Anisotropic long range spin systems

Nicolò Defenu*,[1,2] Stefano Ruffo,[1] and Andrea Trombettoni[1,2]

[1] *SISSA, Via Bonomea 265, I-34136 Trieste, Italy*

[2] *CNR-IOM DEMOCRITOS Simulation Center, Via Bonomea 265, I-34136 Trieste, Italy*

We consider the case of anisotropic long range (LR) interacting spin systems in general dimension d. The system is divided into two subspaces of dimension d_1 and d_2, with $d_1 + d_2 = d$. In the first subspace the interaction between the spins decays with the distance as a power law $r^{-d_1-\sigma}$, while in the other subspace it decays as $r^{-d_2-\tau}$. We introduce a low energy effective action with non analytic power of the momenta. As a function of the two exponents τ and σ we show the system to have three different regimes, two where it is actually anisotropic and one where the isotropy is finally restored. The model we propose is a lattice spin system in dimension d_1, with an arbitrary number of spin components N. The interactions among the spins is LR with different exponents depending on the spatial directions

$$H = -\sum_{i\neq j}\frac{J_\parallel}{2}\frac{\boldsymbol{S}_i\boldsymbol{S}_j}{r_{\parallel,ij}^{d_1+\sigma}}\delta(\boldsymbol{r}_{\perp,ij}) - \sum_{i\neq j}\frac{J_\perp}{2}\frac{\boldsymbol{S}_i\boldsymbol{S}_j}{r_{\perp,ij}^{d_2+\tau}}\delta(\boldsymbol{r}_{\parallel,ij}). \tag{1}$$

where boldface symbols stand for vectors. The $\boldsymbol{S}_i$ are classical N component vectors. The distance $r_{\parallel,ij}$ is calculated on a d_1 dimensional plane, where both the spins $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$ belong, as ensured by the presence of the $\delta(\boldsymbol{r}_{\perp,ij})$. On the same ground $r_{\perp,ij}$ measures the distance between two spins i, j belonging to the same d_2 dimensional plane. Thus any spin of the model belongs to two different subspaces, one of dimension d_1 and the other of dimension d_2, and interacts only with the spins sitting on the same subspaces.

When one of the two exponent goes is infinite $\sigma\backslash\tau \to \infty$ [1] the interaction becomes SR in the correspondent subspace. However, in analogy with the isotropic LR case, two threshold values σ^* and τ^* exist such that for $\sigma > \sigma^*\backslash\tau > \tau^*$ the systems behaves as only SR interactions were present in respectively the d_1 or d_2 dimensional subspace.

This system can look a bit exotic, but it has in fact at least one evident physical realization. We consider a quantum spin system in dimension d' in presence of LR interactions

$$H = -\frac{J}{2}\sum_{i\neq j}\frac{\sigma_i^{(z)}\sigma_j^{(z)}}{|i-j|^{d+\sigma}} - h\sum_i \sigma_i^{(x)}\,, \tag{2}$$

where $\sigma^{(z),(x)}$ is the z, x component of the quantum spin $\boldsymbol{\sigma}$ and J is some positive constant. In the thermodynamic limit a quantum spin system can be mapped into a classical analogous [2, 3]. Thus the quantum phase transition at zero temperature of a quantum spin system in dimension d' lies in the same universality class of a classical system in dimension $d+z$, where z is the dynamic exponent in the quantum case. For the Ising case ($N = 1$) with SR interactions the dynamic exponent is $z = 1$. Then we can map a quantum Ising model in dimension d' with a classical analogous in $d = d' + 1$ [4]. This result is also valid with LR interactions and the mapping is between a quantum Ising model, described in (1), and the classical model with anisotropic interactions with $d_1 = 1$, $d_2 = d'$ and $\sigma > \sigma^*$.

In the general N case we do not know explicitly the z value. However we can in general state that a quantum spin system in dimension d' with LR interactions decaying with exponent τ' has a phase transition which lies in the same universality of the one found in the classical system (1) with $\sigma > 2$, $d_1 = z$, $d_2 = d'$ and $\tau = \tau'$.

The model is then strictly related to the investigation of the critical properties of LR interactions in quantum spin systems that have been recently realized in cold atoms experiments [5, 6].

---

[1] The symbol \ stands for "or"

[2] E. H. Lieb Comm. in Math. Phys. Volume **31**, Issue 4, pp 327-340 (1973).

[3] D. Podolsky, E. Shimshoni, P. Silvi, S. Montangero, T. Calarco, G. Morigi, and S. Fishman Phys. Rev. B 89, 214408 (2014)

[4] G. Mussardo *Statistical Field Theory: An Introduction to Exactly Solved Models in Statistical Physics,* (Oxford, Oxford University Press, 2010).

[5] C. Senko et al., Phys. Rev. X 5 (2015), 021026.

[6] P. Richerme et al., Nature 511 (2014), 198–201.