Modeling of cloud microphysics: from simple concepts to sophisticated parameterizations.

Part I: warm-rain microphysics

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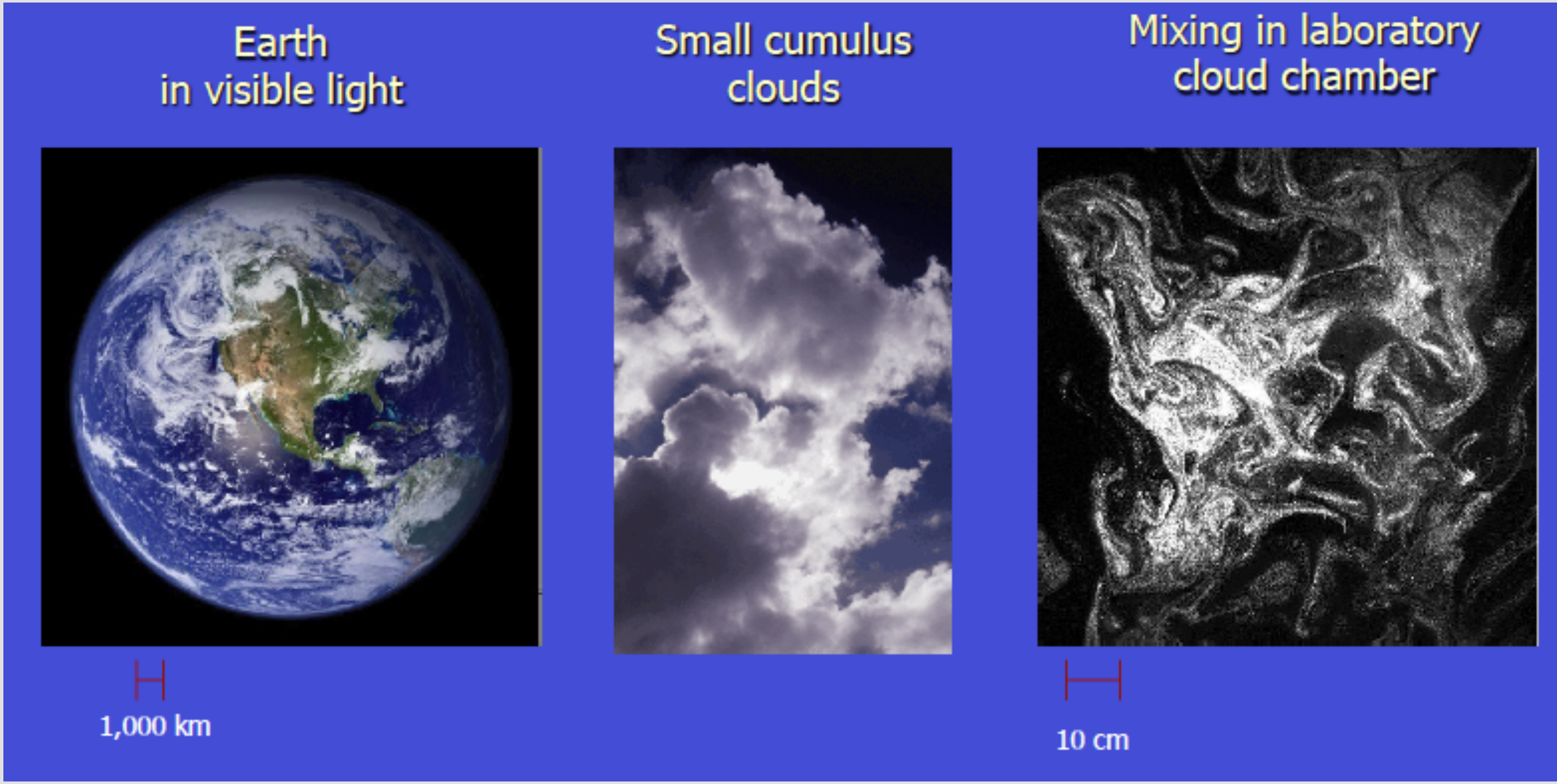
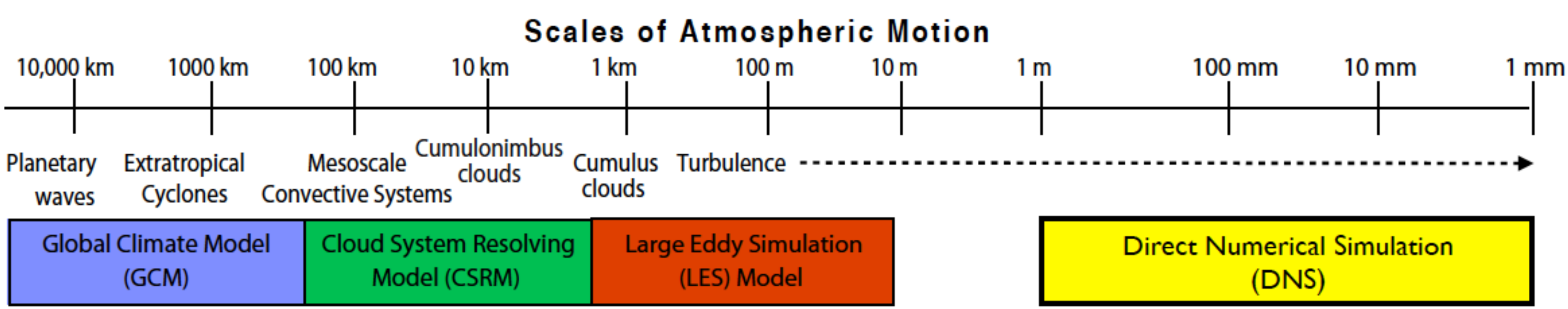
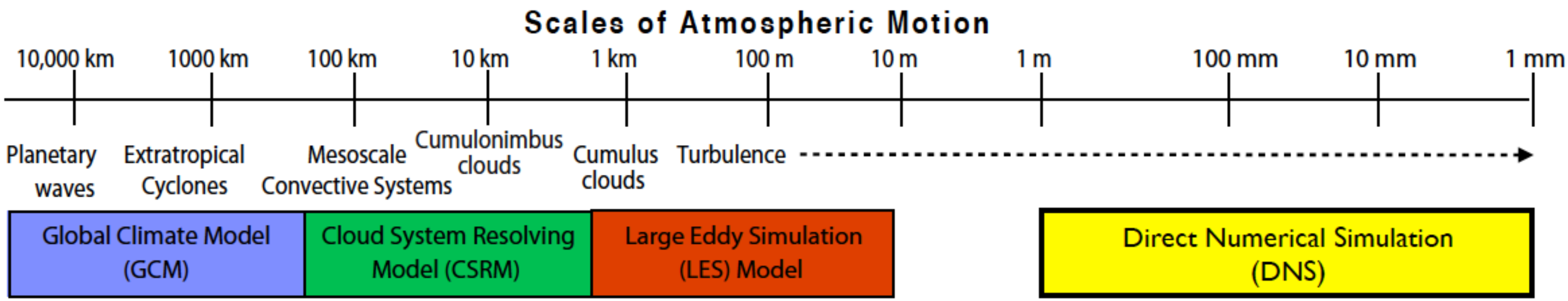
Mesoscale & Microscale Meteorology
MMM
LABORATORY


Earth
in visible light
Small cumulus
clouds
Mixing in laboratory
cloud chamber
1,000 km
10 cm


Scales of Atmospheric Motion
10,000 km
1000 km
100 km
10 km
1 km
100 m
10 m
1 m
100 mm
10 mm
1 mm
Planetary waves
Extratropical Cyclones
Mesoscale Convective Systems
Cumulonimbus clouds
Cumulus clouds
Turbulence
Global Climate Model (GCM)
Cloud System Resolving Model (CSRM)
Large Eddy Simulation (LES) Model
Direct Numerical Simulation (DNS)



parameterization problem:
parameterized microphysics in (under)resolved clouds

parameterization^2 problem:
parameterized microphysics in parameterized clouds

microphysics at its native scale

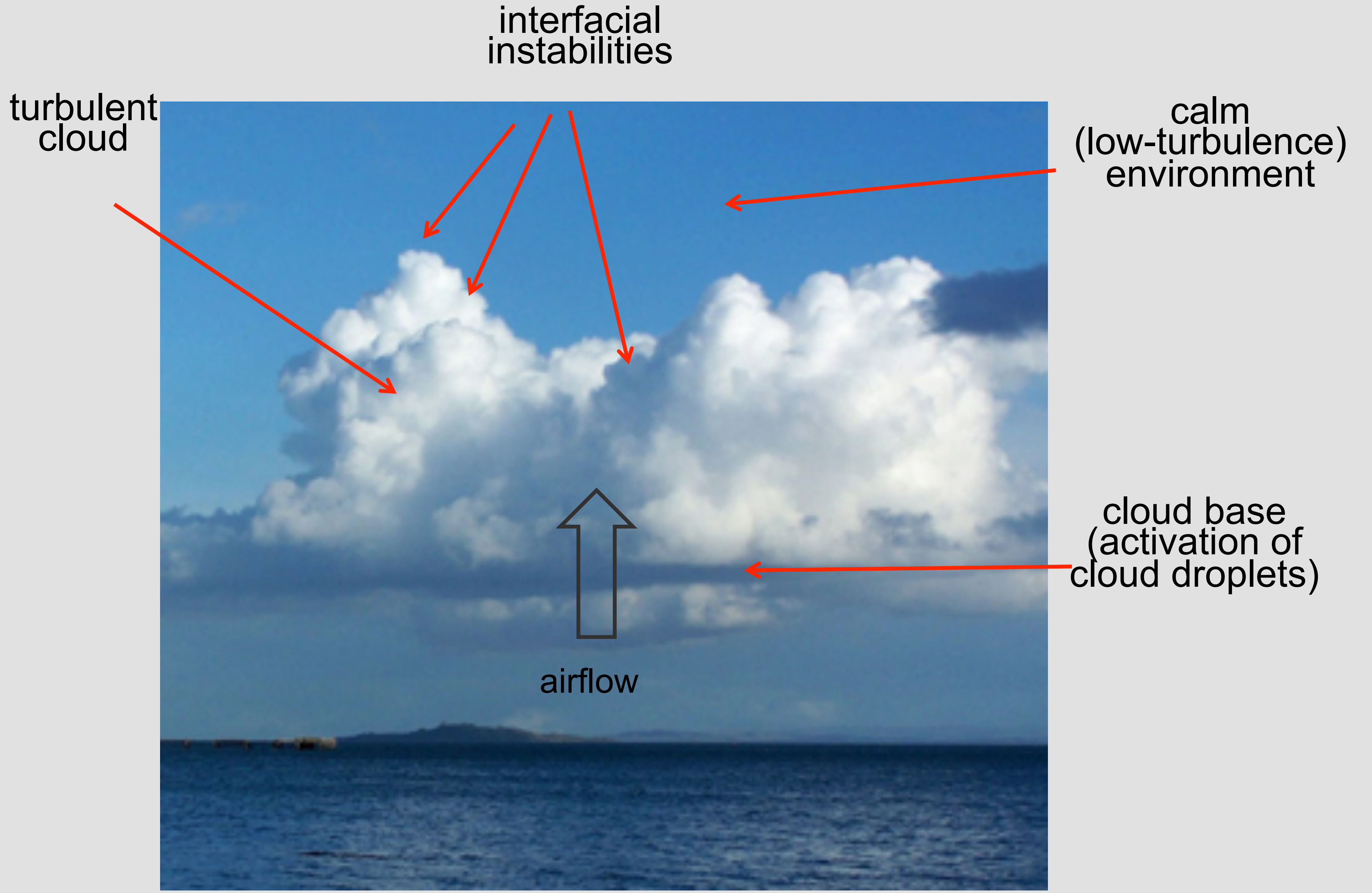
Cloud microphysics across scales

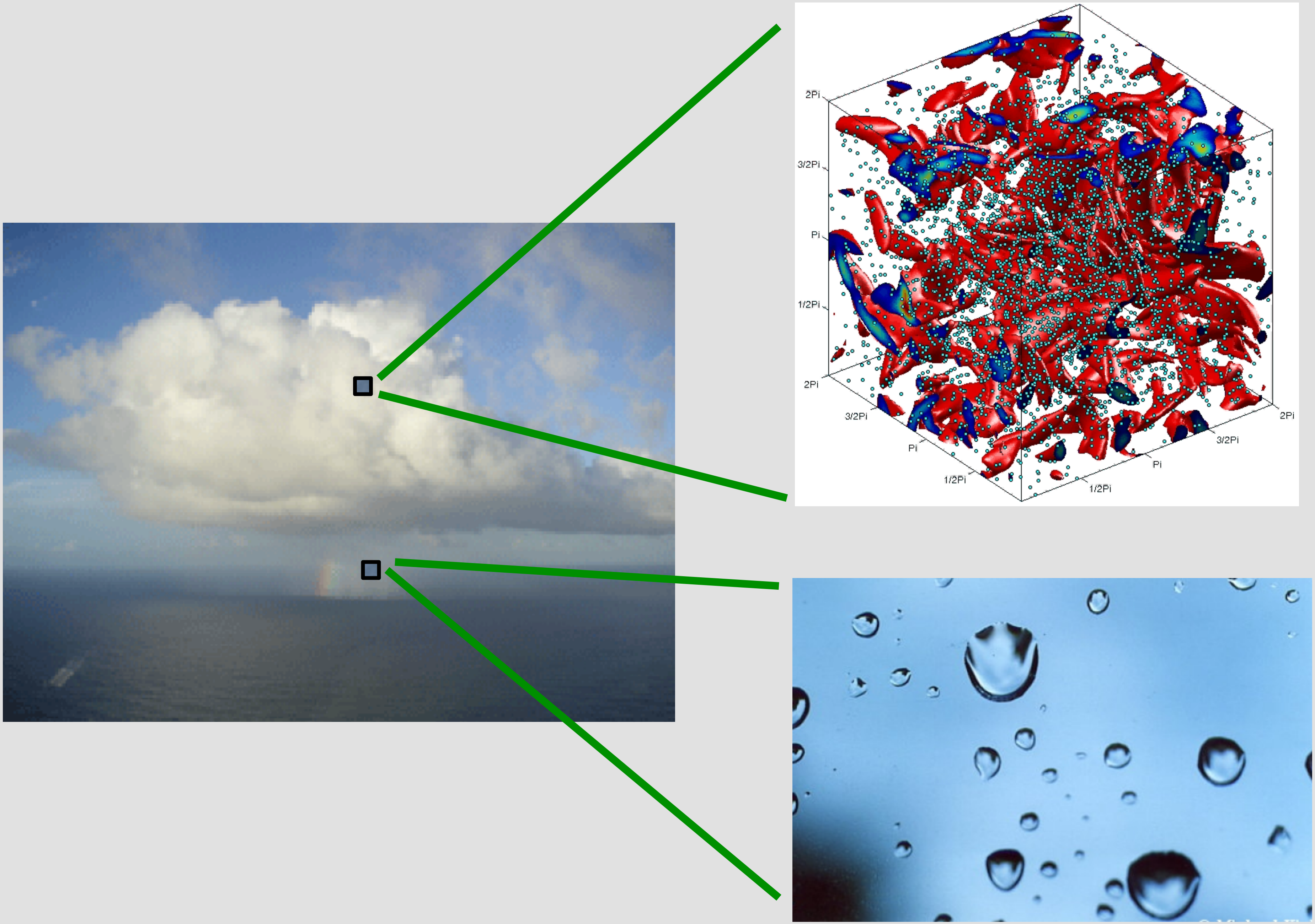
Eulerian versus Lagrangian methodology
(continuous medium versus particle-based)

Warm (no-ice) versus ice-bearing clouds

Understanding of the physics
versus numerical implementation

Precise and complex
versus approximate and easy to apply


interfacial instabilities
turbulent cloud
calm (low-turbulence) environment
cloud base (activation of cloud droplets)
airflow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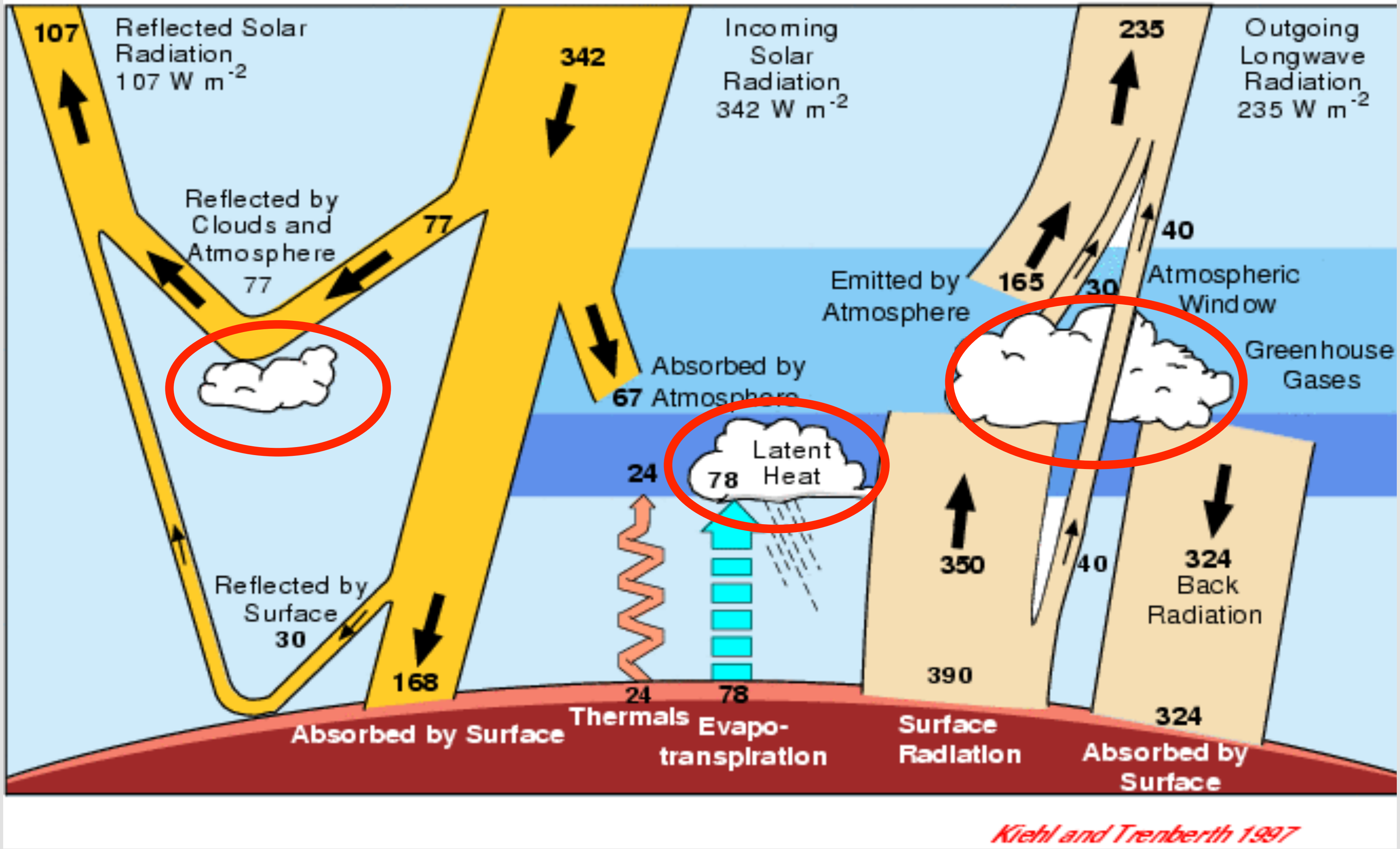


The Earth annual and global mean energy budget

Fundamentals of cloud physics

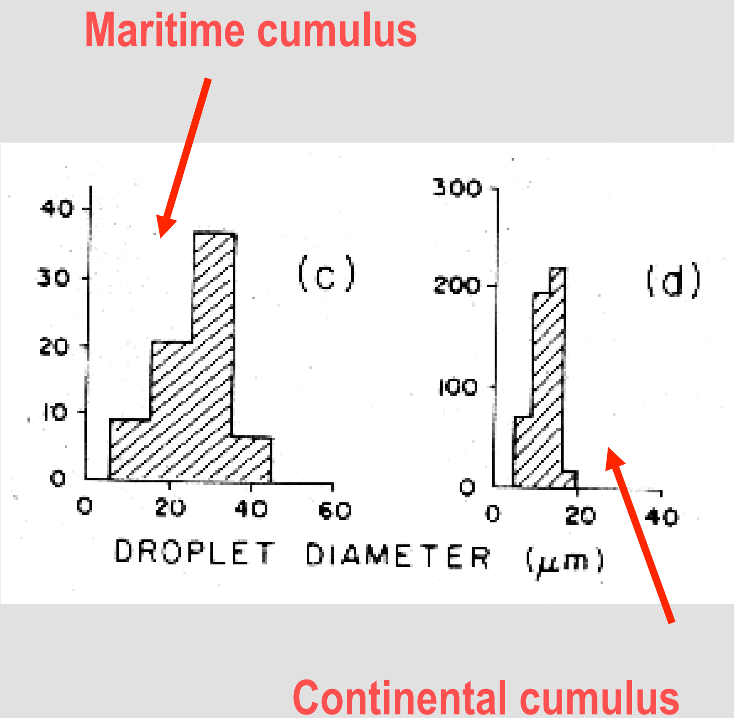
ELEMENTARY CLOUD PHYSICS:

clouds form due to cooling of air (e.g., adiabatic expansion of a parcel of air rising in the atmosphere)

- *condensation:* water vapor ⟶ cloud droplets

heterogeneous nucleation on atmospheric aerosols called Cloud Condensation Nuclei (CCN); typically highly soluble salts (sea salt, sulfates, ammonium salts, nitrates)

typically, only a small percentage of CCN used by clouds (i.e., water clouds form just above saturation)



ELEMENTARY CLOUD PHYSICS, cont.:

- *formation of ice particles*

heterogeneous nucleation on atmospheric aerosols called Ice-forming Nuclei (IN); dominates for temperatures higher than about -40 deg C (233 K); poorly understood; various modes (contact, deposition, condensation-freezing)

IN are typically silicate particles (clays) or other compounds with crystallographic lattice similar to ice, highly insoluble (contact nucleation) or coated with soluble compound (condensation-freezing)

IN are scarce, their number depends strongly on temperature (typically, 1 per liter at -20 deg C, 10 per liter at -25 deg C).

homogeneous freezing is possible once droplet temperature is smaller than about -40 deg C.

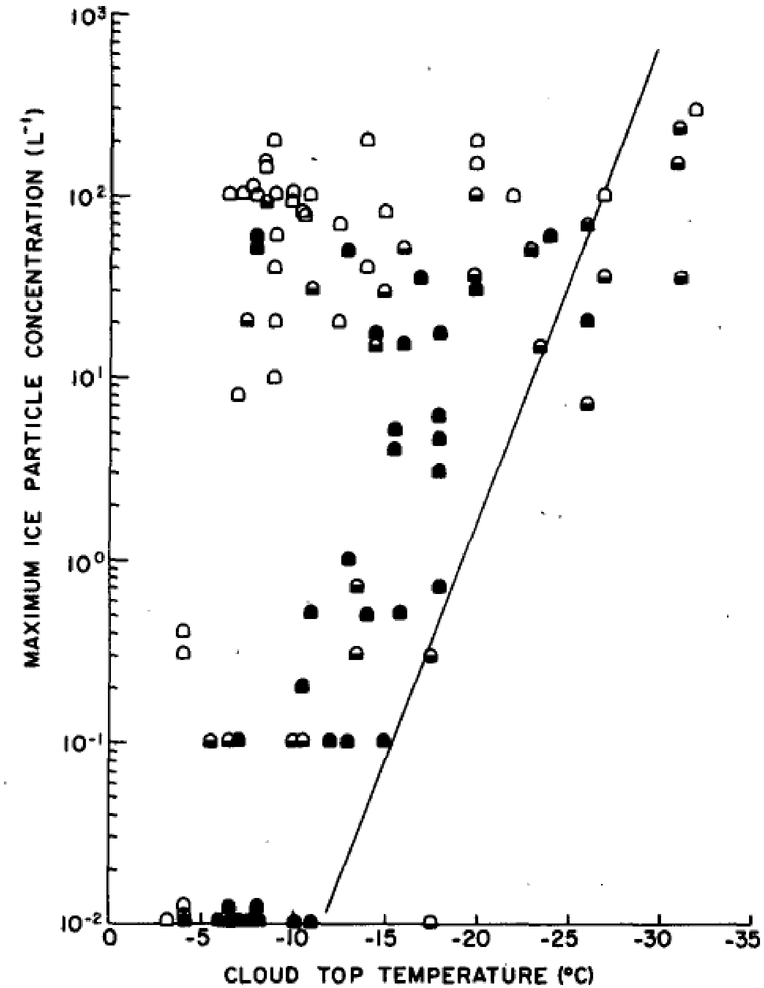


FIG. 2. Measurements of the maximum ice particle concentrations in mature and aging maritime (open humps), continental (closed humps) and transitional (half-open humps) cumuliform clouds. The line represents the concentrations of ice nuclei given by Eq. (1).

From cloud droplets and ice crystals to precipitation:

WARM RAIN:

⟶ gravitational collision and coalescence between cloud droplets

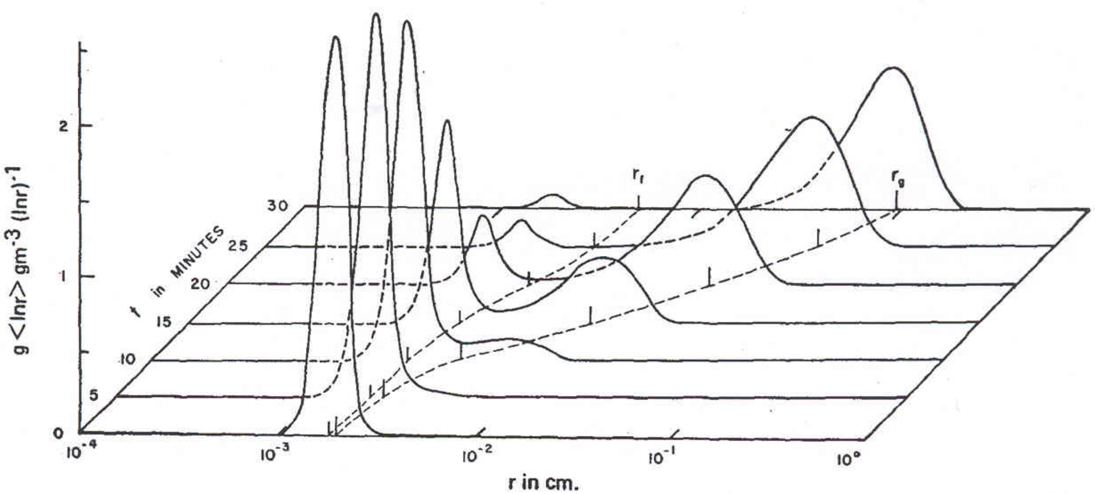


FIG. 5. Time evolution of the initial spectrum for $r_f^0 = 18\ \mu m$, var $x = 0.25$.

Berry and Reinhardt JAS 1974

THE DISTRIBUTION OF RAINDROPS WITH SIZE

By J. S. Marshall and W. McK. Palmer[1]

McGill University, Montreal

(Manuscript received 26 January 1948)

$$N_D = N_0 e^{-\Lambda D}$$

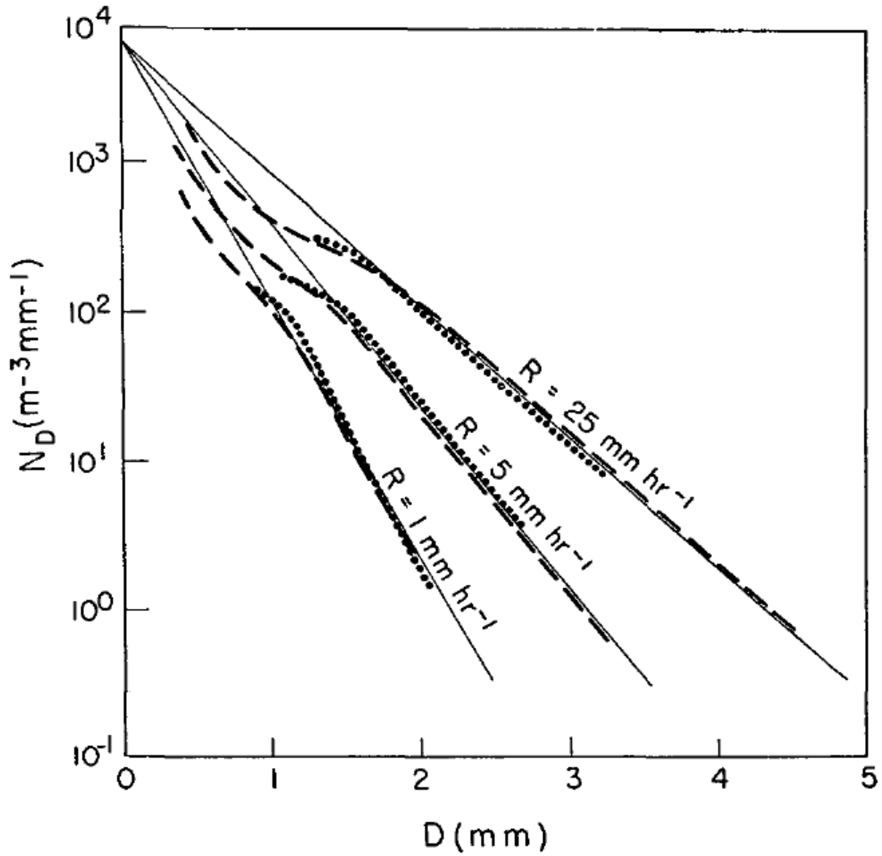


FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

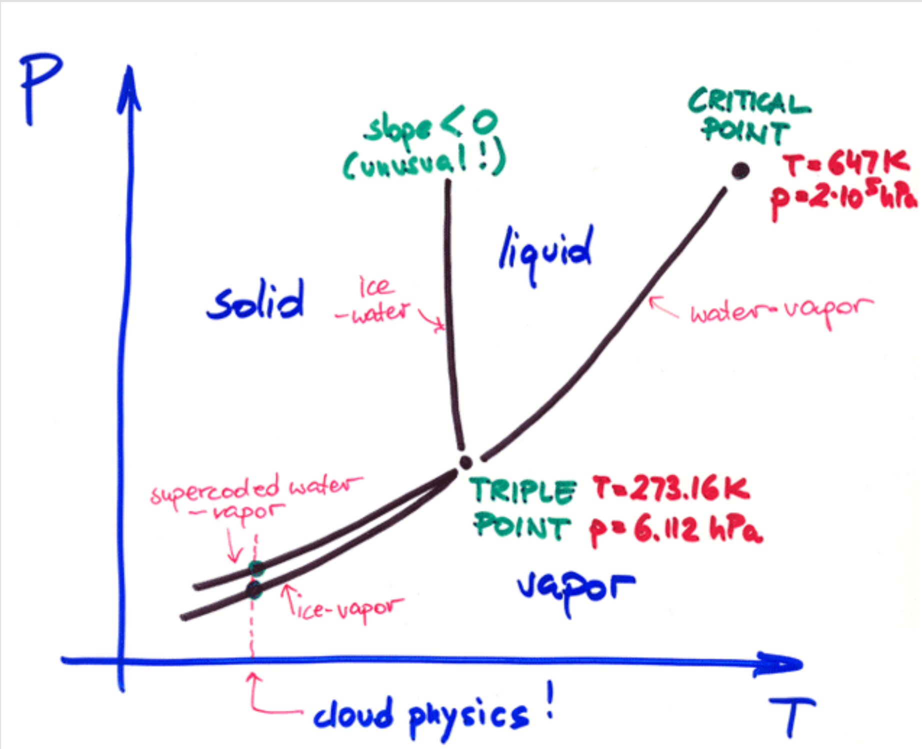
From cloud droplets and ice crystals to precipitation:

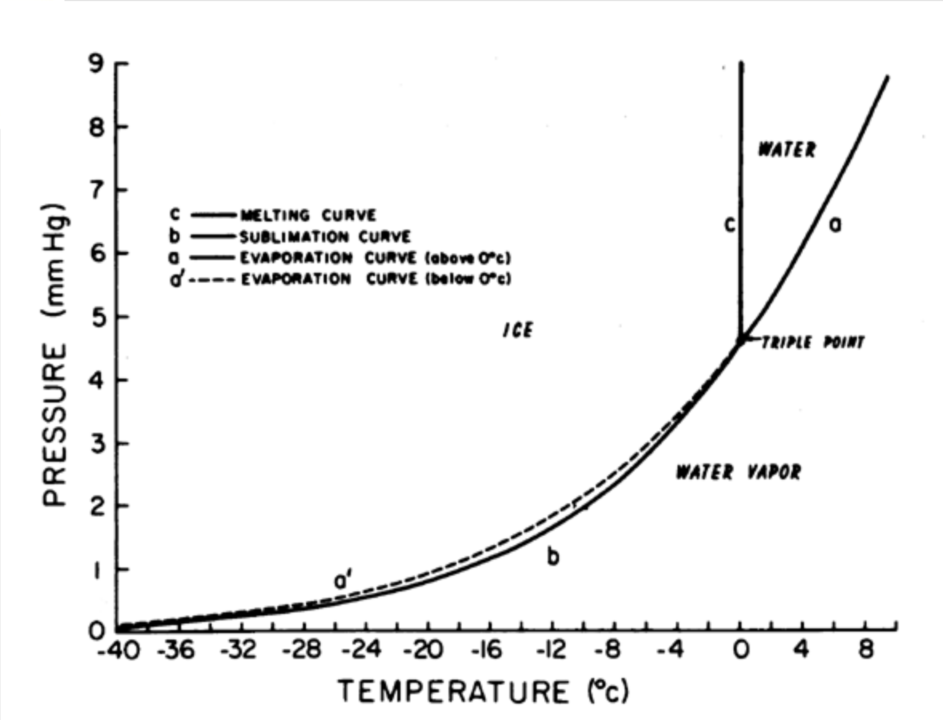
ICE PROCESSES:

$\longrightarrow$ Findeisen-Bergeron process: water vapor pressure at saturation is lower over ice than over water; it follows that once ice crystal is formed from supercooled droplet, it grows rapidly through diffusion of water vapor at the expense of cloud droplets

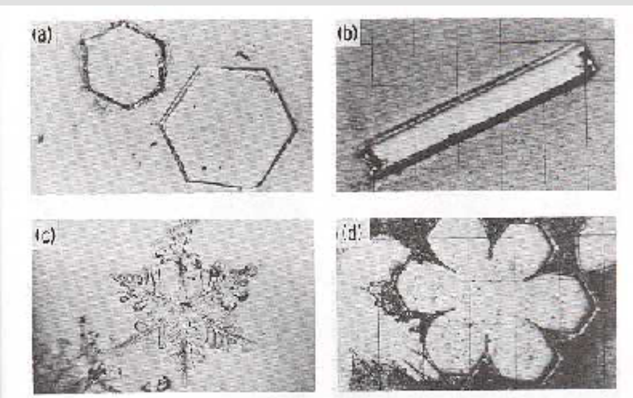
$\longrightarrow$ riming: falling ice crystal collects supercooled droplets that freeze upon contact (graupel, hail, etc).

p-T phase equilibrium diagram for water substance





Pristine ice crystals, grown by diffusion of water vapor (water vapor between ice- and water-saturation)



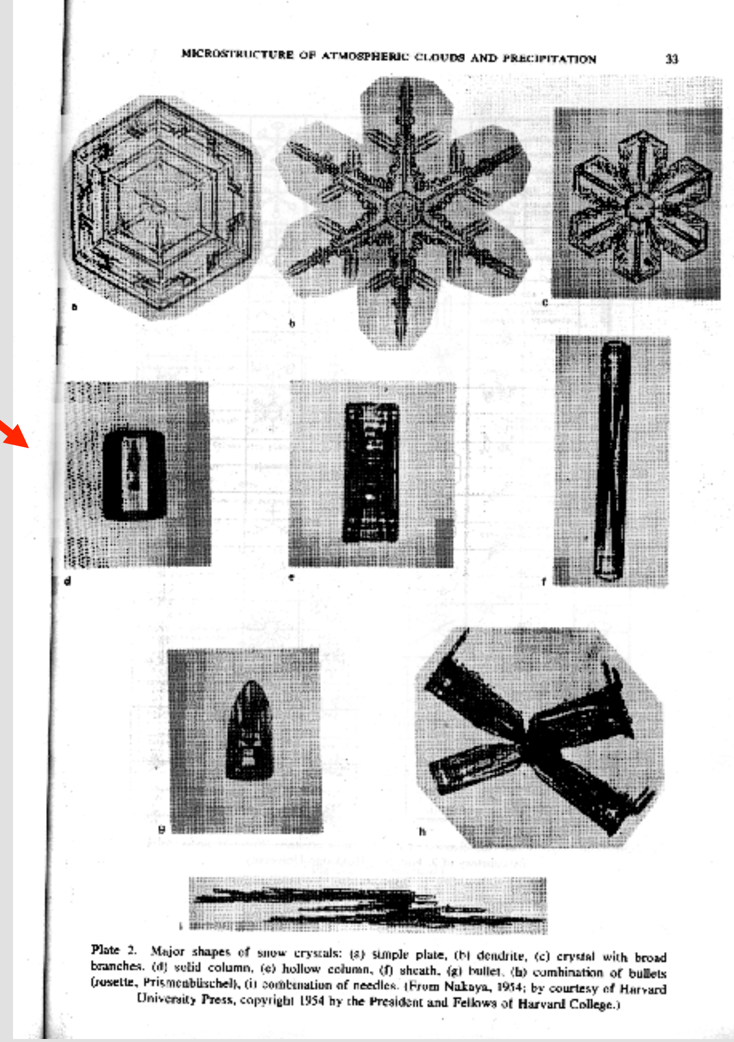
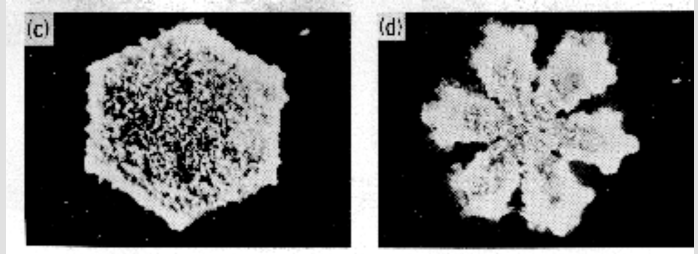
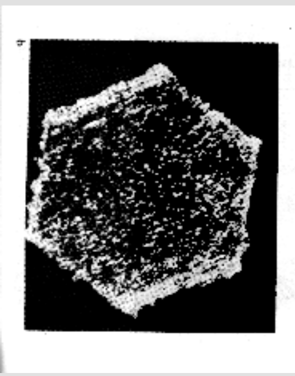

MICROSTRUCTURE OF ATMOSPHERIC CLOUDS AND PRECIPITATION 33

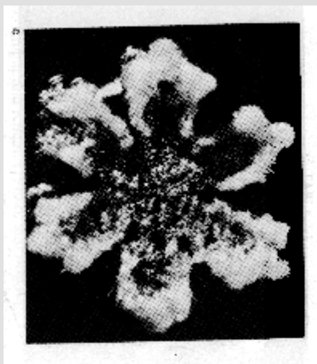
Plate 2. Major shapes of snow crystals: (a) simple plate, (b) dendrite, (c) crystal with broad branches, (d) solid column, (e) hollow column, (f) sheath, (g) bullet, (h) combination of bullets (rosette, Prismenbüschel), (i) combination of needles. (From Nakaya, 1954; by courtesy of Harvard University Press, copyright 1954 by the President and Fellows of Harvard College.)

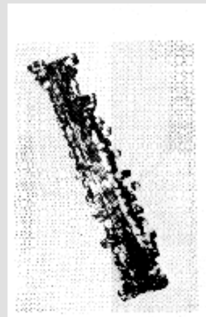
Snowflakes, grown by aggregation

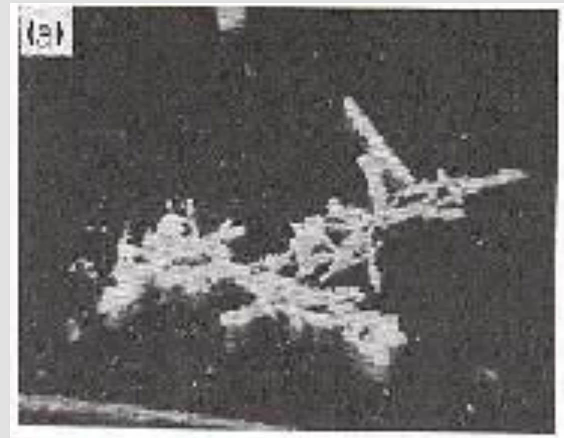
Rimed ice crystals (accretion of supercooled cloud water)



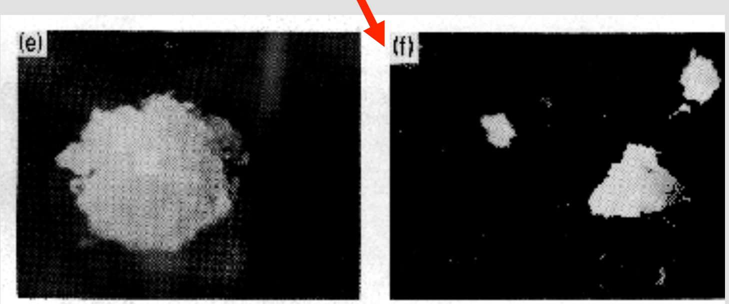




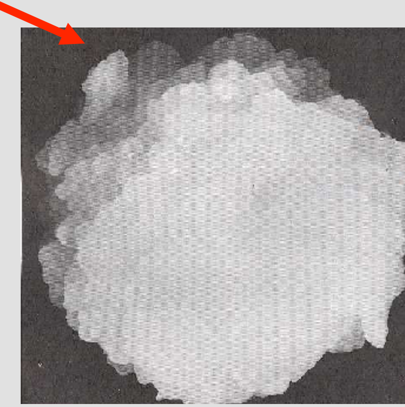




Graupel (heavily rimed ice crystals)



Hail (not to scale)



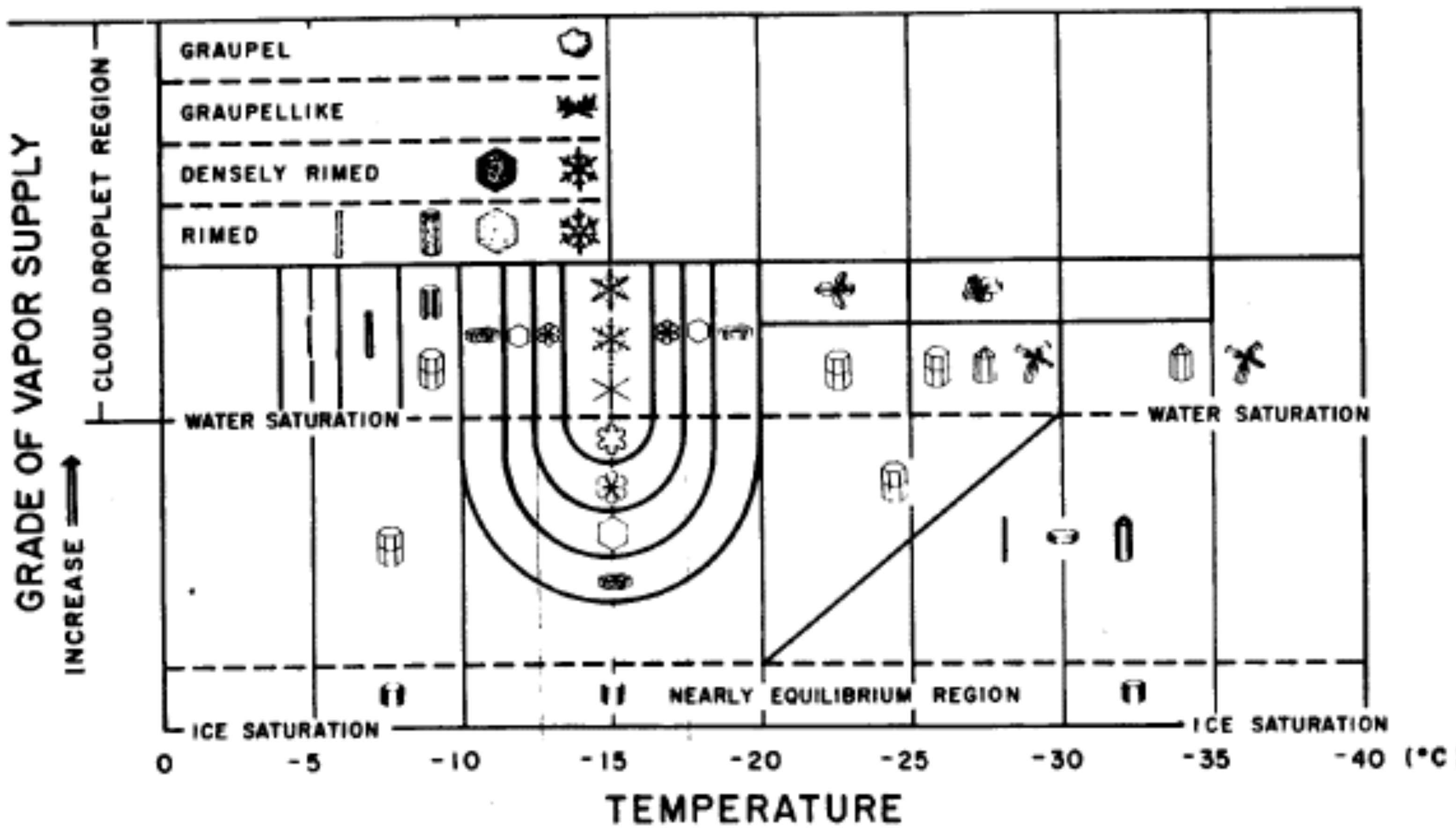


Fig. 2-26. Temperature and humidity conditions for the growth of natural snow crystals of various types. (From Magono and Lee, 1966; by courtesy of *J. Fac. Sci.*, Hokkaido University.)

Magono and Lee (1966) classification of ice crystals and their growth regimes

Magono and Lee Classification of Snow Crystals Part 1

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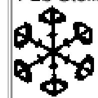
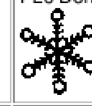
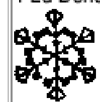
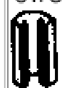
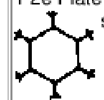
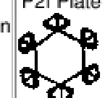
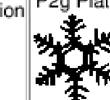
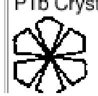
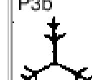
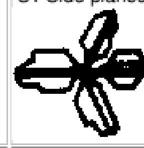
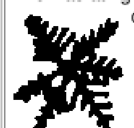
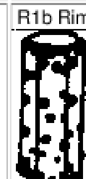
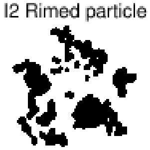
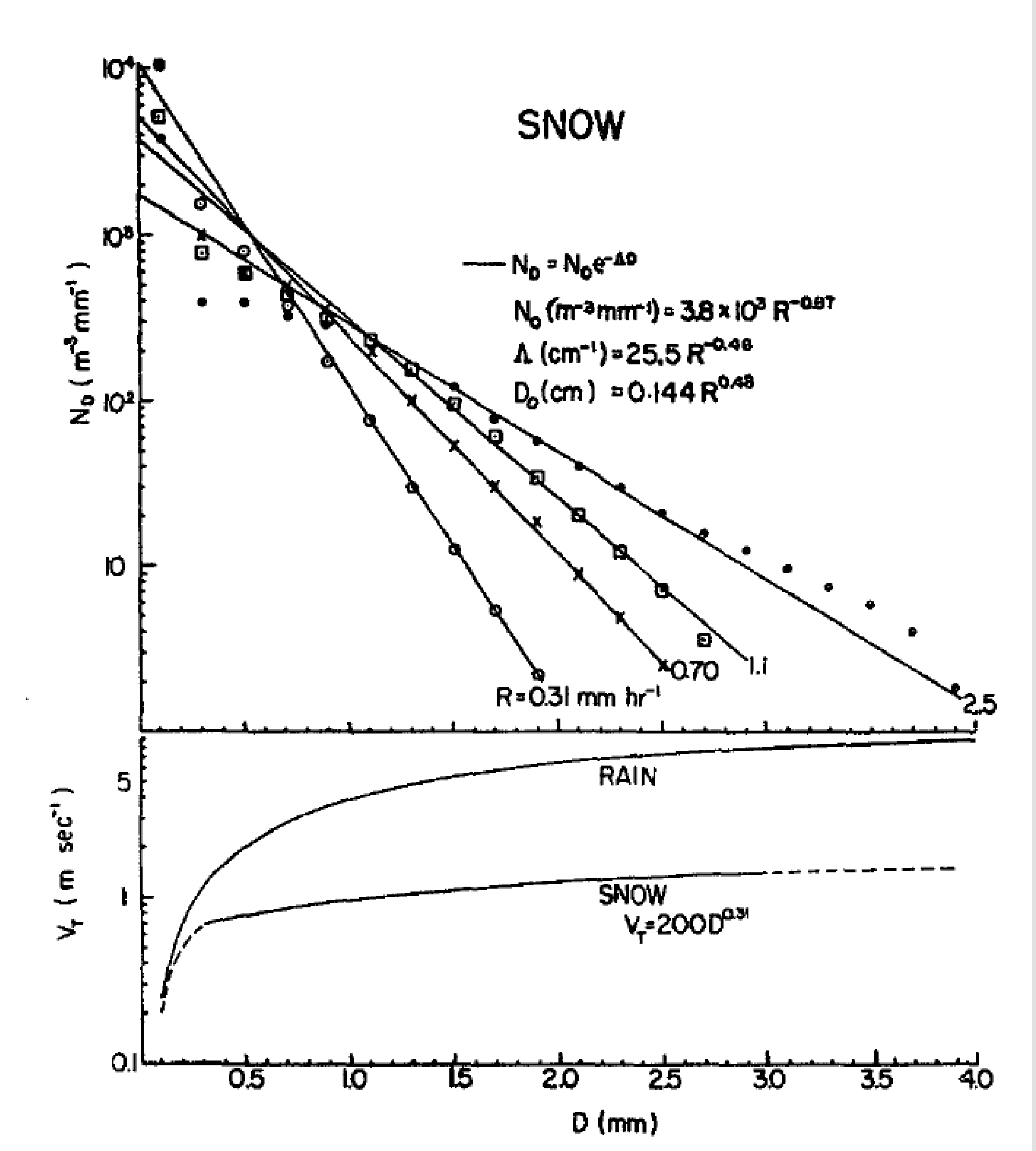
N1a Elementary needle 	C1f Hollow column 	P2b Stellar crystal with  sectorlike ends
N1b Bundle of elementary needles 	C1g Solid thick plate 	P2c Dendritic crystal with plates at end 
N1c Elementary sheath 	C1h Thick plate of  skeleton form	P2d Dendritic crystal with  sectorlike ends
N1d Bundle of elementary sheaths 	C1i Scroll 	P2e Plate with simple  extensions
N1e Long solid column 	C2a Combination of bullets 	P2f Plate with  sectorlike extensions
N2a  Combination of needles	C2b Combination of  columns	P2g Plate with dendritic  extensions
N2b  Combination of sheaths	P1a Hexagonal plate 	P3a  Two-branched crystal
N2c 	P1b Crystal with 	P3b 

Magono and Lee Classification of Snow Crystals Part 2

{return}

P6b Plate with spatial dendrites 	CP3d Plate with scrolls at ends 	R3c Graupellike snow with nonrimed extensions 
P6c Stellar crystal with spatial plates 	S1 Side planes 	R4a Hexagonal graupel 
P6d Stellar crystal with spatial dendrites 	S2 Scalelike side planes 	R4b Lump graupel 
P7a Radiating assemblage of plates 	S3 Combination of side planes, bullets, and columns 	R4c Conelike graupel 
P7b Radiating assemblage of dendrites 	R1a Rimed needle crystal 	I1 Ice particle 
CP1a Column with plates 	R1b Rimed columnar crystal 	I2 Rimed particle 

$$N_D = N_0 e^{-\Lambda D}$$



Gunn and Marshall JAS 1958

Fundamentals of cloud thermodynamics modeling

Water vapor is a minor constituent:
mass loading is typically smaller than 1%; thermodynamic properties (e.g., specific heats etc.) only slightly modified;

Suspended small particles (cloud droplets, cloud ice):
mass loading is typically smaller than a few tenths of 1%, particles are much smaller than the smallest scale of the flow; multiphase approach is not required, but sometimes used (e.g., DNS with suspended droplets, Lagrangian Cloud Model)

Precipitation (raindrops, snowflakes, graupel, hail):
mass loading can reach a few %, particles are larger than the smallest scale the flow; multiphase approach needed only for very-small-scale modeling

Continuous medium approach: *density (i.e., mass in the unit volume) is the main field variable (density of water vapor, density of cloud water, density of rainwater, etc…)*

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + u \cdot \nabla \psi$$

In practice, mixing ratios are typically used. Mixing ratio is the ratio between the density (of water vapor, cloud water…) and the air density.

Mixing ratios versus specific humidities…

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_a}{dt} + \rho_a \nabla \mathbf{u} = 0$$

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

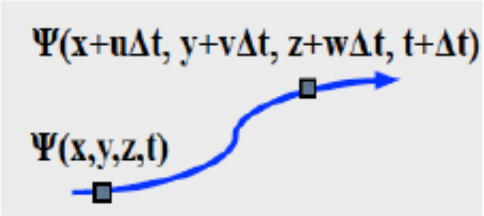
$$\boxed{\text{mixing ratio}: \quad q = \frac{\rho_v}{\rho_a}}$$

$$\frac{dq}{dt} = \frac{S}{\rho_a}$$

$$\boxed{\text{specific humidity}: \quad Q = \frac{\rho_v}{\rho_v + \rho_a}}$$

$$\frac{dQ}{dt} = \frac{\rho_a}{\rho_v + \rho_a} \frac{S}{\rho_v + \rho_a}$$

Lagrangian versus Eulerian formulation



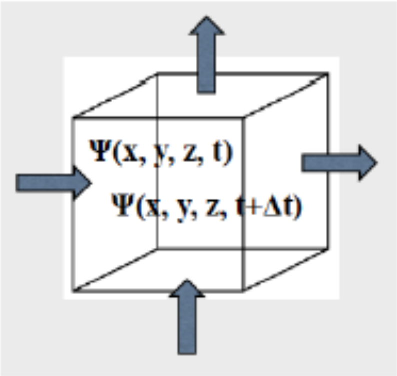
$$\frac{D\Psi}{Dt} = S$$

or

$$\frac{\partial \Psi}{\partial t} + \mathbf{u} \cdot \nabla \Psi = S$$

combined with dry air continuity equation:

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0$$



gives:

$$\frac{\partial \rho_a \Psi}{\partial t} + \nabla(\rho_a \mathbf{u} \Psi) = \rho_a S$$

For the anelastic system:

$$\frac{\partial \Psi}{\partial t} + \frac{1}{\rho_o} \nabla(\rho_o \mathbf{u} \Psi) = S$$

$$\rho_o = \rho_o(z)$$

Modeling of warm-rain microphysics

BULK MODEL OF CONDENSATION:

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} C_d$$
$$\frac{dq_v}{dt} = -C_d$$
$$\frac{dq_c}{dt} = \quad C_d$$

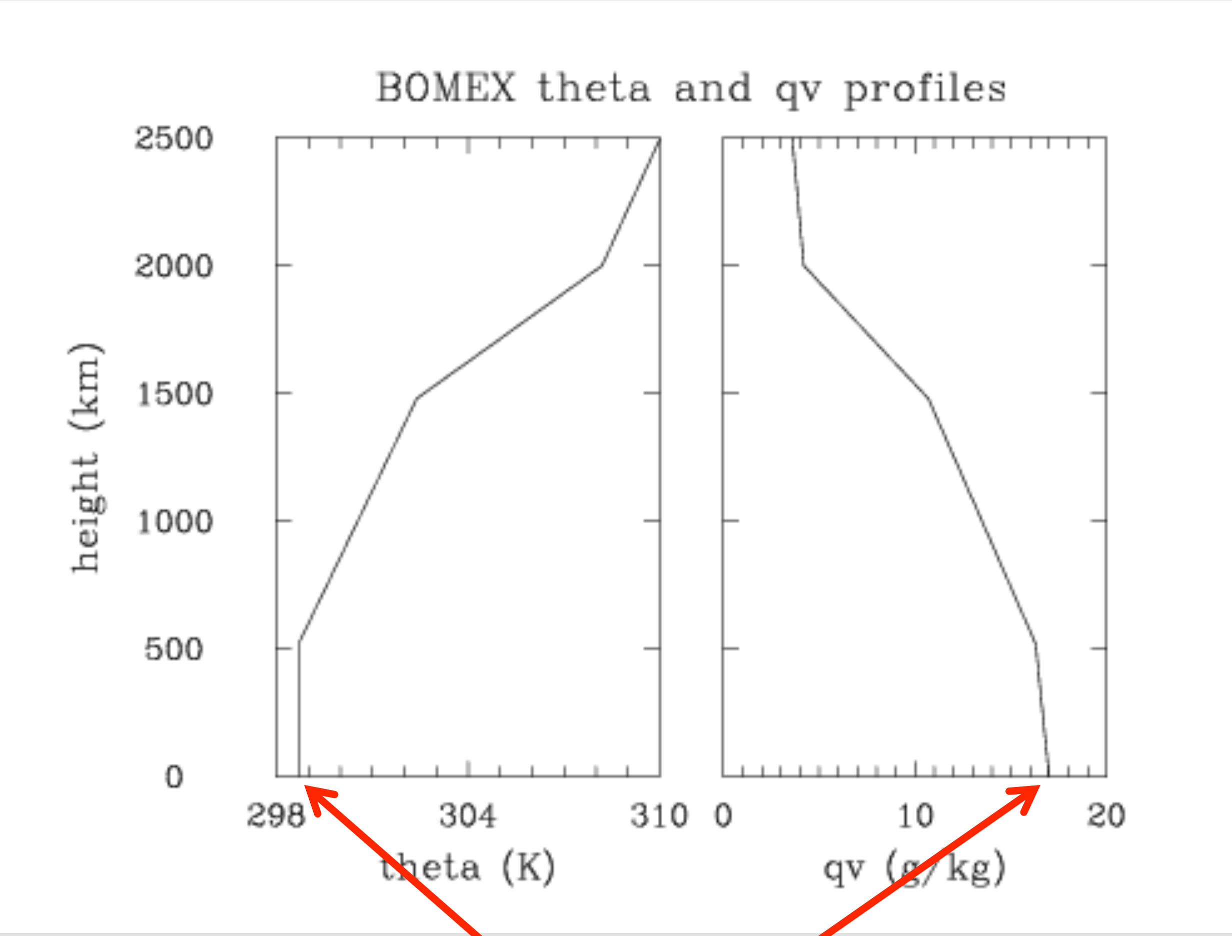
θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
L_v - latent heat of condensation/evaporation
C_d - condensation rate
Note: θ/T function of pressure only ($\approx \theta_o/T_o$) $\quad \frac{L_v}{c_p \Pi_e}$

C_d is defined such that cloud is always at saturation, which is a very good approximation:

$$q_c = 0 \quad \text{if} \quad q_v < q_{vs}$$
$$q_c > 0 \quad \text{only if} \quad q_v = q_{vs}$$

where $q_{vs}(p,T) \approx 0.622 \frac{e_s(T)}{p}$ is the water vapor mixing ratio at saturation

A very simple (but useful) model: rising adiabatic parcel…



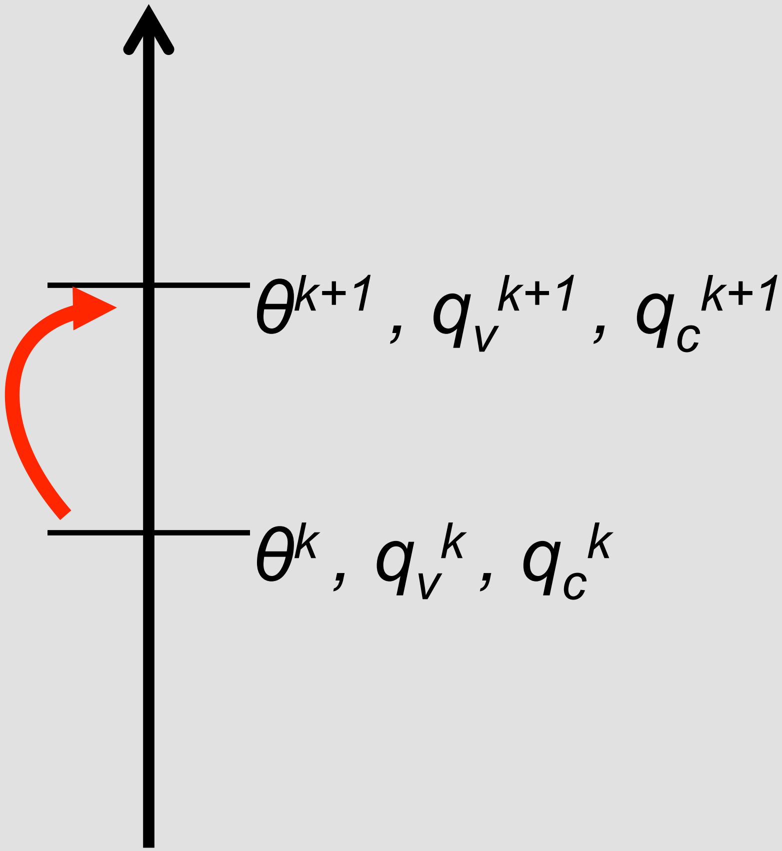
$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$

… by solving these equations.

Take a parcel from the surface and move it up…



$$\theta^{k+1} = \theta^k + \frac{L_v}{c_p \Pi_e} \Delta q$$

$$q_v^{k+1} = q_v^k - \Delta q$$

$$q_c^{k+1} = q_c^k + \Delta q$$

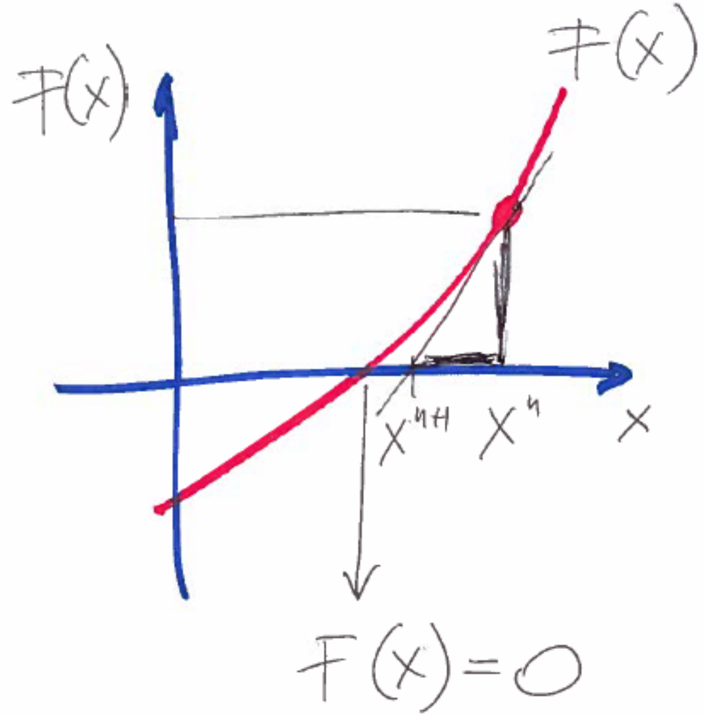
$$\Delta q = ?$$

$$q_v^{k+1} = q_{vs}(\theta^{k+1})$$

$$q_v^k - \Delta q = q_{vs}(\theta^k + \frac{L_v}{c_p \Pi_e} \Delta q)$$

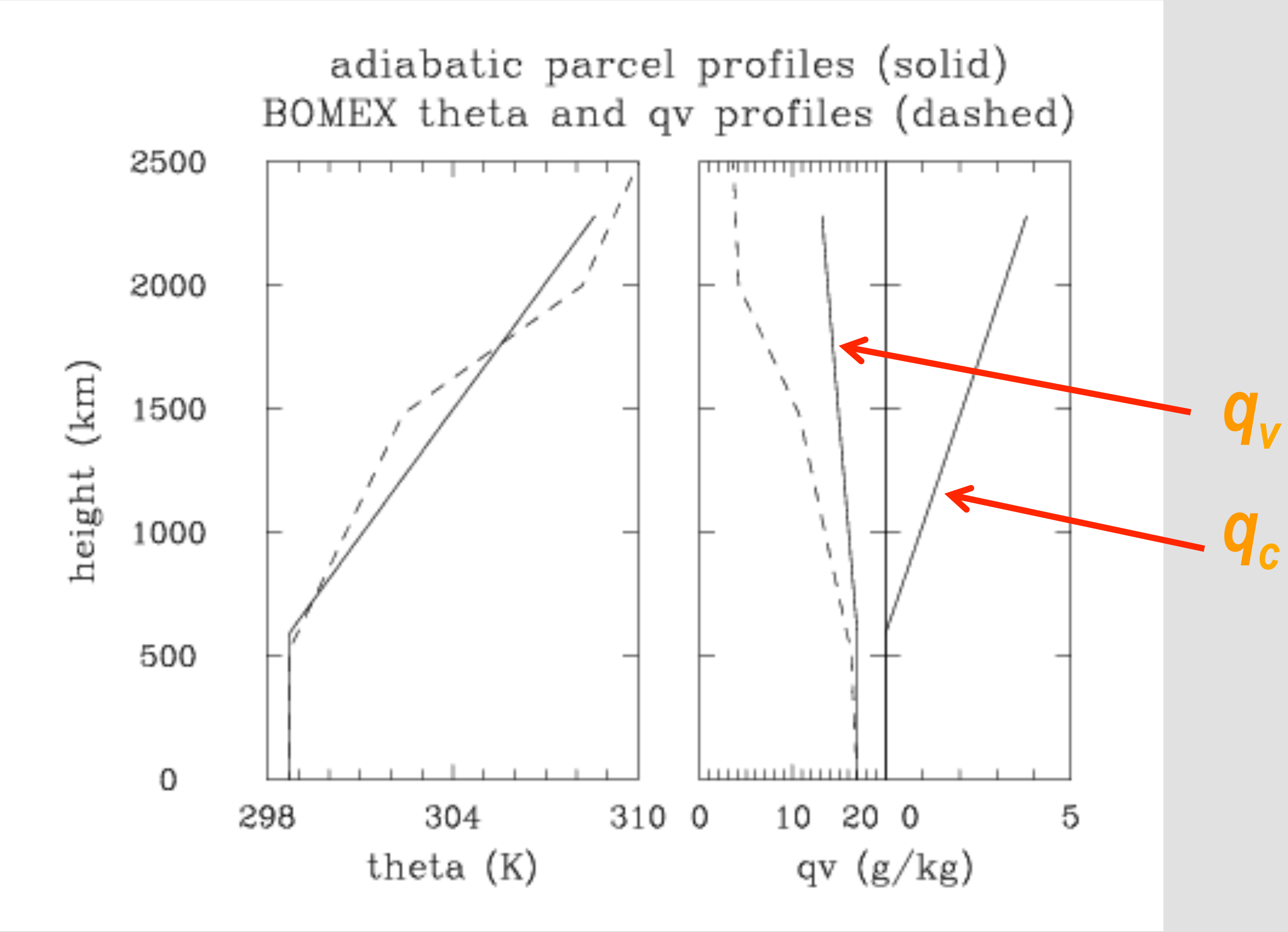
The nonlinear equation for Δq can be solved using the Newton-Raphson method…

F(x) = 0
x = ?



$$F'(x^n) = \frac{F(x^n)}{x^n - x^{n+1}}$$

$$x^{n+1} = x^n - \frac{F(x^n)}{F'(x^n)}$$



Look not only on the patterns (i.e., processes), but also on specific numbers (e.g., temperature change, mixing ratios, etc).

Invariant variables:

total water,

liquid water potential temperature,

equivalent potential temperature.

Note: equivalent potential temperature is closely related to moist static energy, $c_pT + gz + Lq_v\ldots$

If $\theta/T \approx$ const (shallow convection approximation)

$$\frac{D\theta}{Dt} = \frac{L_v\theta}{c_pT}C_d$$

$$\frac{Dq_v}{Dt} = -C_d$$

$$\frac{Dq_c}{Dt} = C_d$$

can be converted into:

$$\frac{D\theta_I}{Dt} = 0$$

$$\frac{DQ}{Dt} = 0$$

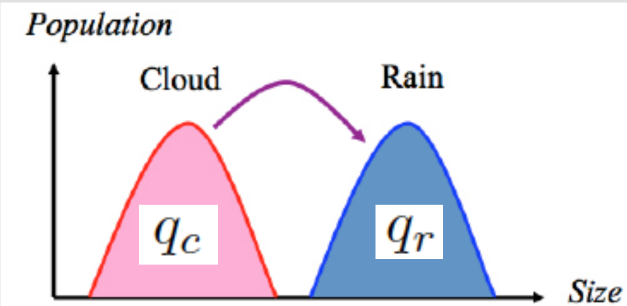
θ_I is one of the two:

$\theta_e = \theta + \frac{L_v\theta}{c_pT}q_v$ - equivalent potential temperature

$\theta_l = \theta - \frac{L_v\theta}{c_pT}q_c$ - liquid water potential temperature

$Q = q_v + q_c$ - total water mixing ratio

Adding rain or drizzle:

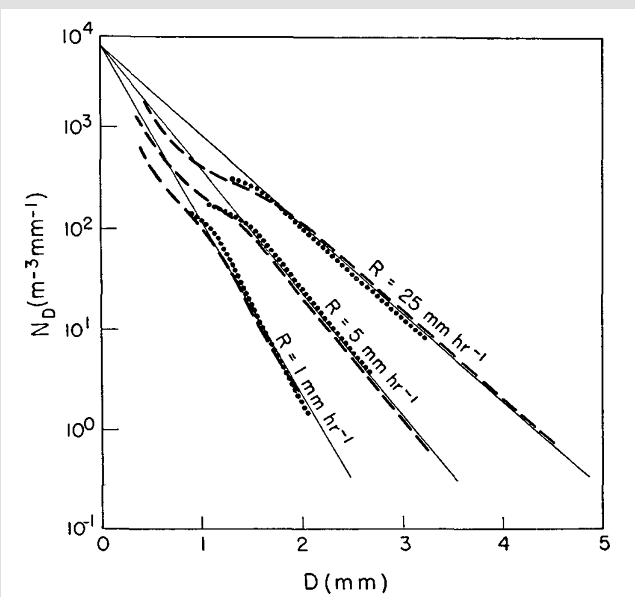


THE DISTRIBUTION OF RAINDROPS WITH SIZE

By J. S. Marshall and W. McK. Palmer[1]

McGill University, Montreal

(Manuscript received 26 January 1948)



WARM RAIN BULK MODEL (Kessler 1969):

$$\frac{D\theta}{Dt} = \frac{L_v \theta}{c_p T}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

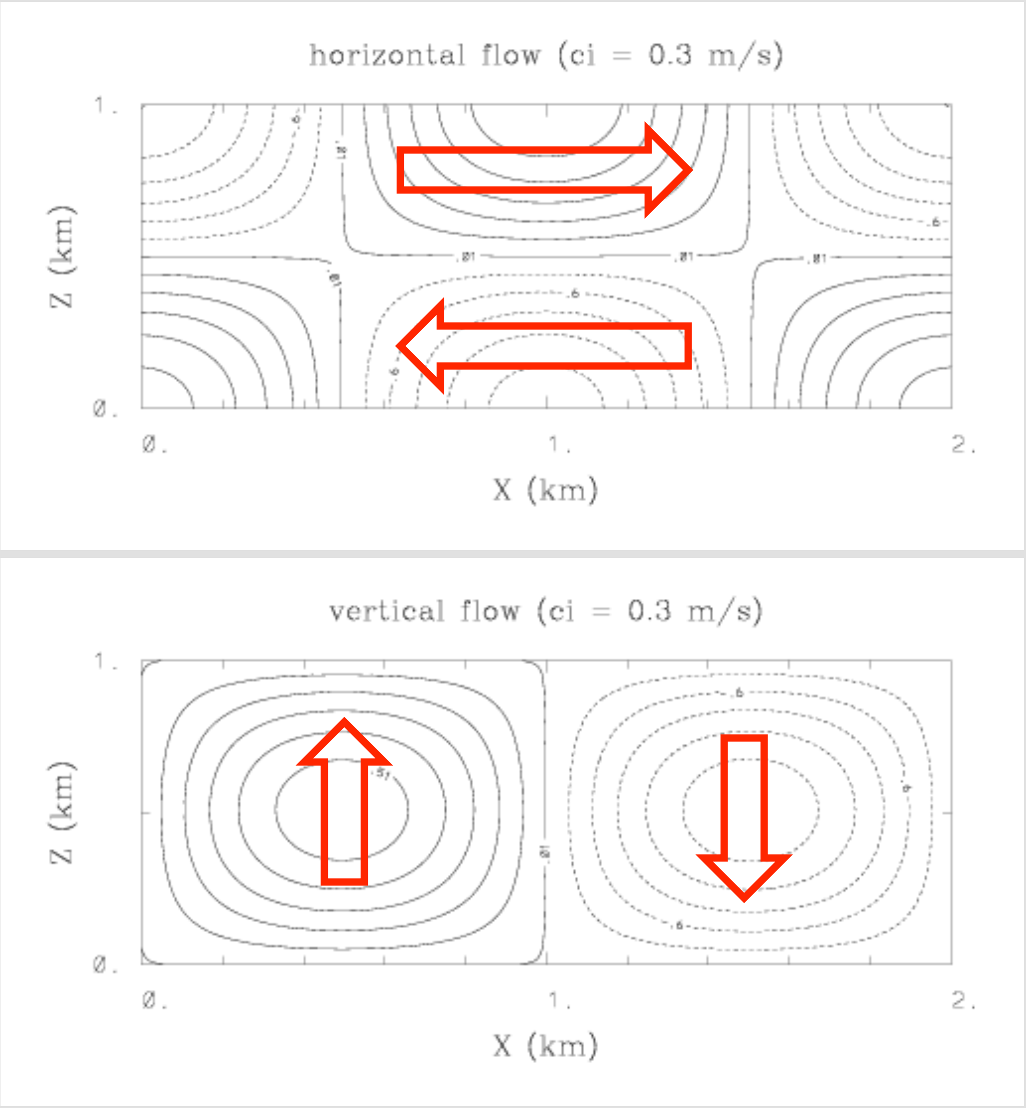
$$\frac{Dq_c}{Dt} = C_d - AUT - ACC$$

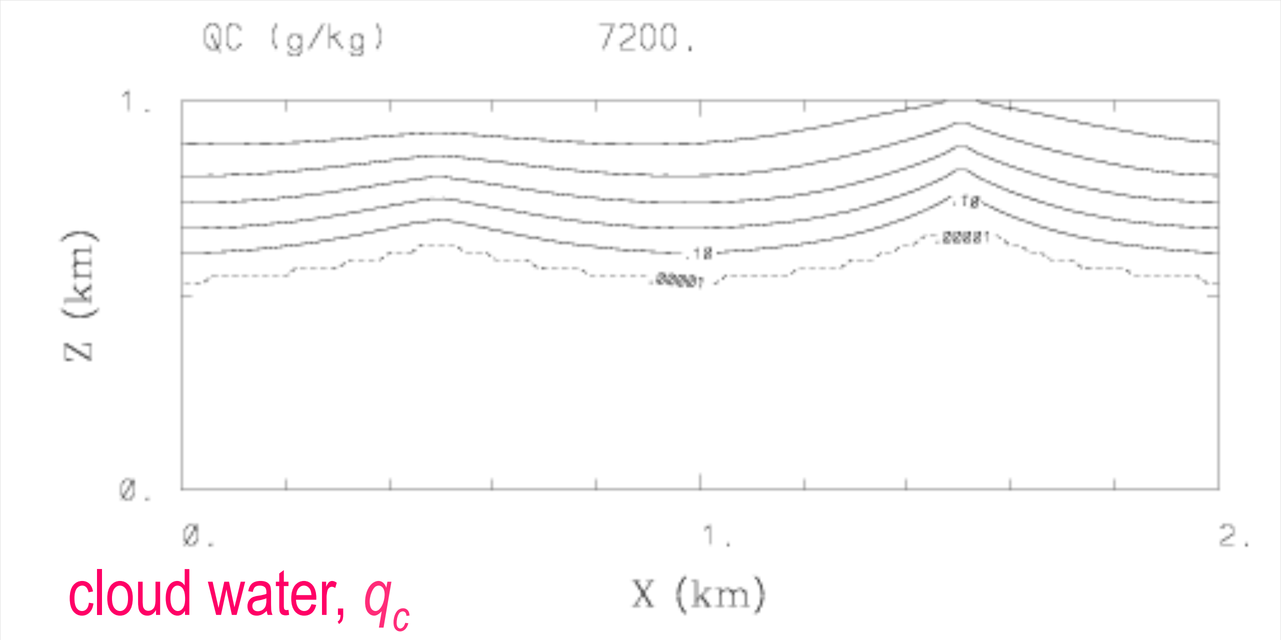
$$\frac{Dq_r}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
q_r - *rain water* mixing ratio
C_d - condensation rate
$EVAP$ - rain evaporation rate
AUT - "autoconversion" rate: $q_c \rightarrow q_r$
ACC - accretion rate: $q_c, q_r \rightarrow q_r$

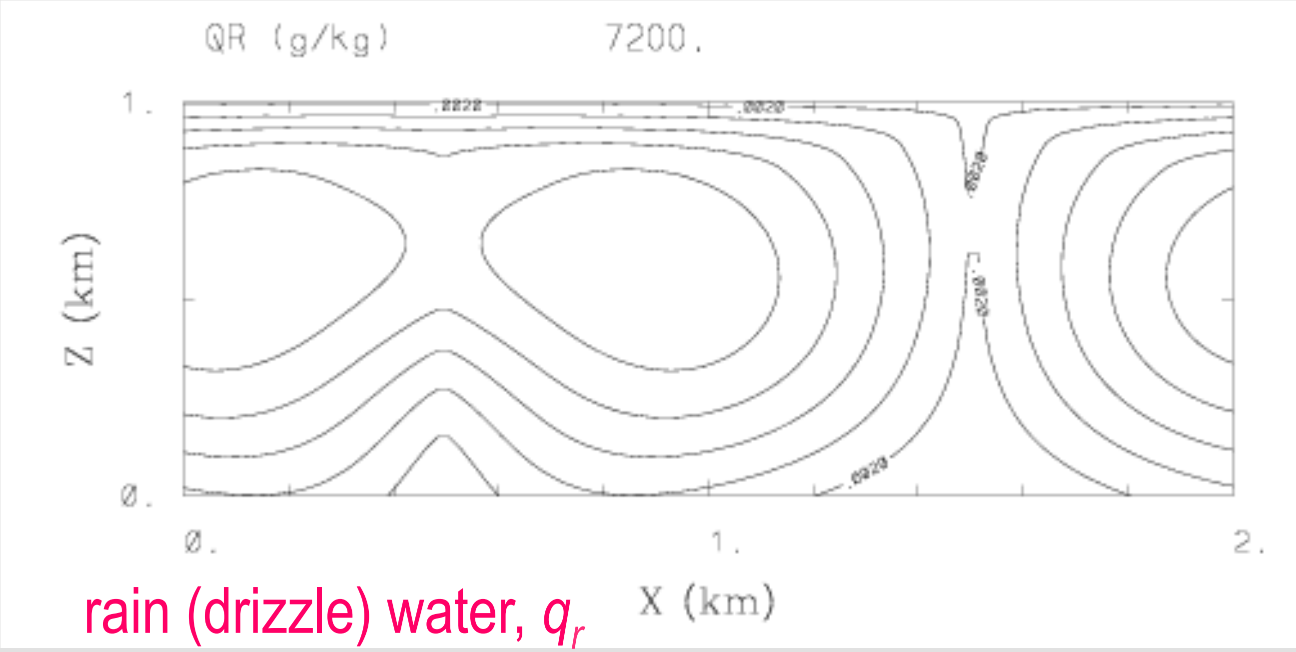
$v_t(q_r)$ - rain terminal velocity (typically derived by assuming a drop size distribution; e.g., the Marshall-Palmer distribution $N(D) = N_o exp(-\Lambda D)$, $N_o = 10^7\ \mathrm{m}^{-4}$).

We need something more complicated than a rising parcel as rain has to fall out. One possibility is to use the kinematic (prescribed flow) framework…



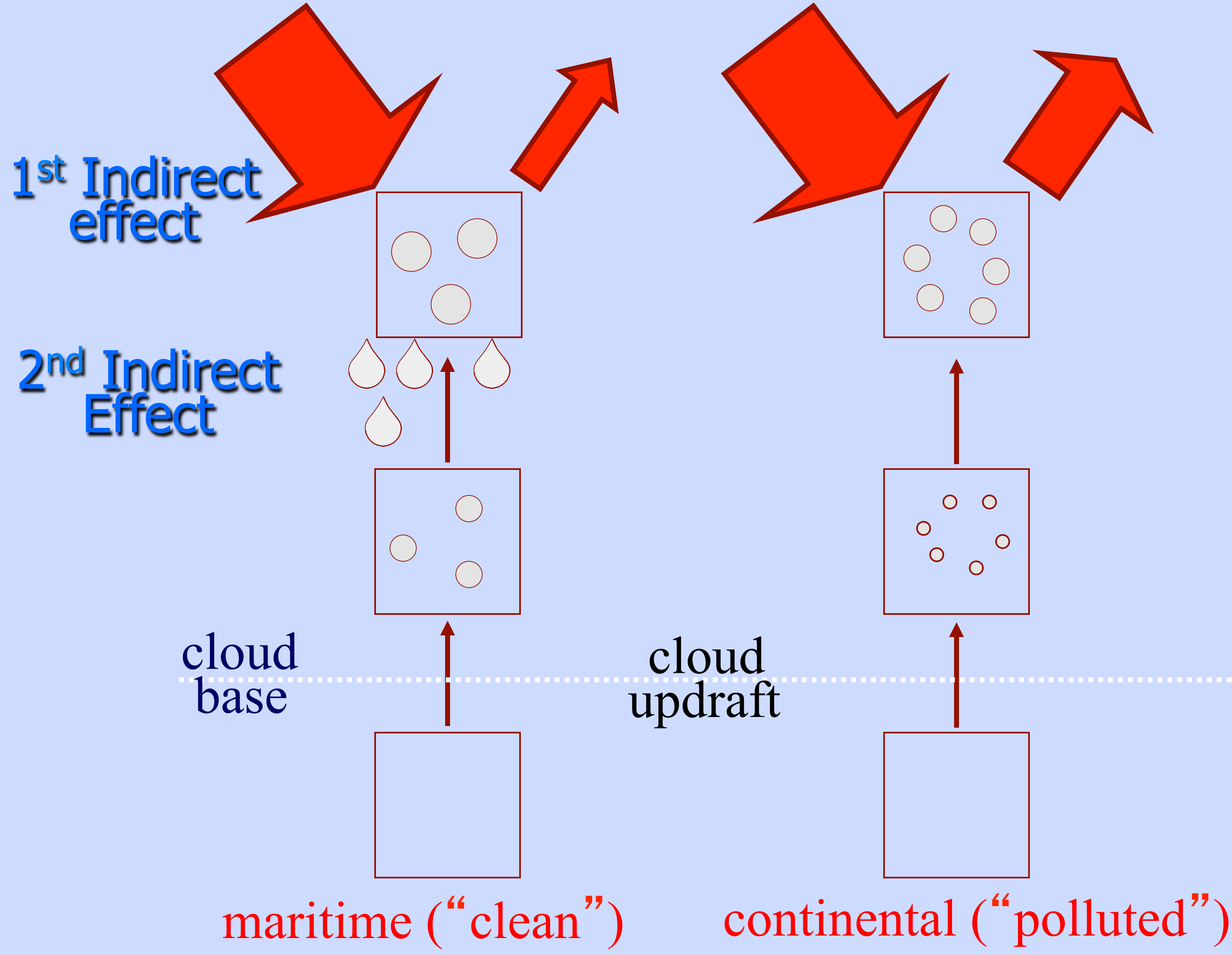


Cloud water and rain (drizzle) fields after 2 hrs (almost quasi-equilibrium…)



So far we only had information about the mass of cloud and precipitation. Do we need to know something about droplet sizes?

Indirect aerosol effects (warm rain only)



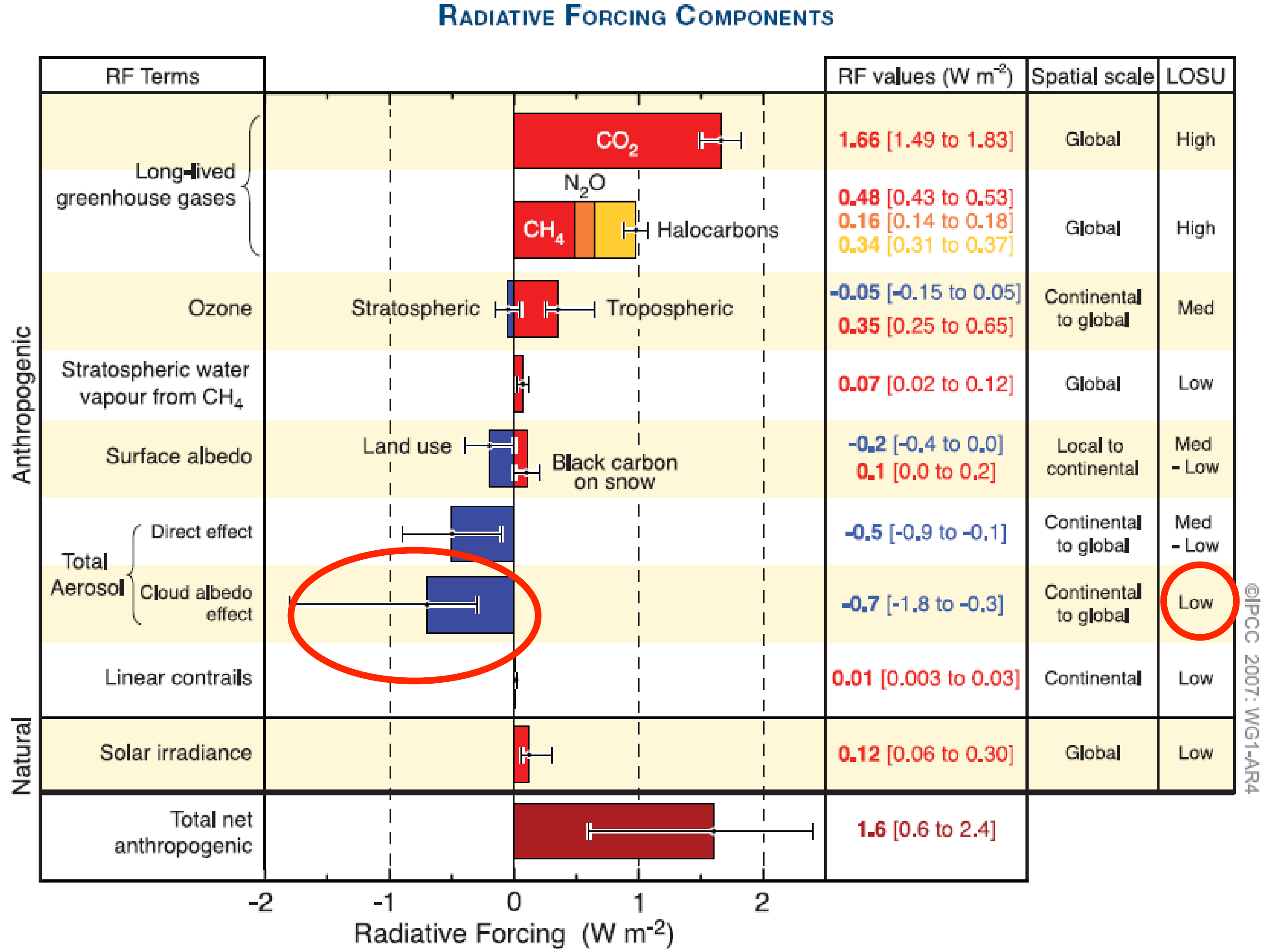


Figure SPM.2. *Global average radiative forcing (RF) estimates and ranges in 2005 for anthropogenic carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and other important agents and mechanisms, together with the typical geographical extent (spatial scale) of the forcing and the assessed level of scientific understanding (LOSU). The net anthropogenic radiative forcing and its range are also shown. These require summing asymmetric uncertainty estimates from the component terms, and cannot be obtained by simple addition. Additional forcing factors not included here are considered to have a very low LOSU. Volcanic aerosols contribute an additional natural forcing but are not included in this figure due to their episodic nature. The range for linear contrails does not include other possible effects of aviation on cloudiness. {2.9, Figure 2.20}*

Intergovernmental Panel on Climate Change (IPCC), Summary for Policymakers, 2007

Stratocumulus topped boundary layer

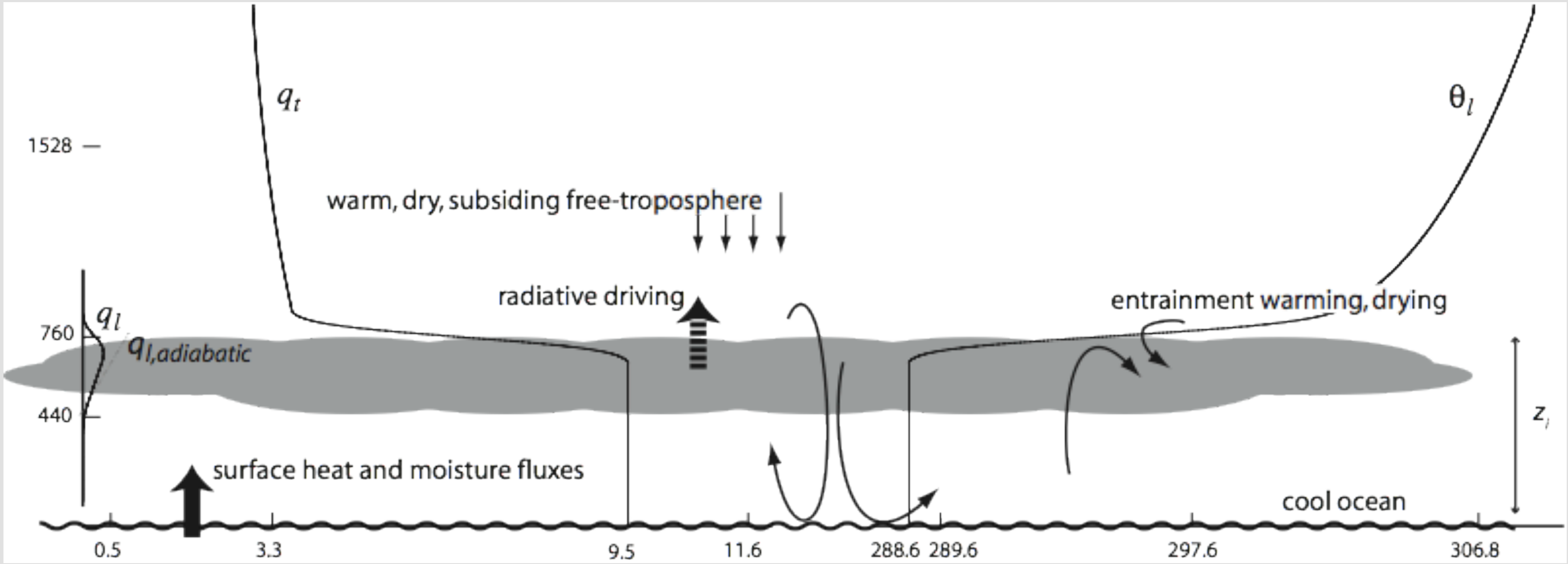


Figure from Bjorn Stevens

What determines the concentration of cloud droplets?

To answer this, one needs to understand formation of cloud droplets, that is, the activation of cloud condensation nuclei (CCN) .

This typically happens near the cloud base, when the rising air parcel approaches saturation.

Saturation ratio:

Saturated water vapor pressure over an aqueous solution droplet with radius *r*

Saturated water vapor pressure over plain water surface

Surface tension (Kelvin) effect

Solute (Raoult) effect

$$\frac{e_r}{e_\infty} = 1 + \frac{a}{r} - \frac{b}{r^3}$$

$$a = \frac{2\sigma}{\rho_l R_v T} \qquad b = \frac{4 i m_s M_v}{3\pi \rho_l M_s}$$

σ - surface tension

ρ_l - water density

R_v – gas constant for water vapor

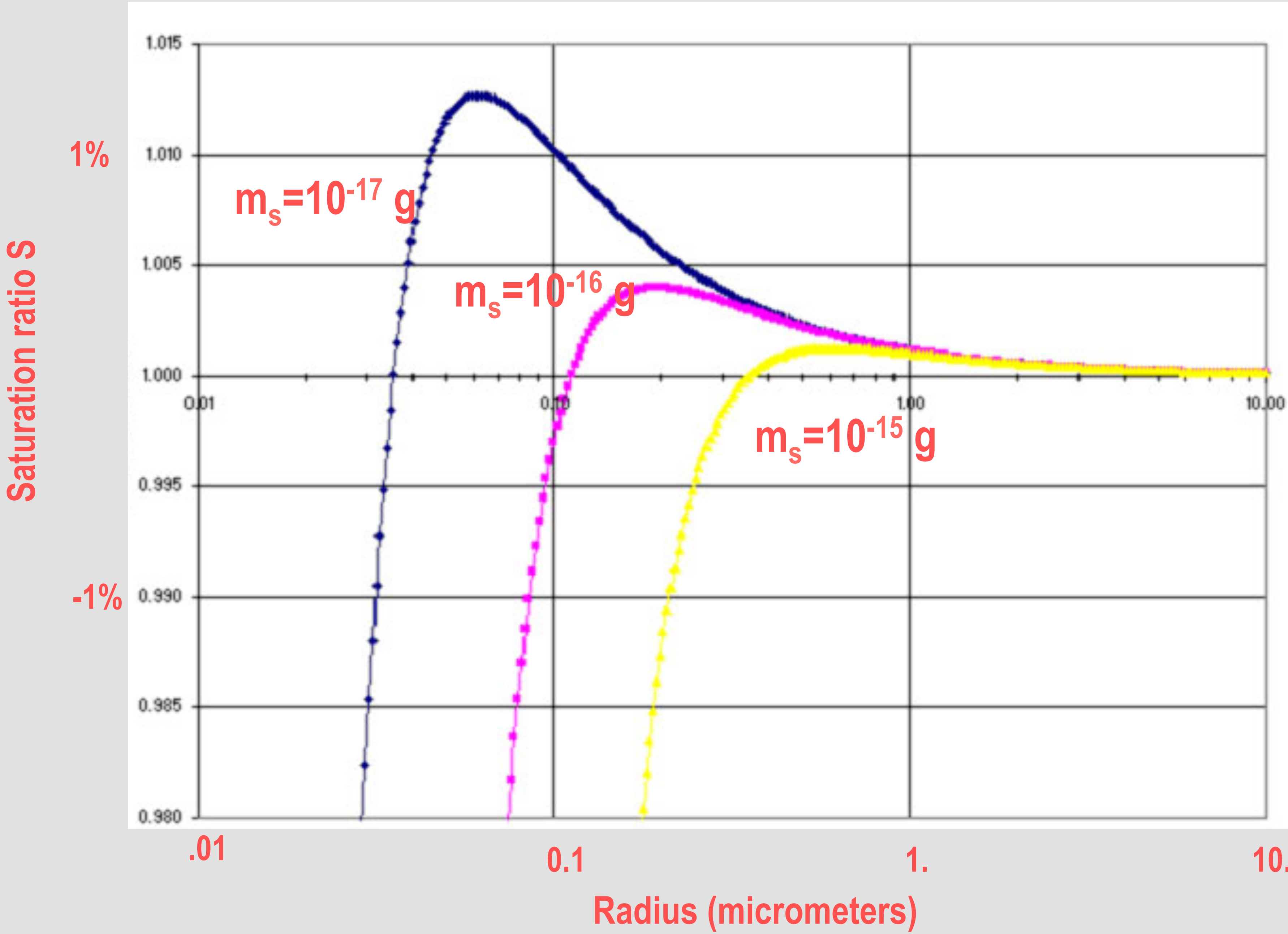
T – air temperature

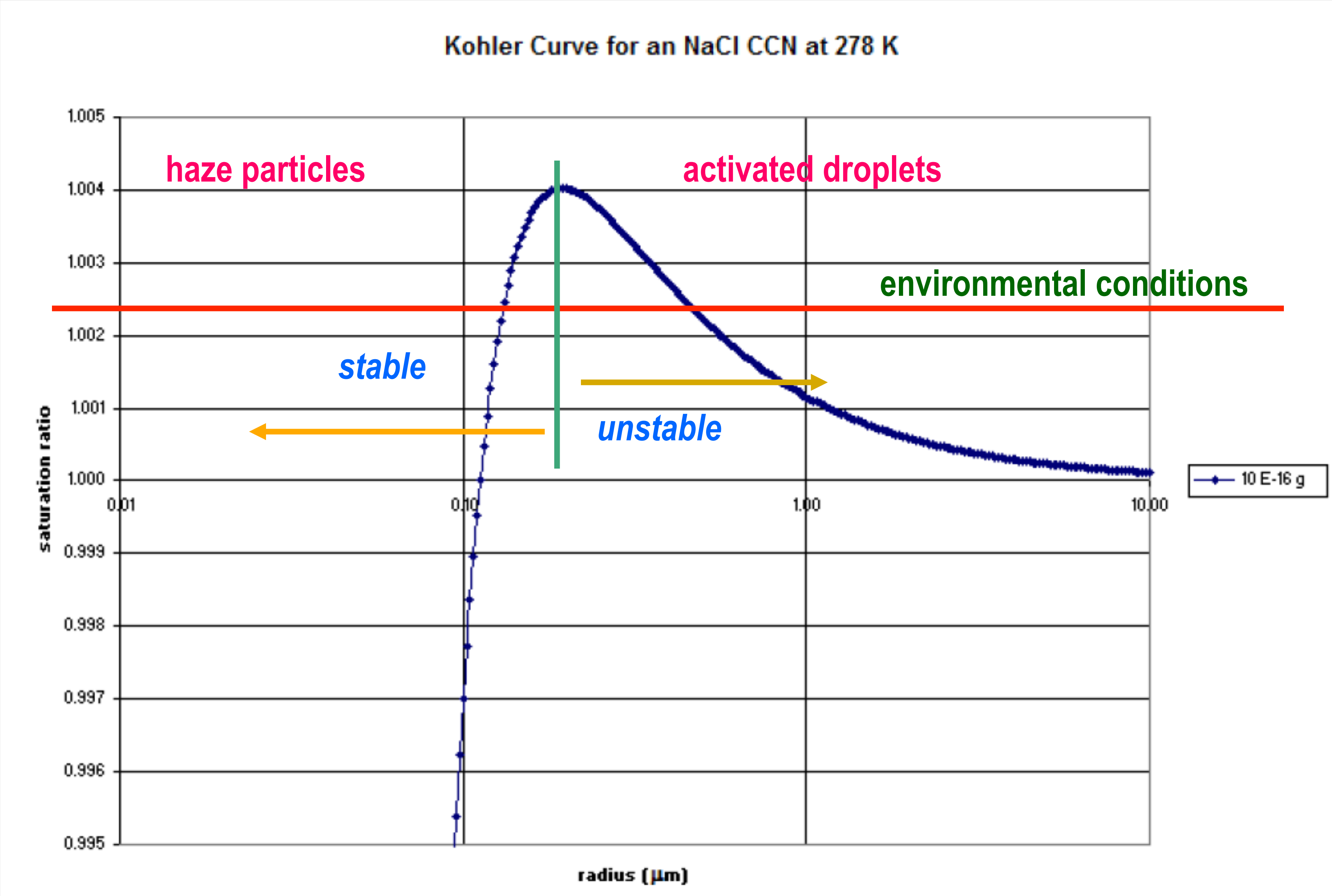
i – van't Hoff factor

m_s – mass of solute

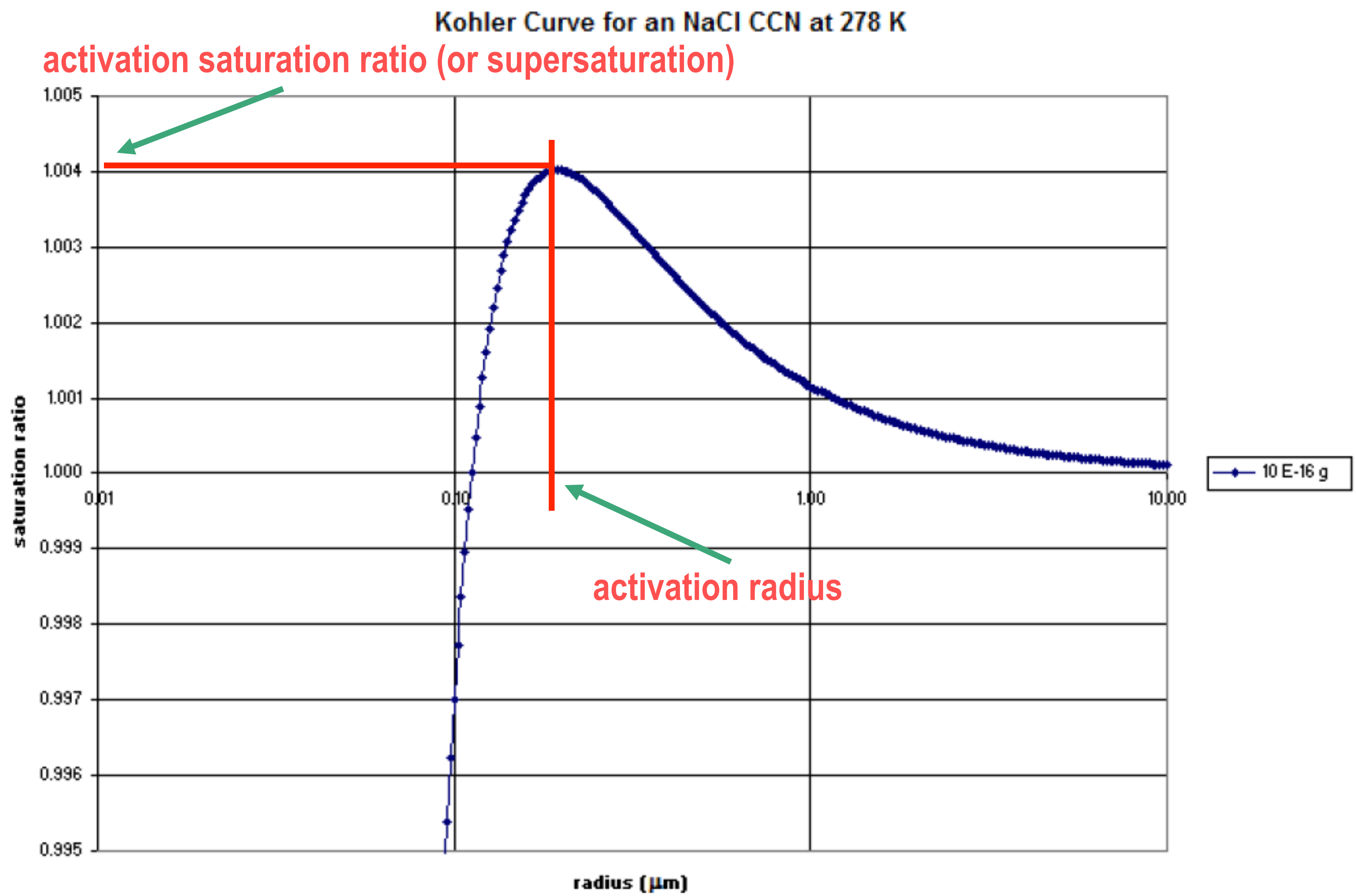
M_v – molar mass of water

M_s – molar mass of solute


Saturation ratio S
1%
-1%
1.015
1.010
1.005
1.000
0.995
0.990
0.985
0.980
0.01
0.10
1.00
10.00
$m_s=10^{-17}$ g
$m_s=10^{-16}$ g
$m_s=10^{-15}$ g
.01
0.1
1.
10.
Radius (micrometers)



Kohler Curve for an NaCl CCN at 278 K
haze particles
activated droplets
environmental conditions
stable
unstable
saturation ratio
1.005
1.004
1.003
1.002
1.001
1.000
0.999
0.998
0.997
0.996
0.995
0.01
0.10
1.00
10.00
radius (µm)
10 E-16 g


Kohler Curve for an NaCl CCN at 278 K
activation saturation ratio (or supersaturation)
activation radius
saturation ratio
radius (µm)
10 E-16 g
1.005
1.004
1.003
1.002
1.001
1.000
0.999
0.998
0.997
0.996
0.995
0.01
0.10
1.00
10.00

CCN, soluble salt particles, have different sizes.

Large CCN are nucleated first, activation of smaller ones follows as the supersaturation builds up.

Once sufficient number of CCN is activated, supersaturation levels off, and activation is completed.

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Large CCN are nucleated first, activation of smaller ones follows as the supersaturation builds up.

Once sufficient number of CCN is activated, supersaturation levels off, and activation is completed.

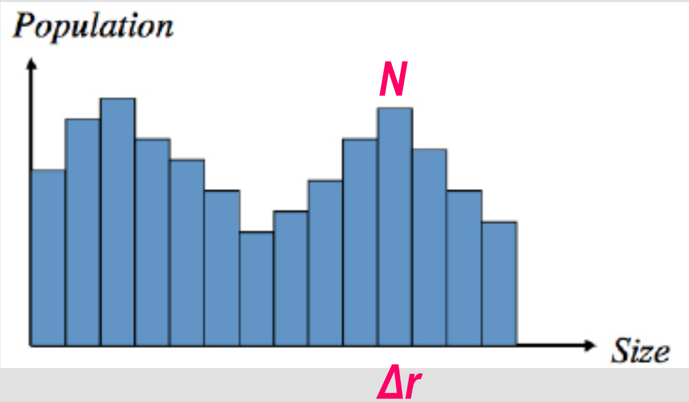
These processes are typically considered in the context of detailed (bin) microphysics…

BIN-RESOLVING WARM MICROPHYSICS:

Introducing *spectral density function* $f(r,t)$:

$$f(r,t) \equiv \frac{dN(r,t)}{dr}$$

$dN(r,t)$ is the concentration (per unit mass as mixing ratio) of droplets smaller than r (cumulative concentration).



$f = N / \Delta r$

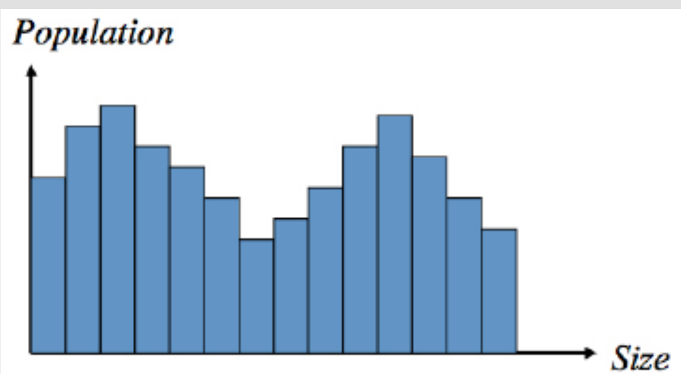
Continuity equation for the growth by condensation:

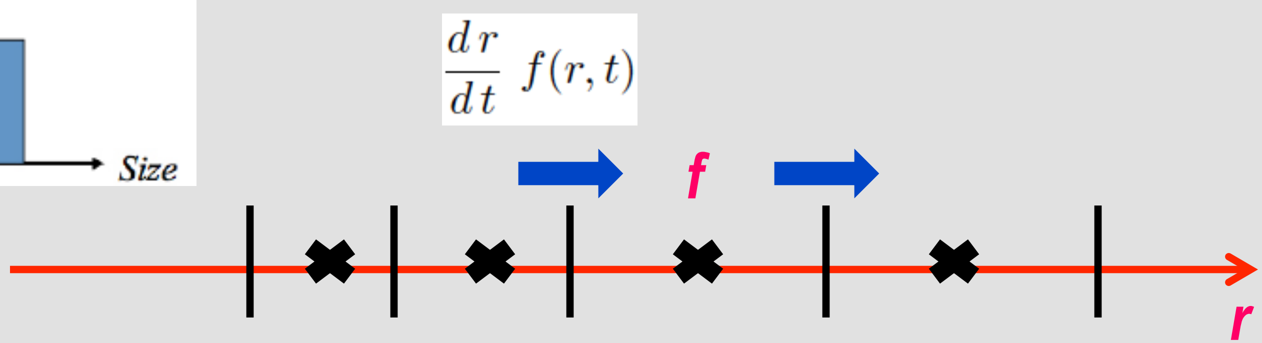
$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{dr}{dt}\, f(r,t)\right) = 0$$

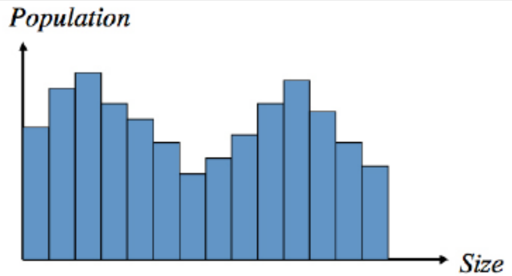
where $\frac{dr}{dt}$ is growth rate of a droplet with radius r:

$$\frac{dr}{dt} = \frac{A(T,p)\, S}{r}$$

$S = \frac{q_v}{q_{vs}} - 1$ is the supersaturation; q_v is the ambient water vapor mixing ratio; $q_{vs}(p,T)$ is the saturated water vapor mixing ratio.







BIN-RESOLVING WARM MICROPHYSICS:

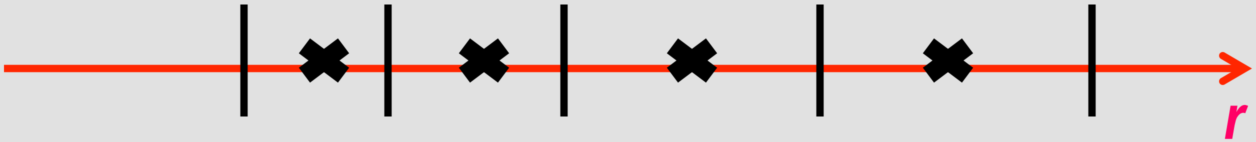
ACTIVATION AND CONDENSATION

Continuity equation for activation and growth by condensation:

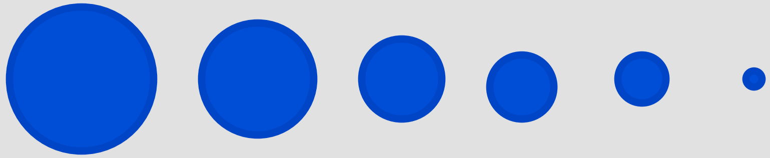
$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{dr}{dt}\, f(r,t)\right) = S_{nucl}$$

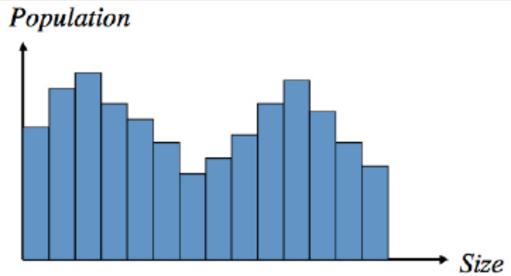
where S_{nucl} is the source associated with activation of cloud droplets (CCN activation).

cloud droplets



aerosols (CCN)





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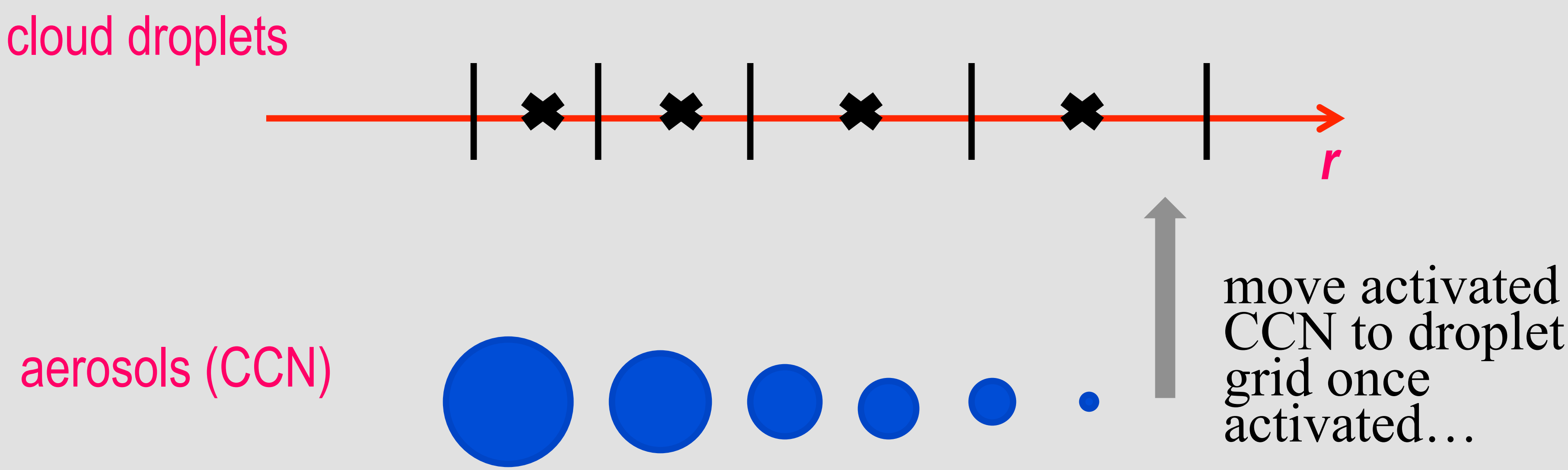


Twomey activation of CCN:

N - total concentration of activated droplets

S – supersaturation

$$N = a\,S^b$$

a, b – parameters characterizing CCN

$0 < b < 1$ (typically, $b=0.5$)

$a \sim 100\ \mathrm{cm}^{-3}$ maritime/clean

$a \sim 1{,}000\ \mathrm{cm}^{-3}$ continental/polluted

Activated CCN are inserted into the first bin (say, $r=1\ \mu m$)

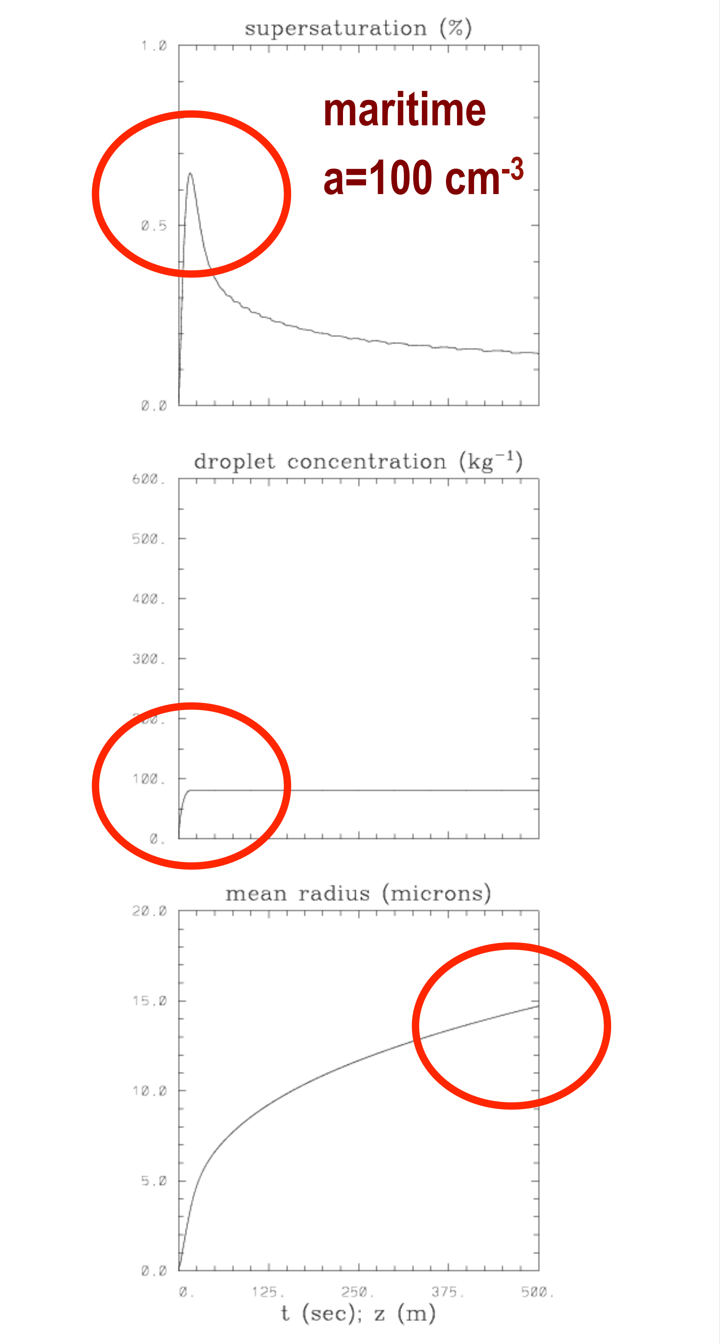
Computational example:

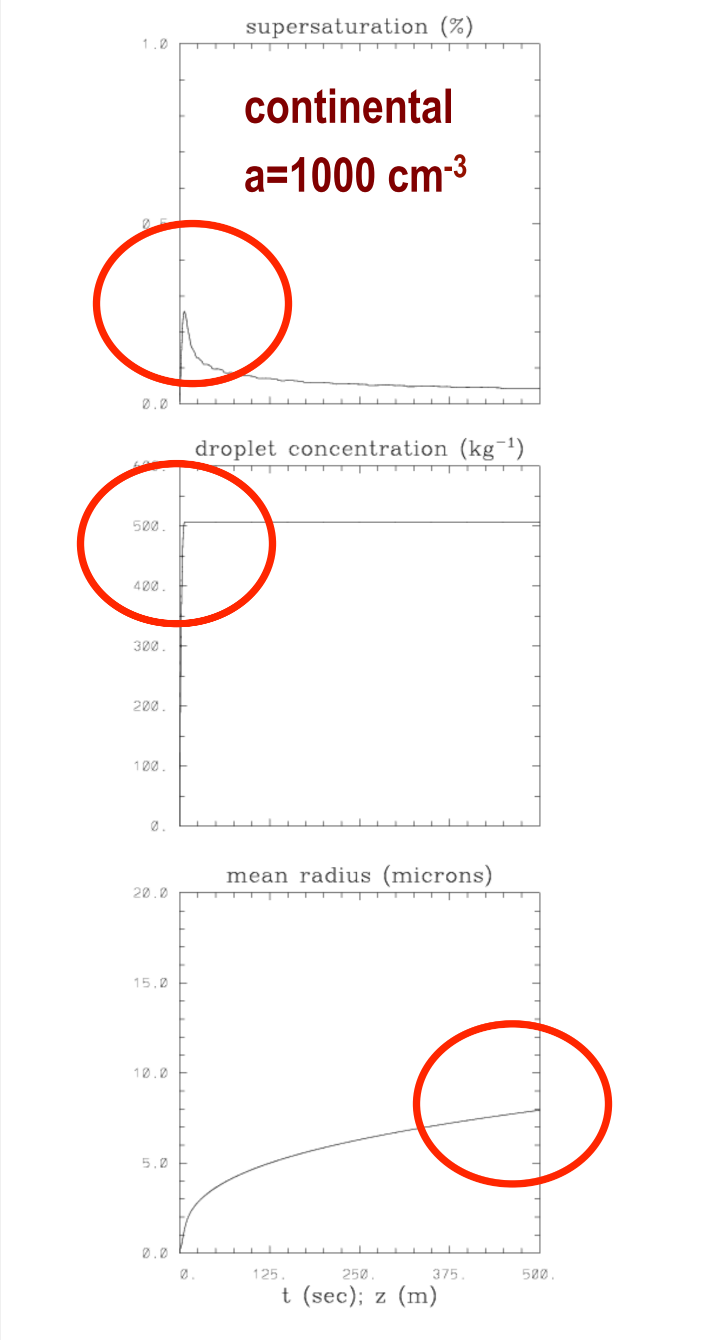
Nucleation and growth of cloud droplets in a parcel of air rising with vertical velocity of 1 m/s;

60 bins used;

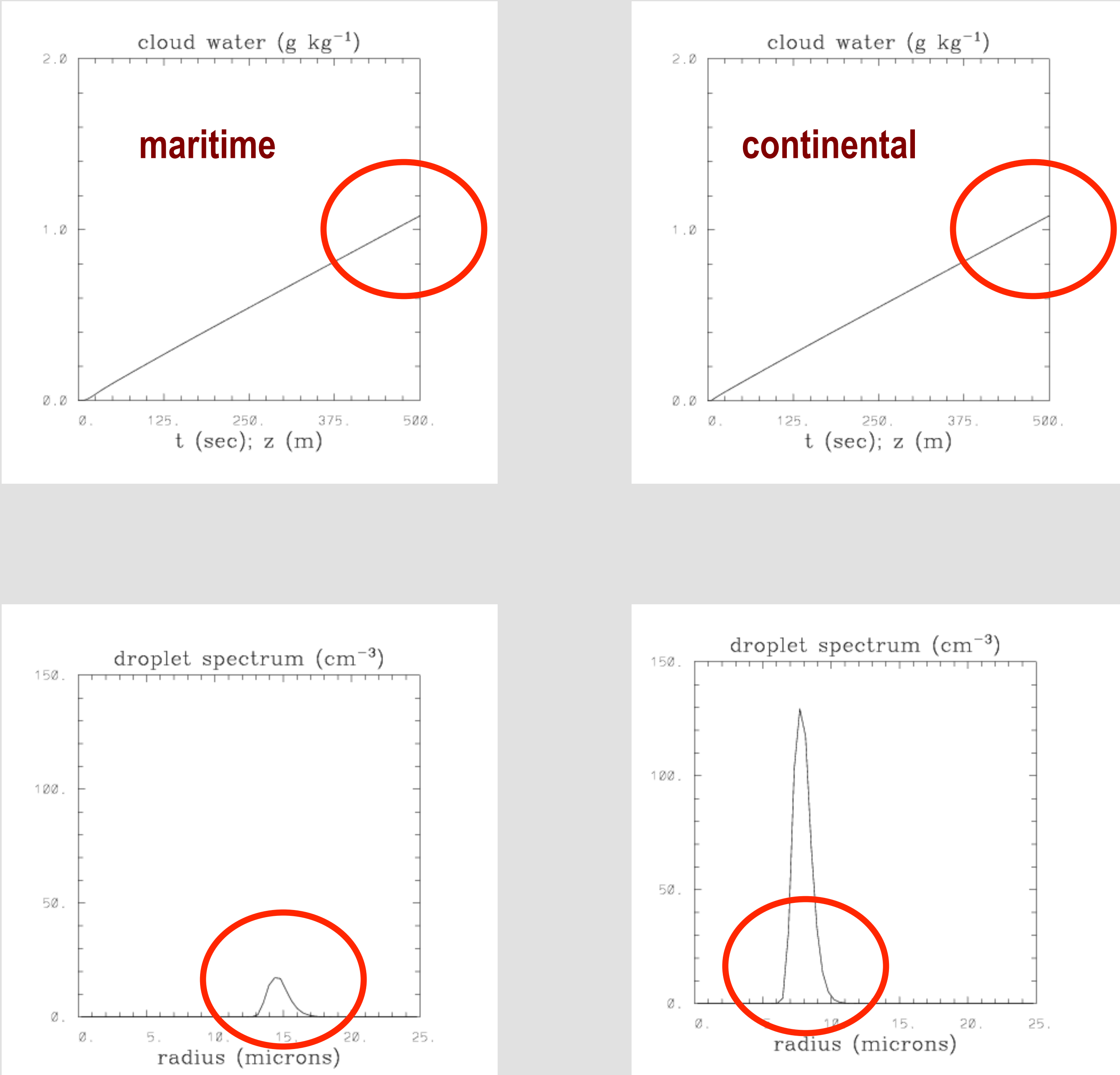
1D flux-form advection applied in the radius space;

Difference between continental/polluted and maritime/ pristine aerosols


supersaturation (%)
maritime
a=100 cm-3
droplet concentration (kg-1)
mean radius (microns)
t (sec); z (m)


supersaturation (%)
continental
a=1000 cm-3
droplet concentration (kg-1)
mean radius (microns)
t (sec); z (m)

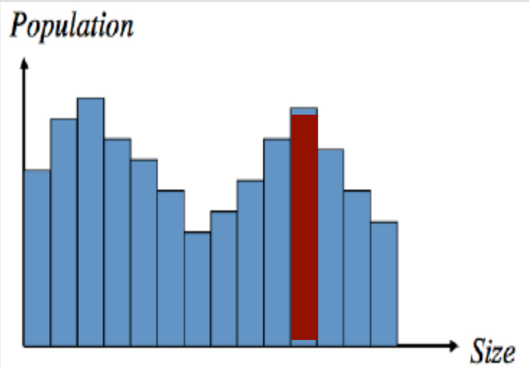
$N = a\,S^{b}$


cloud water (g kg⁻¹)
maritime
2.0
1.0
0.0
0.
125.
250.
375.
500.
t (sec); z (m)
cloud water (g kg⁻¹)
continental
2.0
1.0
0.0
0.
125.
250.
375.
500.
t (sec); z (m)
droplet spectrum (cm⁻³)
150.
100.
50.
0.
0.
5.
10.
15.
20.
25.
radius (microns)
droplet spectrum (cm⁻³)
150.
100.
50.
0.
0.
10
15.
20.
25.
radius (microns)

30 cm

Application of the bin-resolving microphysics to the problem of turbulent mixing between cloudy and clear air: cloud chamber mixing versus DNS simulation

Andrejczuk et al. JAS 2004, 2006, 2009



GROWTH BY COLLISION/COALESCENCE

The Smoluchowski equation (aka *kinetic collection equation*, *stochastic coalescence equation*) for the spectral density function $f(m,t)$:

$$\frac{\partial f(m,t)}{\partial t} =$$

$$= \frac{1}{2}\int_0^m f(m-M,t)\; f(M,t)\; K(m-M,M)\; dM$$

$$- f(m,t) \int_0^\infty f(M,t)\; K(m,M)\; dM$$

m, M - droplet masses

$K(m,M)$ - *collection kernel*; frequency of collisions (per unit volume of air) between droplets with mass m and M

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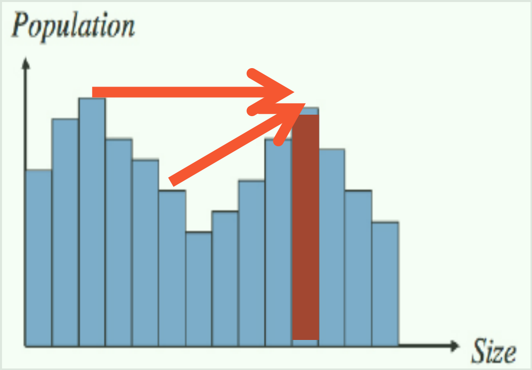

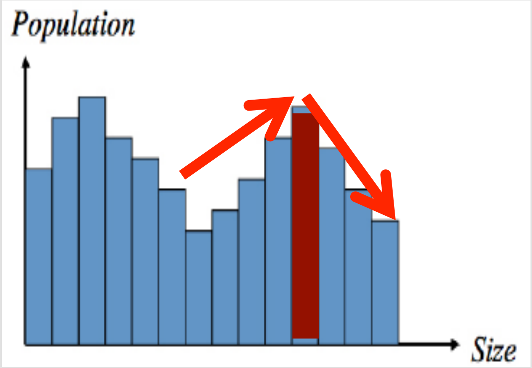

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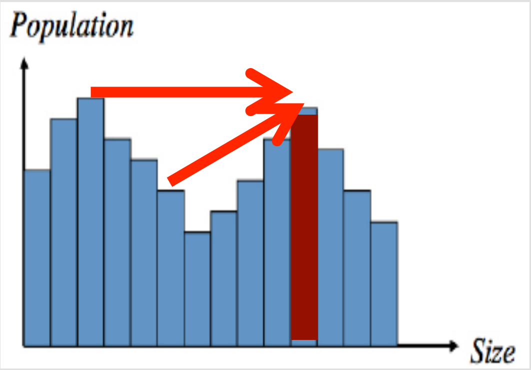

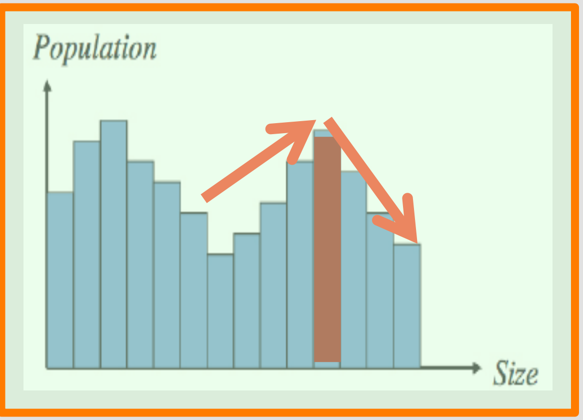

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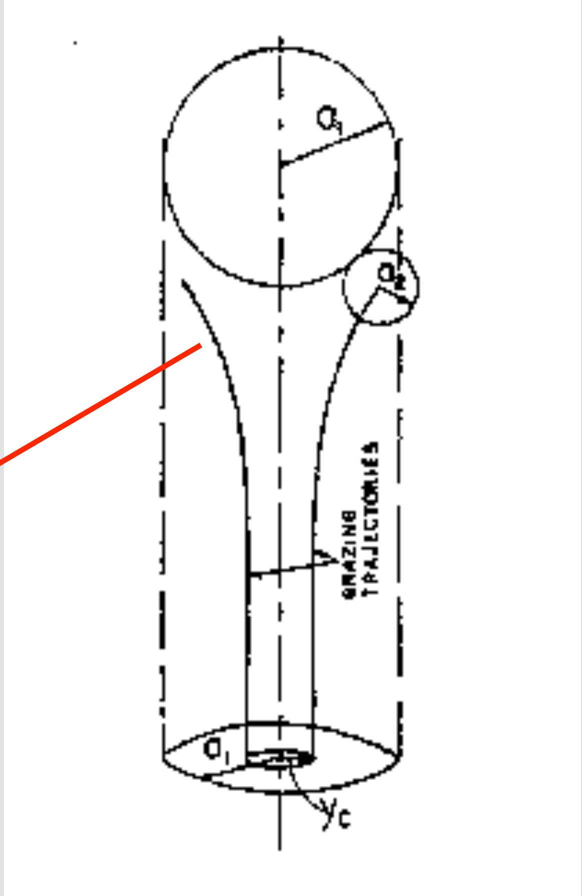
Growth of water droplets by gravitational collision-coalescence:

$$K(m_{a1}, m_{a2}) = E_c \, \pi(a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$



Grazing trajectory

Collision efficiency:

$$E_c = \frac{y_c^2}{(a_1 + a_2)^2}$$

Droplet inertia is the key; without it, there will be no collisions. This is why collision efficiency for droplets smaller than 10 μm is very small.

$$K(m_{a1}, m_{a2}) = E_c \pi(a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$

TABLE 1. Radius ratio r/R.

Collector drop radius (μm)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
300	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
200	0.87	0.96	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	0.77	0.93	0.97	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	0.50	0.79	0.91	0.95	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
70	0.20	0.58	0.75	0.84	0.88	0.90	0.92	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.97	1.0	1.02	1.04	2.3	4.0
60	0.05	0.43	0.64	0.77	0.84	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.91	0.92	0.93	0.95	1.0	1.03	1.7	3.0
50	0.005	0.40	0.60	0.70	0.78	0.83	0.86	0.88	0.90	0.90	0.90	0.90	0.89	0.88	0.88	0.89	0.92	1.01	1.3	2.3
40	0.001	0.07	0.28	0.50	0.62	0.68	0.74	0.78	0.80	0.80	0.80	0.78	0.77	0.76	0.77	0.77	0.78	0.79	0.95	1.4
30	0.0001	0.002	0.02	0.04	0.085	0.17	0.27	0.40	0.50	0.55	0.58	0.59	0.58	0.54	0.51	0.49	0.47	0.45	0.47	0.52
20	0.0001	0.0001	0.005	0.016	0.022	0.03	0.043	0.052	0.064	0.072	0.079	0.082	0.080	0.076	0.067	0.057	0.048	0.040	0.033	0.027
10	0.0001	0.0001	0.0001	0.014	0.017	0.019	0.022	0.027	0.030	0.033	0.035	0.037	0.038	0.038	0.037	0.036	0.035	0.032	0.029	0.027

Hall (*J. Atmos. Sci*. 1980)
(compilation of many theoretical studies and laboratory measurements)

$$K(m_{a1}, m_{a2}) = E_c \pi(a_1 + a_2)^2 \ |(V_{a_1} - V_{a_2}|$$

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300	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
200	0.87	0.96	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	0.77	0.93	0.97	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	0.50	0.79	0.91	0.95	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
70	0.20	0.58	0.75	0.84	0.88	0.90	0.92	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.97	1.0	1.02	1.04	2.3	4.0
60	0.05	0.43	0.64	0.77	0.84	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.91	0.92	0.93	0.95	1.0	1.03	1.7	3.0
50	0.005	0.40	0.60	0.70	0.78	0.83	0.86	0.88	0.90	0.90	0.90	0.90	0.89	0.88	0.88	0.89	0.92	1.01	1.3	2.3
40	0.001	0.07	0.28	0.50	0.62	0.68	0.74	0.78	0.80	0.80	0.80	0.78	0.77	0.76	0.77	0.77	0.78	0.79	0.95	1.4
30	0.0001	0.002	0.02	0.04	0.085	0.17	0.27	0.40	0.50	0.55	0.58	0.59	0.58	0.54	0.51	0.49	0.47	0.45	0.47	0.52
20	0.0001	0.0001	0.005	0.016	0.022	0.03	0.043	0.052	0.064	0.072	0.079	0.082	0.080	0.076	0.067	0.057	0.048	0.040	0.033	0.027
10	0.0001	0.0001	0.0001	0.014	0.017	0.019	0.022	0.027	0.030	0.033	0.035	0.037	0.038	0.038	0.037	0.036	0.035	0.032	0.029	0.027

Hall (*J. Atmos. Sci*. 1980)
(compilation of many theoretical studies and laboratory measurements)

$$K(m_{a1}, m_{a2}) = E_c \pi (a_1 + a_2)^2 \, |(V_{a_1} - V_{a_2}|$$

TABLE 1. Radius ratio r/R.

Collector drop radius (μm)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
300	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
200	0.87	0.96	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	0.77	0.93	0.97	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	0.50	0.79	0.91	0.95	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
70	0.20	0.58	0.75	0.84	0.88	0.90	0.92	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.97	1.0	1.02	1.04	2.3	4.0
60	0.05	0.43	0.64	0.77	0.84	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.91	0.92	0.93	0.95	1.0	1.03	1.7	3.0
50	0.005	0.40	0.60	0.70	0.78	0.83	0.86	0.88	0.90	0.90	0.90	0.90	0.89	0.88	0.88	0.89	0.92	1.01	1.3	2.3
40	0.001	0.07	0.28	0.50	0.62	0.68	0.74	0.78	0.80	0.80	0.80	0.78	0.77	0.76	0.77	0.77	0.78	0.79	0.95	1.4
30	0.0001	0.002	0.02	0.04	0.085	0.17	0.27	0.40	0.50	0.55	0.58	0.59	0.58	0.54	0.51	0.49	0.47	0.45	0.47	0.52
20	0.0001	0.0001	0.005	0.016	0.022	0.03	0.043	0.052	0.064	0.072	0.079	0.082	0.080	0.076	0.067	0.057	0.048	0.040	0.033	0.027
10	0.0001	0.0001	0.0001	0.014	0.017	0.019	0.022	0.027	0.030	0.033	0.035	0.037	0.038	0.038	0.037	0.036	0.035	0.032	0.029	0.027

Hall (*J. Atmos. Sci.* 1980)
(compilation of many theoretical studies and laboratory measurements)

$$K(m_{a1}, m_{a2}) = E_c \pi (a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$

TABLE 1. Radius ratio r/R.

Collector drop radius (μm)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
300	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
200	0.87	0.96	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	0.77	0.93	0.97	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	0.50	0.79	0.91	0.95	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
70	0.20	0.58	0.75	0.84	0.88	0.90	0.92	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.97	1.0	1.02	1.04	2.3	4.0
60	0.05	0.43	0.64	0.77	0.84	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.91	0.92	0.93	0.95	1.0	1.03	1.7	3.0
50	0.005	0.40	0.60	0.70	0.78	0.83	0.86	0.88	0.90	0.90	0.90	0.90	0.89	0.88	0.88	0.89	0.92	1.01	1.3	2.3
40	0.001	0.07	0.28	0.50	0.62	0.68	0.74	0.78	0.80	0.80	0.80	0.78	0.77	0.76	0.77	0.77	0.78	0.79	0.95	1.4
30	0.0001	0.002	0.02	0.04	0.085	0.17	0.27	0.40	0.50	0.55	0.58	0.59	0.58	0.54	0.51	0.49	0.47	0.45	0.47	0.52
20	0.0001	0.0001	0.005	0.016	0.022	0.03	0.043	0.052	0.064	0.072	0.079	0.082	0.080	0.076	0.067	0.057	0.048	0.040	0.033	0.027
10	0.0001	0.0001	0.0001	0.014	0.017	0.019	0.022	0.027	0.030	0.033	0.035	0.037	0.038	0.038	0.037	0.036	0.035	0.032	0.029	0.027

Hall (*J. Atmos. Sci*. 1980)
(compilation of many theoretical studies and laboratory measurements)

Adiabatic parcel model

$$c_p \frac{dT}{dt} = -g\,w + L\,C$$

$$\frac{dq_v}{dt} = -C$$

$$\frac{dp}{dt} = -\rho_o w g$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial r}\left(\frac{dr}{dt}\phi\right) = \left(\frac{\partial \phi}{\partial t}\right)_{\mathrm{act}} + \left(\frac{\partial \phi}{\partial t}\right)_{\mathrm{coal}}$$

Grabowski and Wang, *Atmos. Chem. Phys*. 2009

In the discrete system consisting of $\mathcal{N}$ bins (or classes) of drop sizes, the spectral density function for each bin (i) (radius $r^{(i)}$) is defined as $\phi^{(i)}=n^{(i)}/\Delta r^{(i)}$, where $n^{(i)}$ is the concentration (per unit mass) of drops in the bin i, $\Delta r^{(i)}=r^{(i+1/2)}-r^{(i-1/2)}$ is the width of this bin, and the bin boundaries are defined as $r^{(i+1/2)}=0.5(r^{(i+1)}+r^{(i)})$. This transforms the continuous Eq. (1d) into a system of $\mathcal{N}$ coupled equations:

$$\frac{\partial \phi^{(i)}}{\partial t} = \left(\frac{\partial \phi^{(i)}}{\partial t}\right)_{\text{cond}} + \left(\frac{\partial \phi^{(i)}}{\partial t}\right)_{\text{act}} + \left(\frac{\partial \phi^{(i)}}{\partial t}\right)_{\text{coal}} , \quad \text{for } i = 1, ..., \mathcal{N} \qquad (3)$$

where the first term on the right-hand-side represents the condensational growth term in (1d) (i.e., the transport of droplets from one bin to another due to their growth by diffusion of water vapor) and, as in (1d), the second and the third term represent cloud droplet activation and growth by collision-coalescence. The cloud water mixing ratio in the discrete system is given by $q_c=\sum_{i=1}^{\mathcal{N}} q_i^{(0)}\, \phi^{(i)}\, \Delta r^{(i)}$, where $q_i^{(0)}$ is the mass of a single droplet with radius $r^{(i)}$.

$$N_{\text{CCN}} = C_0\,(100\,S)^k$$

Grabowski and Wang, *Atmos. Chem. Phys*. 2009

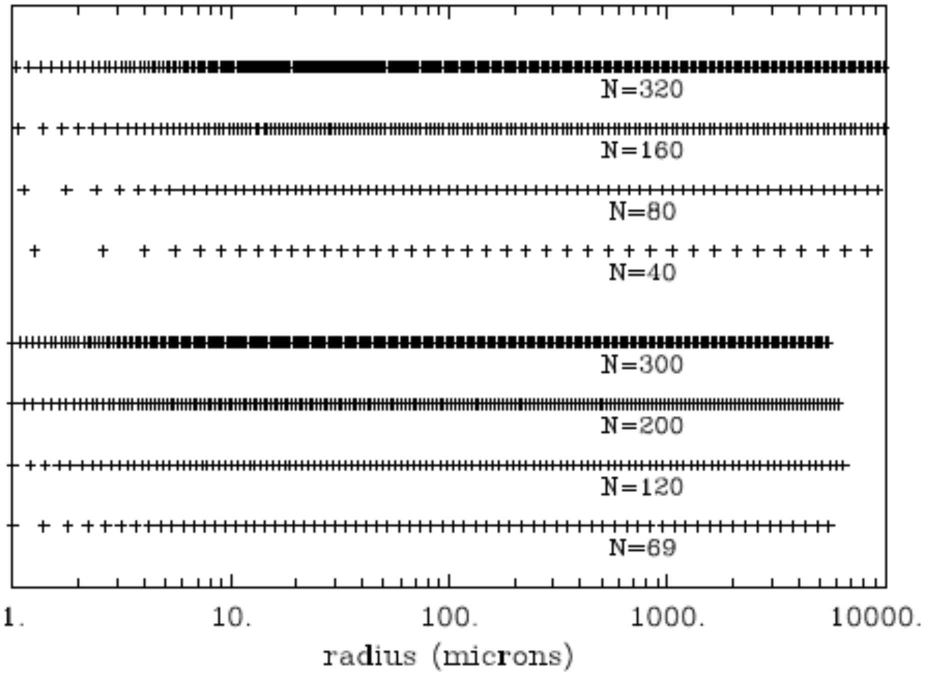


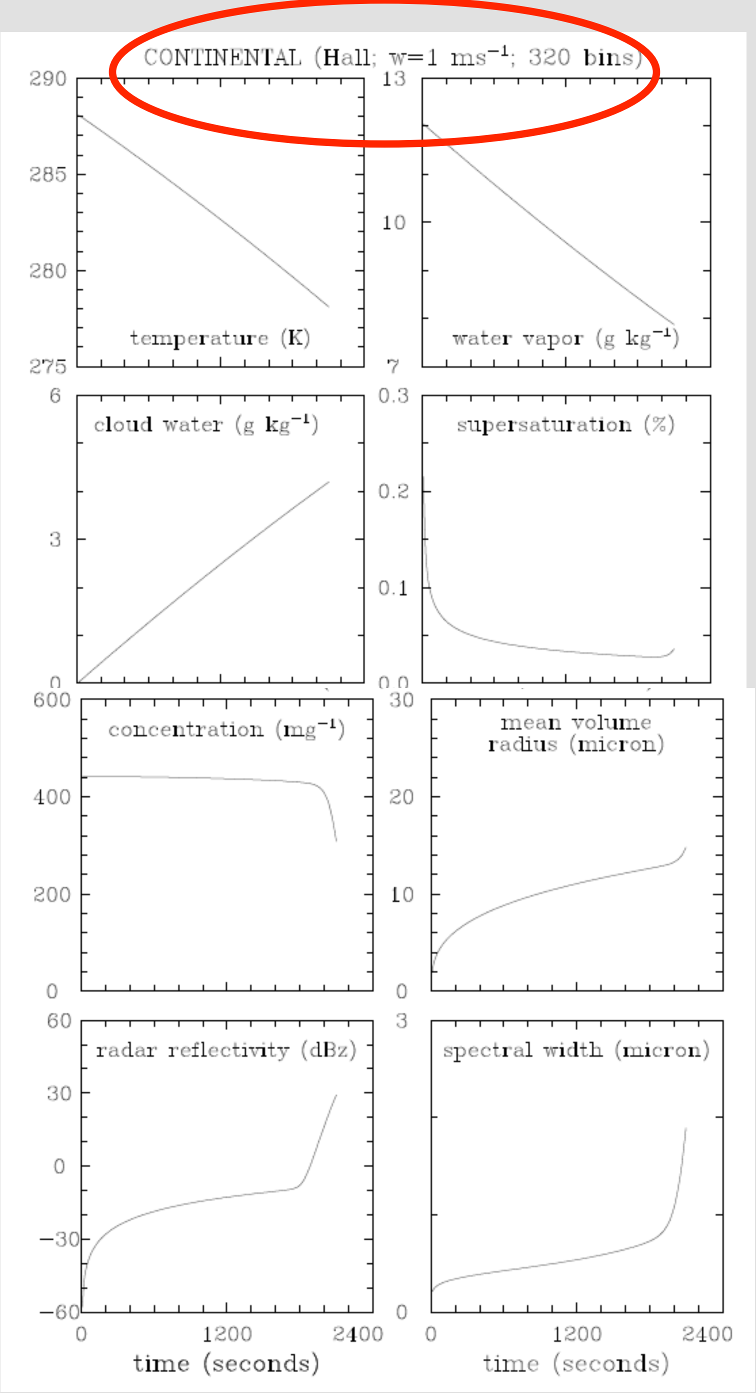
Table 1. Grid formulation parameters and time steps for collisional (Δt_{coll}) and condensational (Δt_{cond}) growth for the case of w=1 m s^{-1}.

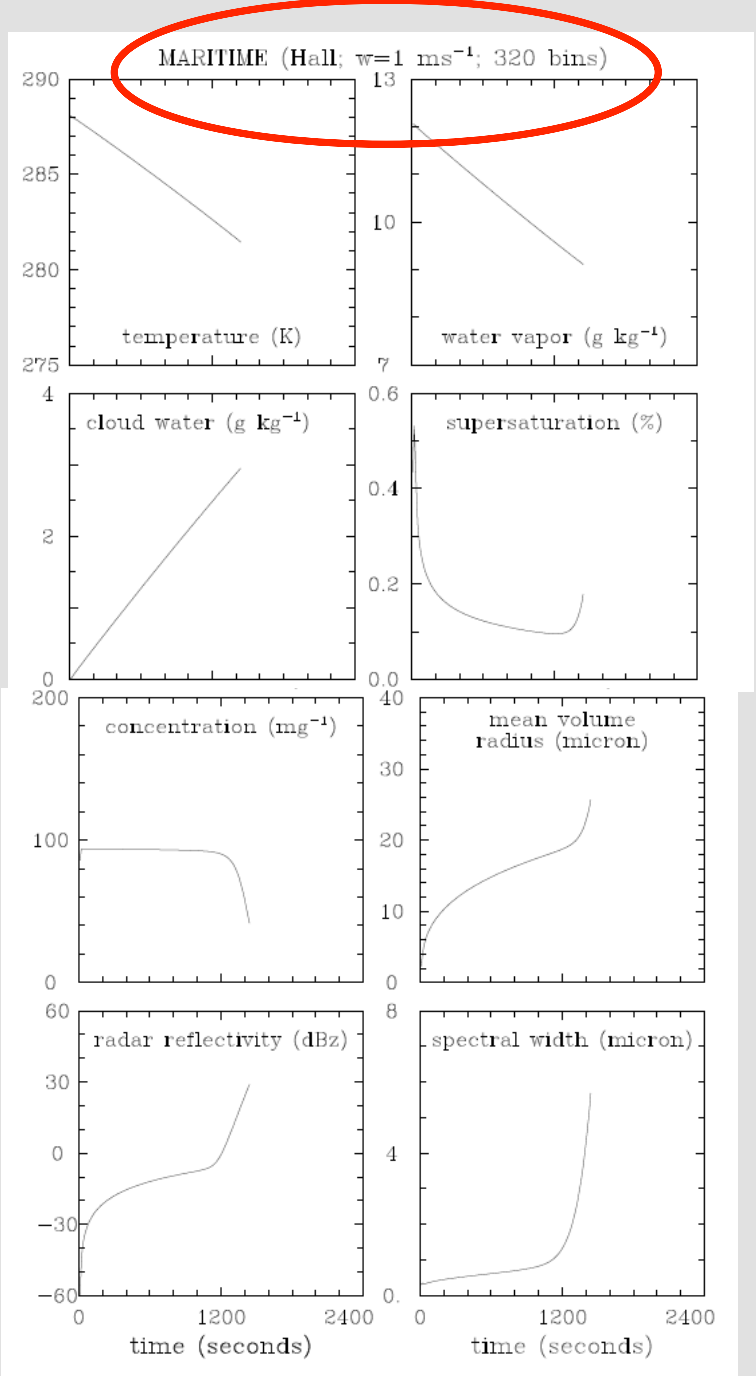
Eq. (6)				
$\mathcal{N}$	α	β	Δt_{coll}	Δt_{cond}
69	0.25	0.055	1 s	0.2 s
120	0.125	0.032	1 s	0.2 s
200	0.075	0.019	0.5 s	0.1 s
300	0.05	0.0125	0. 2s	0.05 s
Eq. (7)				
$\mathcal{N}$	α	s	Δt_{coll}	Δt_{cond}
40	1.0	1	2 s	0.5 s
80	0.5	2	1 s	0.5 s
160	0.25	4	1 s	0.5 s
320	0.125	8	0.5 s	0.1 s

$$r_i = (i-1)\,\alpha \; + \; 10^{(i-1)\,\beta} \quad \text{for } i = 1, ..., \mathcal{N} \quad , \tag{6}$$

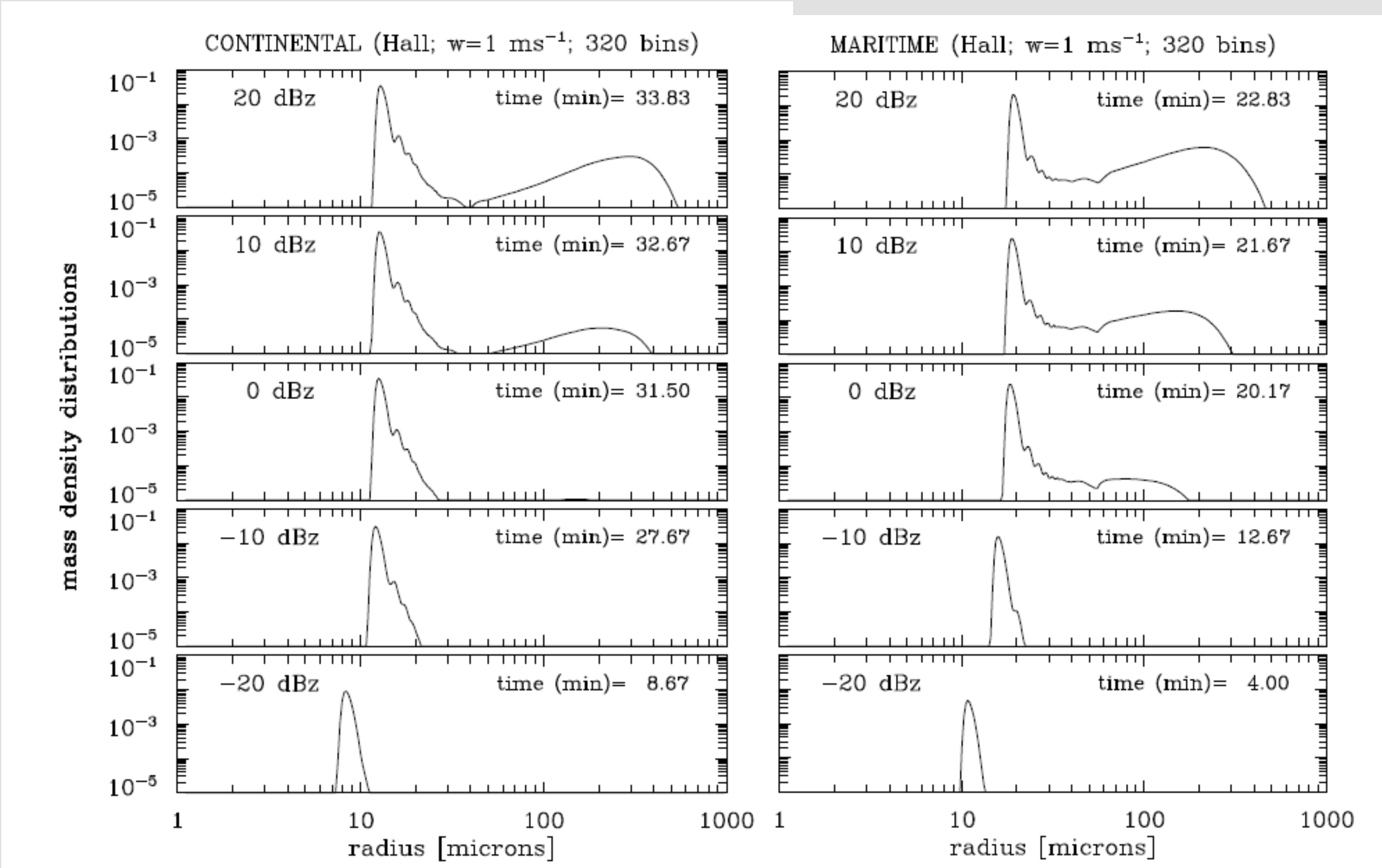
$$r_i = (i-1)\,\alpha \; + \; \left(\frac{3m_i}{4\pi\rho_w}\right)^{1/3} \quad \text{for } i = 1, ..., \mathcal{N} \; , \tag{7}$$

where the mass m_i is given by the recurrence $m_i/m_{i-1}=2^{1/s}$ and m_0 is taken as the mass of a droplet with 1-μm radius.

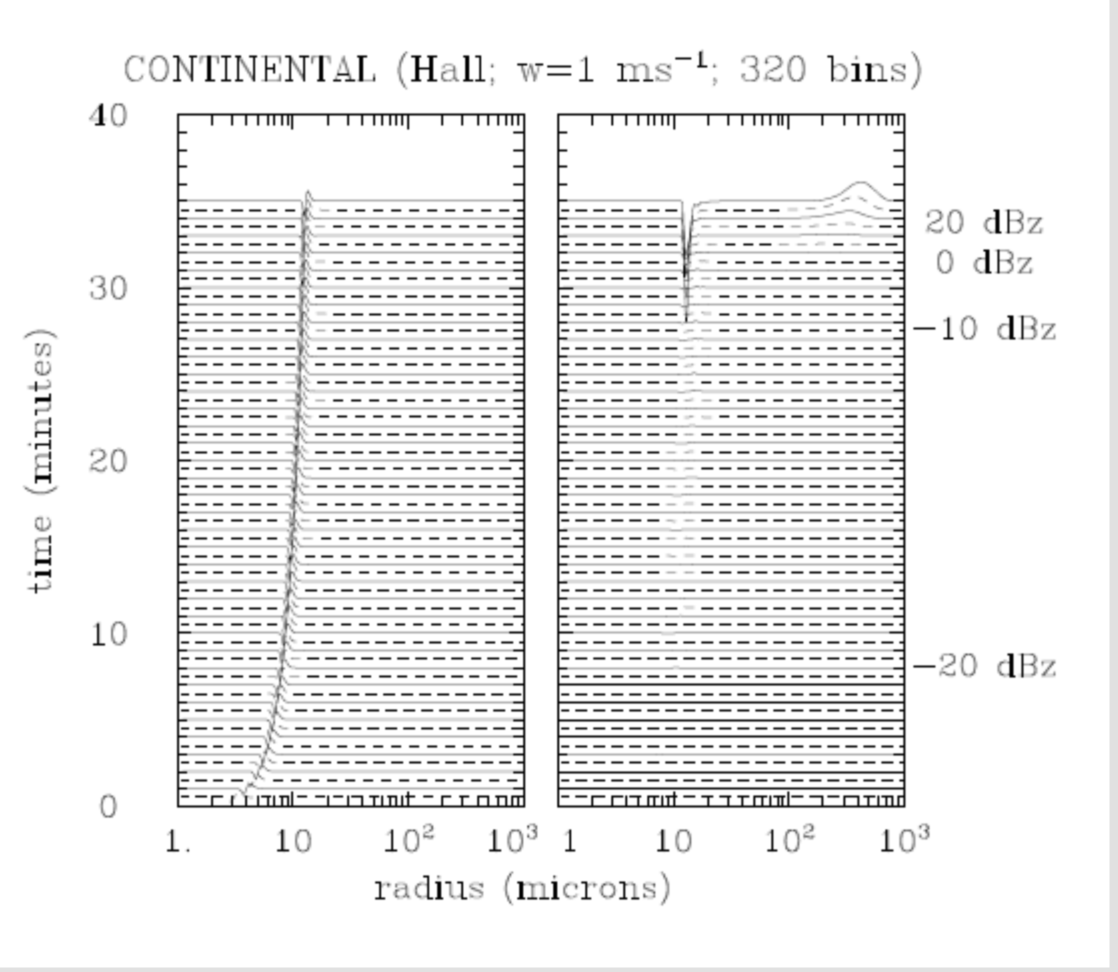




Grabowski and Wang, *Atmos. Chem. Phys.* 2009

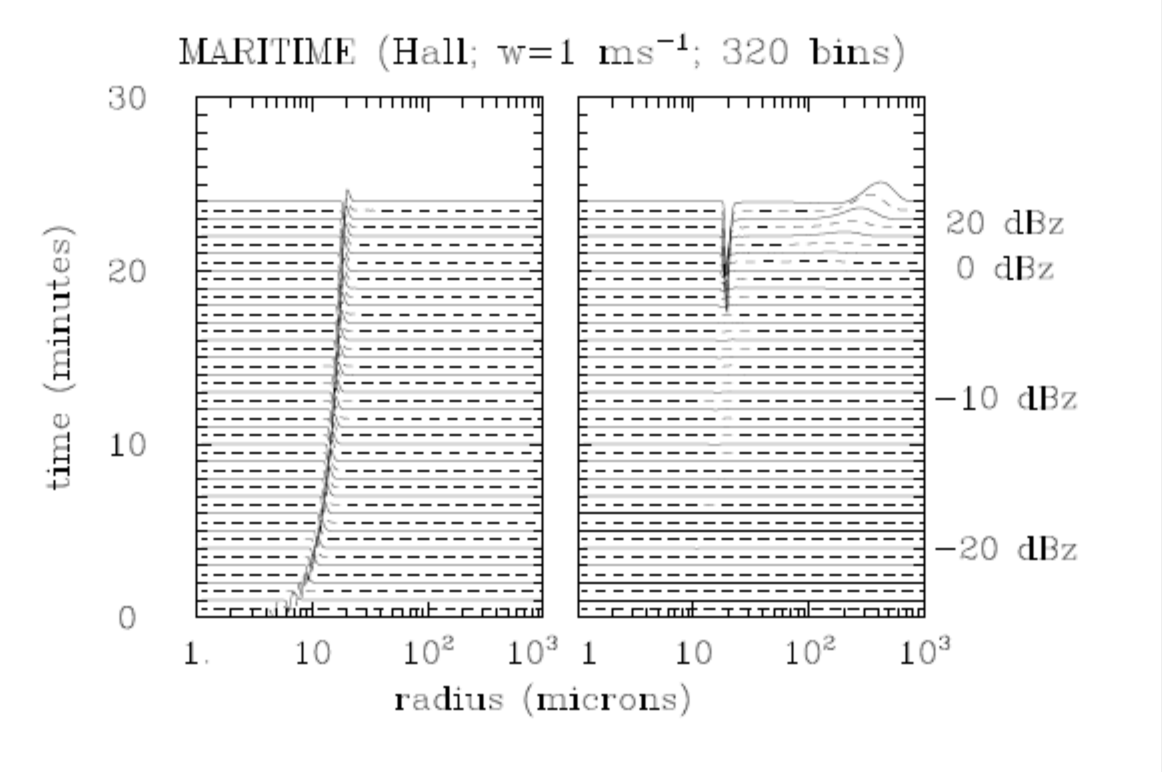


Grabowski and Wang, *Atmos. Chem. Phys*. 2009



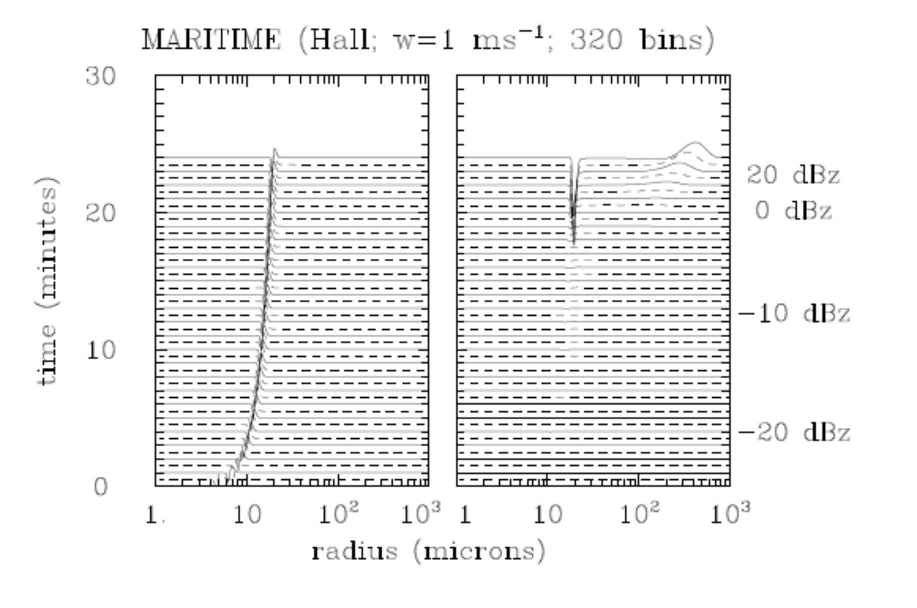
Mass transfer rates (left – condensation, right – collision/coalescence); CONTINENTAL case

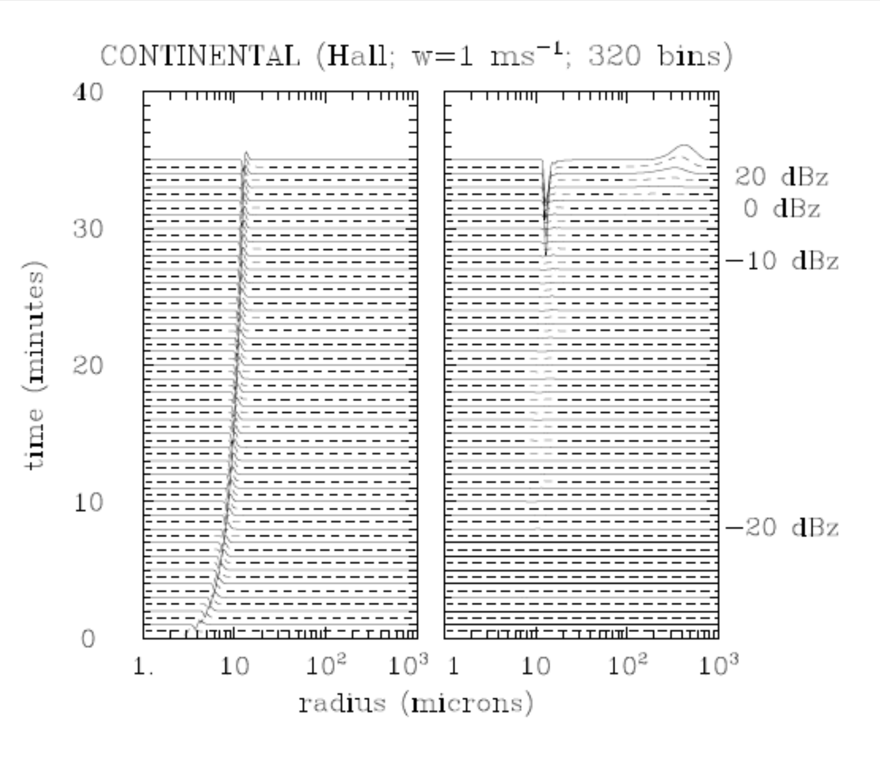
Grabowski and Wang, *Atmos. Chem. Phys*. 2009



Mass transfer rates (left – condensation, right – collision/coalescence); MARITIME case

Grabowski and Wang, *Atmos. Chem. Phys*. 2009





Grabowski and Wang, *Atmos. Chem. Phys*. 2009

BIN-RESOLVING WARM RAIN MODEL:

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} \sum_{i=1}^{N} C_d^{(i)}$$

$$\frac{dq_v}{dt} = - \sum_{i=1}^{N} C_d^{(i)}$$

for $i = 1, N$:

$$\frac{dq_c^{(i)}}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} \left[\rho q_c^{(i)} v_t(r^{(i)}) \right] + C_d^{(i)} + F_+^{(i)} - F_-^{(i)}$$

θ - potential temperature
q_v - *water vapor* mixing ratio
$q_c^{(i)}$ - *cloud water* mixing ratio for drops in size bin i $(i = 1,\ N;\ N \sim 100)$

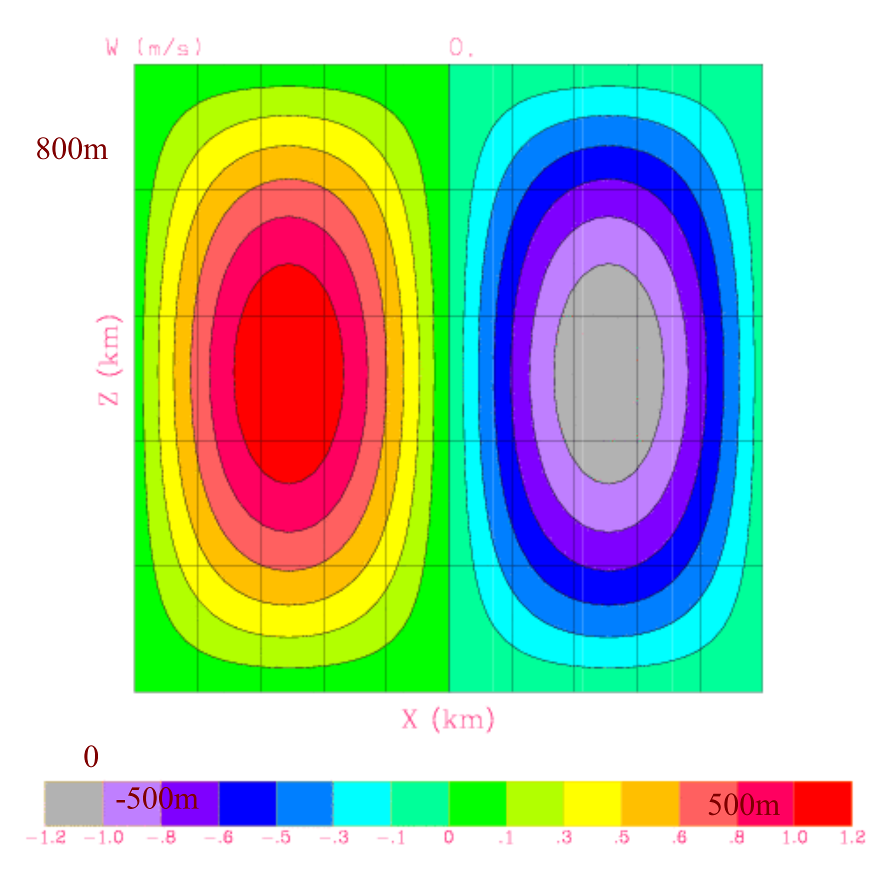
$C_d^{(i)}$ - condensation/evaporation rate for drops in size bin i; depends on super/undersaturation $S = q_v/q_{vs} - 1$ and drop size $r^{(i)}$.

$F_+^{(i)}$ - source due to collisions between j and k resulting in drops in i
$F_-^{(i)}$ - sink due to collisions between i and all other bins

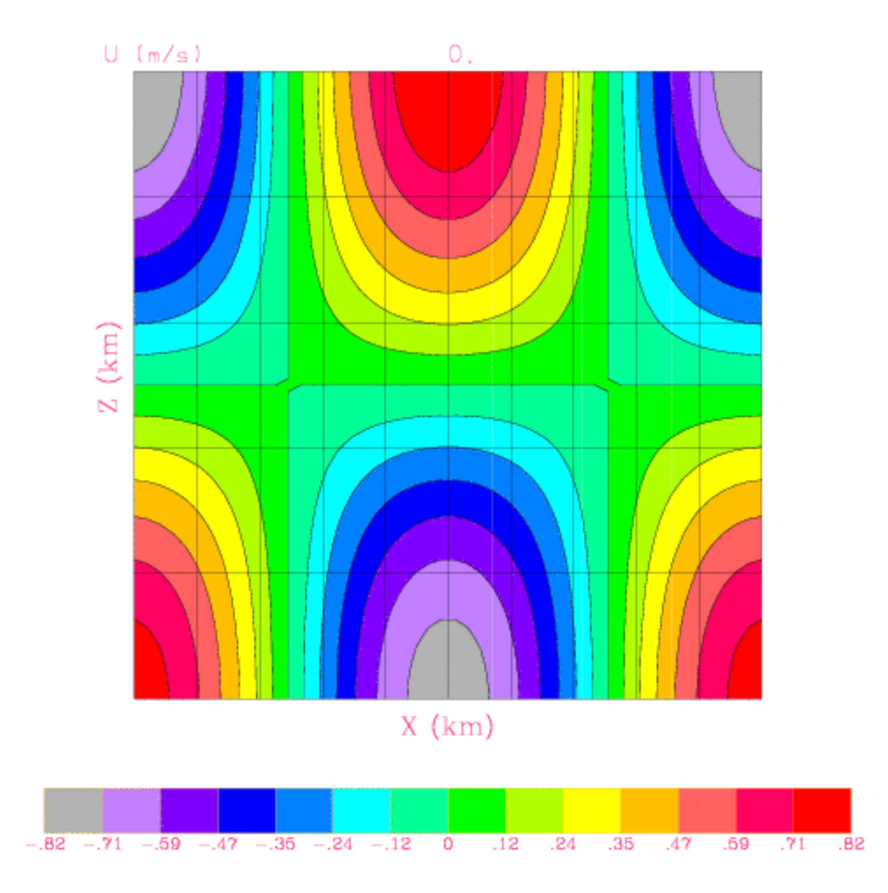
$$\frac{\partial \Psi}{\partial t} + \frac{1}{\rho_o} \nabla(\rho_o \mathbf{u} \Psi) = S$$

$$\rho_o = \rho_o(z)$$

Kinematic (prescribed-flow) model of microphysical processes in Stratocumulus (2D: x-z)



Vertical velocity



Horizontal velocity

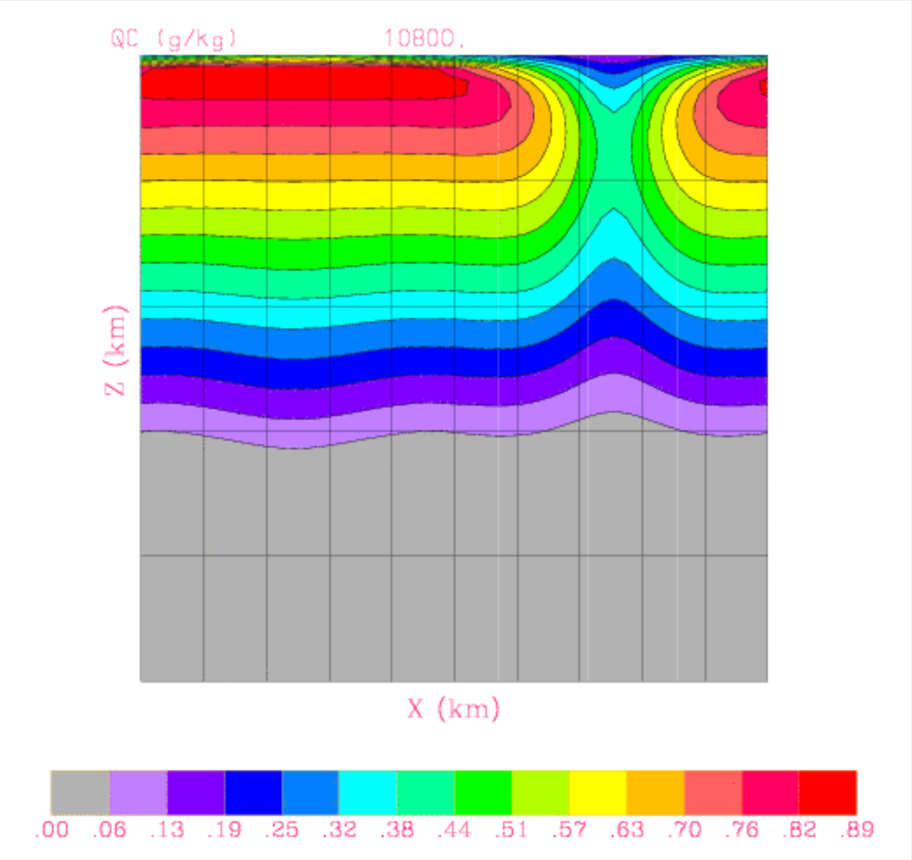
Run up to quasi-steady-state is obtained (typically couple hours)…

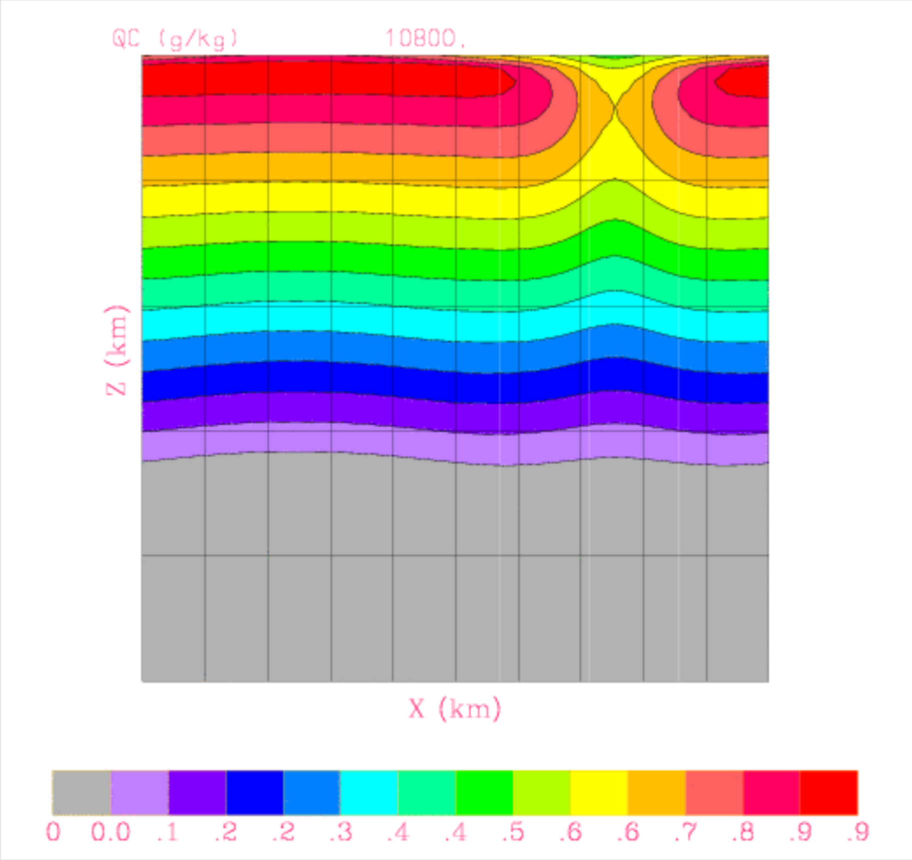
Morrison and Grabowski JAS 2007
Rasinski et al. AR , 2011

Cloud water (after 3hrs)

Maritime (clean)

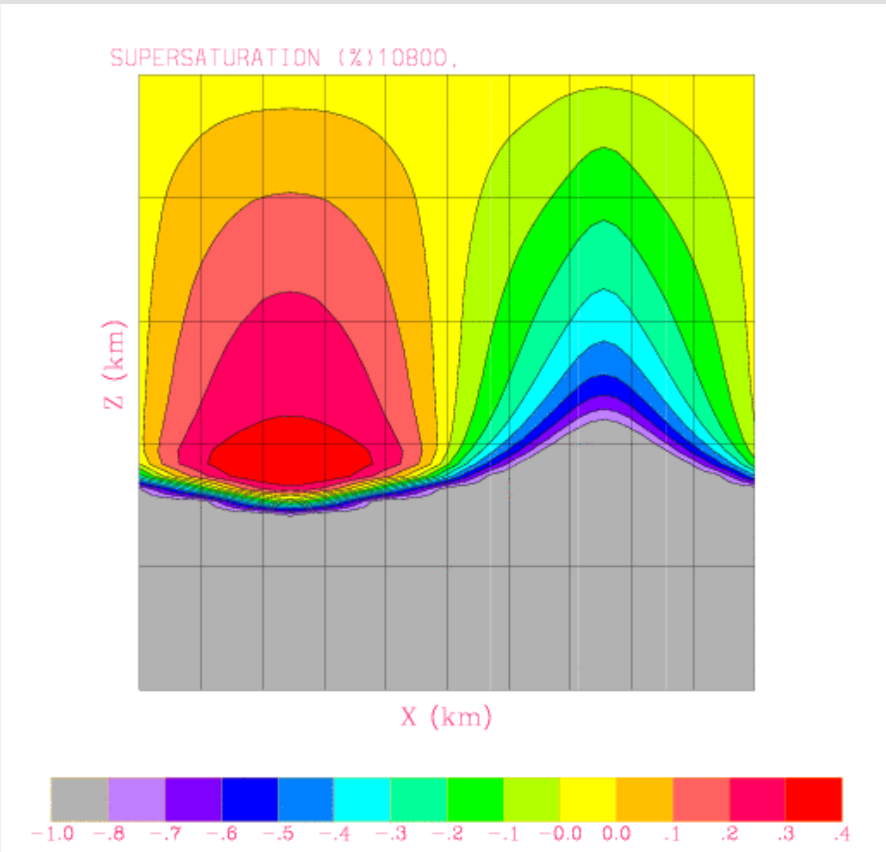
Continental (polluted)



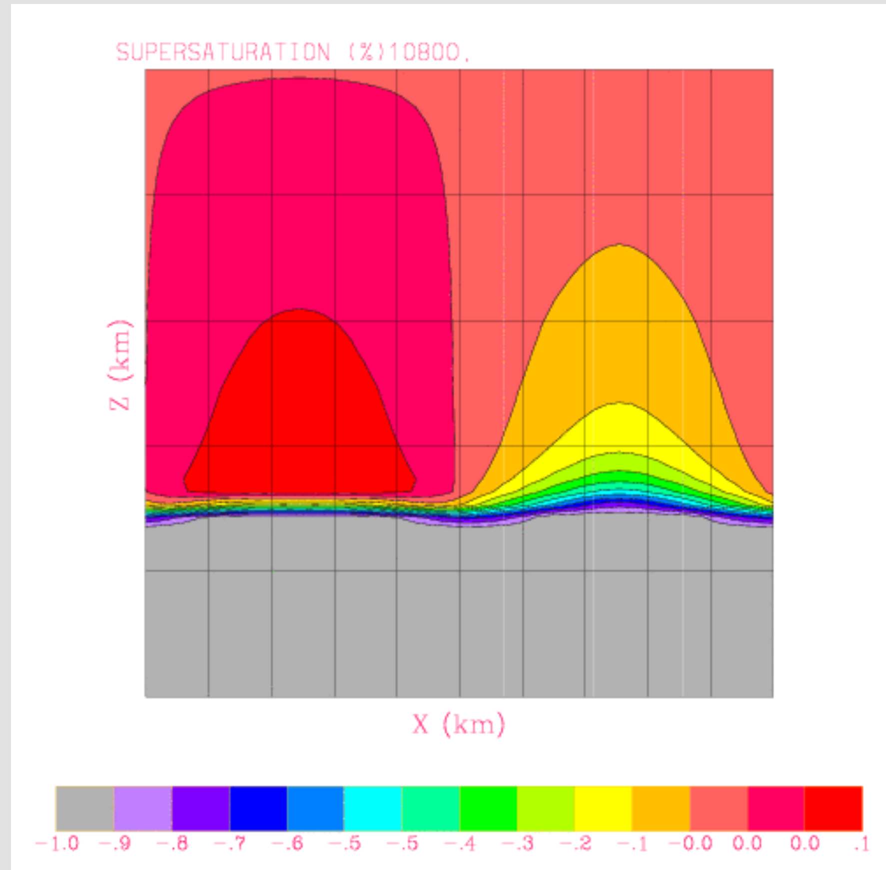


Supersaturation (after 3hrs)

Maritime (clean)

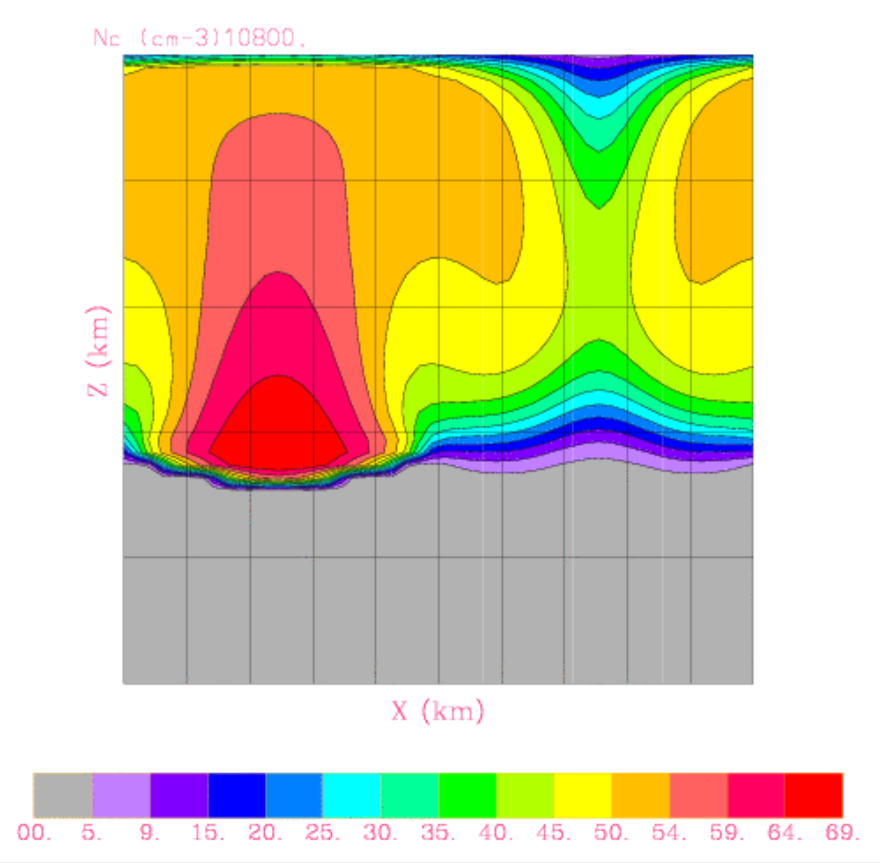


Continental (polluted)

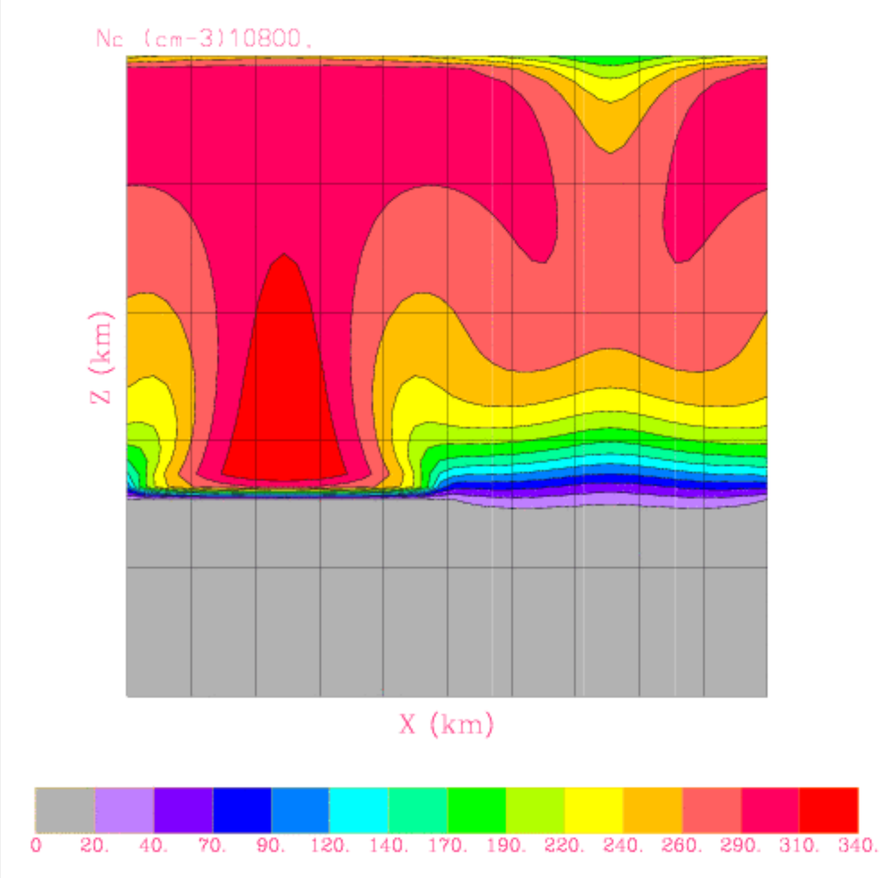


Cloud droplet (r < 20 microns) number concentration

Maritime (clean)

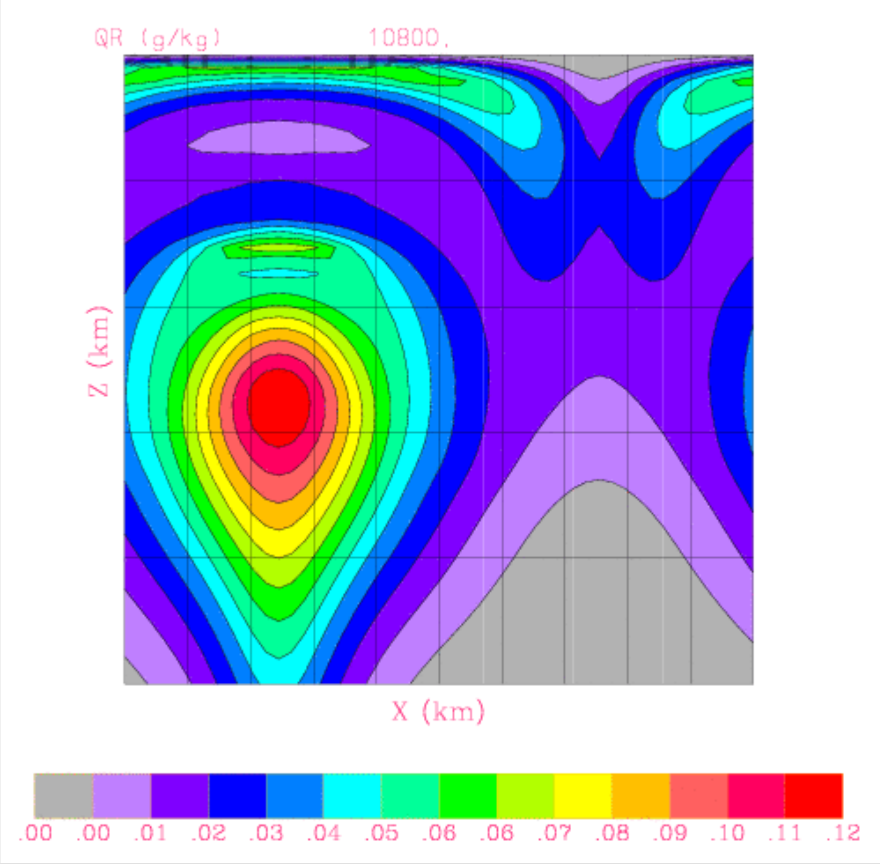


Continental (polluted)

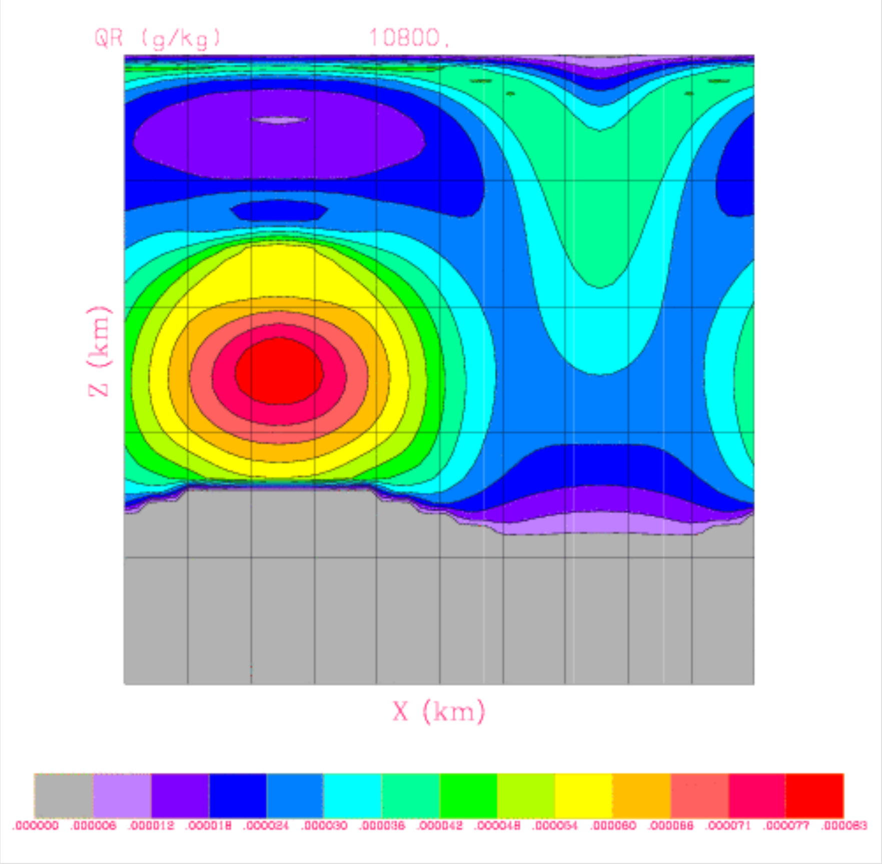


Drizzle (r > 25 microns) mixing ratio

Maritime (clean)



Continental (polluted)



Traditional bulk model is computationally efficient (just 2 variables for condensed water).

Traditional bin-resolving (detailed) microphysics is computationally demanding (~100 variables).

Is there anything between?

YES, a two-moment bulk scheme, i.e., predicting mass and number of cloud droplets and rain/drizzle drops (just 4 variables; e.g., Morrison and Grabowski JAS 2007, 2008). This scheme also predicts supersaturation.

A bulk two-moment, warm-rain microphysics scheme has been developed based on the approach of Morrison et al. (2005). This scheme predicts the number concentrations (N_c, N_r) and mixing ratios (q_c, q_r) of cloud droplets (subscript c) and drizzle/rain (subscript r). Cloud droplets and drizzle/raindrops are assumed to follow gamma size distributions,

$$N(D) = N_0 D^{\mu} e^{-\lambda D}, \tag{1}$$

where D is diameter, N_0 is the "“intercept” parameter, λ is the slope parameter, and $\mu = 1/\eta^2 - 1$ is the spectral shape parameter (η is the relative radius dispersion: the ratio between the standard deviation and the mean radius). Parameters N_0 and λ are derived from the specified μ and predicted number concentration and mixing ratio of the species (see Morrison et al. 2005). Drizzle/raindrops are assumed to follow a Marshall–Palmer (exponential) size distribution, implying $\mu = 0$.

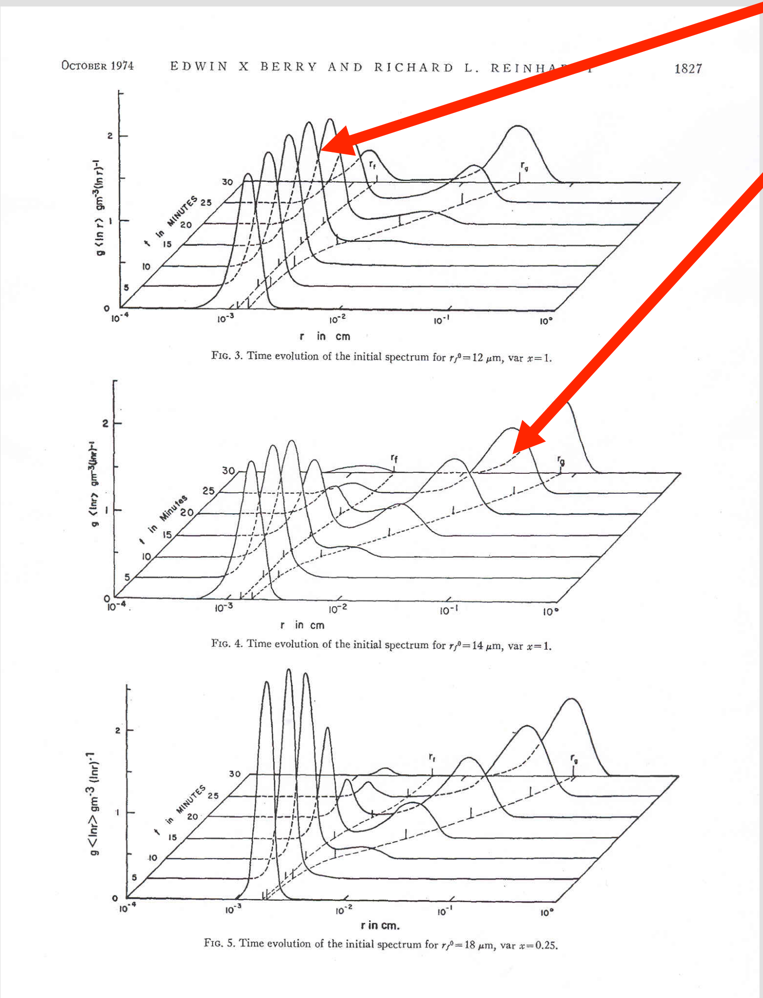
Morrison and Grabowski JAS 2007, 2008

$$\eta = 0.146 - 5.964 \times 10^{-2} \ln\left(\frac{N_c}{2000}\right).$$

WARM-RAIN PHYSICS:

cloud water: q_c , N_c

drizzle/rain water: q_r , N_r



Nucleation of cloud droplets: link to CCN characteristics

Drizzle/rain development: link to mean droplet size

Morrison and Grabowski JAS 2007, 2008

Morrison and Grabowski JAS 2007, 2008

$$\frac{\partial N}{\partial t} + \frac{1}{\rho_a}\nabla\cdot[\rho_a(\mathbf{u} - V_N\mathbf{k})N] = \mathcal{F}_N$$
$$\equiv \left(\frac{\partial N}{\partial t}\right)_{\text{act}} + \left(\frac{\partial N}{\partial t}\right)_{\text{cond}} + \left(\frac{\partial N}{\partial t}\right)_{\text{acc}} + \left(\frac{\partial N}{\partial t}\right)_{\text{auto}}$$
$$+ \left(\frac{\partial N}{\partial t}\right)_{\text{self}} + D(N) \quad \text{and} \tag{3}$$

$$\frac{\partial q}{\partial t} + \frac{1}{\rho_a}\nabla\cdot[\rho_a(\mathbf{u} - V_q\mathbf{k})q] = \mathcal{F}_q$$
$$\equiv \left(\frac{\partial q}{\partial t}\right)_{\text{act}} + \left(\frac{\partial q}{\partial t}\right)_{\text{cond}} + \left(\frac{\partial q}{\partial t}\right)_{\text{acc}} + \left(\frac{\partial q}{\partial t}\right)_{\text{auto}}$$
$$+ D(q), \tag{4}$$

***N_c, q_c* – cloud water concentration and mixing ratio**
***N_r, q_r* – drizzle/rain water concentration and mixing ratio**

$$\frac{\partial N_{\text{act}}}{\partial t} + \frac{1}{\rho_a}\nabla\cdot(\rho_a\mathbf{u}N_{\text{act}}) = \mathcal{F}_{N_{\text{act}}} \equiv \left(\frac{\partial N_c}{\partial t}\right)_{\text{act}} + D(N_{\text{act}}).$$

concentration of activated CCN

The time evolution of absolute supersaturation in Eulerian form is

$$\frac{\partial \delta}{\partial t} + \frac{1}{\rho_a} \nabla \cdot (\rho_a \mathbf{u} \delta) = A - \frac{\delta}{\tau}, \quad (10)$$

$$\delta = q_v - q_s$$

$$\frac{1}{\tau} = \frac{1}{\tau_c} + \frac{1}{\tau_r}$$

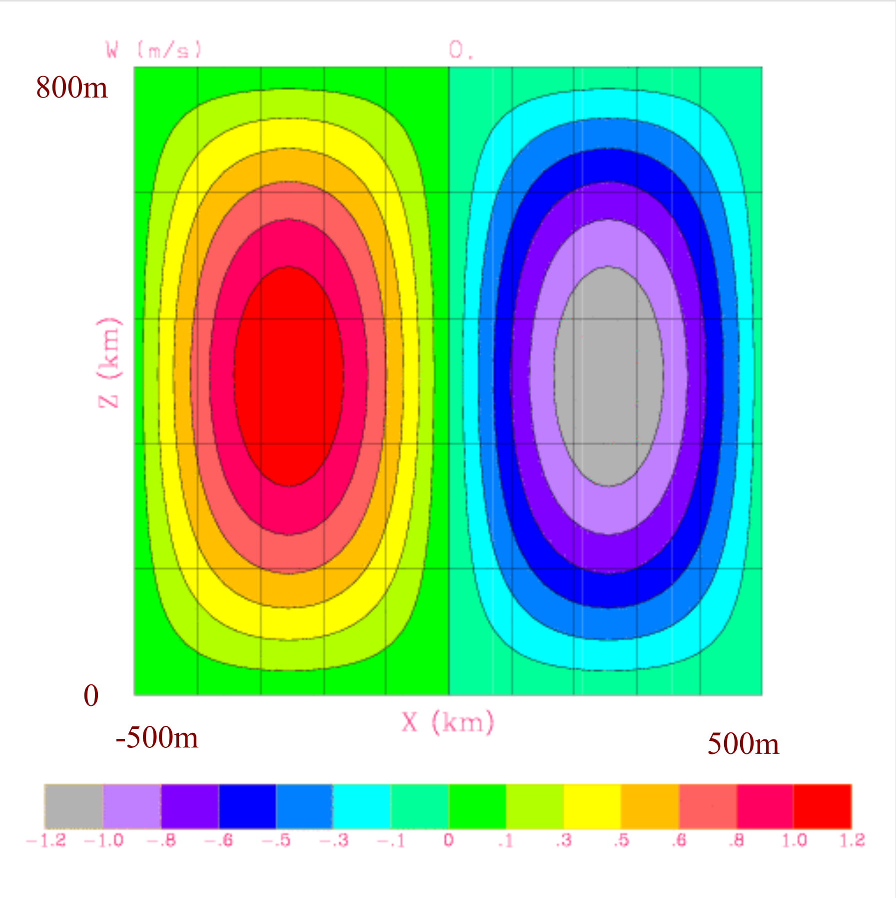
$$\tau_c = (4\pi D_v N_c \langle r \rangle_c)^{-1}, \quad \tau_r = (4\pi D_v N_r \langle r_r f(r) \rangle)^{-1},$$

$$\langle x \rangle \equiv \int x n(r)\, dr / \int n(r)\, dr$$

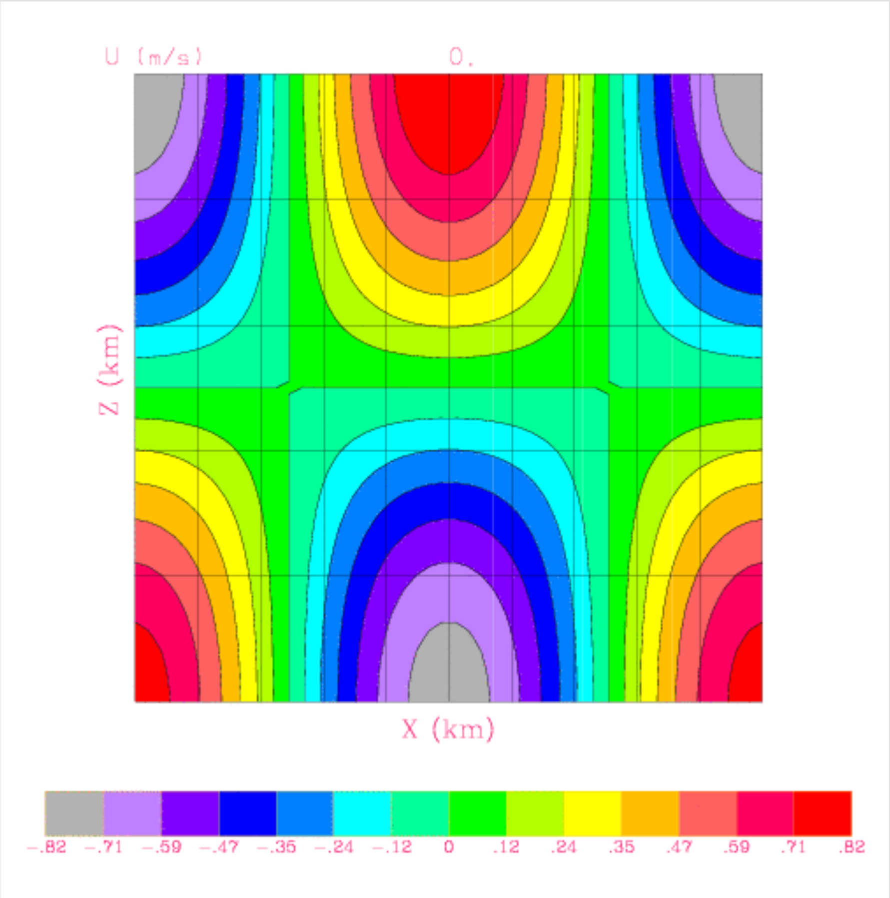
ventilation coefficient

$$A = \left(\frac{dq_v}{dt}\right)_{\text{mix}} - \frac{q_s \rho_a g w}{p - e} - \frac{dq_s}{dT} \times \left[-\frac{gw}{c_p} + \left(\frac{dT}{dt}\right)_{\text{mix}} + \left(\frac{dT}{dt}\right)_{\text{rad}} \right].$$

Kinematic (prescribed-flow) model of microphysical processes in Stratocumulus (2D: x-z)



Vertical velocity



Horizontal velocity

Run up to quasi-steady-state is obtained (typically couple hours)…

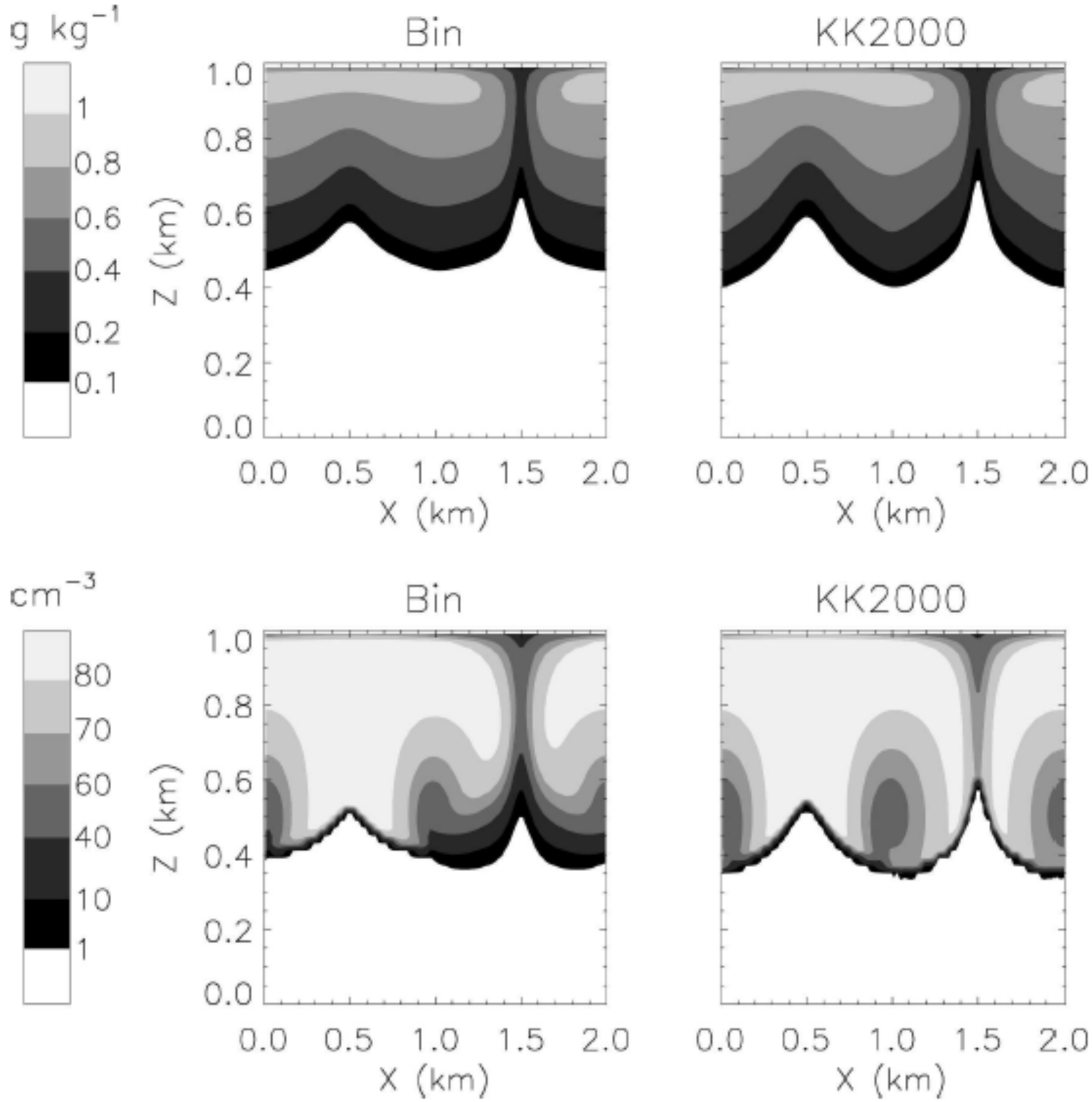


FIG. 4. Plot (x–z) of the (top) equilibrium cloud water mixing ratio and (bottom) droplet number concentration for the bin and bulk (using KK2000) PRISTINE stratocumulus simulations with LHF = 3 W m^{-2}. A similar cloud structure is produced by the bulk model using the SB2001 and B1994 parameterizations.

Morrison and Grabowski JAS 2007

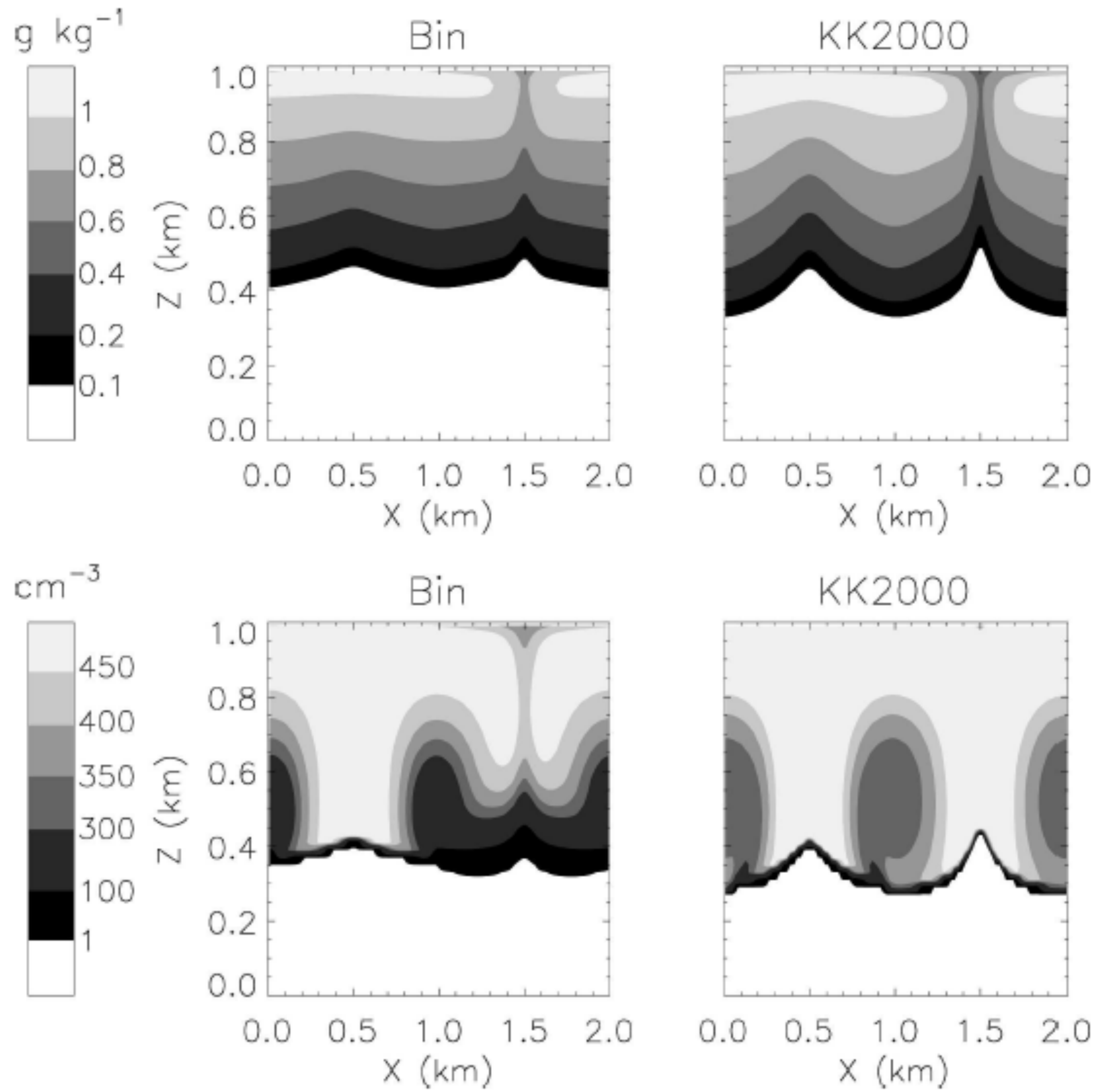
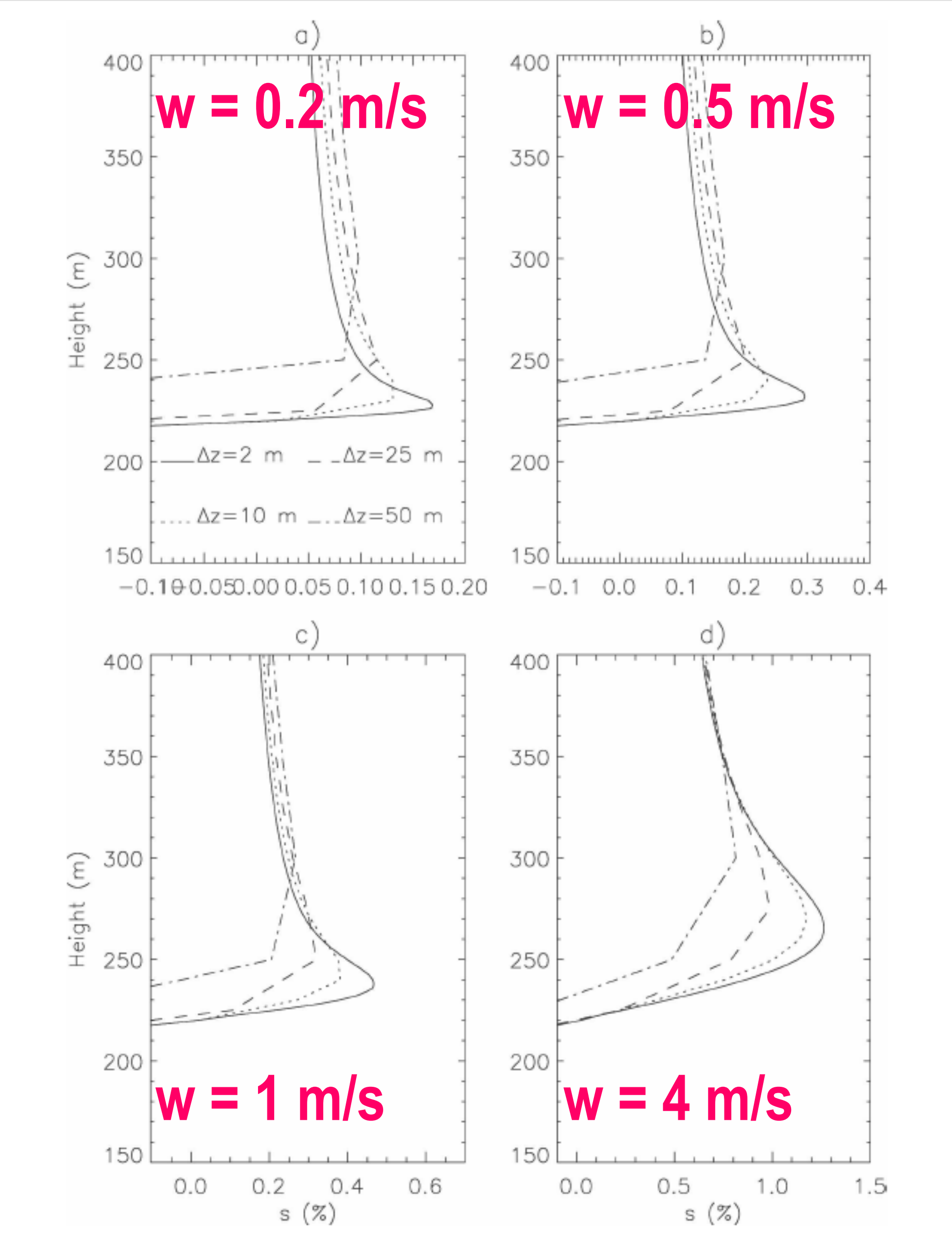


FIG. 5. As in Fig. 4 but for the POLLUTED stratocumulus simulations.

Morrison and Grabowski JAS 2007

1D updraft: profiles of the supersaturation as a function of vertical model resolution

(Morrison and Grabowski JAS 2008)



New trend: Lagrangian treatment of the condensed phase:

The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model

S. Shima,[a]* K. Kusano,[c] A. Kawano,[a] T. Sugiyama[a] and S. Kawahara[b]

Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model

M. Andrejczuk,[1] W. W. Grabowski,[2] J. Reisner,[3] and A. Gadian[1]

Large-Eddy Simulations of Trade Wind Cumuli Using Particle-Based Microphysics with Monte Carlo Coalescence

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A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision

T Riechelmann[1,3], Y Noh[2] and S Raasch[1]

Eulerian dynamics, energy and water vapor transport:

$$\frac{\partial(u\rho)}{\partial t}+\frac{\partial(uu\rho)}{\partial x}+\frac{\partial(wu\rho)}{\partial z}=-\frac{\partial p'}{\partial x}+\Phi_{m,x}+\frac{\partial(\kappa\rho\tau^{11})}{\partial x}+\frac{\partial(\kappa\rho\tau^{13})}{\partial z},$$

$$\frac{\partial(w\rho)}{\partial t}+\frac{\partial(uw\rho)}{\partial x}+\frac{\partial(ww\rho)}{\partial z}=-\frac{\partial p'}{\partial z}-\rho' g+\Phi_{m,z}+\frac{\partial(\kappa\rho\tau^{31})}{\partial x}+\frac{\partial(\kappa\rho\tau^{33})}{\partial z},$$

$$\frac{\partial(\theta\rho)}{\partial t}+\frac{\partial(u\theta\rho)}{\partial x}+\frac{\partial(w\theta\rho)}{\partial z}=\frac{\theta\rho L}{TC_p}f_{cond}+f_{surface-energy}+f_{rad}+\frac{\partial F_{\theta x}}{\partial x}+\frac{\partial F_{\theta z}}{\partial z},$$

$$\frac{\partial(q_v\rho)}{\partial t}+\frac{\partial(uq_v\rho)}{\partial x}+\frac{\partial(wq_v\rho)}{\partial z}=-f_{cond}+f_{surface-gas}+\frac{\partial F_{q_v x}}{\partial x}+\frac{\partial F_{q_v z}}{\partial z},$$

Lagrangian physics of "super-particles"

a single "super-particle" represents a number of the same airborne particles (aerosol, droplet, ice crystal, etc.) with given attributes

$$\frac{dx_i}{dt}=v_i$$

$$\frac{dv_i}{dt}=\frac{1}{\tau_p}(v_i^*-v_i)+g\delta_{i,2}$$

$$\frac{dr}{dt}=\frac{G}{r}(S^*-S_{eq})$$

Coupling

$$\Phi_{m,x}=\sum_{id} m_{id}\frac{M_{id}}{\Delta V}\frac{(u^*-u_{id})}{\tau_{p,id}}$$

$$\Phi_{m,z}=\sum_{id} m_{id}\frac{M_{id}}{\Delta V}\frac{(w^*-w_{id})}{\tau_{p,id}}$$

$$f_{cond}=\sum_{id}\frac{M_{id}}{\Delta V}\frac{dm_{id}}{dt}$$

m_{id} – mass of the super-particle

M_{id} – concentration of super-particles

ΔV – volume of the gridbox

Andrejczuk et al. 2008, 2010

Why Lagrangian SD approach is appealing?

- no numerical diffusion due to advection;

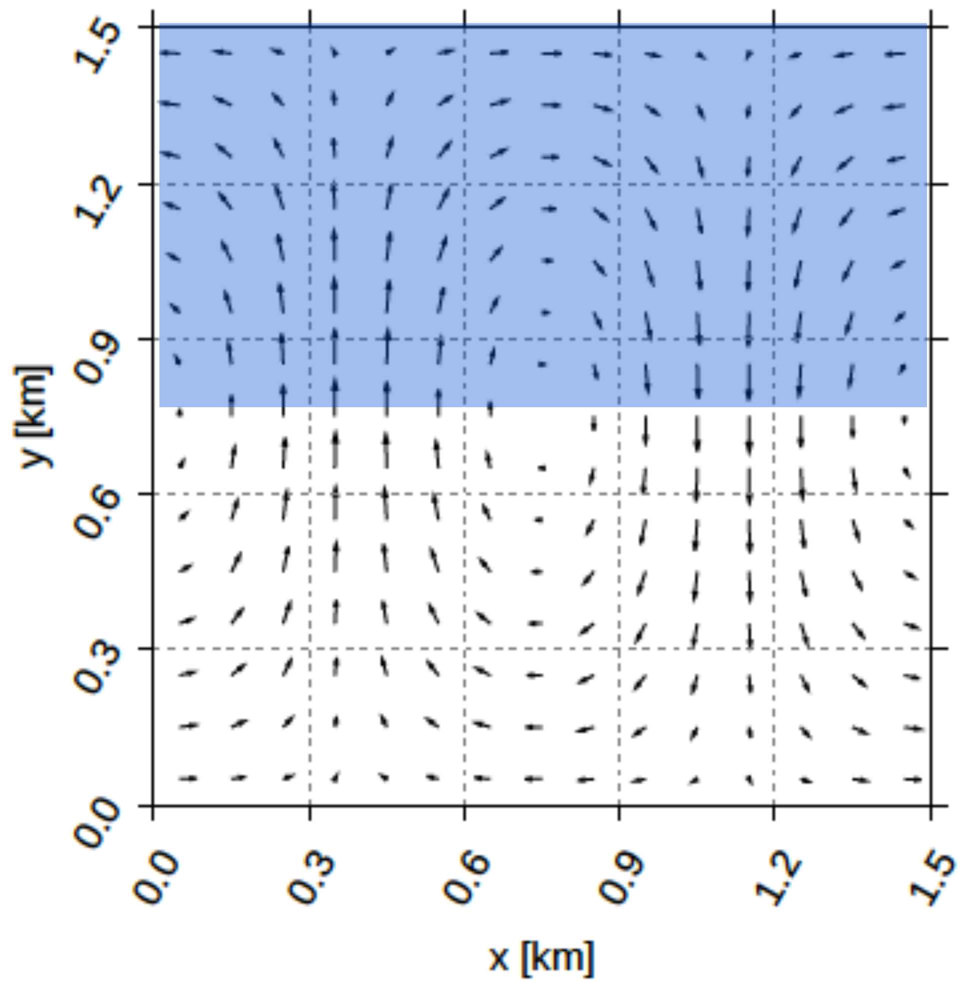
- but sampling errors: one needs ~100 particles per gridbox for simple problems, many more with a longer list of attributes for appropriate sampling of the parameter space;

- straightforward for condensational growth of cloud droplets (initial sampling of the CCN distribution, growth/activation/ evaporation of aerosol/droplet) – *ideal for entrainment/mixing!*

- more complex for collisions (collision of two SDs creates a new SD: two methods in the literature to deal with this…);

- seems ideal to couple with sophisticated subgrid-scale models to represent effects of turbulence (e.g., randomly choose thermodynamic environment within a gridbox, use LEM approach, etc);

- easy representation of ice particle habits and diffusional varsus accretional growth.



y [km]
x [km]
0.0
0.3
0.6
0.9
1.2
1.5

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www.geosci-model-dev.net/8/1677/2015/
doi:10.5194/gmd-8-1677-2015

Geoscientific Model Development
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libcloudph++ 1.0: a single-moment bulk, double-moment bulk, and particle-based warm-rain microphysics library in C++

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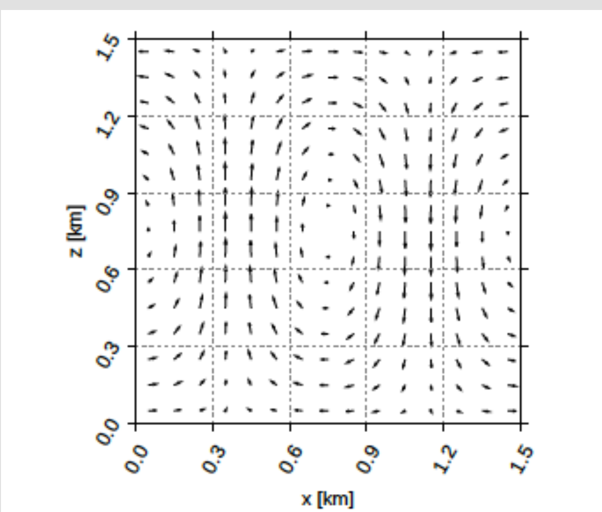
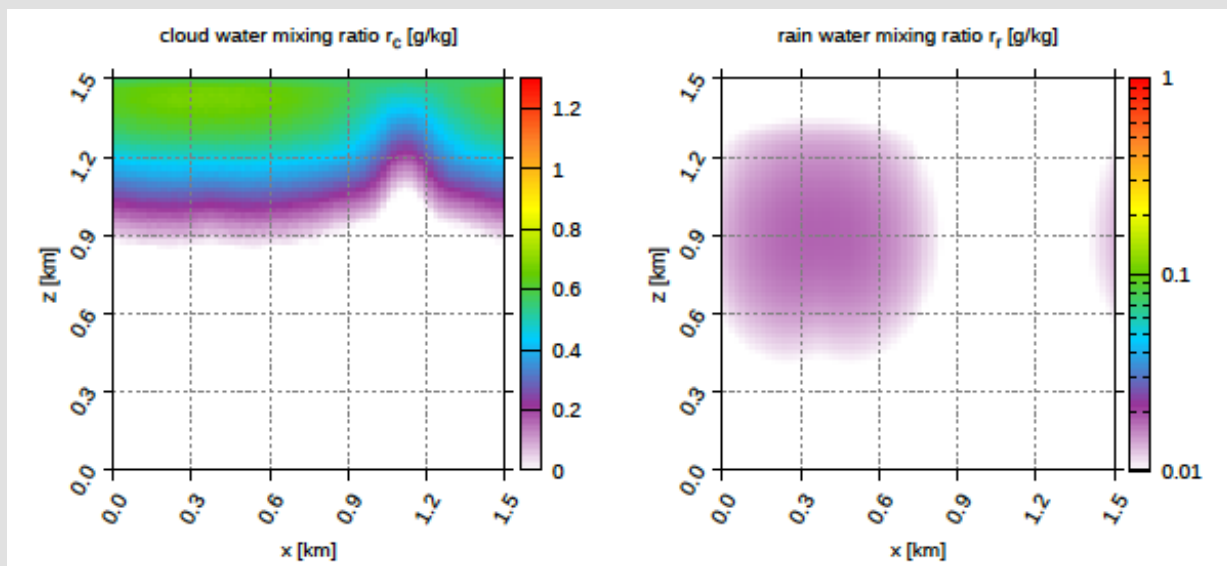
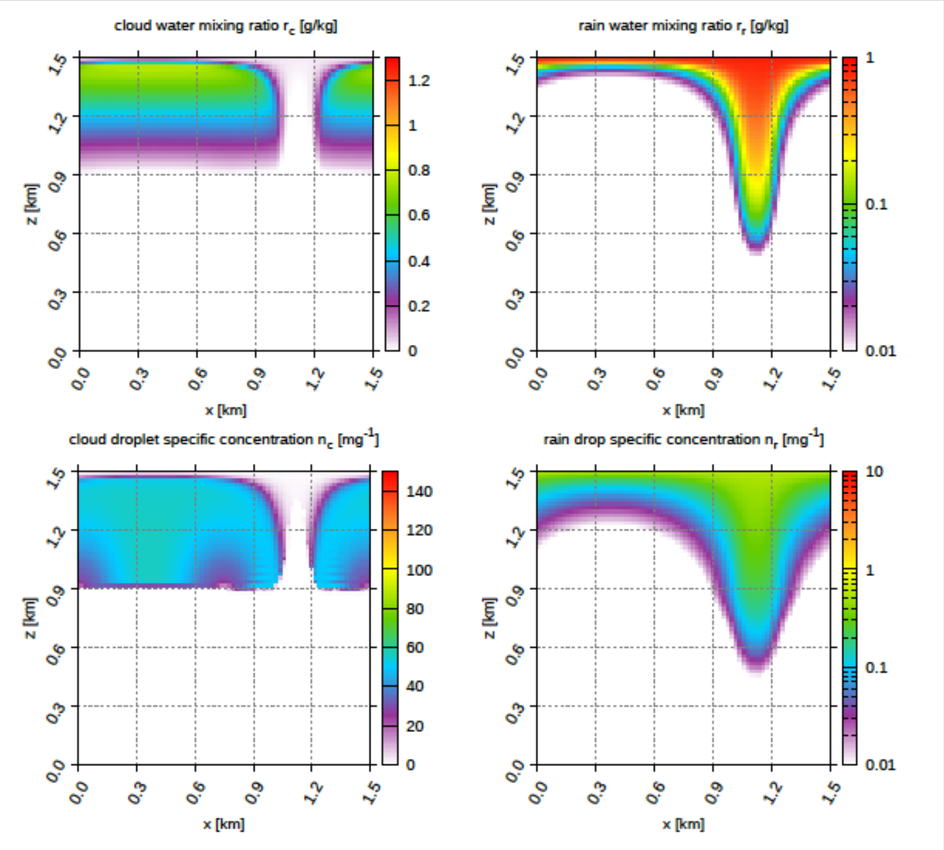
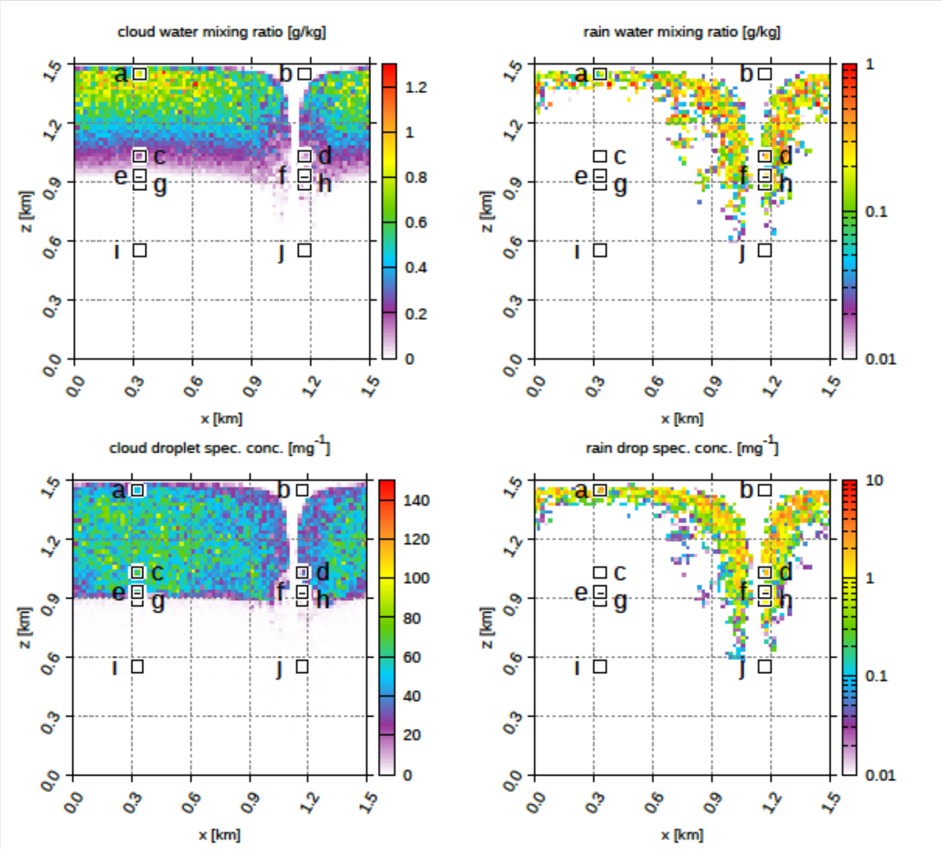
1-moment Eulerian scheme







2-moment Eulerian scheme



Lagrangian scheme (super-droplets)

Arabas et al. GMD 2015

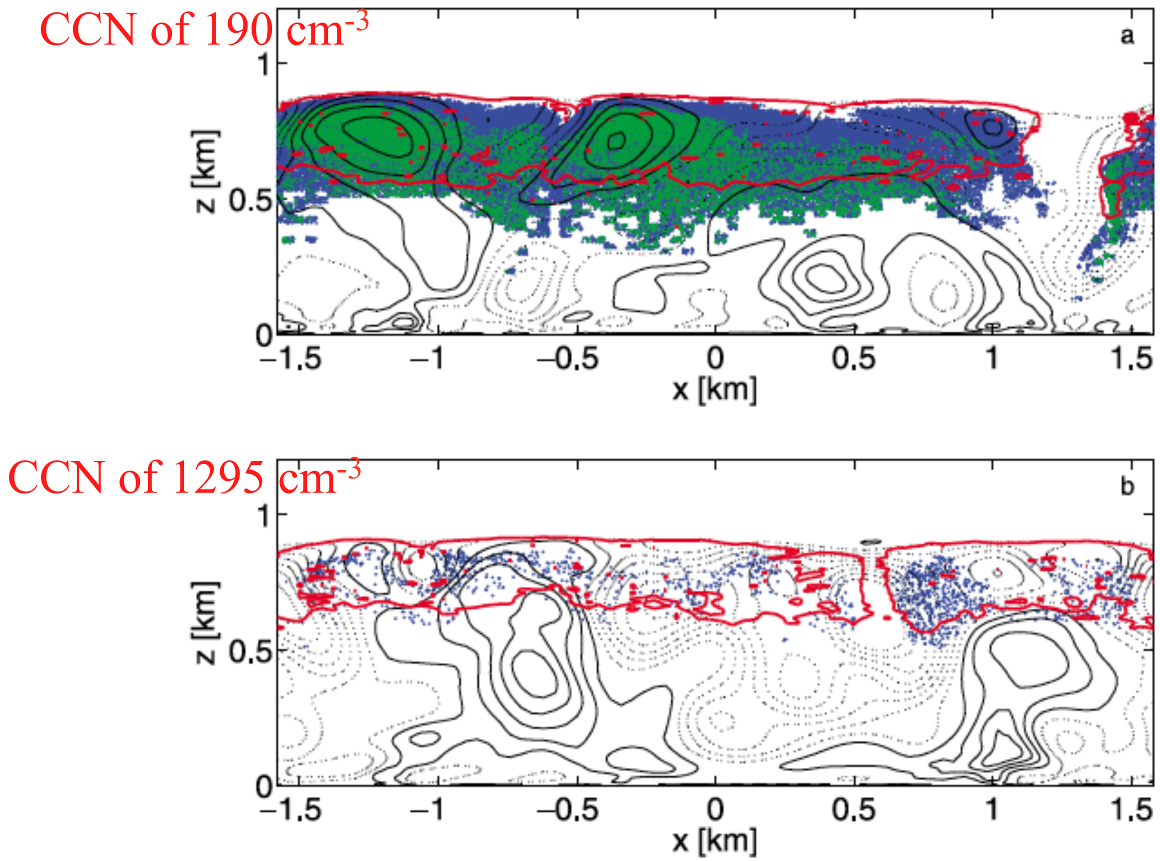
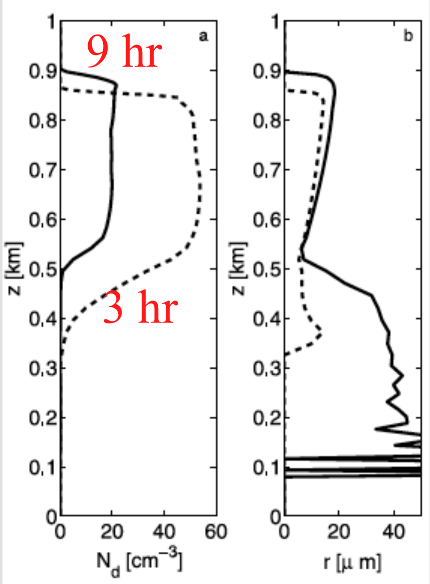
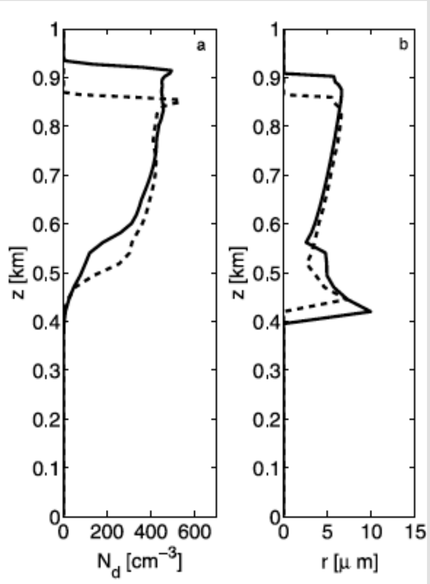


Figure 3. Vertical cross sections through model domain after 9 h of simulations: cross sections at (a) $y = -1180$ m for LOW and (b) $y = 420$ m for HIGH. Solid lines indicate positive velocities starting from 0.1 m/s with contour interval 0.2 m/s; dotted lines represent negative velocity starting from -0.1 m/s with the interval -0.2 m/s. The red line presents condensed water contour of 0.1 g/kg. Blue/green dots represent the locations of droplets bigger than 50/90 μm.





Andrejczuk et al. JGR 2010

Summary:

Warm-rain microphysics: **cloud droplet activation, condensational growth, collisional growth.**

Eulerian modeling warm-rain processes:

- bulk single-moment scheme: mixing ratios for cloud water and drizzle/rain water (activation irrelevant, no information about spectral characteristics, model resolution can be low);

- detailed (bin) microphysics: concentration (per unit mass) of cloud and drizzle/rain drop in each size (mass) category (~100 variables); supersaturation and droplet activation predicted, requires high spatial resolution (especially near cloud base); can be even more complicated if detailed information about aerosols is added;

- double-moment microphysics: mixing ratios and concentrations of cloud and drizzle/rain drops, supersaturation does not have to be predicted (but it can be; e.g., MG scheme), activation either predicted (MG; high resolution needed) or parameterized (e.g., as a function of the updraft speed; lower resolution possible).

Lagrangian modeling of warm-rain processes:

- relatively straightforward simulation of aerosol processing.