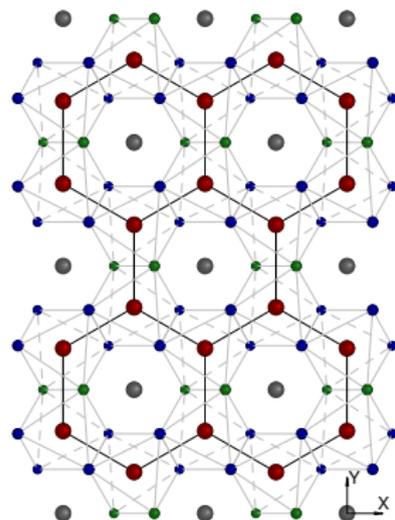
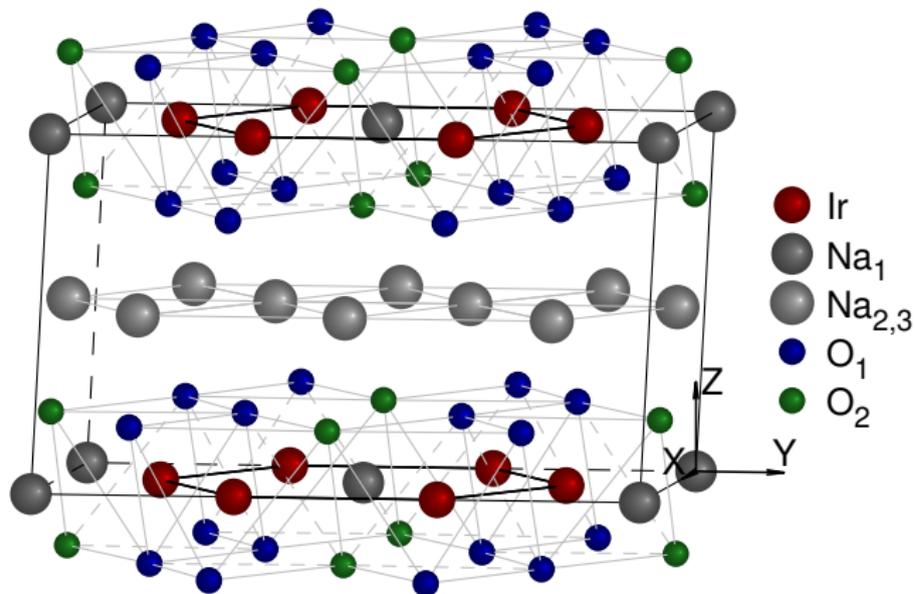
- 1 to calculate the total (band) energy as a function of angle between spins using spin-spiral calculations and/or constraining magnetization direction
- 2 to map  $\varepsilon(\{\phi\})$  onto an appropriate Heisenberg+Kitaev+... model
  - spin-spiral calculations do not work with SOC
  - tricky to impose constraints on magnetization direction in LDA+ $U$  calculations

but we can do:

- calculations for magnetic configurations **constrained by symmetry**
  - limited number of magnetic configurations
  - not all exchange parameters can be determined simultaneously

- 1 motivations:  $j_{\text{eff}}=1/2$  beauty
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- 3 noncollinear ground state in  $\text{Sr}_2\text{IrO}_4$  and  $\text{Sr}_3\text{Ir}_2\text{O}_7$

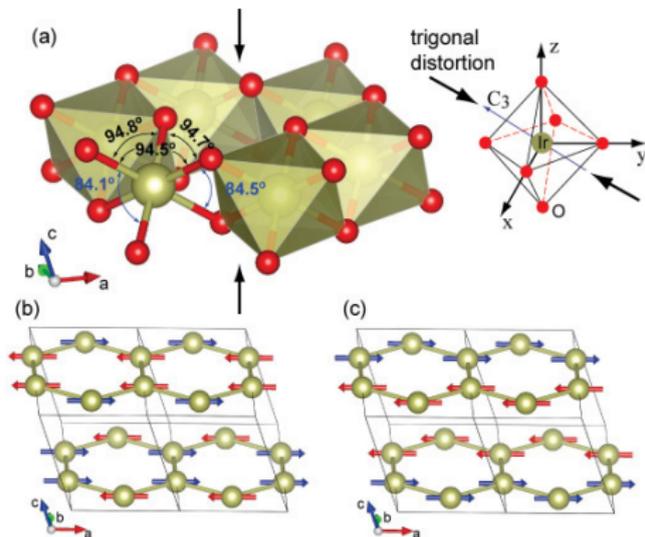
# crystal structure



- monoclinic  $C/2m$  space group
- honeycomb Ir layers separated by triangular Na layers
- trigonally distorted  $\text{IrO}_6$  octahedra;  $\text{Ir}^{4+} d^5$  with half-filled  $j_{\text{eff}}=1/2$  states

S.K. Choi, *et al* PRL **108**, 127204 (2012), F. Ye, *et al* PRB **85**, 180403 (2012)

# experimental magnetic structure



- zigzag order (c)
- ordered Ir moment 0.22  $\mu_B$

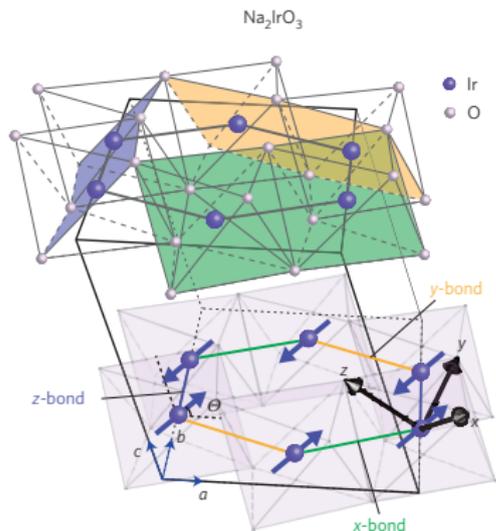
F. Ye, *et al* PRB **85**, 180403 (2012)

explanations:

- isotropic Heisenberg model with long-ranged interactions
- Kitaev-Heisenberg model
- Kitaev-Heisenberg model + additional anisotropic exchanges  $\Gamma$ ,  $\Gamma'$

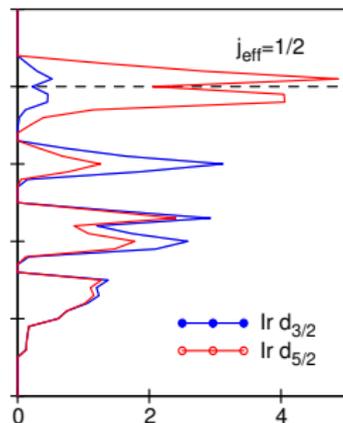
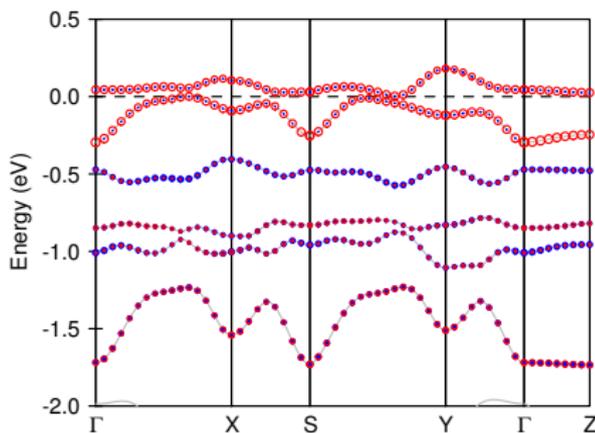
## Direct evidence for dominant bond-directional interactions in a honeycomb lattice iridate $\text{Na}_2\text{IrO}_3$

Sae Hwan Chun<sup>1</sup>, Jong-Woo Kim<sup>2</sup>, Jungho Kim<sup>2</sup>, H. Zheng<sup>1</sup>, Constantinos C. Stoumpos<sup>1</sup>, C. D. Malliakas<sup>1</sup>, J. F. Mitchell<sup>1</sup>, Kavita Mehawat<sup>3</sup>, Yogesh Singh<sup>3</sup>, Y. Choi<sup>2</sup>, T. Gog<sup>2</sup>, A. Al-Zein<sup>4</sup>, M. Moretti Sala<sup>4</sup>, M. Krisch<sup>4</sup>, J. Chaloupka<sup>5</sup>, G. Jackeli<sup>6,7</sup>, G. Khaliullin<sup>8</sup> and B. J. Kim<sup>6\*</sup>

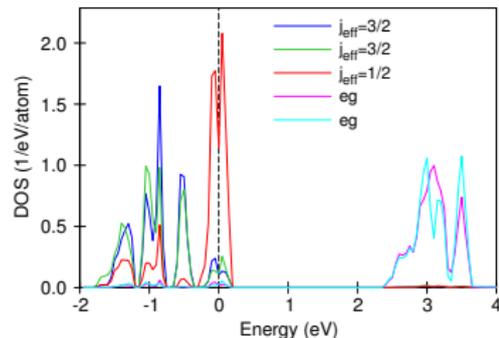


- zig-zag magnetic order
- Ir moments lie in  $ac$  plane
- form angle  $\Theta = 44.3^\circ$  with  $a$  axis
- scattering intensities above  $T_N$  are explained by strongly anisotropic exchange interactions

# relativistic bands for $\text{Na}_2\text{IrO}_3$



- Ir  $t_{2g}$  bands are split into sub-bands due to formation of quasi-molecular orbitals (MO)  
I. Mazin, *et al* PRL **109**, 197201 (2012)
- highest MO are coupled by SOC
- dominant contribution of Ir  $d_{5/2}$  states to bands crossing  $E_F \Rightarrow \sim j_{\text{eff}} = 1/2$  states



## 8 distinct magnetic structures

- 4 inequivalent magnetic structures (magnetic groups) for the  $C2/m$  cell and 4 for a doubled  $P2/m$  cell
- all Na<sub>1,2,3</sub>, Ir, and O<sub>1,2</sub> sites remain equivalent

$C_{2y}$  rotation transforms each Ir site into itself:  $C_{2y}Ir_i = Ir_i$

$$C_{2y}m_x = -m_x, C_{2y}m_y = m_y, C_{2y}m_z = -m_z \Rightarrow m_x = m_z = 0, \mathbf{m}_{Ir} || b$$

$$\hat{\Theta}C_{2y}m_x = m_x, \hat{\Theta}C_{2y}m_y = -m_y, \hat{\Theta}C_{2y}m_z = m_z \Rightarrow m_y = 0, \mathbf{m}_{Ir} || ac$$

	symmetry operations				$\mathbf{m}_{Ir}   $	
$c_f$	$E$	$-C_{2y}$	$I$	$-M_y$	$ac$	ferro
$c_f$	$E$	$C_{2y}$	$I$	$M_y$	$b$	
$c_a$	$E$	$-C_{2y}$	$-I$	$M_y$	$ac$	Néel
$c_a$	$E$	$C_{2y}$	$-I$	$-M_y$	$b$	
$p_z$	$E$	$-C_{2y}$	$-I$	$M_y$	$ac$	zig-zag
$p_z$	$E$	$C_{2y}$	$-I$	$-M_y$	$b$	
$p_s$	$E$	$-C_{2y}$	$I$	$-M_y$	$ac$	stripe
$p_s$	$E$	$C_{2y}$	$I$	$M_y$	$b$	

- “-” means that rotation is followed by time reversal  $\hat{\Theta} = -i\sigma_y\hat{K}$
- $\mathbf{m}_{Ir} || ac$ : Ir magnetization direction is defined by polar angle  $\theta$
- Ir moments are collinear, self-consistency in Ir, O, Na magnetization directions

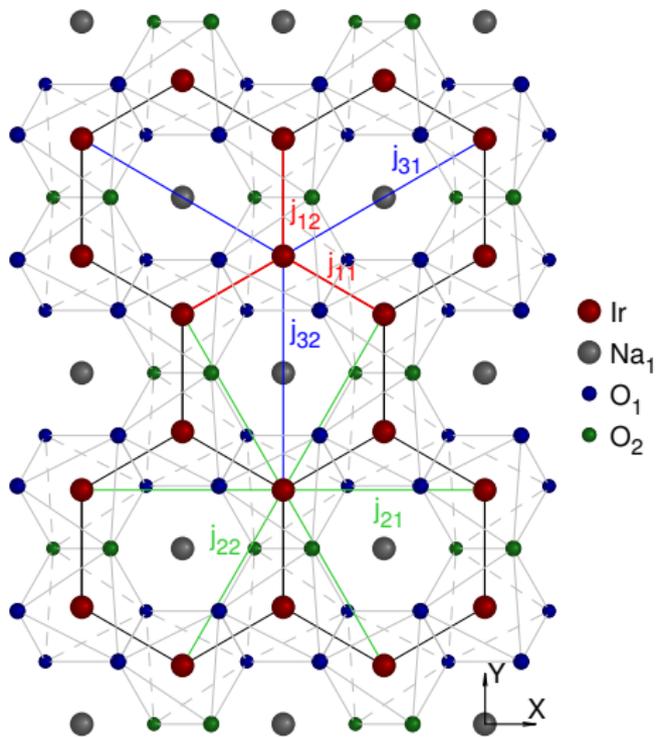
## summary for 8 configurations

- LDA+ $U$  with  $U=2.1$  eV;  $J=0.6$  eV  $\Rightarrow U_{\text{eff}}=U - J=1.5$  eV
- for  $\text{Sr}_2\text{IrO}_4$   $U_{\text{eff}}=1.3$  eV gives good agreement with optical spectra

	<b>M</b>	$\varepsilon$ (meV)	$\theta$	$\phi$	$m_s$ ( $\mu_B$ )	$m_l$ ( $\mu_B$ )
$c_f$	$ac$	-5.5	18.8	180	0.32	0.40
$c_f$	$b$	0.	90.0	90	0.20	0.43
$c_a$	$ac$	-10.9	14.2	180	0.24	0.35
$c_a$	$b$	-12.4	90.0	90	0.14	0.38
$p_z$	$ac$	<b>-16.7</b>	26.4	0	0.25	0.34
$p_z$	$b$	-14.9	90.0	90	0.18	0.39
$p_s$	$ac$	-8.7	138.5	0	0.26	0.36
$p_s$	$b$	0.5	90.0	90	0.16	0.42

- correct ground state with zig-zag order and Ir moments rotated away from  $a$
- insulating solutions for all magnetic orders
- zig-zag ground state also for  $U_{\text{eff}}=1.0$  and 2.0 eV
- $\theta$  does not depend on  $U_{\text{eff}}$  but depends on SOC strength  $\xi$

# Heisenberg model



- interlayer coupling is neglected
- ideal honeycomb lattice is assumed

isotropic exchange:

$$n: J_{11} = J_{12} \equiv J_1$$

$$nn: J_{21} = J_{22} \equiv J_2$$

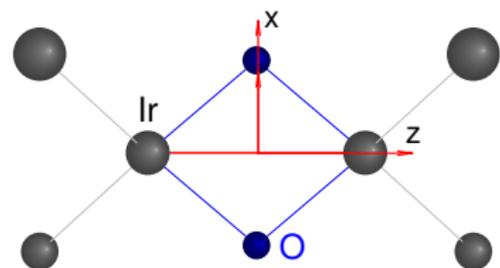
$$nnn: J_{31} = J_{32} \equiv J_3$$

# anisotropic exchanges

Ir-Ir bond in ideal honeycomb lattice:  $D_{2h} (E, C_{2x}, C_{2y}, C_{2z}, I, M_x, M_y, M_z)$

Ir-Ir bond in  $\text{Na}_2\text{IrO}_3$ :

$$C_{2h} (E, C_{2z}, I, M_z)$$



inversion symmetry  $\Rightarrow \mathfrak{J}^- = 0$

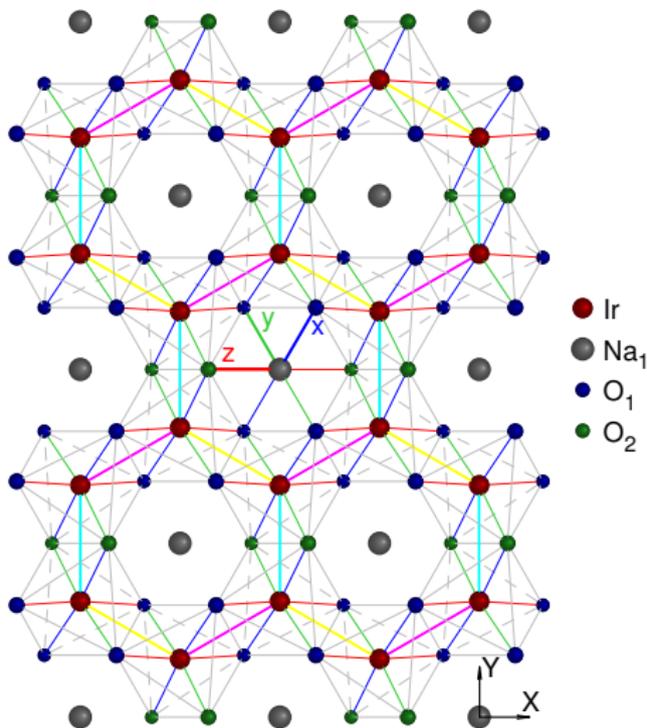
$$\mathfrak{J}^+ = \begin{pmatrix} \mathfrak{J}_{xx}^+ & \mathfrak{J}_{xy}^+ & 0 \\ \mathfrak{J}_{xy}^+ & \mathfrak{J}_{yy}^+ & 0 \\ 0 & 0 & \mathfrak{J}_{zz}^+ \end{pmatrix}, \quad \mathfrak{J}_{xx}^+ \neq \mathfrak{J}_{yy}^+ \neq \mathfrak{J}_{zz}^+$$

or isotropic  $J_0$  + traceless symmetric part

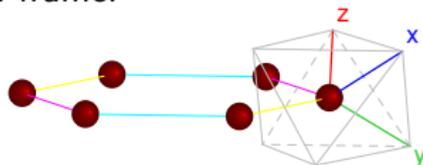
$$\mathfrak{J}^+ = J_0 \cdot \hat{I} + \begin{pmatrix} B & C & 0 \\ C & -B - A & 0 \\ 0 & 0 & A \end{pmatrix}$$

V. Katukuri, *et al* NJP **16**, 013056 (2014)

# anisotropic exchange in a rotated frame



rotated frame:



$x$ ,  $y$ , and  $z$  point to nearest O  
 Ir-Ir bonds along  $xy$ ,  $yz$ ,  $zx$

$$\mathfrak{J}^{xy} = \begin{pmatrix} J & \Gamma & -\Gamma' \\ \Gamma & J & \Gamma' \\ -\Gamma' & \Gamma' & J + K \end{pmatrix}$$

$$K = -\frac{3}{2}(A + B), \quad J = J_0 - K/3,$$

$$\Gamma = (A - B)/2, \quad \Gamma' = C/\sqrt{2}$$

Kitaev-like terms:

$$K_{ij}^{xy} S_i^z S_j^z, \quad K_{ij}^{yz} S_i^x S_j^x, \quad K_{ij}^{zx} S_i^y S_j^y$$

$$K^{xy} \neq K^{yz} = K^{zx} \approx K$$

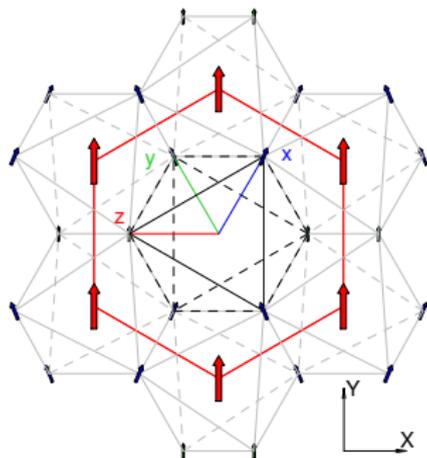
$\Gamma$ -terms:

$$\Gamma_{ij}^{xy} (S_i^x S_j^y + S_i^y S_j^x), \dots$$

J.G. Rau, *et al* PRL **112**, 077204 (2014), J. Chaloupka and G. Khaliullin, PRB **92**, 024413 (2015)

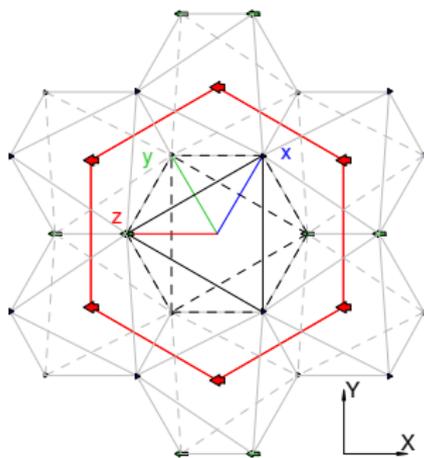
# FM order ( $c_f$ )

$\mathbf{M} \parallel Y$  ( $\mathbf{M} \parallel b$ )



$$\begin{aligned} \varepsilon &= 6J_1 + 12J_2 + 6J_3 \\ &+ 2K \\ \varepsilon &= 0. \end{aligned}$$

$\mathbf{M} \perp Y$  ( $\mathbf{M} \parallel ac$ )

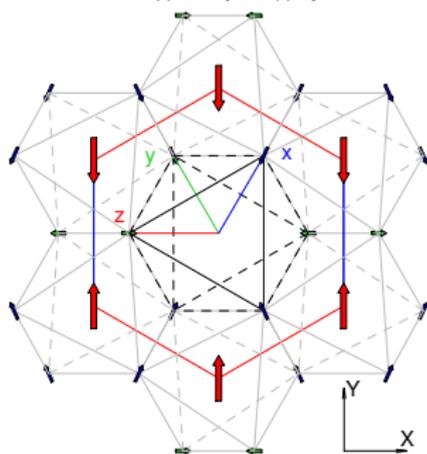


$$\begin{aligned} \varepsilon &= 6J_1 + 12J_2 + 6J_3 \\ &+ 2K \\ \varepsilon &= -5.5 \text{ meV}, \theta = 18.8^\circ, \phi = 180^\circ \end{aligned}$$

- all energies are per f.u. and relative to  $\varepsilon_{cf}(\mathbf{m} \parallel b)$ ; **FM** (**AF**) bonds
- $\theta \neq 0$ ,  $\Delta\varepsilon = \varepsilon(ac) - \varepsilon(b) \neq 0 \Rightarrow \Gamma$ -term or  $K_a \neq K_b$ ?
- noncollinear O moments (0.04–0.09  $\mu_B$ )  $\Rightarrow$  anisotropic interactions

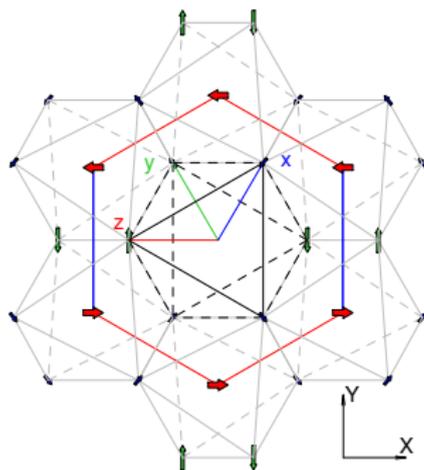
# zig-zag ( $p_z$ )

$M \parallel Y$  ( $M \parallel b$ )



$$\begin{aligned} \varepsilon &= 2J_1 - 4J_2 - 6J_3 \\ &\quad + 2K \\ \varepsilon &= -14.9 \text{ meV} \end{aligned}$$

$M \perp Y$  ( $M \parallel ac$ )



$$\begin{aligned} \varepsilon &= 2J_1 - 4J_2 - 6J_3 \\ &\quad + 2K(\cos 2\theta + 2\sqrt{2} \sin 2\theta)/3 \\ \varepsilon &= -16.7 \text{ meV}, \theta = 26.4^\circ, \phi = 0 \end{aligned}$$

correct ground state with zig-zag order;  $\theta = 26.4^\circ < \theta_{exp} = 45.7^\circ$  ( $90^\circ - 44.3^\circ$ )

# least-square fits to $J_{123}$ - $K$ - $\Gamma$ models

		$\epsilon_{calc}$	$\epsilon_{JK}$	$\epsilon_{JK\Gamma}$	$\epsilon_{123}$	$\epsilon_{123K\Gamma}$
$c_f$	$ac$	-5.5	0.0	-3.8	0.0	-1.9
$c_f$	$b$	0.0	0.0	0.0	0.0	0.0
$c_a$	$ac$	-10.9	-10.5	-9.0	-11.7	-10.7
$c_a$	$b$	-12.4	-10.5	-13.0	-11.7	-12.6
$p_z$	$ac$	-16.7	-8.6	-6.5	-15.8	-14.3
$p_z$	$b$	-14.9	-9.0	-12.4	-15.8	-17.3
$p_s$	$ac$	-8.7	-17.1	-15.8	-4.1	-8.6
$p_s$	$b$	0.5	-1.6	-0.7	-4.1	0.4
	$\chi$		0.30	0.27	0.18	0.10

$JK$   $J_1 = 9.0, K = -16.4$  meV

incorrect  $p_s^{ac}$  ground state (in agreement with the  $J$ - $K$  model)

$JK\Gamma$   $J_1 = 6.7, K = -12.8, \Gamma = 5.6$  meV

the  $p_s^{ac}$  ground state is still incorrect

123  $J_1 = -0.1, J_2 = 1.0, J_3 = 3.9$  meV

correct  $p_z$  ground state but:  $\epsilon(ac) = \epsilon(b); J_3 > J_2 \gg J_1$

123K $\Gamma$   $J_1 = 2.7, J_2 = 0.5, J_3 = 3.2, K = -8.0, \Gamma = 2.8$  meV

best fit although  $\epsilon(p_z^b) > \epsilon(p_z^{ac})$

# least-square fits to $J_{123}$ - $K$ - $\Gamma$ models

		$\epsilon_{calc}$	$\epsilon_{JK}$	$\epsilon_{JK\Gamma}$	$\epsilon_{123}$	$\epsilon_{123K\Gamma}$
$c_f$	$ac$	-5.5	0.0	-3.8	0.0	-1.9
$c_f$	$b$	0.0	0.0	0.0	0.0	0.0
$c_a$	$ac$	-10.9	-10.5	-9.0	-11.7	-10.7
$c_a$	$b$	-12.4	-10.5	-13.0	-11.7	-12.6
$p_z$	$ac$	-16.7	-8.6	-6.5	-15.8	-14.3
$p_z$	$b$	-14.9	-9.0	-12.4	-15.8	-17.3
$p_s$	$ac$	-8.7	-17.1	-15.8	-4.1	-8.6
$p_s$	$b$	0.5	-1.6	-0.7	-4.1	0.4
$\chi$			0.30	0.27	0.18	0.10

- $123K\Gamma$ :  $J_1 = 2.7$ ,  $J_2 = 0.5$ ,  $J_3 = 3.2$ ,  $K = -8.0$ ,  $\Gamma = 2.8$  meV
- $(J, K, D = -\Gamma) = (1.1, -0.7, -0.7)$  meV for  $\phi_{(IrOIr)} = 90^\circ$   
 $(J, K, D) = (1.4, -10.9, -2.1)$  meV for  $\phi_{(IrOIr)} = 98.5^\circ$   
V. Katukuri, *et al* NJP **16**, 013056 (2014)
- $J = 5.3$  meV,  $K = -7$  meV,  $\Gamma = 9.3$  meV,  $\sqrt{2}\Gamma' = -6.6$  meV  
J. Chaloupka and G. Khaliullin, PRB **92**, 024413 (2015)

- LSDA+ $U$  calculations reproduce correct zig-zag magnetic order in  $\text{Na}_2\text{IrO}_3$  although the Ir magnetization direction seems to be too far away from  $ab$  plane compared to the experiment
- Calculated total energies and magnetization directions cannot be explained using the isotropic  $J_1$ - $J_3$  Heisenberg model
- Best fit is obtained when the anisotropic  $K$  and  $\Gamma$  terms are added

- 1 motivations:  $j_{\text{eff}}=1/2$  beauty
- 2 anisotropic exchange in honeycomb  $\text{Na}_2\text{IrO}_3$
- 3 noncollinear ground state in  $\text{Sr}_2\text{IrO}_4$  and  $\text{Sr}_3\text{Ir}_2\text{O}_7$

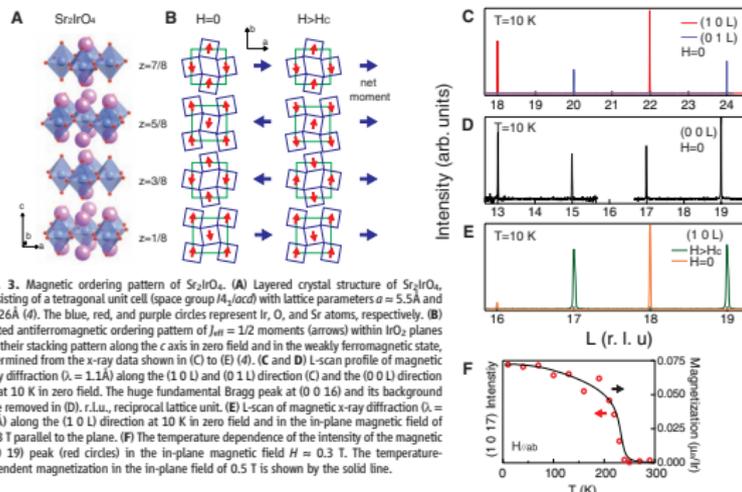
## Phase-Sensitive Observation of a Spin-Orbital Mott State in $\text{Sr}_2\text{IrO}_4$

B. J. Kim,<sup>1,2\*</sup> H. Ohsumi,<sup>3</sup> T. Komesu,<sup>3</sup> S. Sakai,<sup>3,4</sup> T. Morita,<sup>3,5</sup> H. Takagi,<sup>1,2\*</sup> T. Arima<sup>3,6</sup>

Measurement of the quantum-mechanical phase in quantum matter provides the most direct manifestation of the underlying abstract physics. We used resonant x-ray scattering to probe the relative phases of constituent atomic orbitals in an electronic wave function, which uncovers the unconventional Mott insulating state induced by relativistic spin-orbit coupling in the layered  $5d$  transition metal oxide  $\text{Sr}_2\text{IrO}_4$ . A selection rule based on intra-atomic interference effects establishes a complex spin-orbital state represented by an effective total angular momentum =  $1/2$  quantum number, the phase of which can lead to a quantum topological state of matter.

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with Ir moments aligned in  $ab$  plane:



with Ir moments aligned along  $c$ :**Dimensionality Driven Spin-Flop Transition in Layered Iridates**J. W. Kim,<sup>1</sup> Y. Choi,<sup>1</sup> Jungho Kim,<sup>1</sup> J. F. Mitchell,<sup>2</sup> G. Jackeli,<sup>3</sup> M. Daghofer,<sup>4</sup>  
J. van den Brink,<sup>4</sup> G. Khaliullin,<sup>3</sup> and B. J. Kim<sup>2,\*</sup><sup>1</sup>Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA<sup>2</sup>Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA<sup>3</sup>Max Planck Institute for Solid State Research, Heisenbergstrasse 1, D-70569 Stuttgart, Germany<sup>4</sup>Institute for Theoretical Solid State Physics, IFW Dresden, Helmholtzstrasse 20, 01069 Dresden, Germany  
(Received 14 May 2012; published 17 July 2012)

Using resonant x-ray diffraction, we observe an easy  $c$ -axis collinear antiferromagnetic structure for the bilayer  $\text{Sr}_3\text{Ir}_2\text{O}_7$ , a significant contrast to the single layer  $\text{Sr}_2\text{IrO}_4$  with in-plane canted moments. Based on a microscopic model Hamiltonian, we show that the observed spin-flop transition as a function of number of  $\text{IrO}_2$  layers is due to strong competition among intra- and interlayer bond-directional pseudodipolar interactions of the spin-orbit entangled  $J_{\text{eff}} = 1/2$  moments. With this we unravel the origin of anisotropic exchange interactions in a Mott insulator in the strong spin-orbit coupling regime, which holds the key to the various types of unconventional magnetism proposed in  $5d$  transition metal oxides.

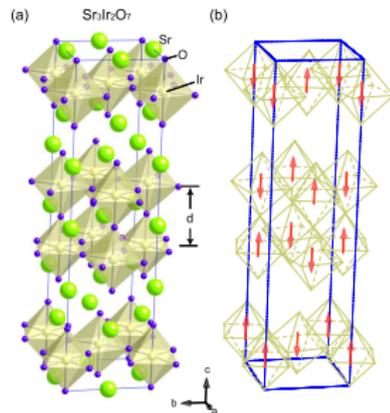


FIG. 1 (color online). (a) Crystal structure of  $\text{Sr}_3\text{Ir}_2\text{O}_7$  as reported in Ref. [17]. Every neighboring  $\text{IrO}_6$  octahedra are rotated in opposite sense about the  $c$  axis by  $\approx 12^\circ$ . (b) Magnetic order has a  $c$ -axis collinear  $G$ -type antiferromagnetic structure. The up and down magnetic moments correlate with counterclockwise and clockwise rotations of the  $\text{IrO}_6$  octahedra, respectively.

# Sr<sub>3</sub>Ir<sub>2</sub>O<sub>7</sub>: crystal structure

*I4/mmm* No. 139 space group:  $a=3.896 \text{ \AA}$ ,  $c=20.879 \text{ \AA}$

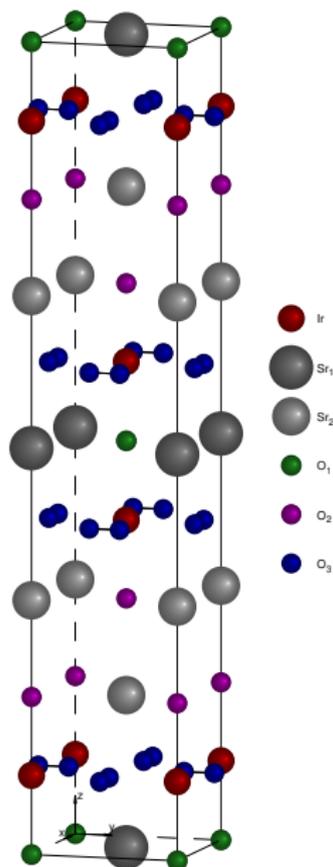
ion	W	x	y	z	occ.
Ir <sub>1</sub> <sup>4+</sup>	4e	0	0	0.09743	1.
Sr <sub>1</sub> <sup>2+</sup>	2b	0.5	0.5	0	1.
Sr <sub>2</sub> <sup>2+</sup>	4e	0.5	0.5	0.1872	1.
O <sub>1</sub> <sup>2-</sup>	2a	0	0	0	1.
O <sub>2</sub> <sup>2-</sup>	4e	0	0	0.1939	1.
O <sub>3</sub> <sup>2-</sup>	16n	0.1043	0.5	0.0960	0.5

M. A. Subramanian, . . . MRB **29**, 645 (1994)

Clock- or counterclockwise rotations of IrO<sub>6</sub> octahedra around  $c$  ( $\phi=11.8^\circ$ )  $\Rightarrow$  averaged O<sub>3</sub> positions

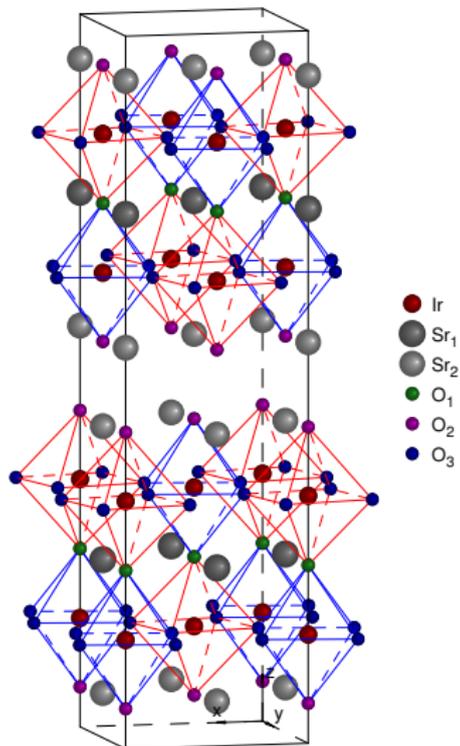
TEM: *Bbcb* ( $\equiv Acaa$ ) No. 68 space group  
rotations in opposite senses in each bilayer

H. Matsuhata, . . . JSSC **177**, 3776 (2004)

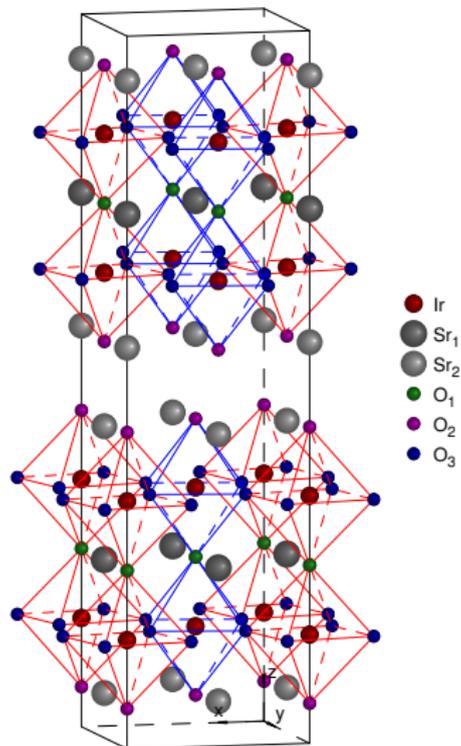


# two structural models for $\text{Sr}_3\text{Ir}_2\text{O}_7$

$Acaa D_{2h}$  No. 68 ( $A_o$ )



$Acam D_{2h}$  No. 64 ( $F_o$ )



with **clockwise** or **counterclockwise** rotated octahedra

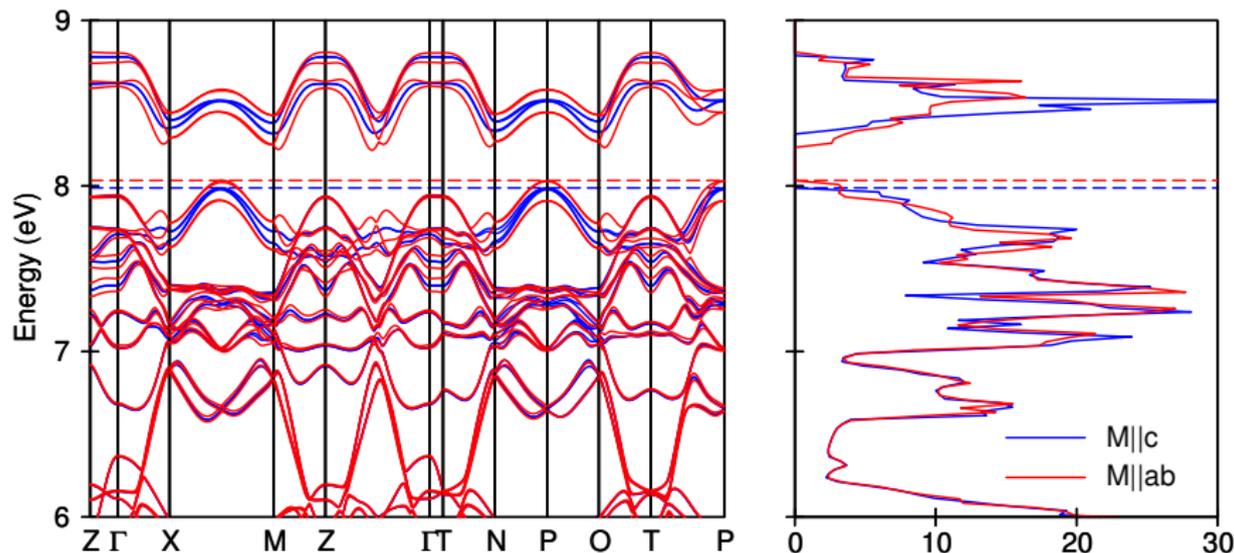
# total energies

$\text{Sr}_3\text{Ir}_2\text{O}_7$			$U_{\text{eff}}=1.0 \text{ eV}$		$U_{\text{eff}}=1.5 \text{ eV}$		$U_{\text{eff}}=2.0 \text{ eV}$	
OO	MO	M	$\Delta E$	$E_g$	$\Delta E$	$E_g$	$\Delta E$	$E_g$
$A_o$	$AA_c$	$c$	0	0.13	0	0.33	0	0.53
		$ab$	7.4	0	6.8	0.20	6.3	0.41
	$AF_c$	$c$	10.0	0	20.8	0.08	19.8	0.31
		$ab$	6.6	0.05	9.1	0.23	10.6	0.43
$F_o$	$AA_c$	$c$	54.8	0.14	50.4	0.33	48.1	0.52
		$ab$	52.9	0.15	49.0	0.33	47.5	0.51
	$AF_c$	$c$	67.7	0	71.3	0.12	68.5	0.35
		$ab$	67.0	0	69.6	0.13	67.7	0.35
$\text{Sr}_2\text{IrO}_4$								
	$A$	$c$	0	0.14	0	0.35	0	0.58
		$ab$	-1.3	0.15	-1.1	0.36	-0.7	0.58

- $\text{Sr}_2\text{IrO}_4$ : lowest energy for  $\mathbf{M}||ab$ ;  $\text{Sr}_3\text{Ir}_2\text{O}_7$ : lowest energy for  $\mathbf{M}||c$
- $\text{Sr}_3\text{Ir}_2\text{O}_7$ :  $A_o$  order of octahedra and  $A_c$  order in a bilayer are always favorable
- $\text{Sr}_3\text{Ir}_2\text{O}_7$ : for  $F_o$  order of octahedra  $\mathbf{M}||ab$  gives lower energy

# Sr<sub>3</sub>Ir<sub>2</sub>O<sub>7</sub>: bilayer splitting

AF order in a bilayer:



- $\mathbf{M}||c$ : Ir moments are antiparallel  $\Rightarrow$  small bilayer splitting
- $\mathbf{M}||ab$ : the angle between moments  $\phi_c = 137^\circ$   
 $\Rightarrow$  bilayer splitting is much stronger

- LSDA+U calculations reproduce magnetic ground states for  $\text{Sr}_2\text{IrO}_4$  and  $\text{Sr}_3\text{Ir}_2\text{O}_7$
- in  $\text{Sr}_3\text{Ir}_2\text{O}_7$  bilayer splitting of unoccupied Ir  $t_{2g}$  bands increases strongly when Ir moments are in  $ab$  plane