Derivation of the Vlasov equation

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31. March 2016

Motivation

- Dynamics of N particles with interaction
  Control analytically and numerically often impossible
  QM: $N \gtrsim 5$
  Physicists use simplified, effective description
  example: gas laws
- Proof of validity of effective description
  in particular influence of interaction
- Here: dynamical questions

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Most obvious system

- N Newtionian particles
- Two different "cases"
  collissions
  long range potential
- Here: Coulomb potential
- e.g. a galaxy of volume N
- rescale such that volume one, coupling constant: N^{-1}
  Here: focus on Singularity

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The microscopic system

- Trajectory in phase space:
  $(Q, P) = (q_1, q_2, \dots q_N, p_1, p_2, \dots p_N) \in \mathbb{R}^{6N}$
- q_j: position of particle j
  p_j momentum (=speed) of particle j
- Newtonian dynamics: $\dot{Q} = P$
  $\dot{P} = F(Q)$
  Force on j^{th} particle: $(F)_j = N^{-1} \sum_{k \neq j} f(q_j - q_k)$
- Macroscopic: law of motion for particle density

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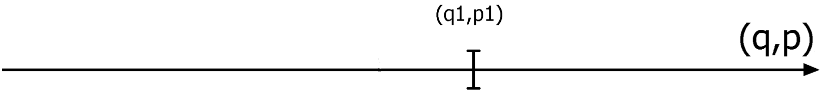

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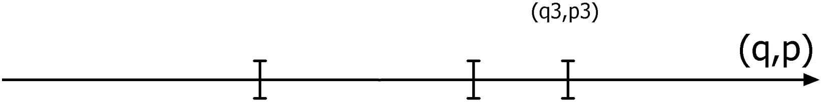

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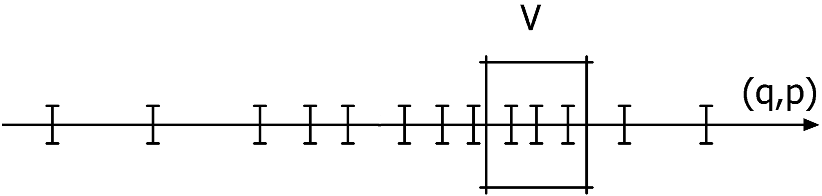
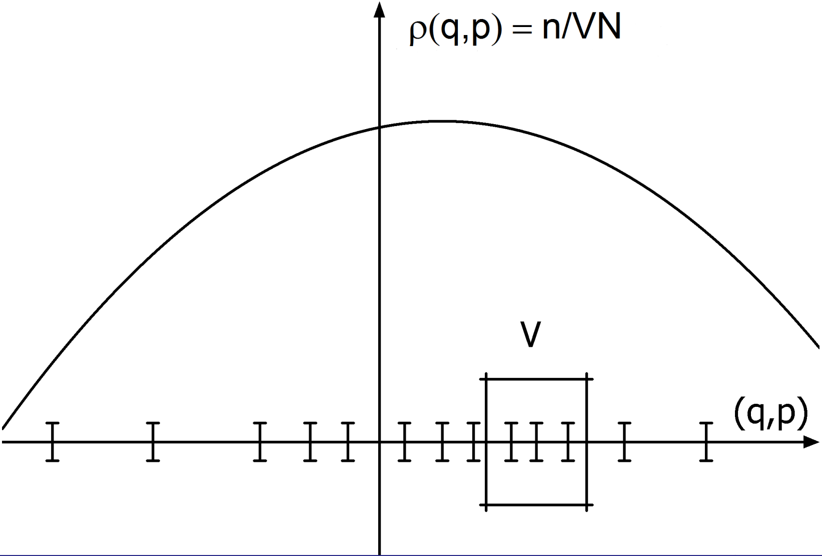
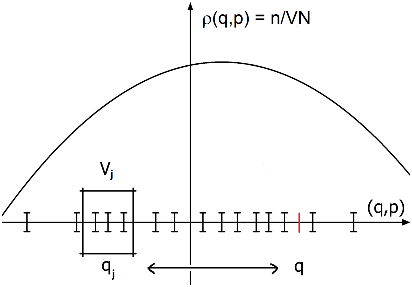


Empirical density ρ



Estimate of the force at q



$$\overline{f}(q) = \sum_j n_j N^{-1} f(q - q_j) = \sum_j V \rho(t, q_j, p_j) f(q - q_j)$$

$$\approx \int \rho(t, q, p) f(q - q_j) d^3p d^3q_j = \rho \star_q f(t, q)$$

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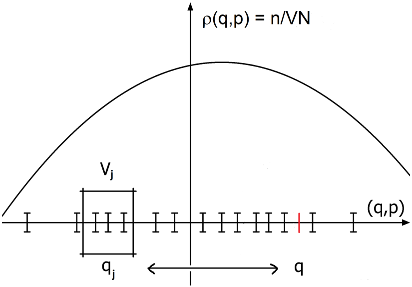

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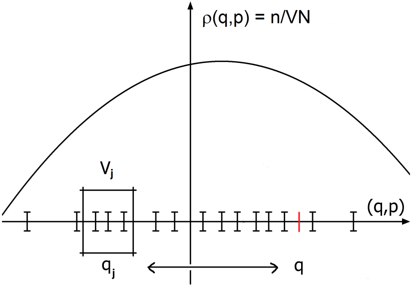

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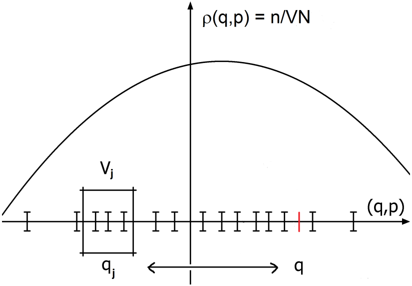

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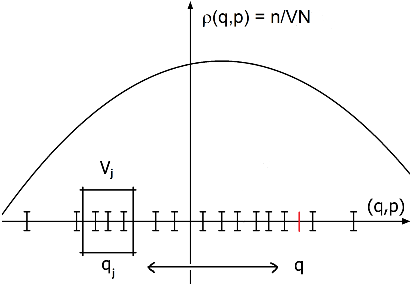

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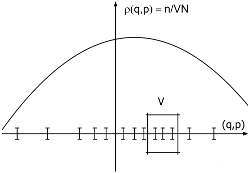
Dynamics of ρ (heuristics)



- $\rho(t, q, p) \approx$ empirical density of particles
- Particles move $\rightarrow \rho$ time dependent
- Conservation of phase space volumes ($n = \text{const} \Rightarrow V = \text{const}$)
- $\rho(t, q(t), p(t)) \approx \frac{n}{VN} = \text{const}$

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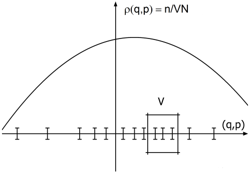

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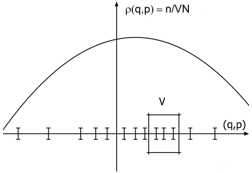

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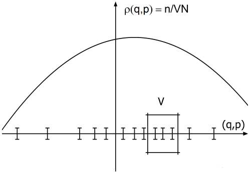

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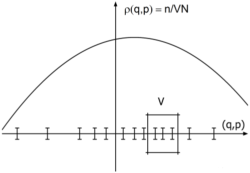

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Continuity equation

- $\rho(t, q(t), p(t)) = \text{const}$
- $\frac{d}{dt}\rho(t, q(t), p(t)) = 0$
- $\frac{\partial}{\partial t}\rho + \nabla_q \rho \cdot \dot{q} + \nabla_p \rho \cdot \dot{p} = 0$
- Continuity equation: $\frac{\partial}{\partial t}\rho + \nabla_q \rho \cdot p + \nabla_p \rho \cdot \overline{f} = 0$
- Vlasov equation: $\frac{\partial}{\partial t}\rho + \nabla_q \rho \cdot p + \nabla_p \rho \cdot (\rho \star_q f) = 0$

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Comparison micro - macro

- For smooth forces f (globally Lipschitz) many results (Neunzert and Wick (1974), Braun and Hepp (1977), ...)
  Deterministic results: $\rho_0^{emp} \to \rho_0 \quad \Rightarrow \quad \rho_t^{emp} \to \rho_t$
- Physically interesting: Coulomb-case (plasma, galaxy)
  $f(q) = \pm \frac{q}{|q|^3}$
- Hauray, Jabin (2014): $f(q) = \pm \frac{q}{|q|^{3-\delta}}$, cut-off: $N^{-1/6}$ i.e.
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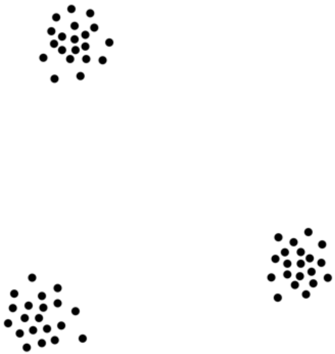
Dynamics of clusters

ϵ^{-1} cluster, ϵN particles each



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- Vlasov equation: $\frac{\partial}{\partial t}\rho + \nabla_q \rho \cdot p + \nabla_p \rho \cdot (\rho \star_q f) = 0$
- Remember: $\dot{Q} = P \qquad \dot{P} = F(Q)$
- Compare with $\dot{\overline{Q}} = \overline{P} \qquad \dot{\overline{P}} = \overline{F}(Q)$
- $(\overline{F})_j = \rho \star_q f(t, q_j)$
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  Precise formulation will follow

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- Vlasov equation: $\frac{\partial}{\partial t}\rho + \nabla_q \rho \cdot p + \nabla_p \rho \cdot (\rho \star_q f) = 0$
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Our results

- Niklas Boers, P.P. (2015):
  $f(q) = \pm \frac{q}{|q|^{3-\delta}}$, cut-off: $N^{-1/3}$ (= distance to nearest neighbour)
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- $(q_j(0), p_j(0))$ independent and identically distributed
- Sample space $\Omega = \mathbb{R}^{6N}$, Probability measure $\mathbb{P}$ with density $\prod_{j=1}^{N} \rho(0, q_j, p_j)$
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- For any t: $(\overline{q}_j(t), \overline{p}_j(t))$ independent and identically distributed (Probability density $\prod_{j=1}^{N} \rho(t, q, p)$)!
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Sub-Coulomb case

- Theorem: for any t holds:
  $\lim_{N\to\infty} \mathbb{P}\left(\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty < N^{-1/3}\right) = 1$
  $\|\cdot\|_\infty$ maximum norm on $\mathbb{R}^{6N}$.
- $(\mathbb{P}(\dots) \geq 1 - C_\gamma N^{-\gamma}$ for any $\gamma > 0)$
- Define: $J(t) = min\left\{N^{1/3}\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty, 1\right\}$
- Lemma: $\frac{d}{dt}\mathbb{E}(J_t) \leq C\left(\mathbb{E}(J_t) + o_N(1)\right)$
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Ingredients of proof

$$J(t) = min\left\{N^{1/3}\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty, 1\right\}$$

- Expectation value easier to control than $\mathbb{P}$: $\mathbb{P}(A) = \mathbb{E}(\chi_A)$
- J_t has reached its maximum when $\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty = N^{-1/3}$.
- We only need to consider the case $\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty < N^{-1/3}$ (boundary condition)

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- Use law of large numbers argument
- $\left\|F(Q) - \overline{F}(\overline{Q})\right\|_\infty \leq \left\|F(Q) - F(\overline{Q})\right\|_\infty + \left\|F(\overline{Q}) - \overline{F}(\overline{Q})\right\|_\infty$
- "Boundary condition" $\left\|(Q,P) - (\overline{Q},\overline{P})\right\|_\infty < N^{-1/3}$, cutoff at $N^{1/3}$
- $\Rightarrow f'(q_j - q_k)$ and $f'(\overline{q}_j - \overline{q}_k)$ of same order
- Taylor: $\left|f(q_j - q_k) - f(\overline{q}_j - \overline{q}_k)\right| \leq Cf'(\overline{q}_j - \overline{q}_k)\|Q - \overline{Q}\|$
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$J(t) = min\left\{N^{1/3}\left\|(Q,P)-(\overline{Q},\overline{P})\right\|_\infty, 1\right\}$

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- Taylor: $\left|f(q_j-q_k)-f(\overline{q}_j-\overline{q}_k)\right| \leq Cf'(\overline{q}_j-\overline{q}_k)\|Q-\overline{Q}\|$
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- Derivation of Vlasov-Maxwell
- Derivation of other equations, e.g. Keller-Segel equation
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  Problem: Coulomb-interaction, first order
  Interesting effective equation, phase transition

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