# Dynamics and Thermodynamics of Stellar Black-Hole Nuclei

Jihad Rachid Touma

Department of Physics
American University of Beirut

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# What's in a Title?

- Dynamics of (Stellar)-Disks around Massive Central Bodies [with Mher Kazandjian (U. Leiden), S. Sridhar(RRI, India)].
- Maximum Entropy Equilibria of (Stellar)-Disks around Massive Central Bodies [with Scott Tremaine (IAS, Princeton)].
- Non-equlibrium Thermodynamics of Stellar Clusters around Massive Central Bodies [with S. Sridhar (RRI, India)].

# Astrophysical Motivation

- Supermassive Black Holes in Centers of Galaxies:$10^6 - 10^{10}$ $M_\odot$
- Nuclear Stellar Clusters: History of Mergers, Black Holes Included
- Close by: Puzzling Galactic Center, Double Nucleus of M31

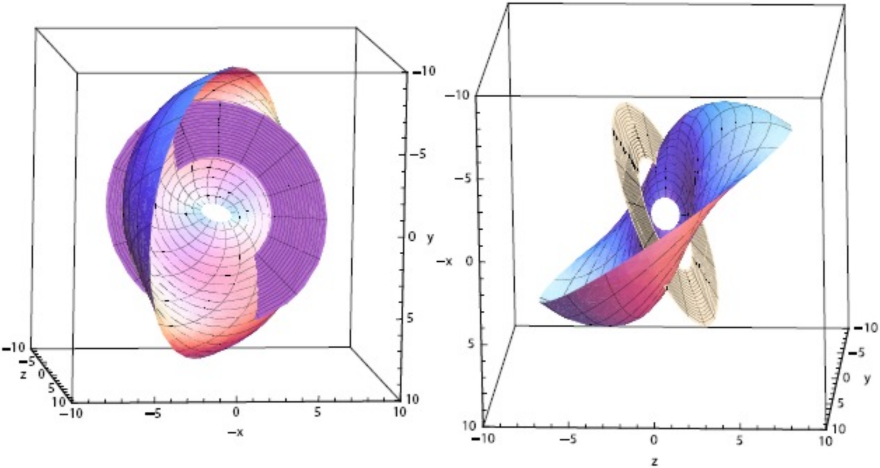
# The Galaxy's Youthful Nucleus: A Paradox

- SMBH: $M_\bullet \sim 4.2 \times 10^6 \mathrm{M}_\odot$ ($r_{sphere} \simeq 2$ pc) [Yelda et. al 2011]

- Kinematically Hot 'Disk(s)' of Young WR, O and B stars: $3 - 10$Myrs, $M_{stars} \sim 10^4 - 10^5 \mathrm{M}_\odot$, $0.032 \leq r \leq 0.15$ pc [Paumard et. al. 2006, Bartko et. al. 2009, Yelda et. al. 2014]

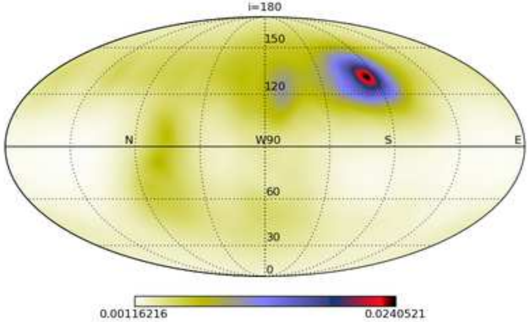
- Mean eccentricity $\sim 0.27$ [Yelda et. al 2014]

# Two CR Disks, or




[Credit: Bartko et. al, 2009]

# A Single Thick Disk?




[Credit: Yelda et. al, 2014]

# The Triple Nucleus of M31: Embarassment of Riches

- SMBH: $M_{\bullet} \sim 1.2 \times 10^{8} \mathrm{M}_{\odot}$ ($r_{sphere} \sim 16$pc) [Bender et. al. 2005]
- Double Nucleus (P1-P2), Old stars: $M \sim 2 \times 10^{7} \mathrm{M}_{\odot}$
- Disk of A Stars (P3): $M_{Disk} =\sim 4200 \mathrm{M}_{\odot}$, $r \leq 1$ pc, $\sim 100 - 200$ Myrs

[Credit: Bender et. al, 2005]

# Pertinent Observations and Associated Puzzles

- Milky Way's Nucleus: Hot Disk(s) of Young Stars: In Situ Formation? If so how do you excite them? If not, how do you transport them in time?
- M31's Triple Nucleus: Origin of the double nucleus? If aligned Keplerian orbits, how do you get them to align? What confines the inner disk? Any links between P1-P2 and P3?

# Double Nucleus of M31

# M31’s Double Nucleus: Tremaine’s Model

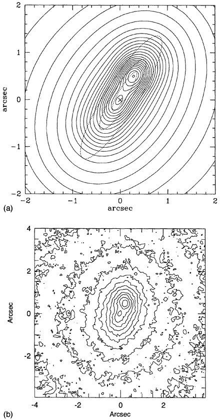



FIG. 2. (a) Contour map of the surface brightness of the best-fit model. The contour interval is $0^{m}\!.1$, and the vertical axis points 70° counterclockwise from north. The origin, which coincides with the black hole at $P2$, is marked by a cross, and the projected locations of the three ringlets in Eq. (2) are shown as dotted lines. (b) Deconvolved $V$-band surface-brightness contours of the nucleus of M31 (Fig. 2 of L93). The orientation and origin are the same as in (a) but the contour interval of $0^{m}\!.25$ is larger and a larger area is shown.

[Credit: Tremaine, 1995 (see Peiris and Tremaine, 2003]

# Stellar Black Hole Nuclei: Sphere of Influence

In the sphere of influence, $r_{sphere} \sim \frac{GM_\bullet}{\sigma^2}$, a hierarchy of time scales: $t_{orbit} \ll t_{secular} \ll t_{rr} \ll t_{relax}$ where:

- $t_{orbit} \sim \left(\frac{r^3}{GM_\bullet}\right)^{\frac{1}{2}}$ , Keplerian orbital time;
- $t_{sec} \sim \frac{M_\bullet}{M_c} t_{orbit}$, precessional time;
- $t_{rr} \sim \frac{M_\bullet}{m} t_{orbit}$; resonant relaxation time;
- $t_{relax} \sim \frac{M_\bullet^2}{Nm^2} t_{orbit}$, two-body relaxation time;

Plus: External Perturber, Dynamical Friction, General Relativistic Corrections.

# Sphere of Influence: Stellar Dynamical Processes

- Black-Hole Dominated, Nearly-Keplerian Motion: Orbit averaged into (Gaussian) Wires, with Constant Keplerian Energy [Sridhar and Touma (1999)]
- Resonant Relaxation of Gaussian Wires Dominates Two Body Relaxation [Rauch and Tremaine (1996)]
- Secular Instabilities of Disks and Spheres [Touma (2002, Tremaine (2005), Polyachenko et al. (2007)]
- Kozai-Lidov instability, driven by massive distant perturbers, sculpting eccentricity inclination distributions [e.g. Blaes et. al. (2003), Lockmann et. al. (2008), Chang(2008)]

# Progress Report

- Counter-Rotating Nearly-Keplerian stellar disks are unstable: They evolve into lopsided uniformly precessing configurations [Touma (MNRAS, 2002), Sridhar and Saini (MNRAS, 2009), Touma and Sridhar (MNRAS, 2012), Kazandjian and Touma (MNRAS, 2013)]

- Microcanonical Thermal equilibria of narrow, ring-like, disks are, more often than not, lopsided [Touma and Tremaine (J. Phys. A, 2014)];

- First-Principles theory of "Resonant Relaxation" lays bare the kinetics of collisional relaxation onto thermal equilibria [Sridhar and Touma (MNRAS, 2016)]

# Self-Consistent, Collisionless Dynamics

Evolution governed by CBE-Poisson system of equations:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial \mathbf{f}}{\partial \mathbf{r}} - \nabla\phi \cdot \frac{\partial \mathbf{f}}{\partial \mathbf{v}} = \mathbf{0},$$

where: $\phi(\mathbf{r}, \mathbf{t}) = \phi_{\mathbf{self}}(\mathbf{r}, \mathbf{t}) + \phi_{\mathbf{ext}}(\mathbf{r}, \mathbf{t})$,

$$\phi_{self}(\mathbf{r}, \mathbf{t}) = -\mathbf{G} \int \mathbf{d^3 r' d^3 v'} \frac{\mathbf{f}(\mathbf{r}', \mathbf{v}', \mathbf{t})}{|\mathbf{r} - \mathbf{r}'|}$$

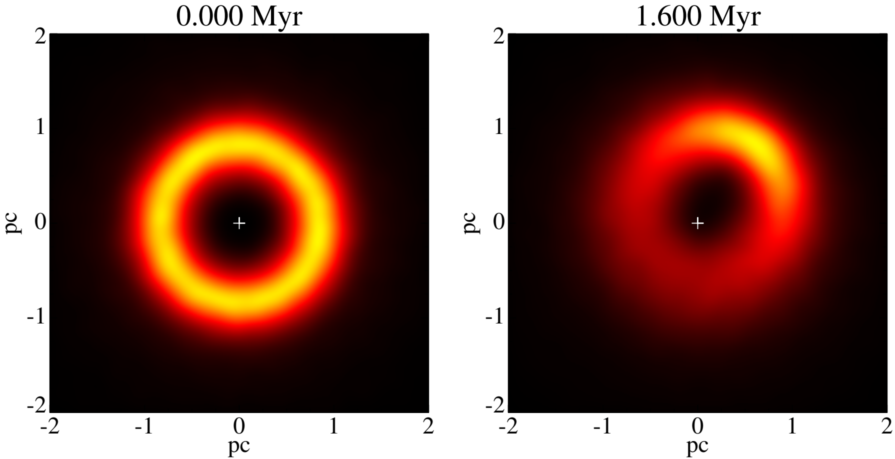
and $\phi_{ext}(\mathbf{r}, \mathbf{t}) = \frac{-\mathbf{GM_\bullet}}{\mathbf{r}} + \phi_{\mathbf{c}}(\mathbf{r}, \mathbf{t})$. Note:

- Black-Hole Dominated, Nearly-Keplerian Motion: Orbits averaged into (Gaussian) Rings
- Consequence of Averaging: $L = \sqrt{GM_\bullet a}$ conserved.

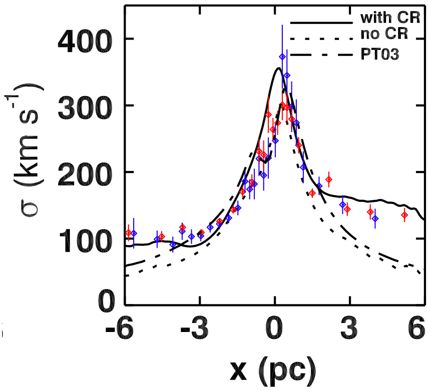
# Numerical Clusters

- Black Hole, $10^8 M_\odot$, Dominating Disk with $10^7 M_\odot$, perturbed by Counter-Rotating Disk with $10^6 M_\odot$;
- Disk: Kuzmin Disk (ring) Radial Scale of 1pc, $\sigma_v \simeq 200$km/s;
- $5 \times 10^5$ Particles, Softening Length: $10^{-3}$pc Particle-Particle, and $10^{-5}$pc for Particle-SMBH interactions;
- Parallel run with Tree Code (Gadget's Parallel Version), Errors: $10^{-4}$ in Energy, and $10^{-5}$ in Angular Momentum over 1 Myr, $(10 T_{prec})$.

# Before and After



# M31's Nucleus in the Looking Glass: Modeling P1 and P2



# Secular (Orbit Averaged) Dynamics

- Counter-Rotating Disks of Stars around SMBH: $N \gg 1$, $M_{\rm disk} \ll M_*$;
- Black-Hole Dominated, Nearly-Keplerian Motion: Separation of Scales $\rightarrow$ Orbits averaged into (Gaussian) Wires;
- Consequence of Averaging: $L = \sqrt{GM_\bullet a}$ conserved; $N$ Gaussian Wires of equal mass $m$, and semi-major axis $a$
- Sense of Rotation $s$: $+1$ for prograde and $-1$ for retrograde
- Coordinates: $e$, $\varpi$, or $\mathbf{e} \equiv (k, h) \equiv e(\cos\varpi, \sin\varpi)$

# 2-Wire Potential

- Orbit Averaged Potential:

$$\Phi(\mathbf{e}, \mathbf{e}') = -Gm^2 \langle |\mathbf{r} - \mathbf{r}'|^{-1} \rangle \equiv (Gm^2/a)\phi(\mathbf{e}, \mathbf{e}')$$

- Equal $a$ and up to $\mathrm{O}(e^2, e^2 \log e)$:

$$\phi(\mathbf{e}, \mathbf{e}') \equiv \phi_L(\mathbf{e}, \mathbf{e}') \equiv -4 \log 2/\pi + (2\pi)^{-1} \log(\mathbf{e} - \mathbf{e}')^2$$

- Eccentricities can grow quite large: High eccnetricity expansion, Interpolation over Grid, but results qualitatively similar, hence stick to Logaritmic interactions

# Continuum Limit

- Distribution functions:

$$n(\mathbf{e}) \equiv n_+(\mathbf{e}) + n_-(\mathbf{e}) \quad \text{or} \quad f(\mathbf{E}) = f_+(\mathbf{E}) + f_-(\mathbf{E});$$

- Transform:

$$n_\pm(\mathbf{e})d\mathbf{e} = f_\pm(\mathbf{E})d\mathbf{E},$$

with

$$d\mathbf{E} = dKdH = \tfrac{1}{2}dk\,dh/\sqrt{1-e^2} = \tfrac{1}{2}d\mathbf{e}/\sqrt{1-e^2},$$

hence

$$n_\pm(\mathbf{e}) = \tfrac{1}{2}f_\pm(\mathbf{E})/\sqrt{1-e^2}.$$

# Wire in Mean Field

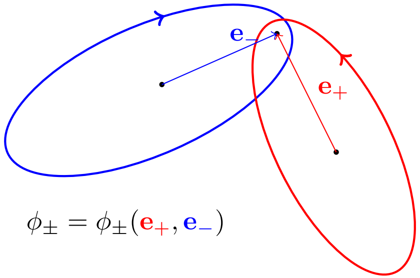
Mean Field Potential:

$$\Gamma(\mathbf{e}) = \frac{1}{N}\int n(\mathbf{e}')\phi(\mathbf{e},\mathbf{e}')d\mathbf{e}' = \frac{1}{N}\int f(\mathbf{E}')\phi(\mathbf{e},\mathbf{e}')d\mathbf{E}'.$$

Particle Equation of Motion:

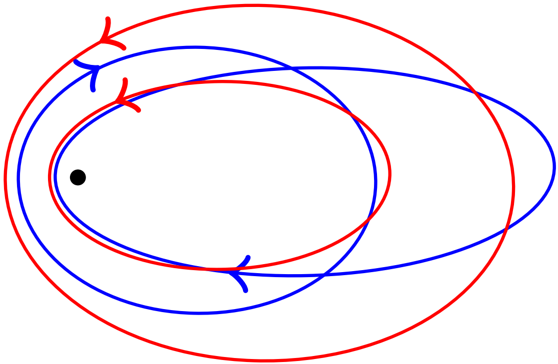
$$\frac{dK}{d\tau} = s\frac{\partial\Gamma}{\partial H},\ \frac{dH}{d\tau} = -s\frac{\partial\Gamma}{\partial K} \ \text{ with } \ \tau = \frac{M_{\rm disk}}{2M_*}\left(\frac{GM_*}{a^3}\right)^{1/2} t.$$

# Coupled Gauss Wires




$$\phi_\pm = \phi_\pm(\mathbf{e}_+, \mathbf{e}_-)$$

# Aligned Counter-Rotating Gauss Wires



# "Maximize" Entropy at fixed N, L, and U

- Gibbs' Microcanonical Ensemble: Ensemble of Particles sharing same $N$, $L$ and $U$;

- Entropy, Measure of Multiplicity:

$$S = -\int [f_+(\mathbf{E}) \log f_+(\mathbf{E}) + f_-(\mathbf{E}) \log f_-(\mathbf{E})]\, d\mathbf{E}$$

- Maximize $S$ at constant:

$$N \equiv \int n(\mathbf{e})\, d\mathbf{e} = \int f(\mathbf{E})\, d\mathbf{E}$$

$$L = m\sqrt{GM_\star a} \int [n_+(\mathbf{e}) - n_-(\mathbf{e})] \sqrt{1 - e^2}$$

$$U = \tfrac{1}{2}(Gm^2/a) \int n(\mathbf{e}) n(\mathbf{e}') \phi(\mathbf{e}, \mathbf{e}')\, d\mathbf{e}\, d\mathbf{e}'$$

# Thermal Equilibria

- Distribution of prograde and retrograde rings:

$$f(\mathbf{E}) = f_+(\mathbf{E}) + f_-(\mathbf{E});$$

- Entropy:

$$S = -\int [f_+(\mathbf{E}) \log f_+(\mathbf{E}) + f_-(\mathbf{E}) \log f_-(\mathbf{E})] \, d\mathbf{E}$$

- Maximize $S$ at constant $N, L, U$.

# Themal Equilibria: Integral Form

- Distribution Function of Thermal Equilibria:

$$f^0_\pm(\mathbf{E}) = \frac{N\alpha}{\beta}\exp[-\beta\Gamma^0(\mathbf{e}) + s\gamma(1-E^2)]$$

- Mean Field of Thermal Equilibrium:

$$\Psi(\mathbf{e}) = 2\alpha\int d\mathbf{E}'\phi(\mathbf{e},\mathbf{e}')\exp[-\Psi(\mathbf{e}')]\cosh\gamma(1-{E'}^2),$$

with $\quad \mathbf{E} = \sqrt{1-\sqrt{1-e^2}}\,\mathbf{e}/e$.

# General Book Keeping

Work with dimensionless conserved quantities:

- Dimensionless Angular Momentum:

$$\ell \equiv \frac{L}{Nm\sqrt{GM_\star a}} = \frac{\int d\mathbf{E}\,(1-E^2)\exp[-\Psi(\mathbf{e})]\sinh\gamma(1-E^2)}{\int d\mathbf{E}\exp[-\Psi(\mathbf{e})]\cosh\gamma(1-E^2)}$$

- Dimensionless Energy:

$$u \equiv \frac{aU}{G(Nm)^2} = \frac{\int d\mathbf{E}\,d\mathbf{E}'\,W(\mathbf{e})W(\mathbf{e}')\phi(\mathbf{e},\mathbf{e}')}{2\left[\int d\mathbf{E}\exp[-\Psi(\mathbf{e})]\cosh\gamma(1-E^2)\right]^2}$$

with $W(\mathbf{e}) = \exp[-\Psi(\mathbf{e})]\cosh\gamma(1-E^2)$.

# Themal Equilibria: The Program

- Solve for Axisymmetric Thermal Equilibria
- Are they thermally stable? Entropy Maxima? Saddle?
- Are they dynamically stable?
- If thermally unstable, what are the global entropy maxima?
- If dynamically unstable, what are the saturated states?
- How do the global entropy maxima relate to saturated states?

# Axisymmetric Equilibria: Formulation

Working with Logarithmic limit of $\phi(\mathbf{e}, \mathbf{e}')$, Differentiate Potential Equation to get:

$$\nabla^2_{\mathbf{e}} \Psi = \frac{2\alpha}{\sqrt{1-e^2}} \exp[-\Psi(\mathbf{e})] \cosh \gamma \sqrt{1-e^2}.$$

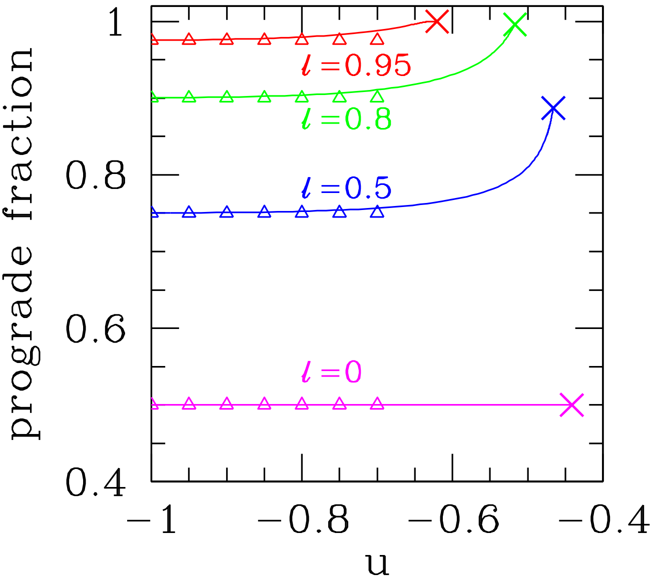
Under axial symmetry

$$\nabla^2_{\mathbf{e}} \Psi = \frac{2\alpha}{\sqrt{1-e^2}} \exp[-\Psi(\mathbf{e})] \cosh \gamma \sqrt{1-e^2},$$

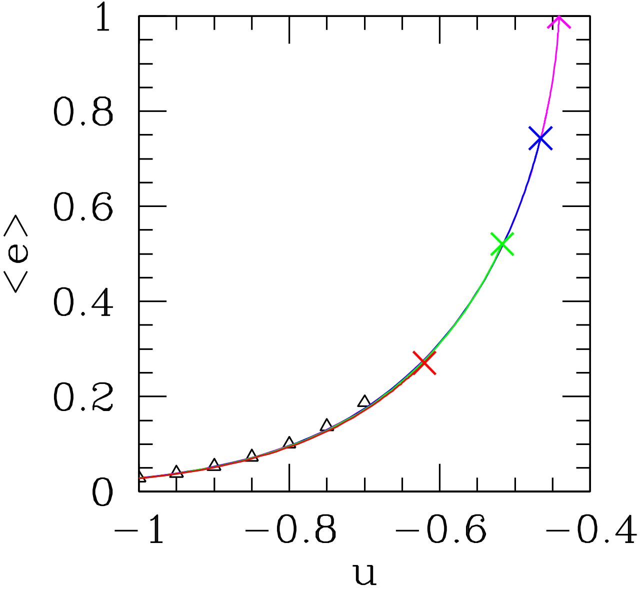
turns into

$$\frac{d^2\Psi}{de^2} + \frac{1}{e}\frac{d\Psi}{de} = \frac{2\alpha}{\sqrt{1-e^2}} \exp[-\Psi(e)] \cosh \gamma \sqrt{1-e^2};$$

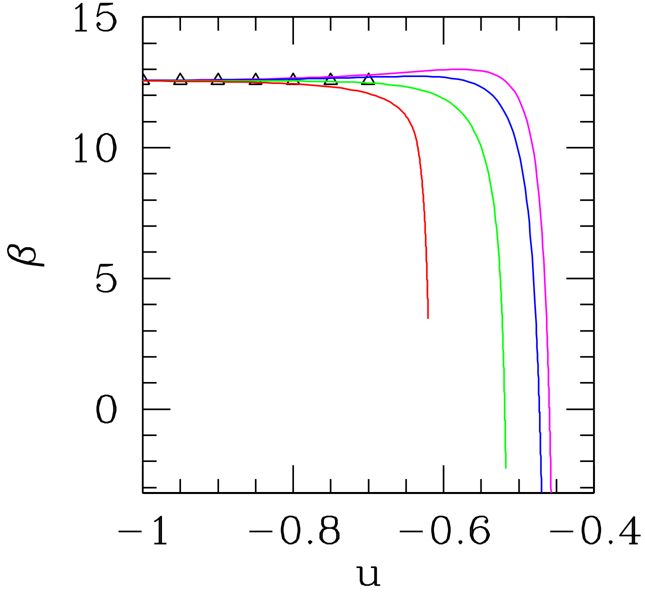
# Axisymmetric Thermal Equilibria: Prograde Fraction



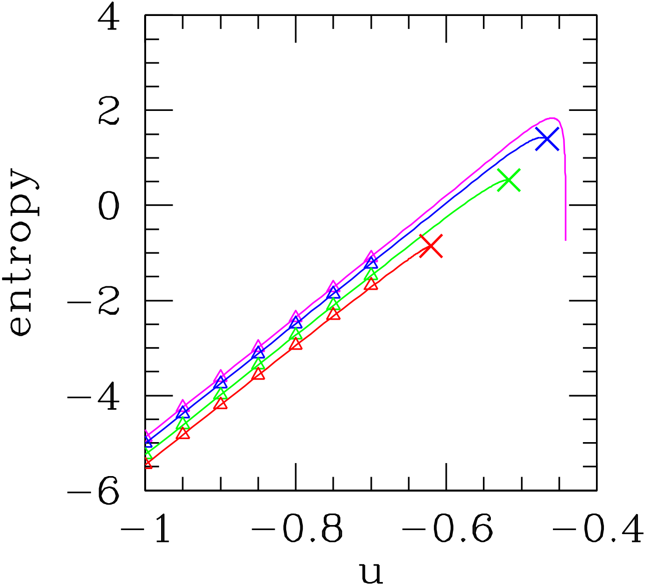
# Axisymmetric Thermal Equilibria: Mean Eccentricity



# Axisymmetric Thermal Equilibria: Inverse Temperature



# Axisymmetric Thermal Equilibria: Entropy

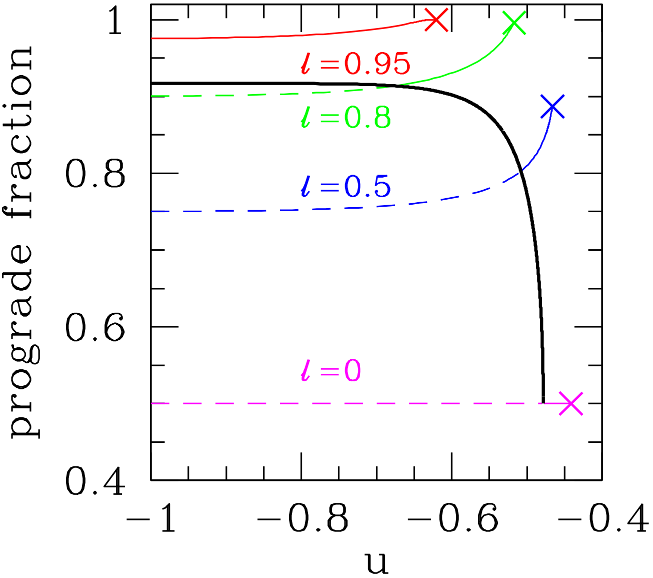


# Thermal Instability

Question: Are Axisymmetric Equilibria Thermally Stable?

- Condition for Non-Axisymmetric Perturbations of Equilibria
- Condition for Thermal Instability: When is Entropy Extremum a Saddle?

# Stability of Axisymmetric Thermal Equilibria



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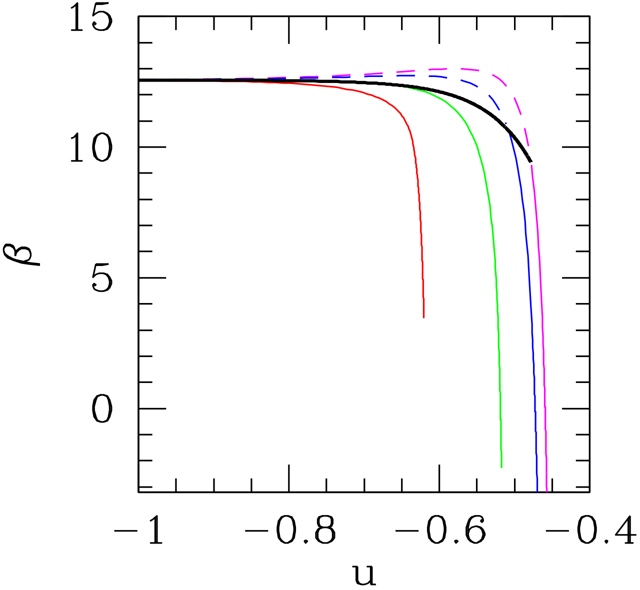

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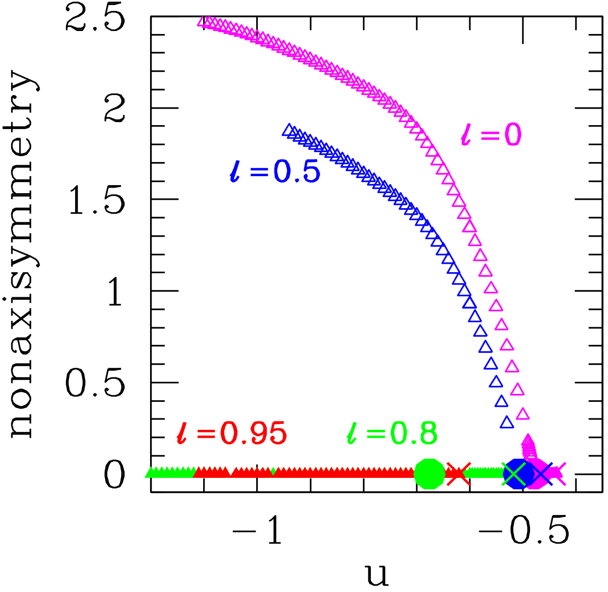
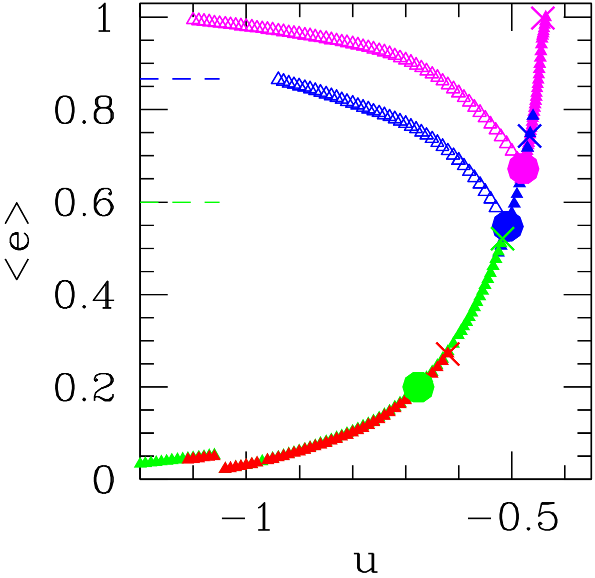
- For $\ell < 0.833$, critical energy below which equilibria are thermally unstable $\rightarrow$ Entropy Maximum is a saddle
- Lopsided Equilibria Are Natural Byproduct of Resonant Relaxation

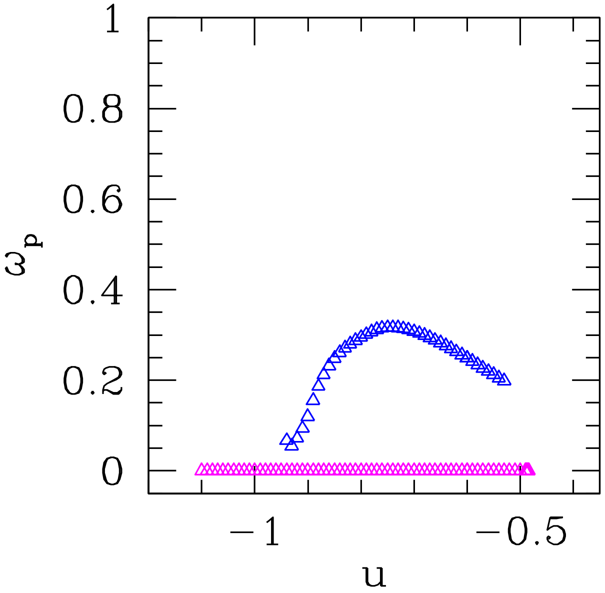
# Global Thermal Equilibria:Non-Axisymmetry



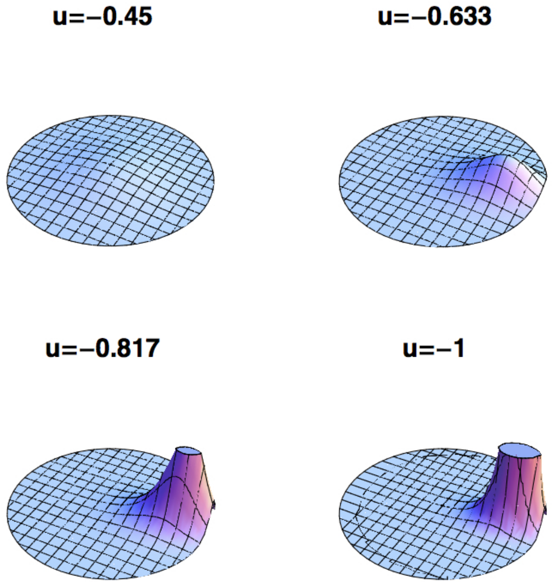
# Global Thermal Equilibria: Mean Eccentricity



# Global Thermal Equilibria: Angular Velocity



# Global Thermal Equilibria: Lopsided Density



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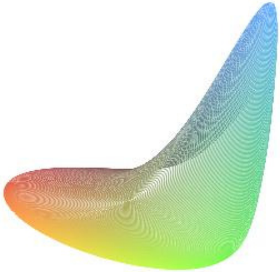

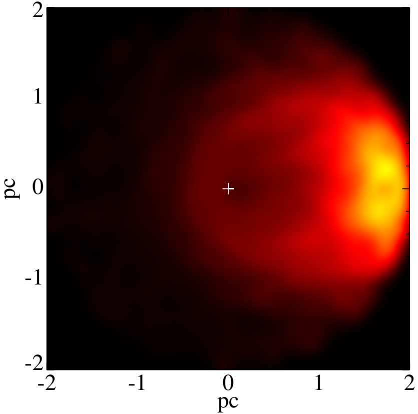

# Dynamical Stability

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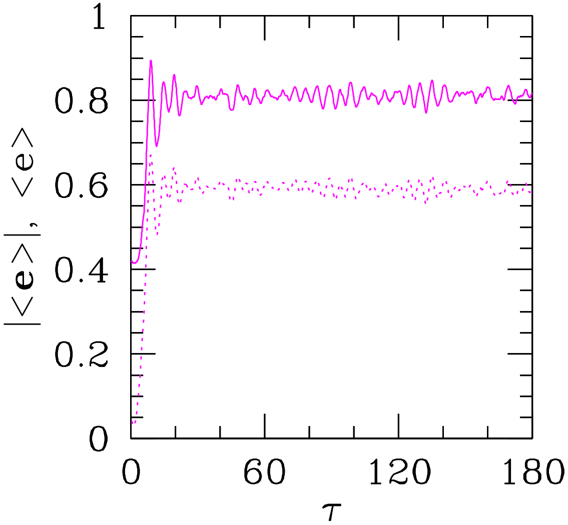
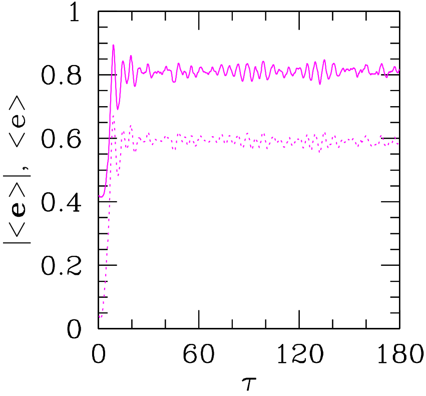
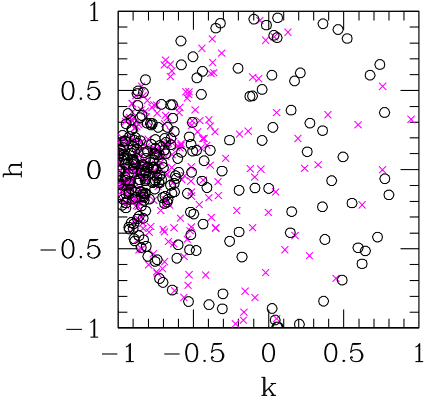
- Sample Equilibrium Distributions and Simulate Their Dynamical Evolution
- Seek the Saturated States of Unstable Configurations
- Confront "Collisionless" Saturated states with "Collisional" Equilibria

# Road to Saturation: $\ell = 0$, $u = -0.55$

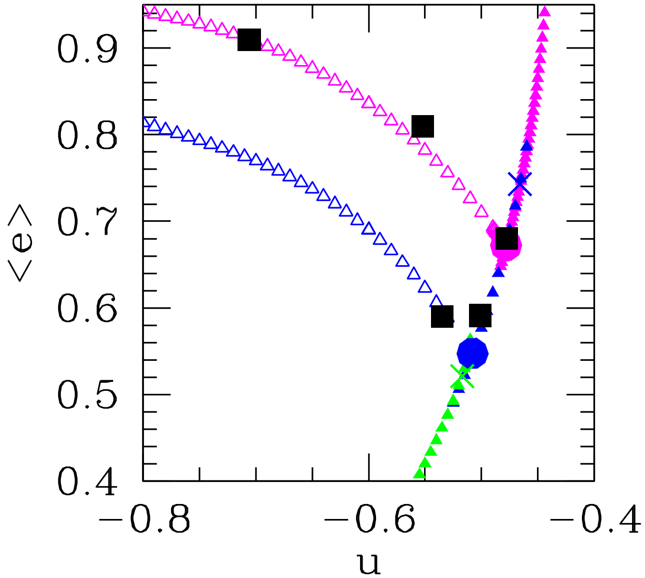


# Phase-Space around Saturation: $\ell = 0$, $u = -0.55$

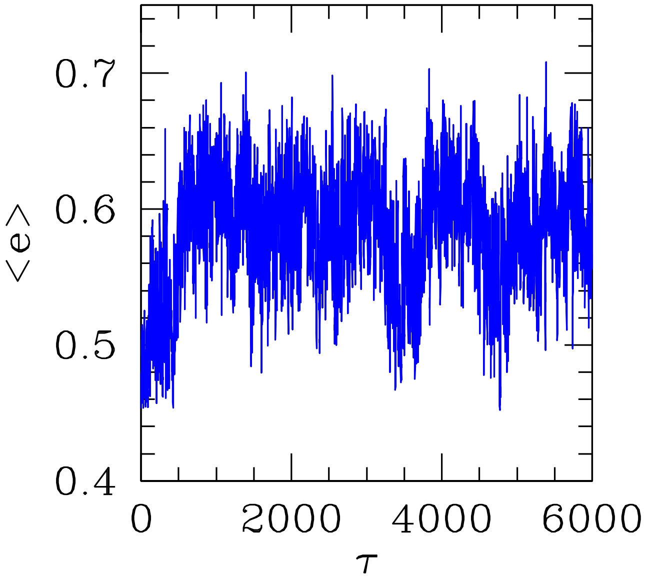


# Mean Eccentricity around Saturation



# Dispersion Around the Mean: $\ell = 0.5$



# Kinetics: Relaxation to Lopsidedness

- Resonant Relaxation Drives Nearly-Keplerian Disks to Lopsided Maximum Entropy Equilibria
- Collisionless Dynamical Instability Drives Nearly-Keplerian Disks to Lopsided Uniformly Precessing Equilibria
- The Full Story Involves the Complementary Action of Both Collisionless and Collisional Relaxation
- A Theory for Both is Lacking, though End States Can be "Securely" Characterized

# Thermal Equilibria: The Report

- Axisymmetric Equilibria are Prone to Lopsided, m=1 Deformations, over a Broad Range of Energy and Angular Momenta.
- Resonant Relaxation Drives Nearly-Keplerian Disks to Lopsided Maximum Entropy Equilibria.
- All Thermally Unstable Disks are Dynamically Unstable.
- Dynamical Instability Drives Nearly-Keplerian Disks to Lopsided Uniformly Precessing Equilibria.
- The Full Story Involves the Complementary Action of Both Collisionless and Collisional Relaxation.

# The Resolution

- Counter-Rotating Nearly-Keplerian stellar disks are unstable: They evolve into lopsided uniformly precessing configurations.
- Microcanonical Thermal equilibria of narrow, ring-like, disks are, more often than not, lopsided.
- Life cycle of a self-gravitating Keplerian cluster: relaxation onto instability, then saturation onto relaxation.