temperature inversion in long-range interacting systems

from astrophysical to atomic scales

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Long-range-interacting many-body systems: from atomic to astrophysical scales

ICTP Trieste, July 25, 2016

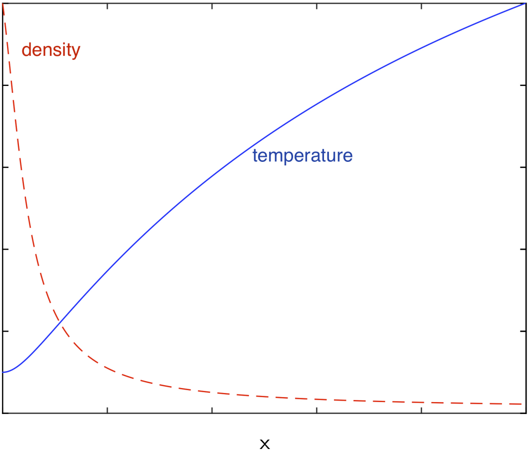
joint work with Pierfrancesco Di Cintio, Shamik Gupta, and Tarcísio N. Teles

LC & Gupta European Physical Journal B **87**, 91 (2014)
Teles, Gupta, Di Cintio & LC Physical Review E **92**, 020101(R) (2015)
Teles, Gupta, Di Cintio & LC Physical Review E **93**, 066102 (2016)
Di Cintio, Gupta & LC (in preparation); Gupta & LC (in preparation)

introduction & motivation

- temperature inversion
  - anticorrelation between density & temperature
    the sparser the hotter, the denser the colder
  - nonequilibrium stationary states
    spontaneously appear with long-range interactions: Quasi-Stationary States (QSSs)
  - from astrophysical scales...
    solar corona, interstellar molecular clouds, (some) cD galaxies, hot gas in galaxy clusters...
  - ...to atomic scales
    cold atoms in a cavity
- minimal ingredients & basic physical mechanism
  - long-range interactions & inhomogeneous states
    "universality" $\longrightarrow$ Julien Barré's talk
  - spontaneous temperature inversion
    after perturbing thermal equilibrium or quenching a field
  - interplay between spatial inhomogeneity & wave-particle interaction
    $\Longrightarrow$ velocity filtration

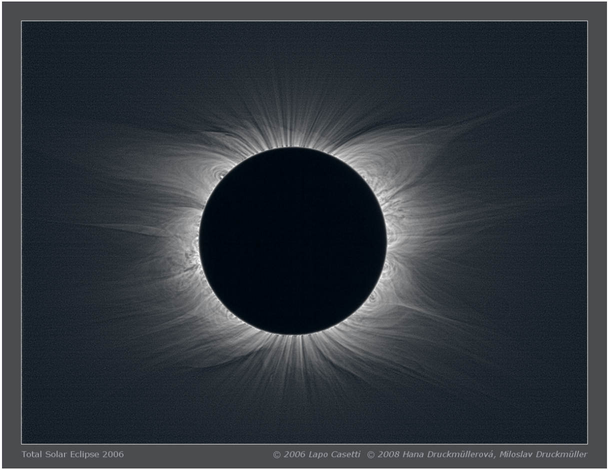
temperature inversion



temperature $\propto$ locally averaged kinetic energy $\propto$ squared velocity dispersion

temperature inversion

solar corona



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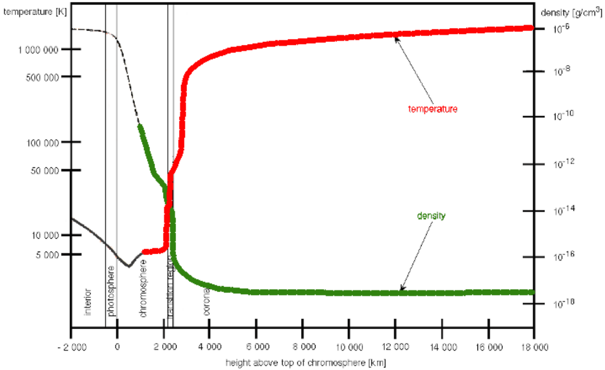

[NASA]

temperature inversion

molecular clouds

[John Corban & the ESA/ESO/NASA Photoshop FITS Liberator]

temperature inversion

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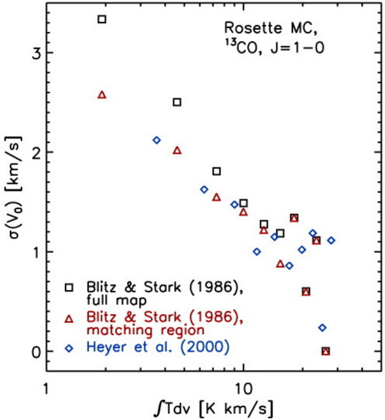

[P. Padoan *et al.*, ApJ 2001]

temperature inversion

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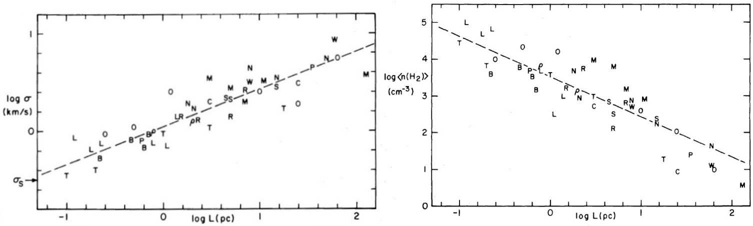
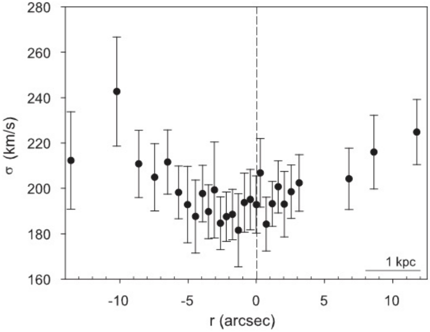


$$\sigma^2 \propto \varrho^{-0.8}$$

[R. P. Larson, MNRAS 1981]

temperature inversion

cD galaxies

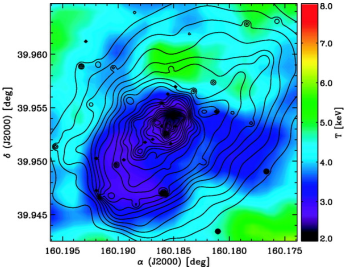


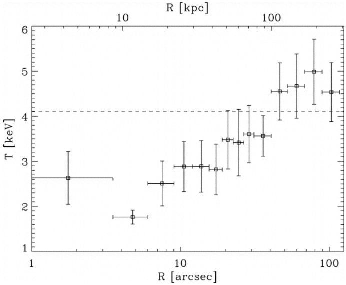
NGC 3311 in Hydra

[S. I. Loubser *et al.*, MNRAS 2008]

temperature inversion

hot gas in galaxy clusters





[M. W. Wise *et al.*, ApJ 2004]

temperature inversion models & theories

no general explanation

a specific mechanism is invoked for each case

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solar corona

- energy injection in the low-density regions

  dissipation of Alfvén waves, magnetic field lines reconnection,...

- velocity filtration

  more soon...

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  simulations ≈ work but no clear physical mechanism

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cD galaxies

- dynamical effects

  resonances, anisotropy, dark matter, varying M/L...

gas in galaxy clusters

- dissipative effects

velocity filtration

1990s: J. D. Scudder to explain coronal heating [J. D. Scudder, ApJ 1992 & 1994]

...without great success in the solar physics community...

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- noninteracting particles in an external field, e.g. gravity
  one dimension: x height above ground
- stationary boundary condition at $x = 0$

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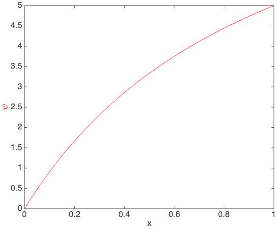
collisionless Boltzmann equation for $f(x, p, t)$

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial x} - \frac{d\psi}{dx}\frac{\partial f}{\partial p} = 0$$

...just single-particle energy conservation in this case...



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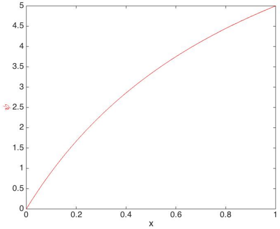

velocity filtration

only particles with kinetic energy $k(0) \geq \psi(x)$ reach x where $k(x) = k(0) - \psi(x)$

velocity filtration thermal boundary condition

density profile

$$n(x) = \int_{-\infty}^{\infty} dp\, f(x, p)$$

decreasing function of x

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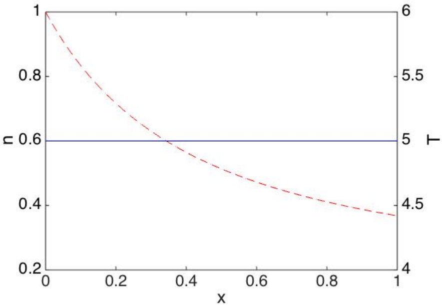
constant temperature profile

$$T(x) = \frac{1}{n(x)} \int_{-\infty}^{\infty} dp\, p^2 f(x,p) \equiv T_0$$

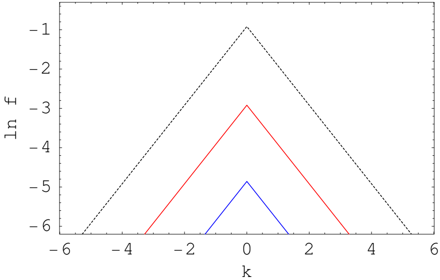
velocity filtration thermal boundary condition



only with thermal boundary condition $f_0{}^M$

velocity filtration how does it work?

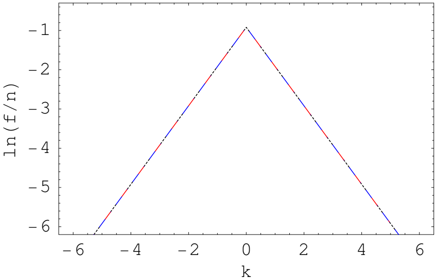
plot ln f as a function of (signed) kinetic energy k



$\cdots\ x = 0$ — $x = 0.25$ — $x = 0.65$

velocity filtration how does it work?

rescale f with n

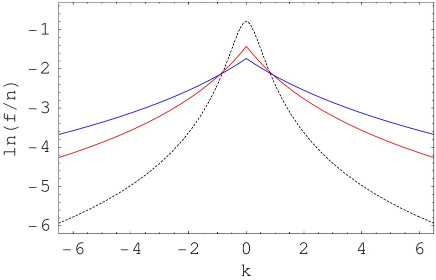


$\cdots$ $x = 0$ — $x = 0.25$ — $x = 0.65$

velocity filtration suprathermal boundary condition

suprathermal f_0, i.e., with tails fatter than a Maxwellian

$$f_0(p) = \frac{\sqrt{2}}{\pi\left(1+p^4\right)}$$



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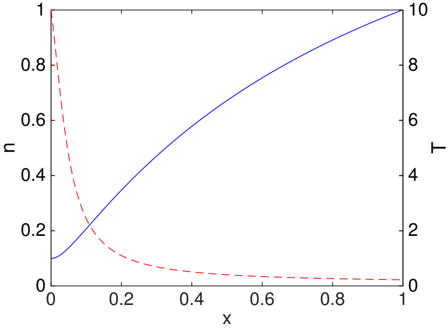

temperature inversion

velocity filtration summary

pros

- simple and general mechanism for temperature inversion
- needs no active energy injection in sparser regions of the system
- makes no use of specific ingredients (magnetic fields, turbulence,...)

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cons

- what about interactions?
- needs a non-thermal boundary condition for all times
  a very strong assumption, seems to rule out isolated systems
- who keeps the system in a non-thermal state at the boundary?
  still an *ad hoc* "active" ingredient

a toy model

N unit mass particles, generic long-range interaction V

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{N} \sum_{i=1}^{N} \sum_{j<i}^{N} V\left(\left|\mathbf{r}_i - \mathbf{r}_j\right|\right)$$

restrict to $d = 1$ and expand V in a Fourier series to the lowest order

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{J}{N} \sum_{i=1}^{N} \sum_{j<i}^{N} \left[1 - \cos\left(\vartheta_i - \vartheta_j\right)\right]$$

Hamiltonian Mean Field (HMF) model

particles on a ring with all-to-all interactions

XY spins on a complete graph (mean-field interactions)

$J > 0$ attractive/ferromagnetic interactions; $J < 0$ repulsive/antiferro

$J > 0$ equilibrium phase transition breaking the $O(2)$ symmetry at small energy (temperature)

broken symmetry phase: clustered/magnetized

[M. Antoni & S. Ruffo PRE 1995]

HMF dynamics

$t < \tau_{coll} \Longrightarrow$ Vlasov equation for $f(\vartheta, p, t)$

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial \vartheta} - \frac{\partial\left(\langle u\rangle + \psi\right)}{\partial \vartheta}\frac{\partial f}{\partial p} = 0$$

self-consistent interaction

$$\langle u\rangle(\vartheta, t) = \int d\vartheta' \int dp'\, u(\vartheta - \vartheta') f(\vartheta', p', t)$$

+ (possibly) external field ψ

for the HMF model

$$\begin{aligned} u(\vartheta - \vartheta') &= J\left[1 - \cos(\vartheta - \vartheta')\right] \\ \psi(\vartheta) &= -h\cos\vartheta \end{aligned}$$

initial conditions ⟶ "violent relaxation" ⟶ QSS (stable stationary Vlasov solution) ⟶ thermal equilibrium
$t = 0$ | $t = \mathcal{O}(1)$ | $t < \tau_{coll}$ | $t > \tau_{coll}$

HMF velocity filtration and temperature inversion

- if f is stationary (QSS)
- if the net effect of $\langle u \rangle + \psi$ is attractive (clustered QSS)

Vlasov equation for HMF $\approx$ collisionless Boltzmann equation of the Scudder model

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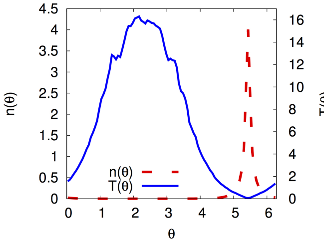
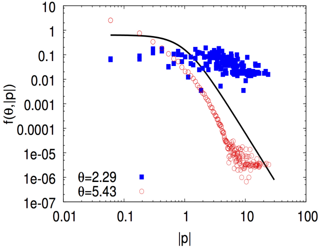
...and it does!





[LC & S. Gupta EPJB 2014]

temperature inversion in long-range systems

what we learned so far

- long-range systems are naturally found in nonequilibrium “steady” states (QSSs)
  due to lack of collisions for $t < \tau_{coll}$
- velocity filtration works also with long-range interactions of arbitrary strength, yielding temperature inversion
  provided the system is in a clustered QSS
- no need of a stationary nonthermal boundary condition
  a suprathermal velocity distribution can be given just as initial condition ($t = 0$)
  temperature inversion via velocity filtration also in isolated long-range-interacting systems

far beyond Scudder’s model, towards a basic mechanism for temperature inversion

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far beyond Scudder's model, towards a basic mechanism for temperature inversion

...still...

- who prepares the system with a suprathermal velocity distribution?
  much weaker assumption than Scudder's boundary condition, yet not very natural...

disturbing the equilibrium

a simple question

- what happens if an isolated macroscopic system in thermal equilibrium is suddenly disturbed?

  by an impulsive force or a "kick" (or by quenching a field)

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long-range systems

- a system with long-range interactions will settle in a QSS

  which one of the infinitely many possible stationary states? nobody knows, in general...

  ⟶ Fernanda P. da C. Benetti's and Yan Levin's forthcoming talks will show that sometimes we do know
- a simpler question: how different from equilibrium is this state?

  can we characterize it by some general features?
- surprisingly, the answer is yes

  the QSS typically exhibits nonuniform temperature and temperature inversion

  even more surprisingly, we can also understand why...

kicking a long-range system away from equilibrium

- prepare a HMF model in a clustered (magnetized) thermal equilibrium at $t = 0$
- evolve until $t = t_0$ then switch on an external field for a short time τ
- look what happens next...

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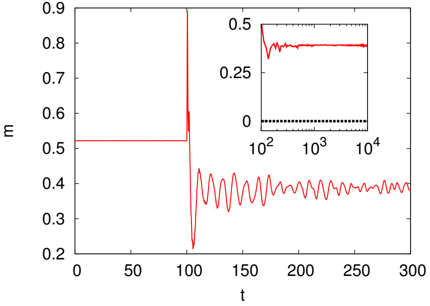

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

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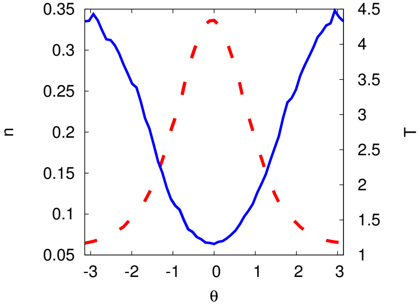

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

quenching a long-range system from equilibrium to nonequilibrium

- prepare a HMF model in a clustered (magnetized) thermal equilibrium at $t = 0$ with external field h
- evolve until $t = t_0$ then quench the external field to another value h'
- look what happens next...

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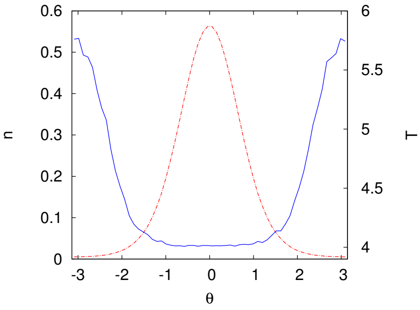


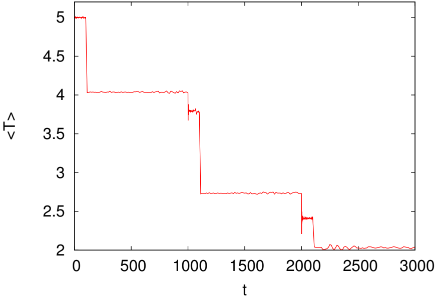
- the same also by quenching J

[S. Gupta & LC in preparation]

bonus: you can cool the system

...with many thanks to Andrea Trombettoni who suggested we could make it...

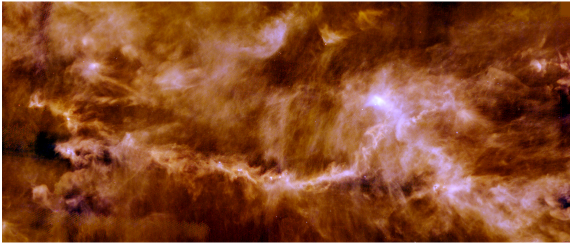
- quench as before
- remove hot particles
- quench again...



[S. Gupta & LC in preparation]

beyond toy models: from astrophysical scales...

filaments in molecular clouds



[esa/Herschel]

model: cylindrical symmetry $\Longrightarrow$ two-dimensional self-gravitating system (2DSGS)

$$\mathcal{H} = \sum_{i=1}^{N} \frac{|\mathbf{p}_i|^2}{2m} + G m^2 \sum_{i=1}^{N} \sum_{j<i}^{N} \ln |\mathbf{r}_i - \mathbf{r}_j|$$

[Toci & Galli MNRAS 2014]

beyond toy models: from astrophysical scales...

- prepare a 2DSGS model in a thermal equilibrium state at $t = 0$
- evolve until $t = t_0$ then apply a radial perturbation
- look what happens next...

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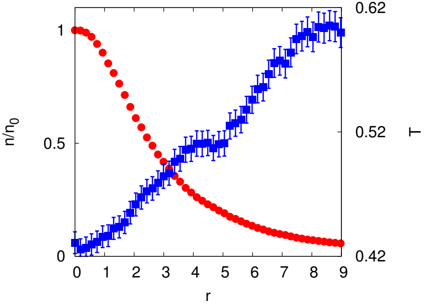


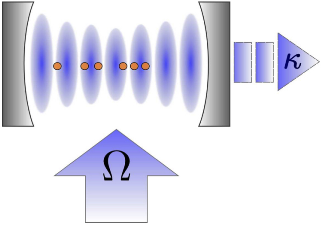
[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

- the same also in a cold collapse with or without magnetic fields

[P. Di Cintio, S. Gupta & LC in preparation]

beyond toy models: ...to atomic scales

cold atoms trapped in a 1-d single-mode optical cavity



model: semiclassical + dissipationless limit $\Longrightarrow$ mean-field Hamiltonian dynamics

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{J}{N} \left[\sum_{j=1}^{N} \cos\left(k x_j\right) \right]^2 \quad \text{with } J \propto \text{laser intensity}$$

[Schütz & Morigi PRL 2014]

$\longrightarrow$ Igor Lesanovsky's and Romain Bachelard's forthcoming talks

beyond toy models: ...to atomic scales

- take $k = 1$ and prepare the system in an inhomogeneous thermal equilibrium state at $t = 0$
- evolve until $t = t_0$ then quench J by suddenly changing the laser intensity
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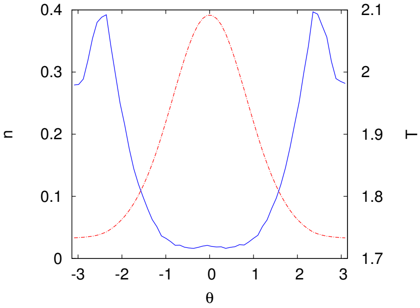

- and also cooling works as in the HMF case

[S. Gupta & LC in preparation]

the physical picture

long-range-interacting system in thermal equilibrium

for 2DSGS also cold nonequilibrium initial state

$\Downarrow$

perturbation/quench

or whatever brings the system far from equilibrium with oscillating mean field

$\Downarrow$

inhomogeneous QSS

$\Downarrow$

temperature inversion in the QSS

- generic feature of long-range-interacting systems
  both mean-field & slowly decaying forces, attractive/repulsive with confining external field, 1-d & 2-d (hopefully 3-d too)
- robust w.r.t. changes in the parameters & in the protocol
  temperature inversion is always there!
- what is going on?
  a general mechanism at work...

wave-particle interaction

- the perturbation induces a wave in the system
  after the kick the system gains energy and m oscillates
- the wave is damped and the system settles in a QSS
  how can it be? no collisions!

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  particles interact with the oscillating mean field

  rigorous theory for small perturbations of homogeneous states [Mohout & Villani Acta Math. 2011]

  occurs also in clustered states [Barré *et al.* JSTAT 2010 & J. Phys. A 2011]

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  occurs also in clustered states [Barré *et al.* JSTAT 2010 & J. Phys. A 2011]
- wave-particle interactions selective in velocity

  only particles with $v \simeq v_w$ exchange energy with the wave

  if $v \lesssim v_w$ the particle gains energy

  if $v \gtrsim v_w$ the particle loses energy

wave-particle interaction

- the perturbation induces a wave in the system

  after the kick the system gains energy and m oscillates

- the wave is damped and the system settles in a QSS

  how can it be? no collisions!

- wave-particle interactions! (Landau damping)

  particles interact with the oscillating mean field

  rigorous theory for small perturbations of homogeneous states [Mohout & Villani Acta Math. 2011]

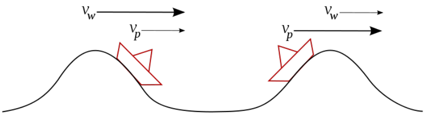
  occurs also in clustered states [Barré *et al.* JSTAT 2010 & J. Phys. A 2011]

- wave-particle interactions selective in velocity

  only particles with $v \simeq v_w$ exchange energy with the wave

  if $v \lesssim v_w$ the particle gains energy

  if $v \gtrsim v_w$ the particle loses energy



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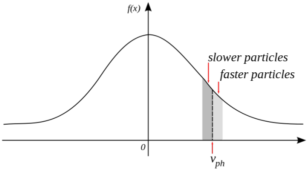

net effect: damping of the wave

wave-particle interaction

- wave-particle interaction ≈ locally changes $f(v)$ after the kick

  let's check it...

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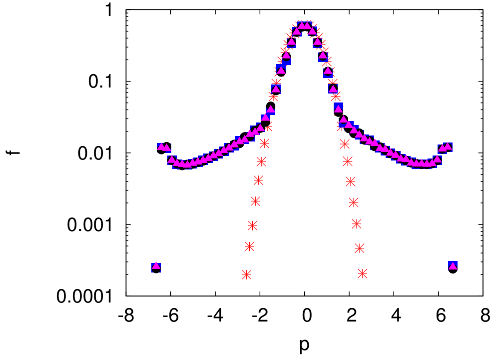

cumulative momentum distribution $f(p)$ as a function of time ($*$ t = 0)

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

wave-particle interaction and velocity filtration

velocity filtration is back!

- $f(v)$ has suprathermal tails in the QSS
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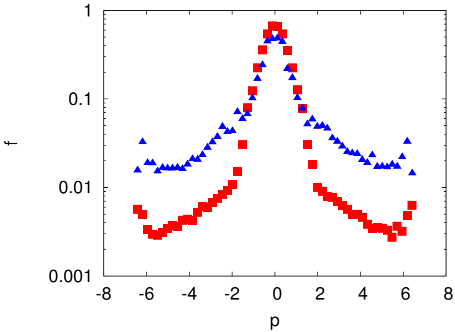

distribution function $f(\vartheta, p)$ ■ $\vartheta = 0$ ▲ $\vartheta = \pi$

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

summary & outlook

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- temperature inversion from astrophysical to atomic scales
  astrophysics: examples of a general phenomenon rather than a collection of unrelated phenomena
  may occur in any system with long-range interactions, also at atomic scales: "universality"
- minimal ingredients of temperature inversion
  long-range interactions, clustered steady nonequilibrium state, fat-tailed velocity distributions
- basic and general physical mechanism
  temperature inversion spontaneously appears after disturbing thermal equilibrium
  interplay between wave-particle interaction and spatial inhomogeneity leading to velocity filtration
- long-living quasi-stationary states obtained disturbing equilibrium
  typically show nonuniform temperature profiles and temperature inversion

summary & outlook

what next?

- 3-d self-gravitating systems
  cD galaxies, Larson's power laws for molecular clouds... (work in progress)
- trapped ions and particle beams
  close to (antiferrro) HMF and 2-d Coulomb systems, respectively (work in progress)
- experiments (hopefully)
  atoms in a cavity, trapped ions, particle beams...
  $\longrightarrow$ Igor Lesanovsky's talk (this afternoon)
  $\longrightarrow$ Romain Bachelard's talk on Friday
- physical picture $\Longrightarrow$ theory?
  hints from Julien Barré's group theory of Landau damping in inhomogeneous states?
  hints from Yan Levin's group theory for self-gravitating systems?
  $\longrightarrow$ Fernanda P. da C. Benetti's talk (next)
  $\longrightarrow$ Yan Levin's talk on Friday