ICTP 2016 Michel Campillo Passive Imaging

1. Seismograms: ballistic waves, coda, ambient noise.

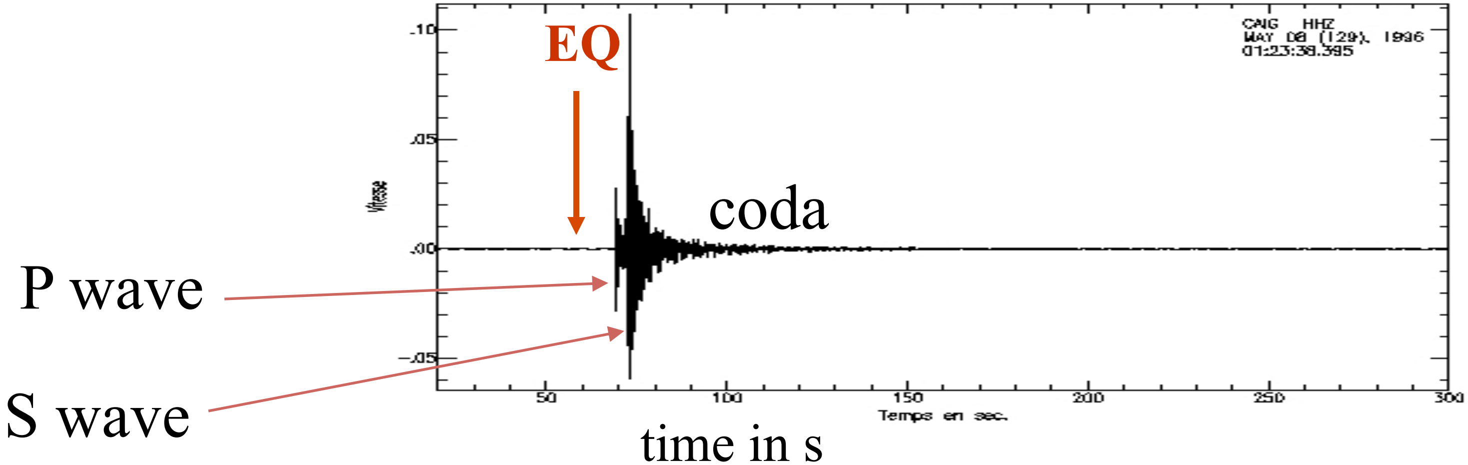
2. Green function and correlations

3. Imaging: surface waves and body waves. Applications.

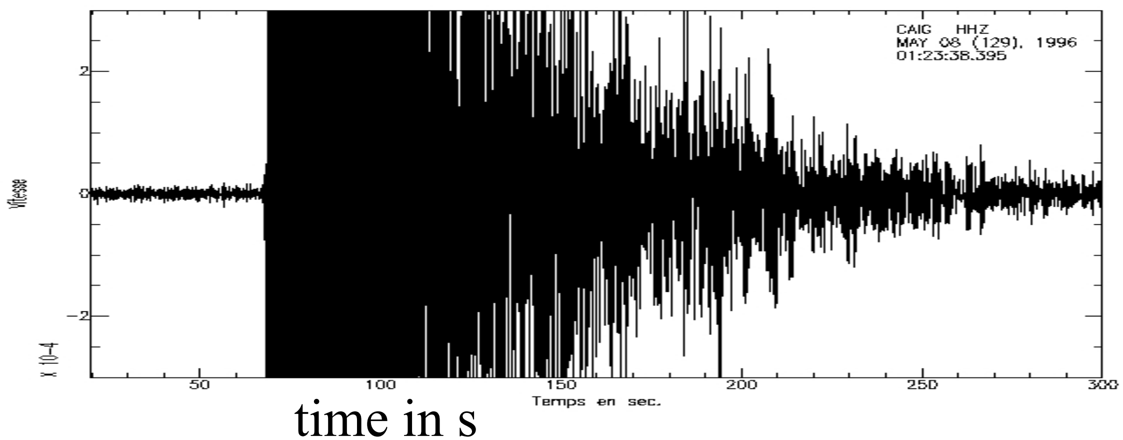
4. Monitoring: sensitivity of direct and scattered waves. Applications.

Analysis of continuous records

Example of a record of a local earthquake in the band .5-20Hz

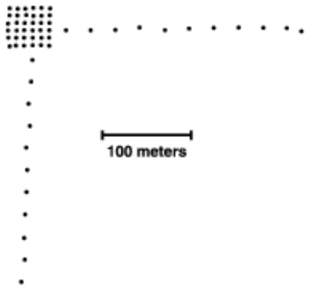


scale x 1000:



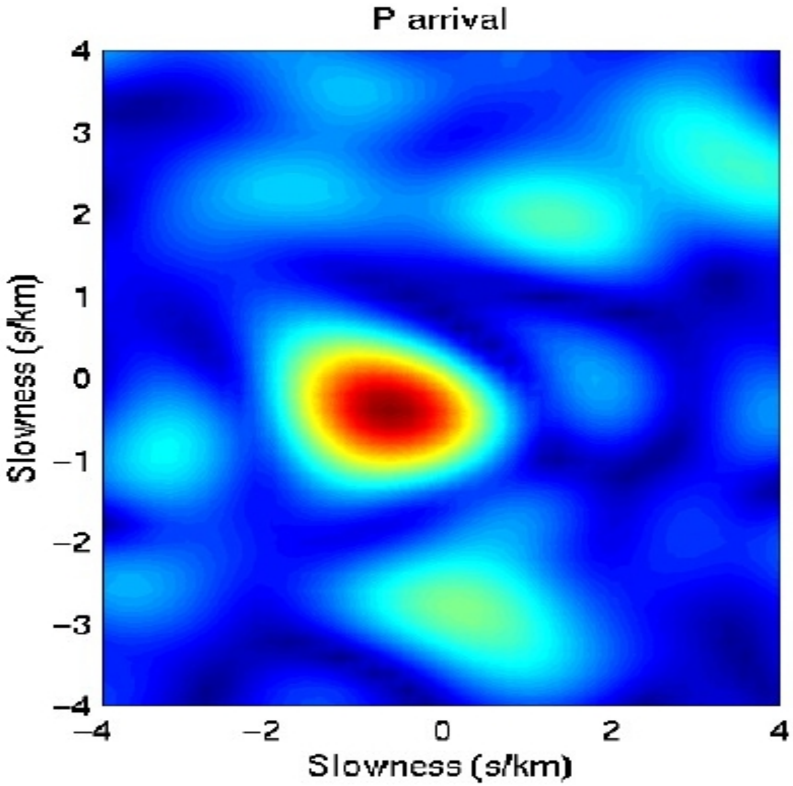
Coda: tail, end of a piece of music….

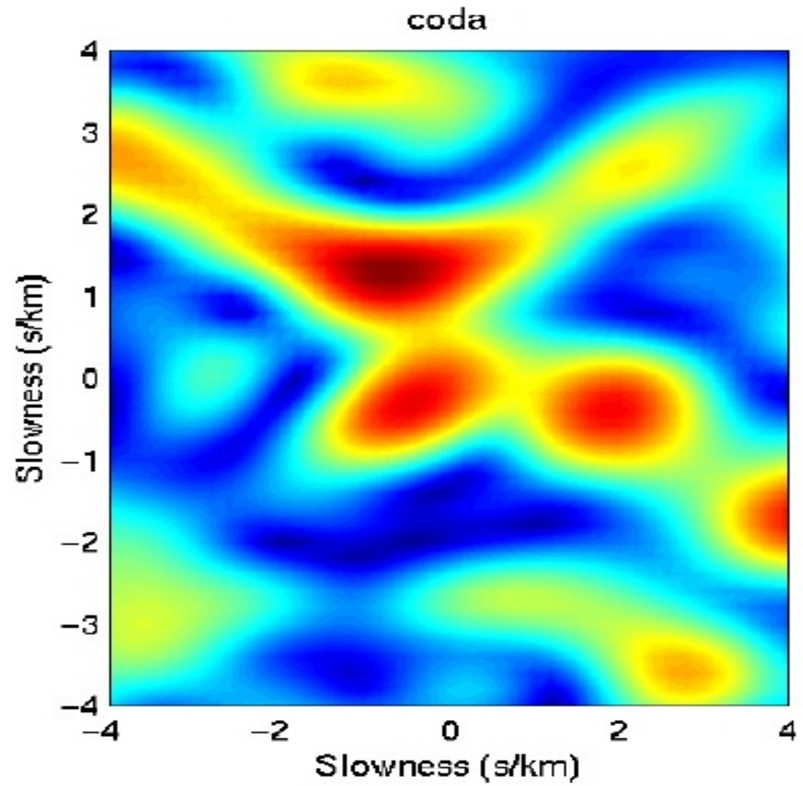
Frequency-wavenumber analysis



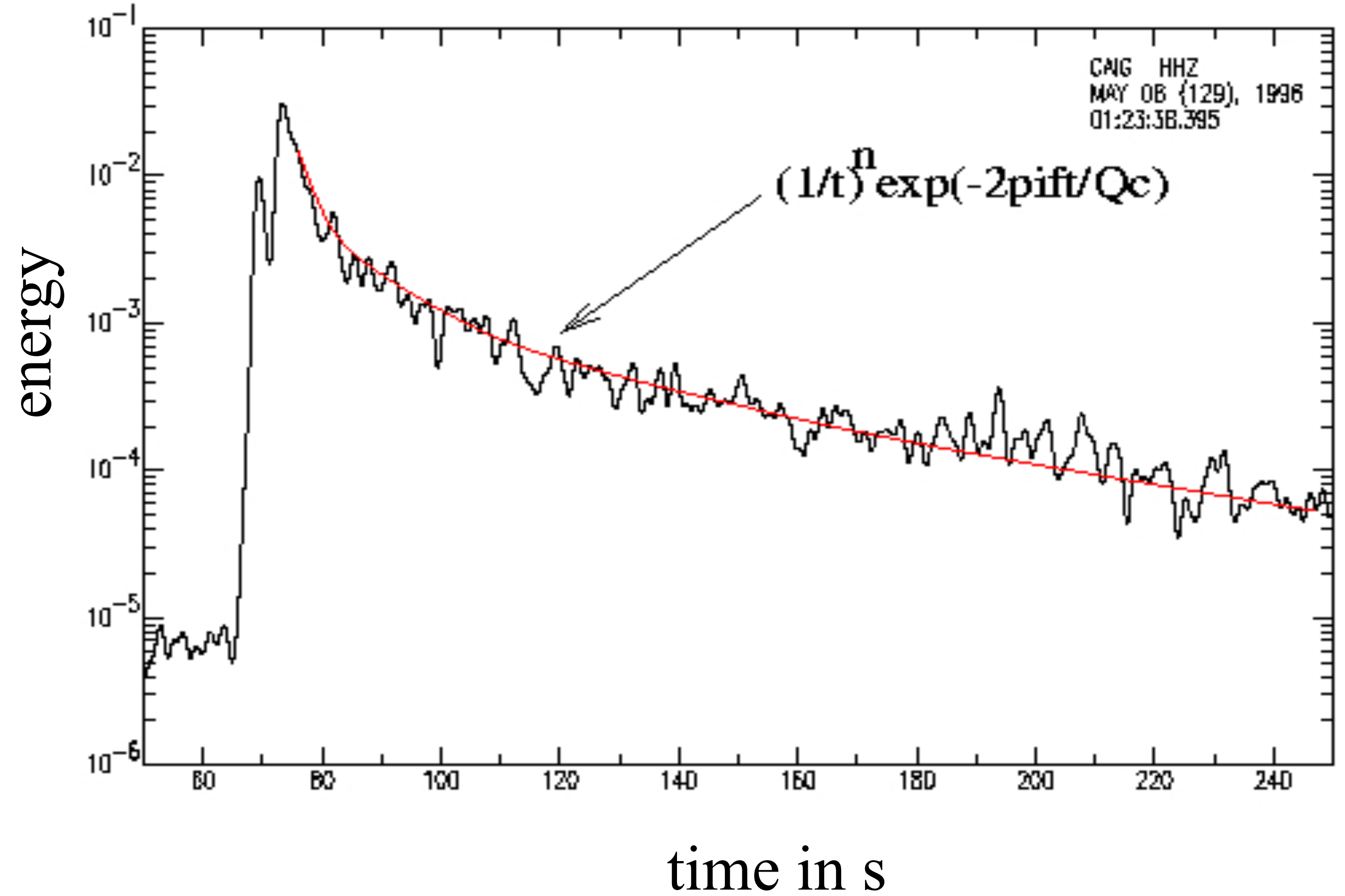
(Pinon Flat Seismometer Array)

$$u(x,y) \rightarrow u(k_x,k_y)$$



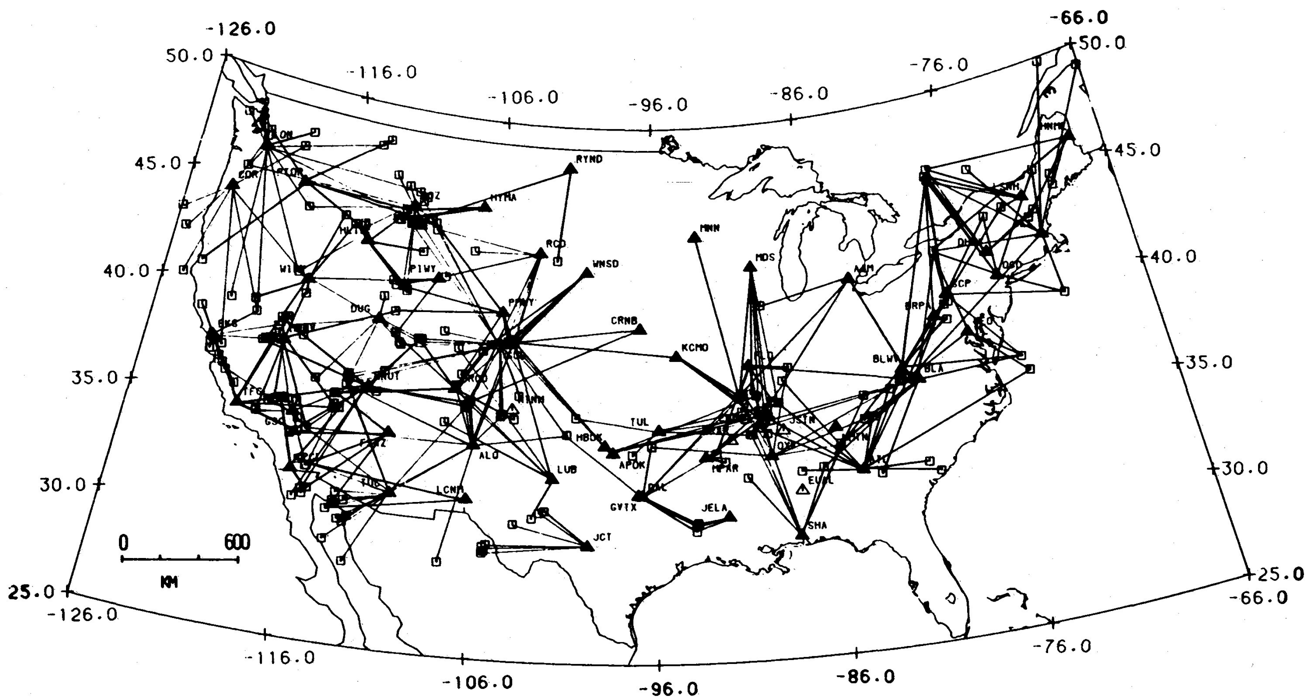


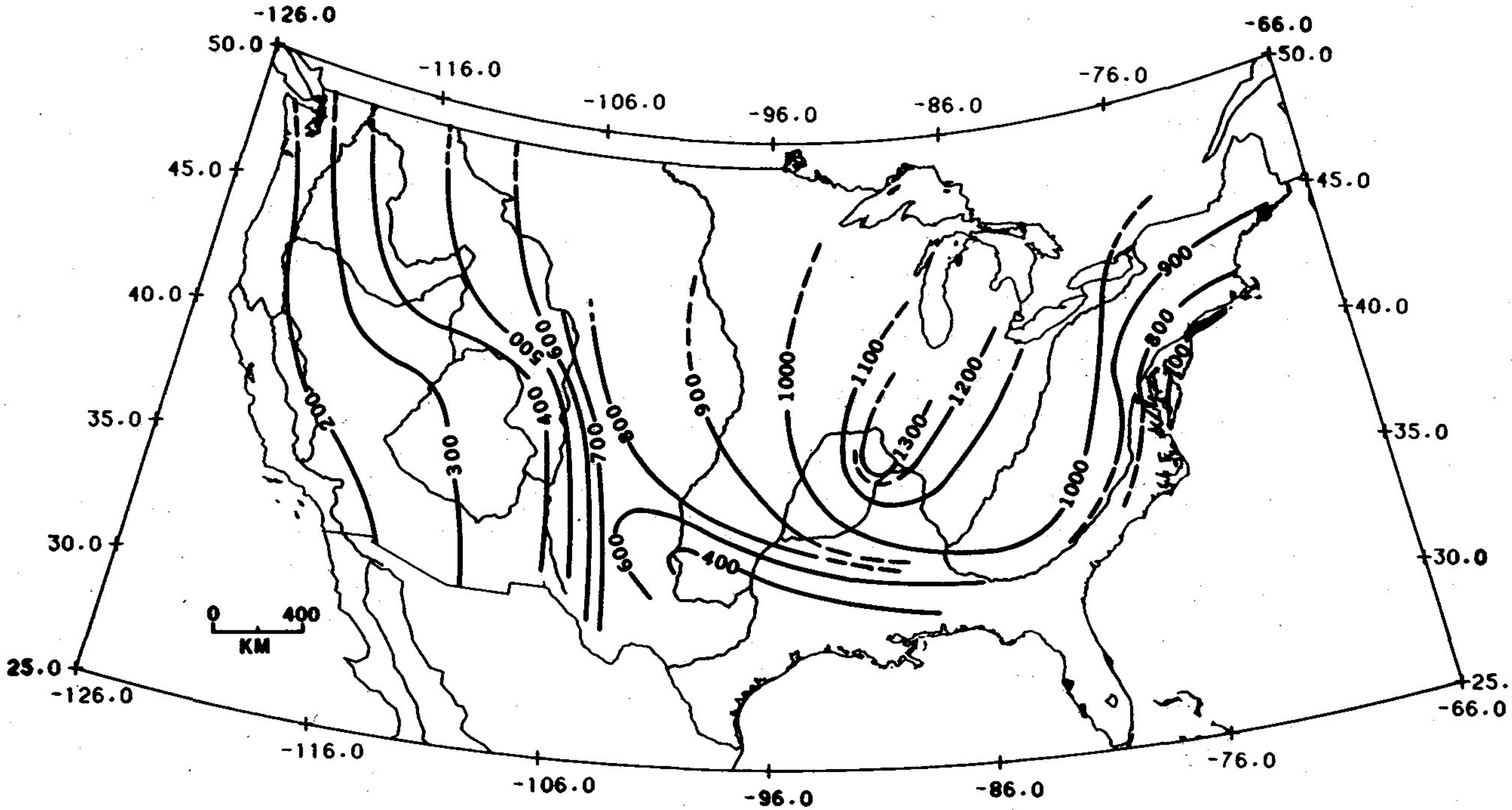
Energy decay in the coda (Aki and Chouet, 1975)



The decay is constant in a region, independently of source and receiver: Qcoda

Coda Q in US (Singh and Herrmann, 1983)




-126.0
-116.0
-106.0
-96.0
-86.0
-76.0
-66.0
50.0
45.0
40.0
35.0
30.0
25.0
0 400
KM
200
300
400
500
600
700
800
900
1000
1100
1200
1300

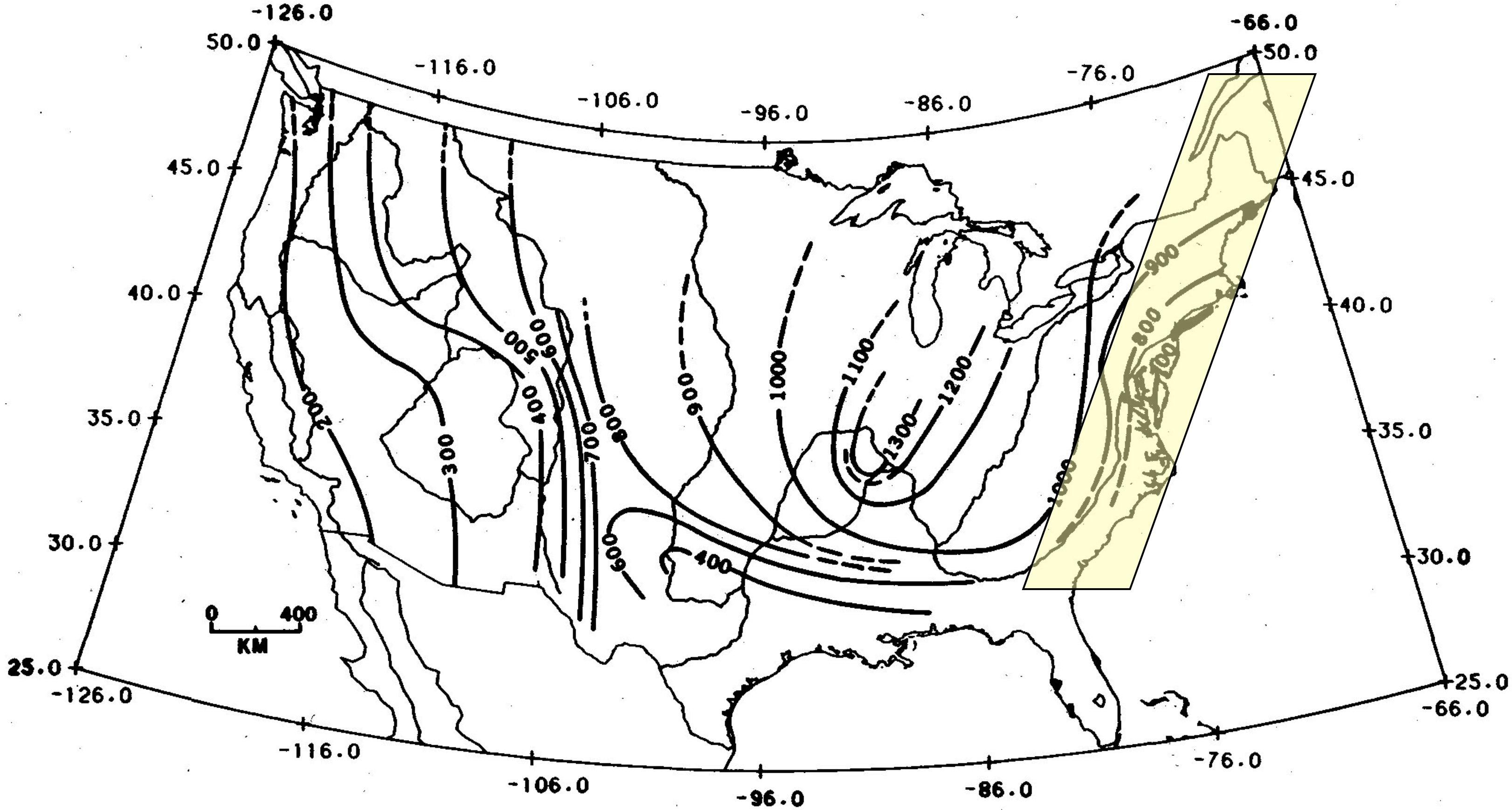


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Appalachian (Hercynian) belt : Qc ~ 600

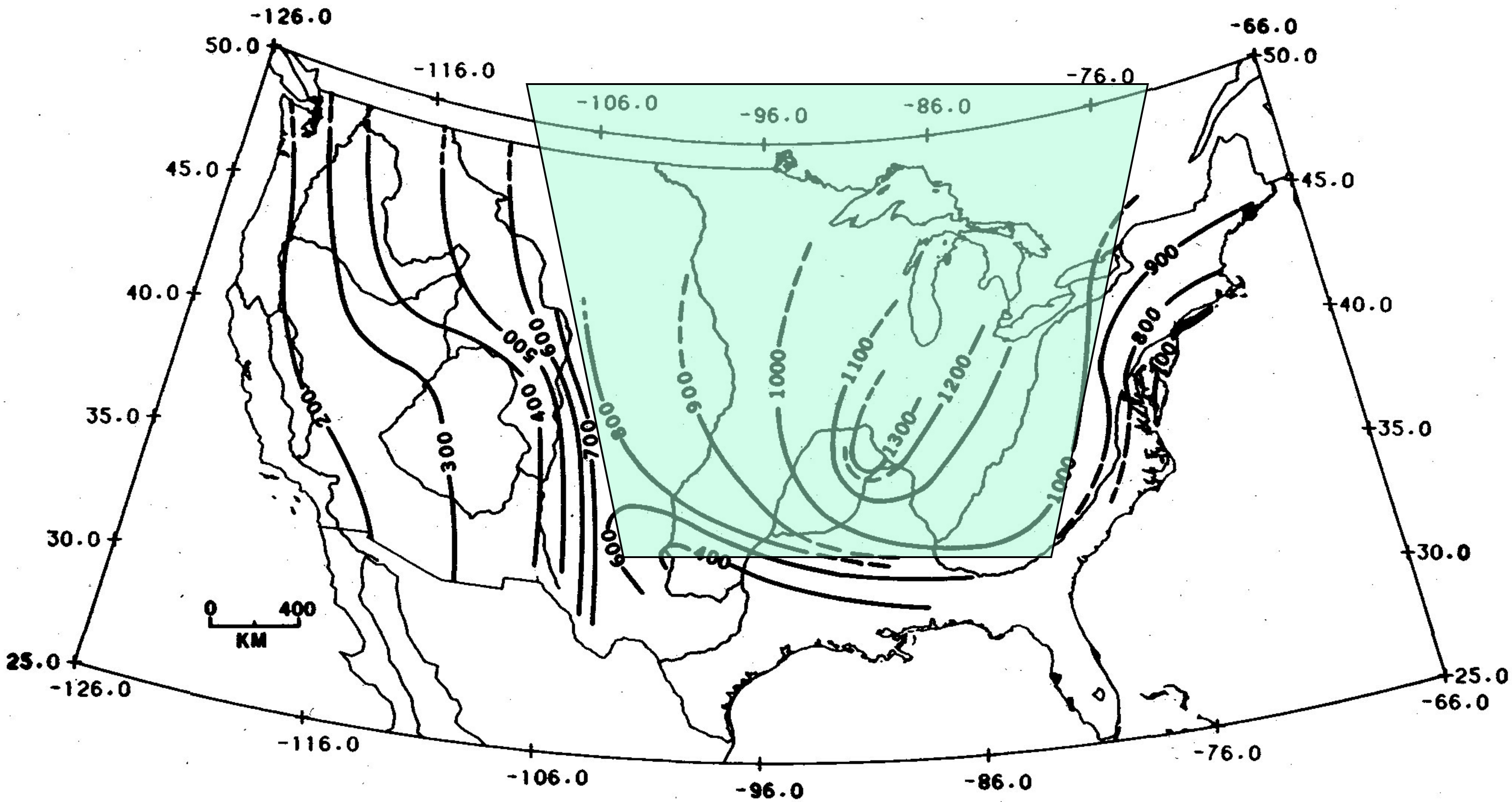


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Central shield : Qc ~ 1000

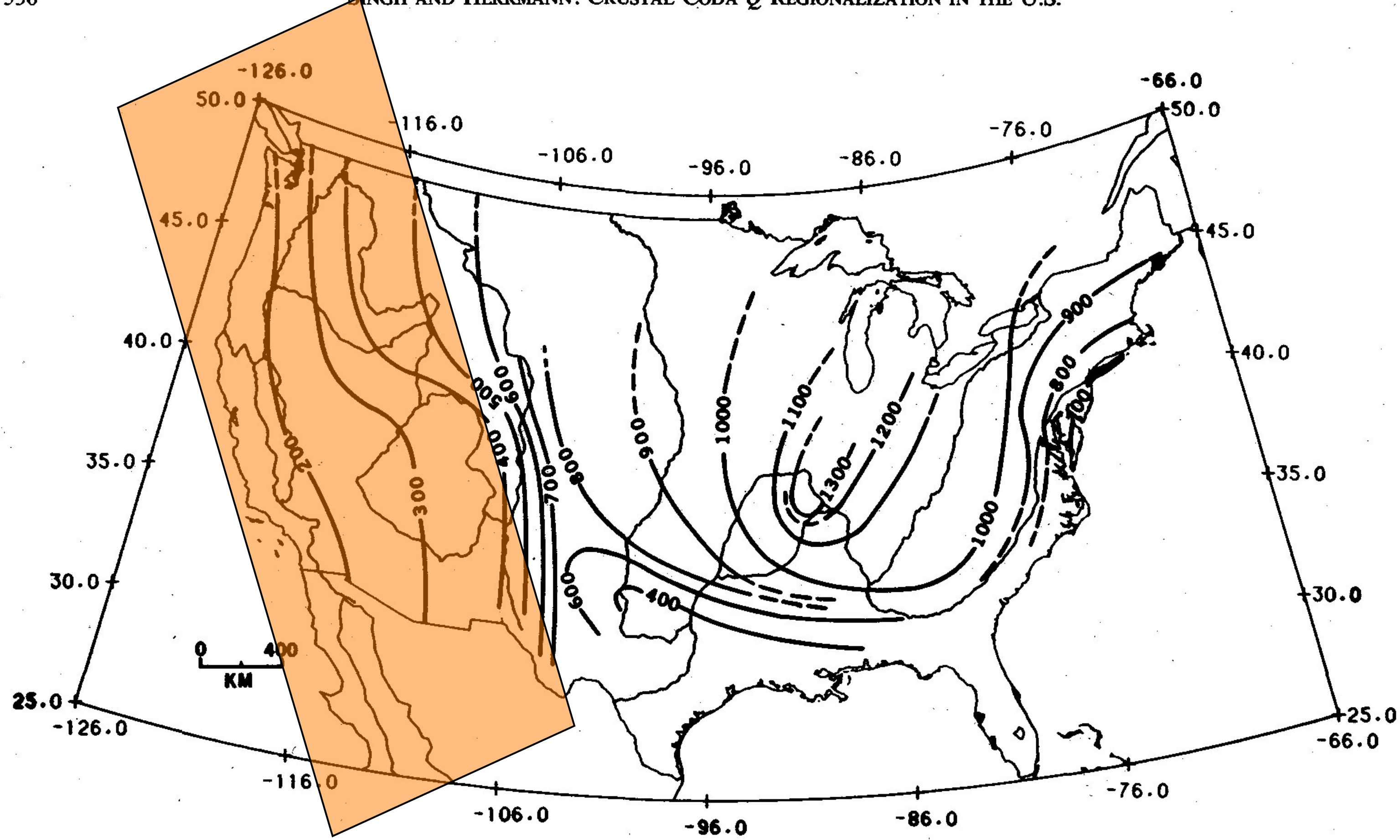
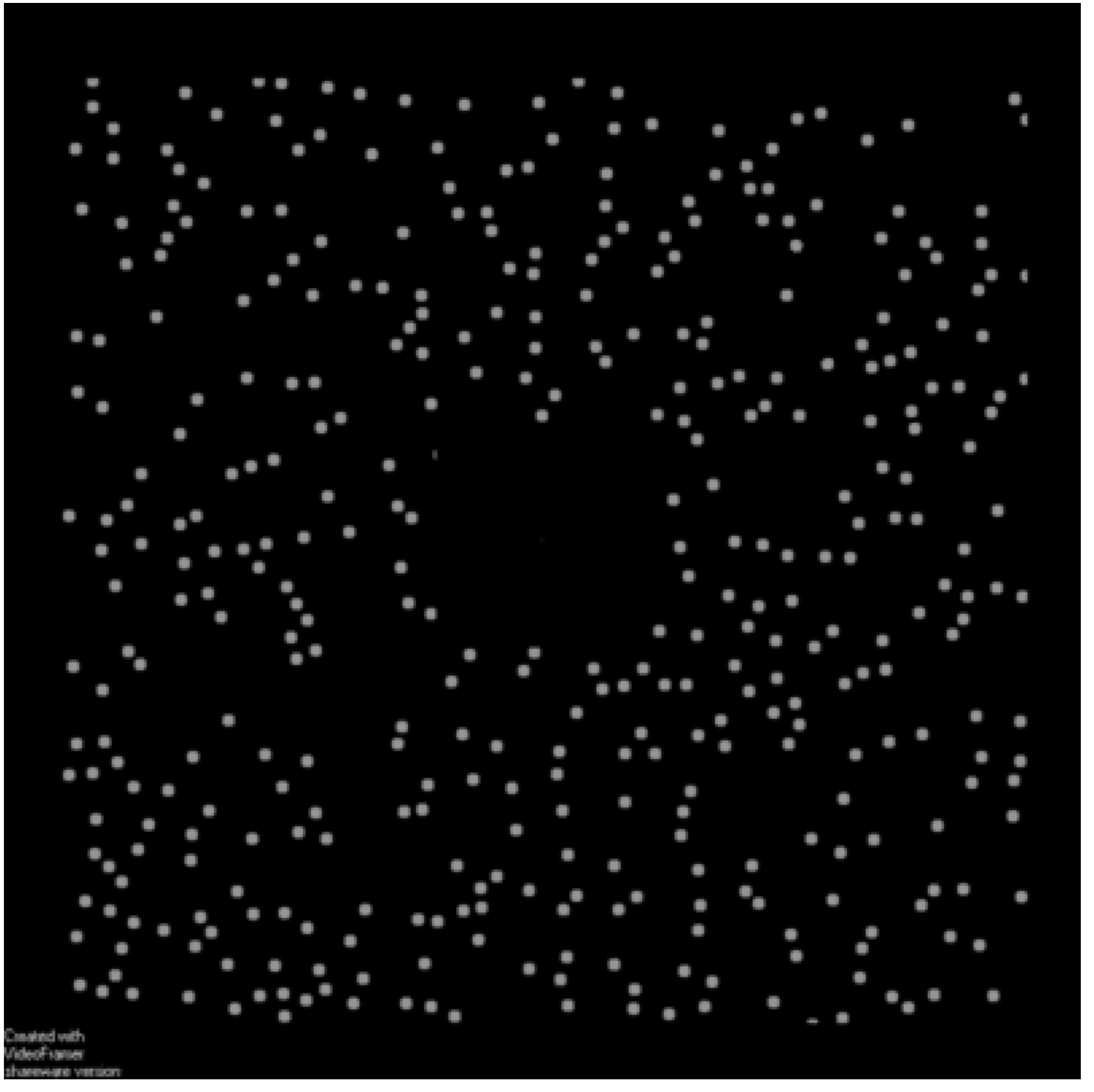


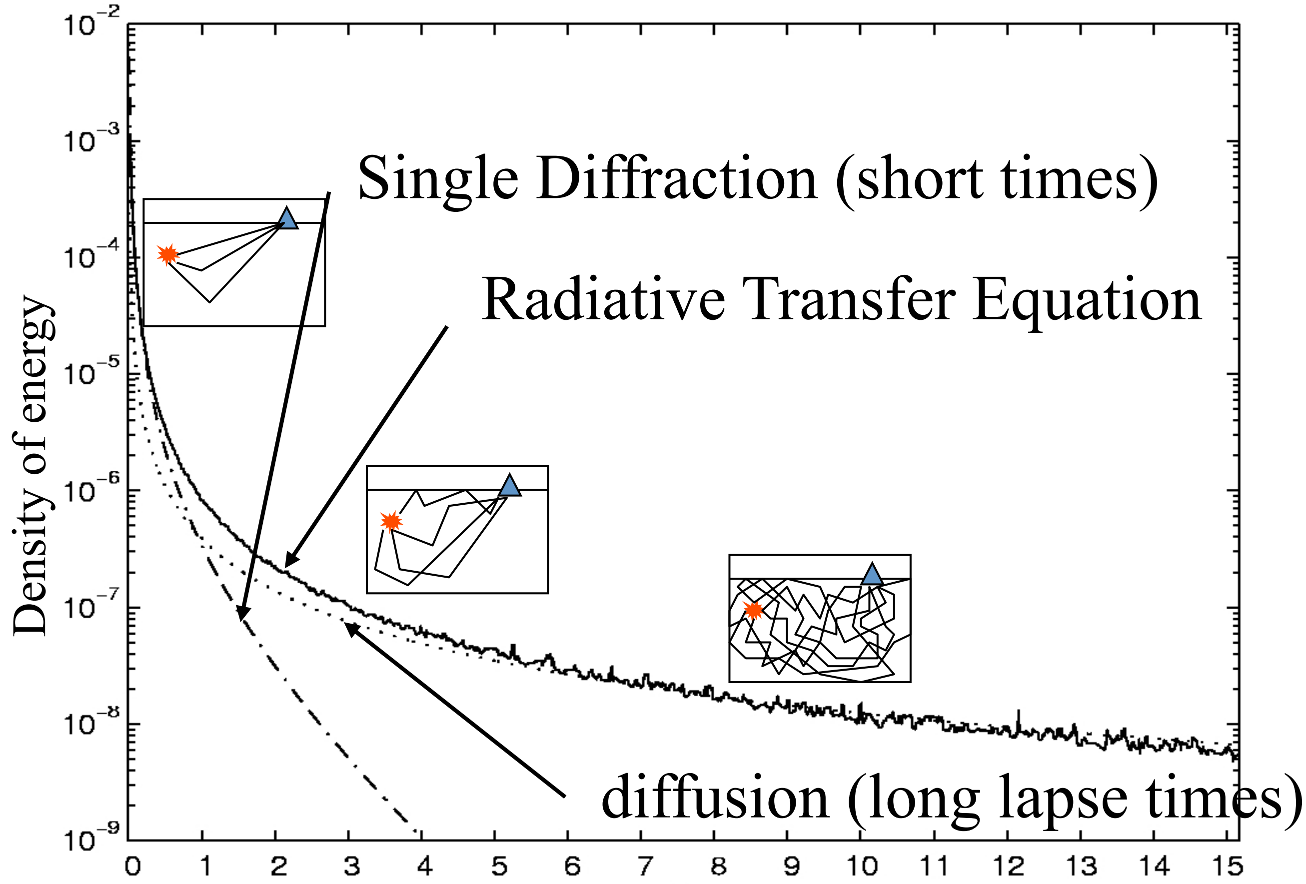
Fig. 15. Contour map of coda Q_0 for the entire continental United States.

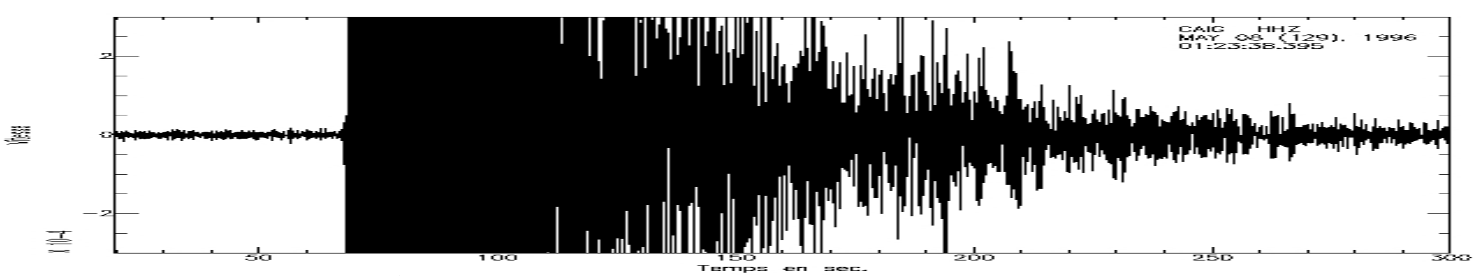
Tectonically active western US: Qc=100-300

Transient signals in a complex medium…….



Propagation regimes and description of **energy**





time

The Diffusion Approximation

General Idea:

- Each scattering distributes energy over all space directions
- After several scatterings the intensity becomes almost isotropic

$$I(t,\vec{r},\vec{\Omega}) = \text{Angularly Averaged Intensity} + \text{constant} \times \vec{J}(t,\vec{r}) \cdot \vec{\Omega}$$

The current density $\vec{J}(\vec{r},t)$, points in the direction of maximum energy flow.
Integrating the RT Eq over all space directions leads to:

$$\partial_t \rho(t,\vec{r}) - D\nabla^2 \rho(t,\vec{r}) = \delta(t,\vec{r})$$

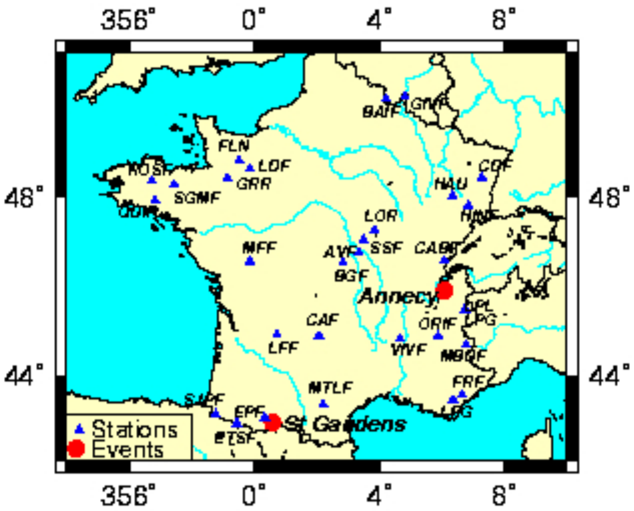
where rho is the local energy density.

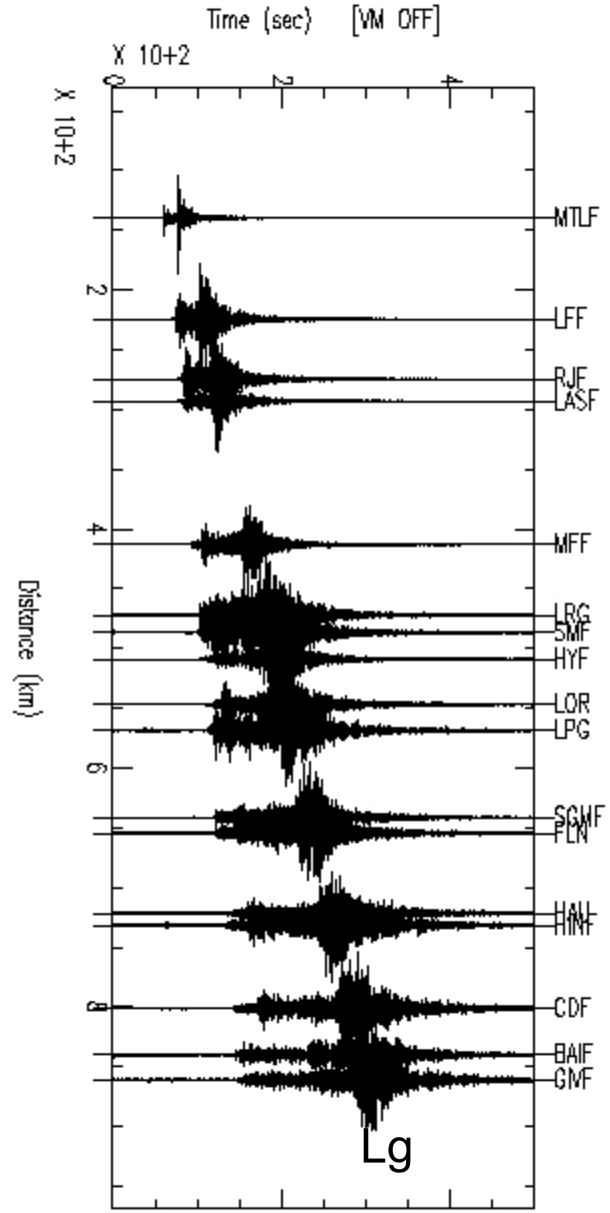
$$\boldsymbol{\rho}(\boldsymbol{r}|\boldsymbol{r}',t) = \frac{1}{(4\pi Dt)^{d/2}} e^{-|\boldsymbol{r}-\boldsymbol{r}'|^2/4Dt}$$

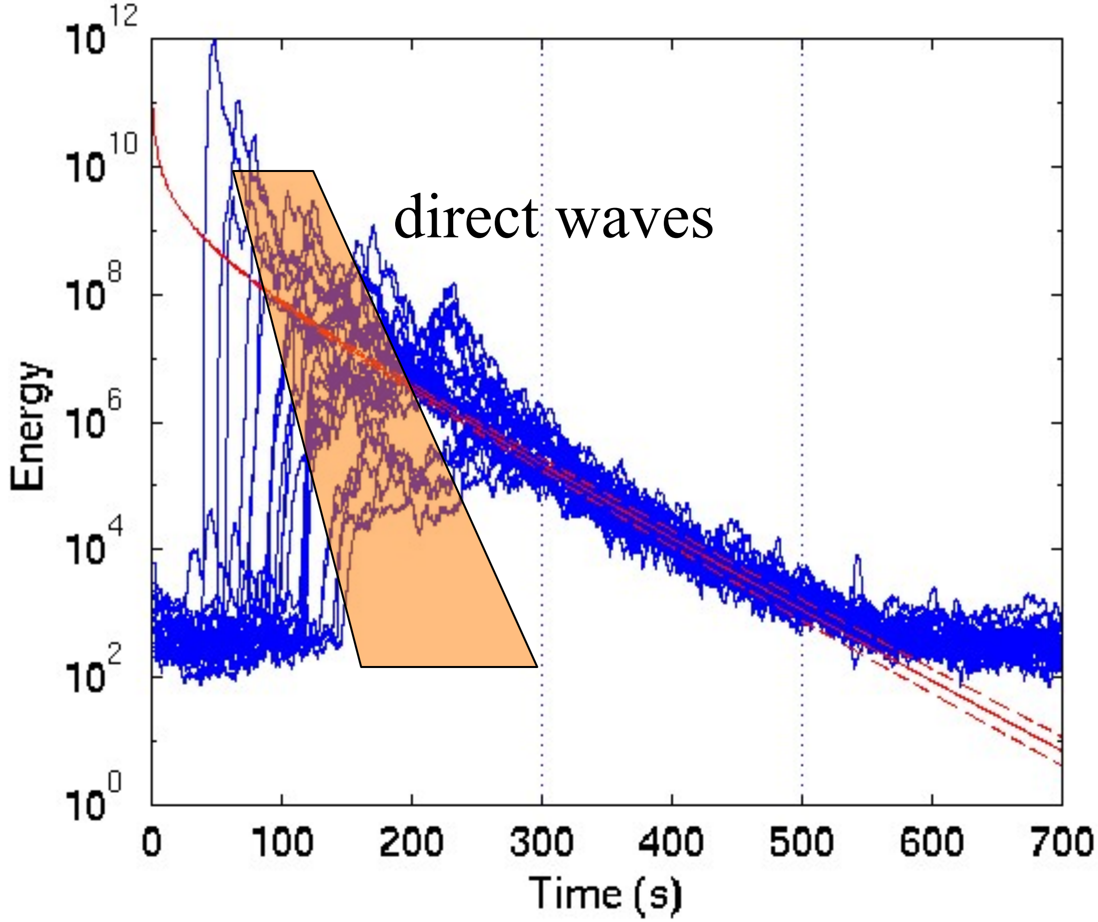
$\rho(t,\vec{r}) \sim \frac{1}{(Dt)^{3/2}}$ for large t.

$D = vl/3$ is the diffusion constant of the waves.

Regional seismograms



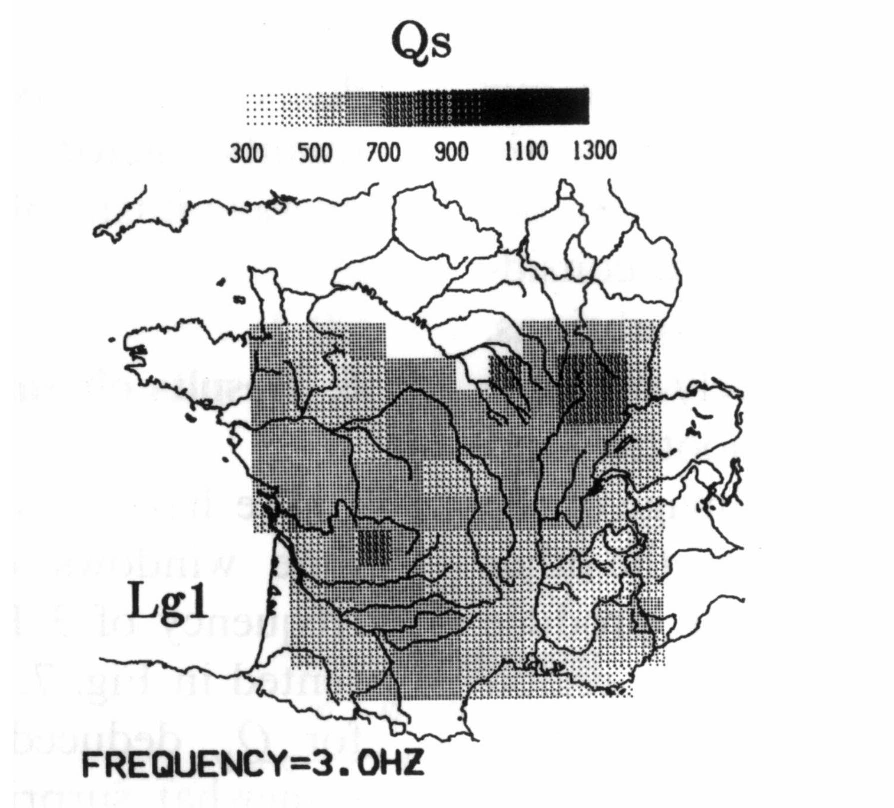




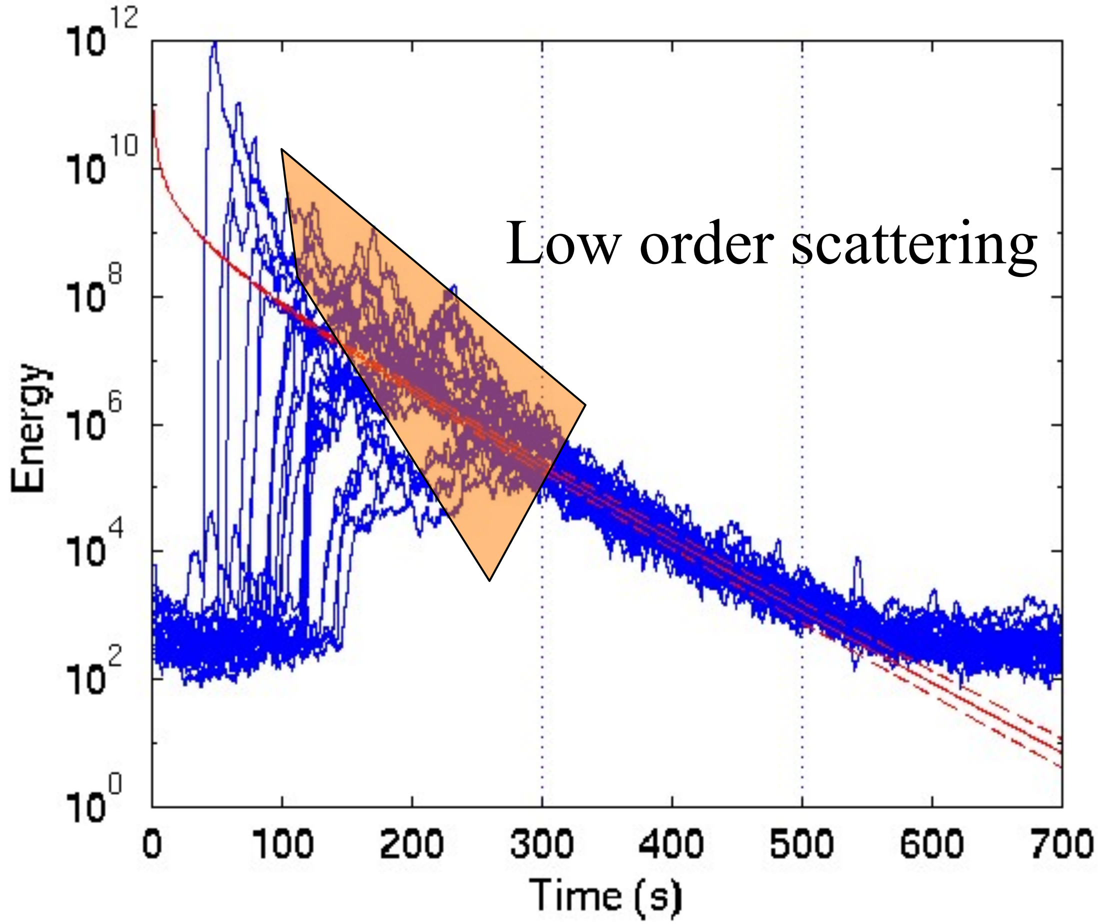
Rapid decay with distance

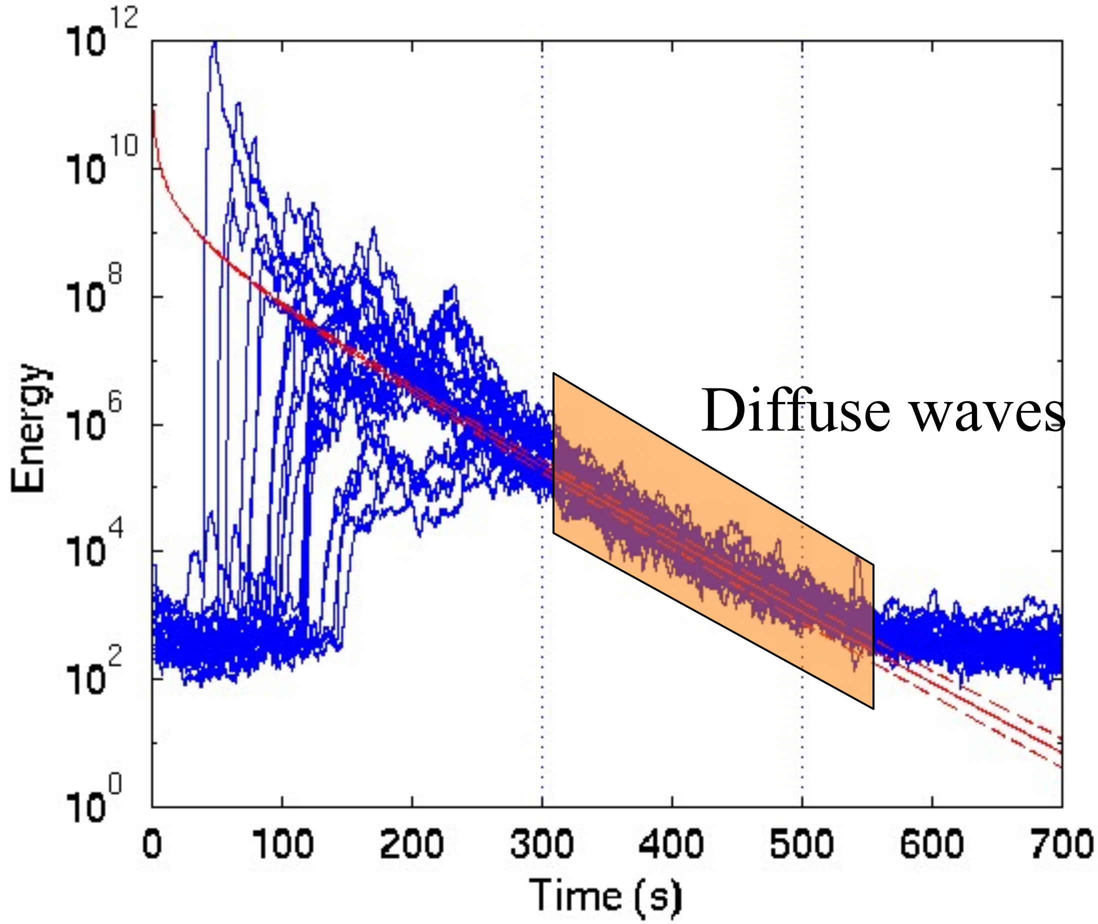
Observations:

l_{total} (km) = 160 / f $^{1/2}$ for S waves



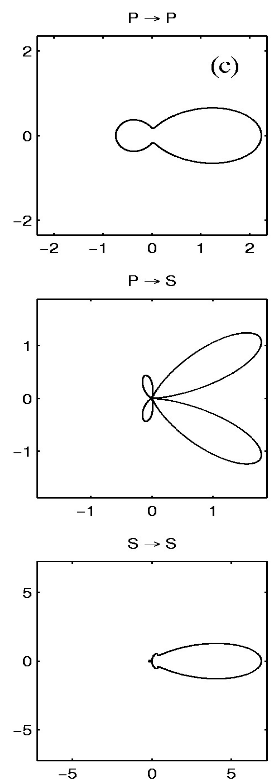
$1/\ l_{\text{total}} = 1/l_S + 1/l_A$


Low order scattering
Energy
Time (s)
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
0
100
200
300
400
500
600
700

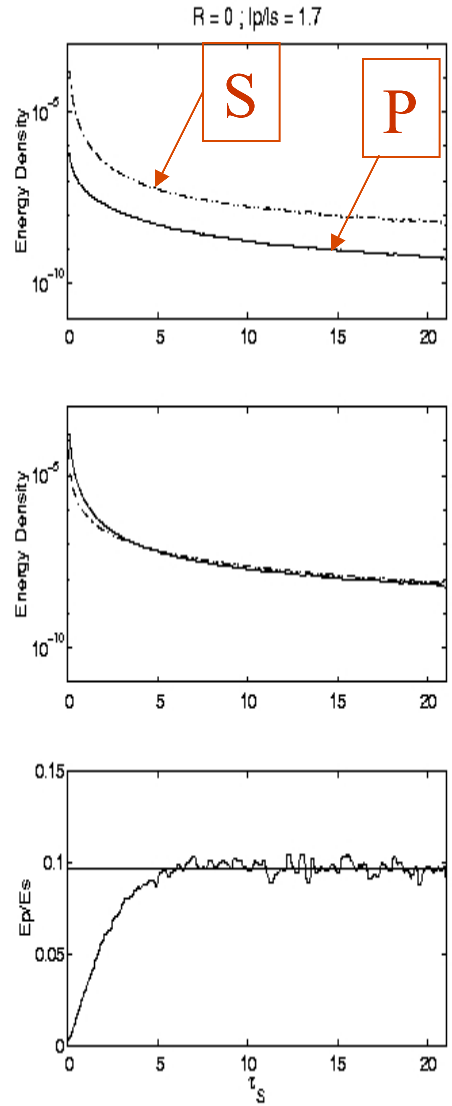


Diffuse waves
Energy
Time (s)

Cross sections



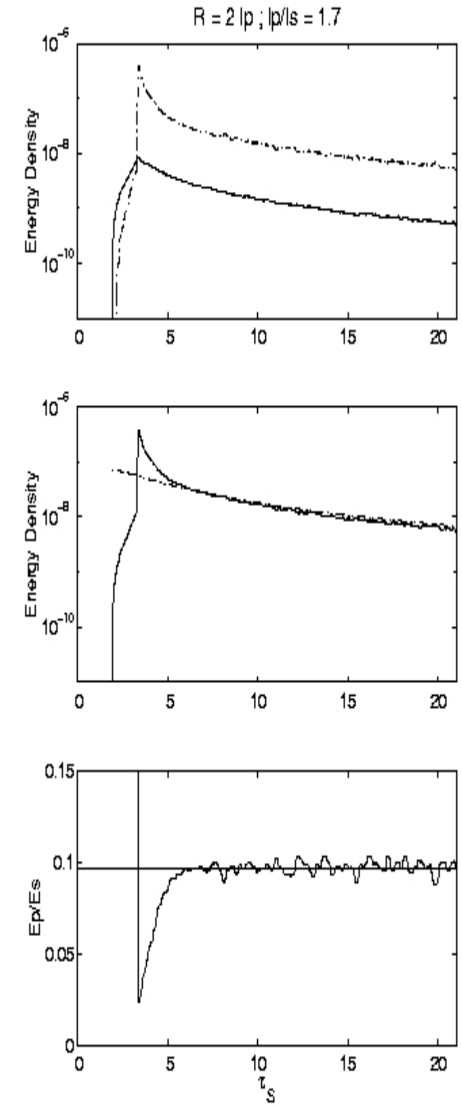
ka=1.6



total energy
vs
diffusion app.

Energy ratio

time



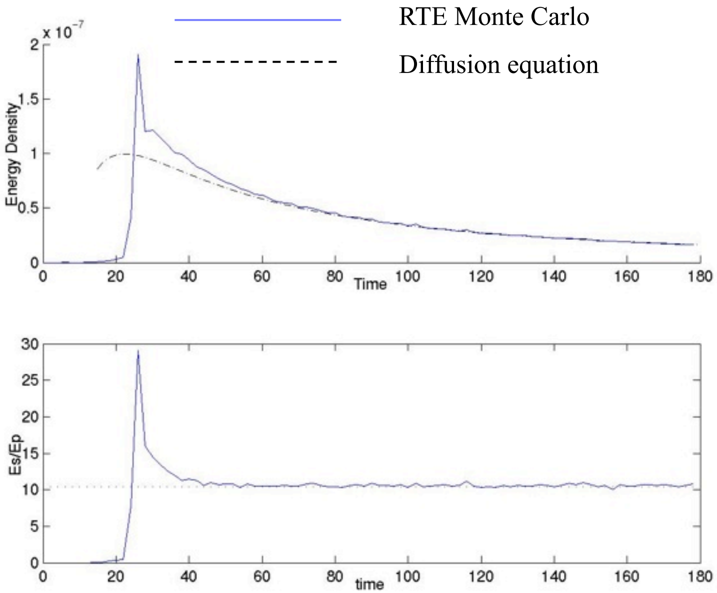
time

Searching for a marker of the regime of scattering...

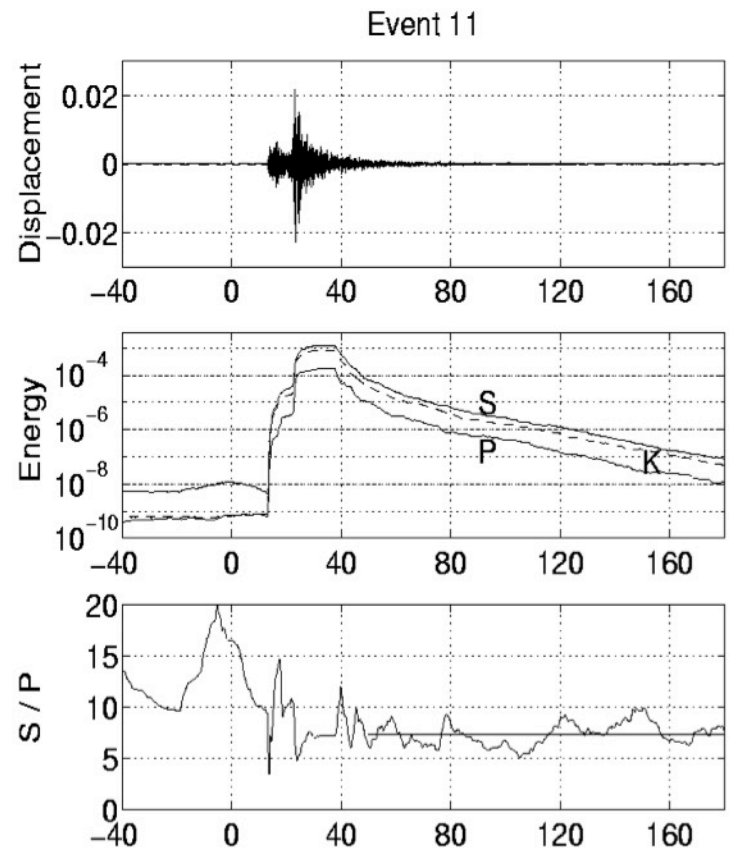
Equipartion principle for a completely randomized (diffuse) wave-field: in average, all the modes of propagation are excited to equal energy.

Implication for elastic waves (Weaver, 1982, Ryzhik et al., 1996): P to S energy ratio stabilizes at a value independant of the details of scattering!

Numerical simulation



Observations



Partition of energy (Full elastic space)

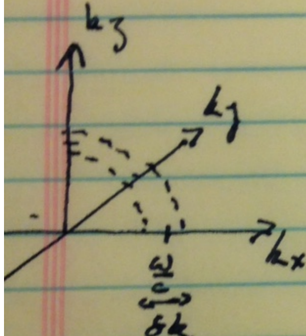
Multiple scattering, large t

$\rightarrow$ "equipartition"

[reference medium + disorder]

Phase space of the full space elastic problem

$\rightarrow$ all propagating plane waves excited at same level of energy



Energy in a band $\omega \pm \frac{\delta\omega}{2}$

$\rightarrow$ Volume for P waves

$$\delta k = \frac{\delta\omega}{\alpha}$$

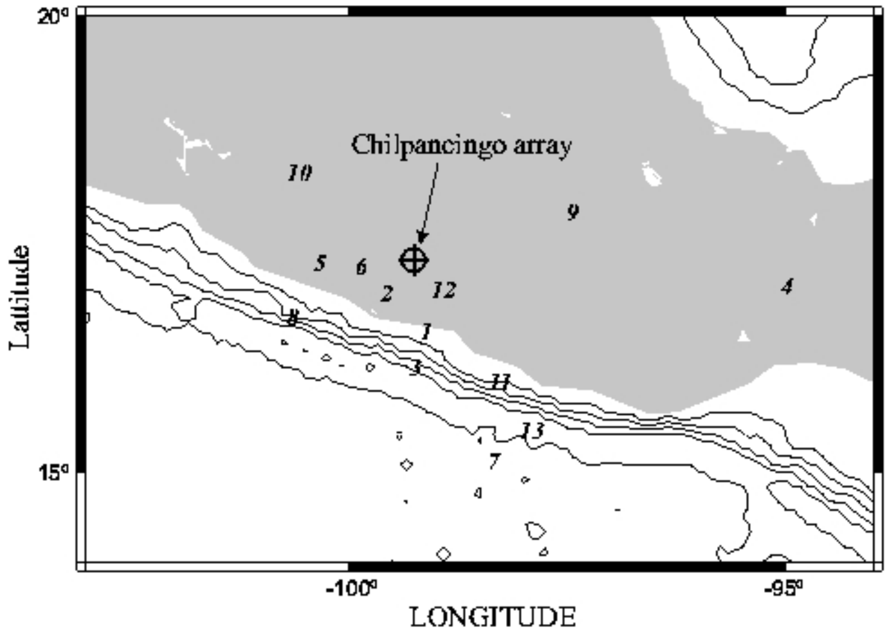
$$V_P = 4\pi \left(\frac{\omega}{\alpha}\right)^2 \frac{\delta\omega}{\alpha} = 4\pi\,\delta\omega\,\omega \frac{1}{\alpha^3}$$

Volume for each S polarisation: $V_{SH/SV} = 4\pi\,\delta\omega\,\omega \frac{1}{\beta^3}$

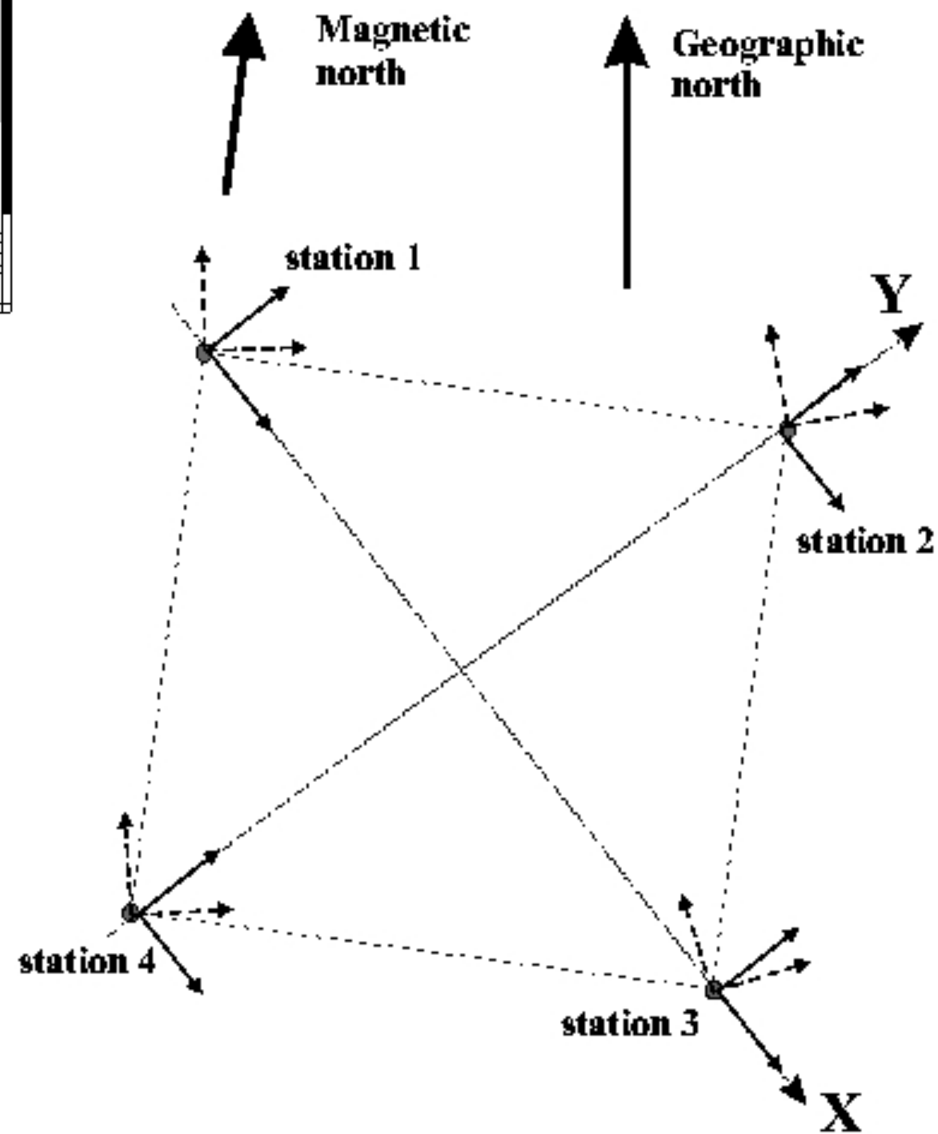
$\Rightarrow V_S = 2 \times 4\pi\,\delta\omega\,\omega \frac{1}{\beta^3}$

Equal excitation $\Rightarrow \dfrac{E_S}{E_P} = \dfrac{V_S}{V_P} = 2\dfrac{\alpha^3}{\beta^3}$

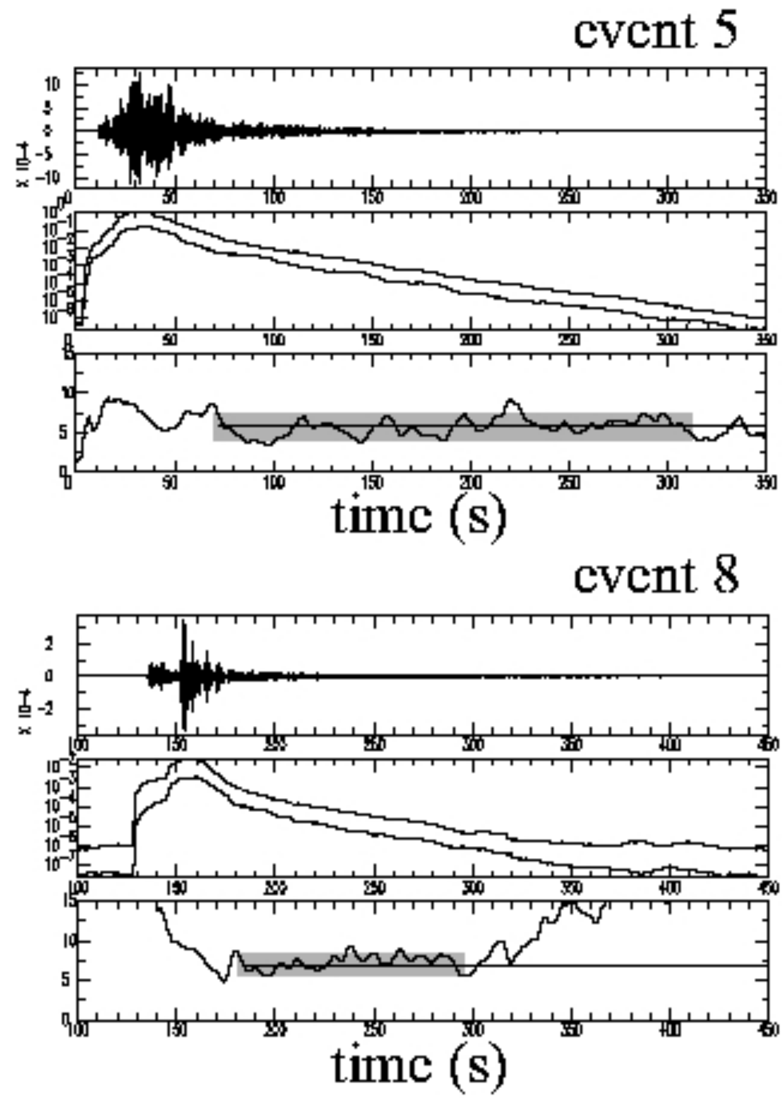
[Note $\lambda = \mu \Rightarrow \dfrac{E_S}{E_P} \sim 10.4 \Rightarrow$ see numerical simulation



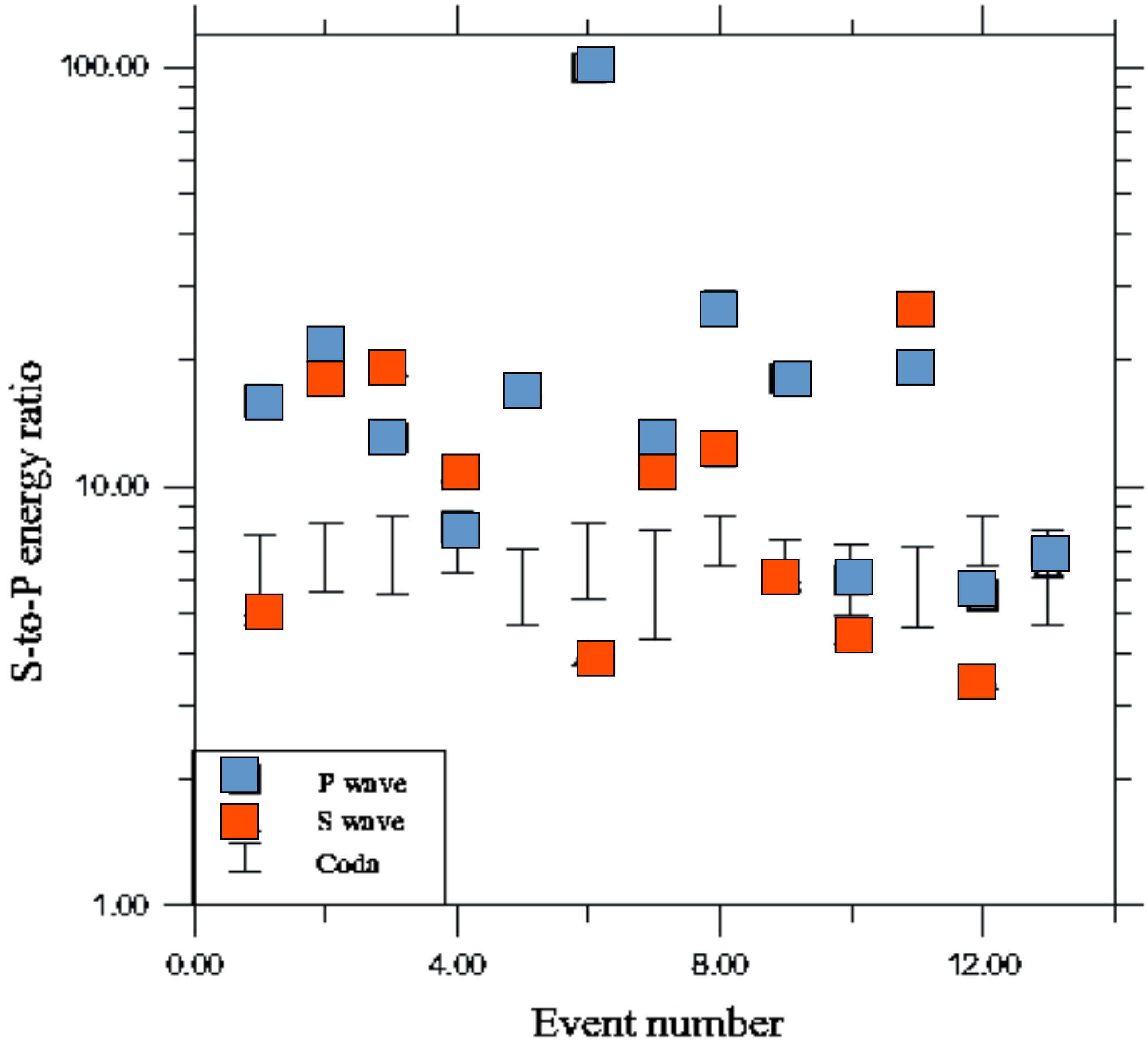
20°
15°
-100°
-95°
Latitude
LONGITUDE
Chilpancingo array
10
9
5
6
2
12
4
8
1
3
11
13
7



Magnetic north
Geographic north
station 1
station 2
station 3
station 4
Y
X



event 5
time (s)
event 8
time (s)

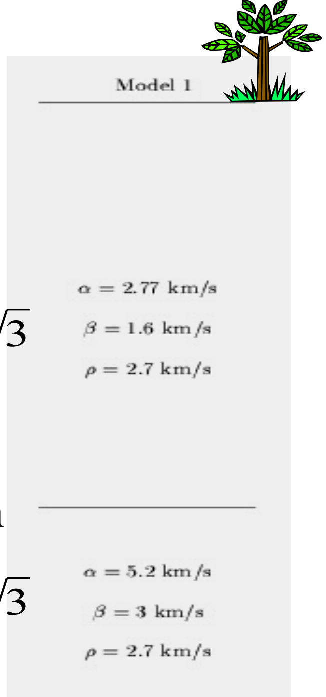


100.00
10.00
1.00
0.00
4.00
8.00
12.00
S-to-P energy ratio
Event number
P wave
S wave
Coda

ENERGY RATIO	DATA	THEORY FULL SPACE	THEORY HALF SPACE BULK WAVES	THEORY HALF SPACE with RAYLEIGH WAVES
S/P	7.3	10.39	9.76	7.19
K/(S+P)	0.65	1	1.19	0.534
I/(S+P)	-0.62	0	-0.336	-0.617

Hennino et al., 2001

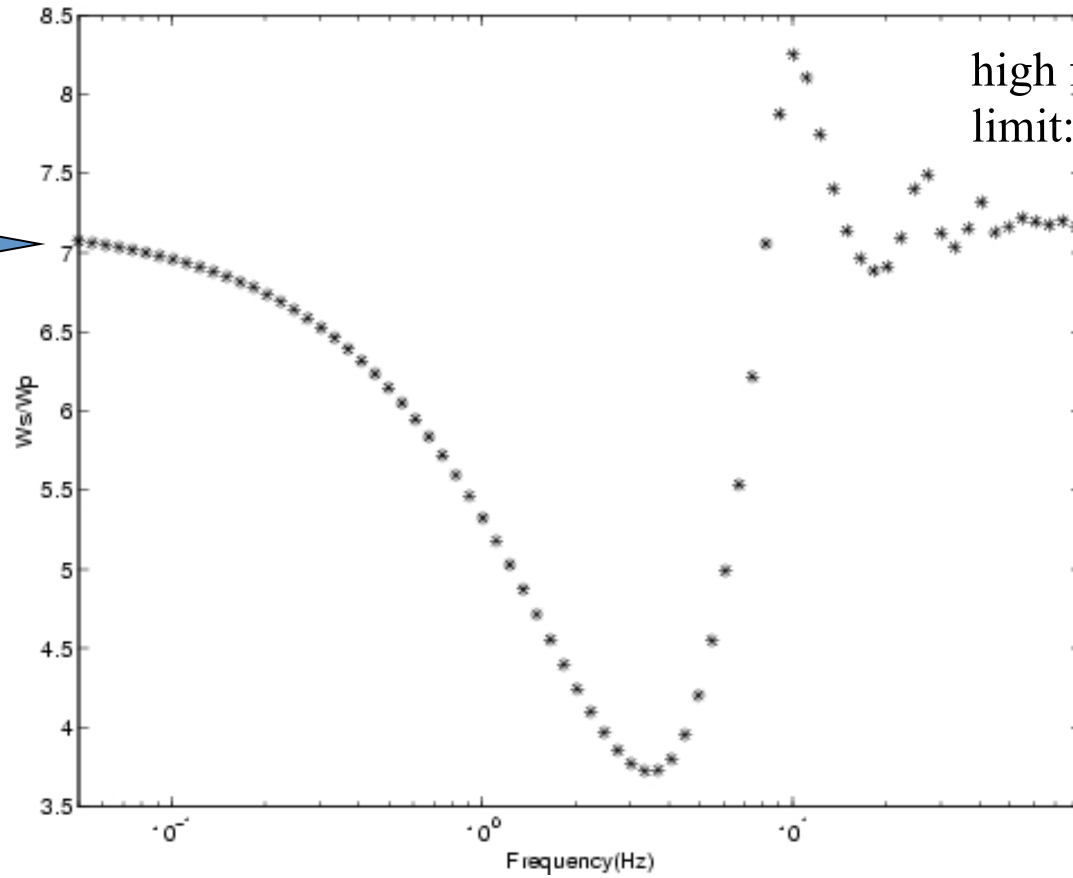
Effect of a 'thin' layer: Love and Rayleigh



$\alpha/\beta = \sqrt{3}$

65 m

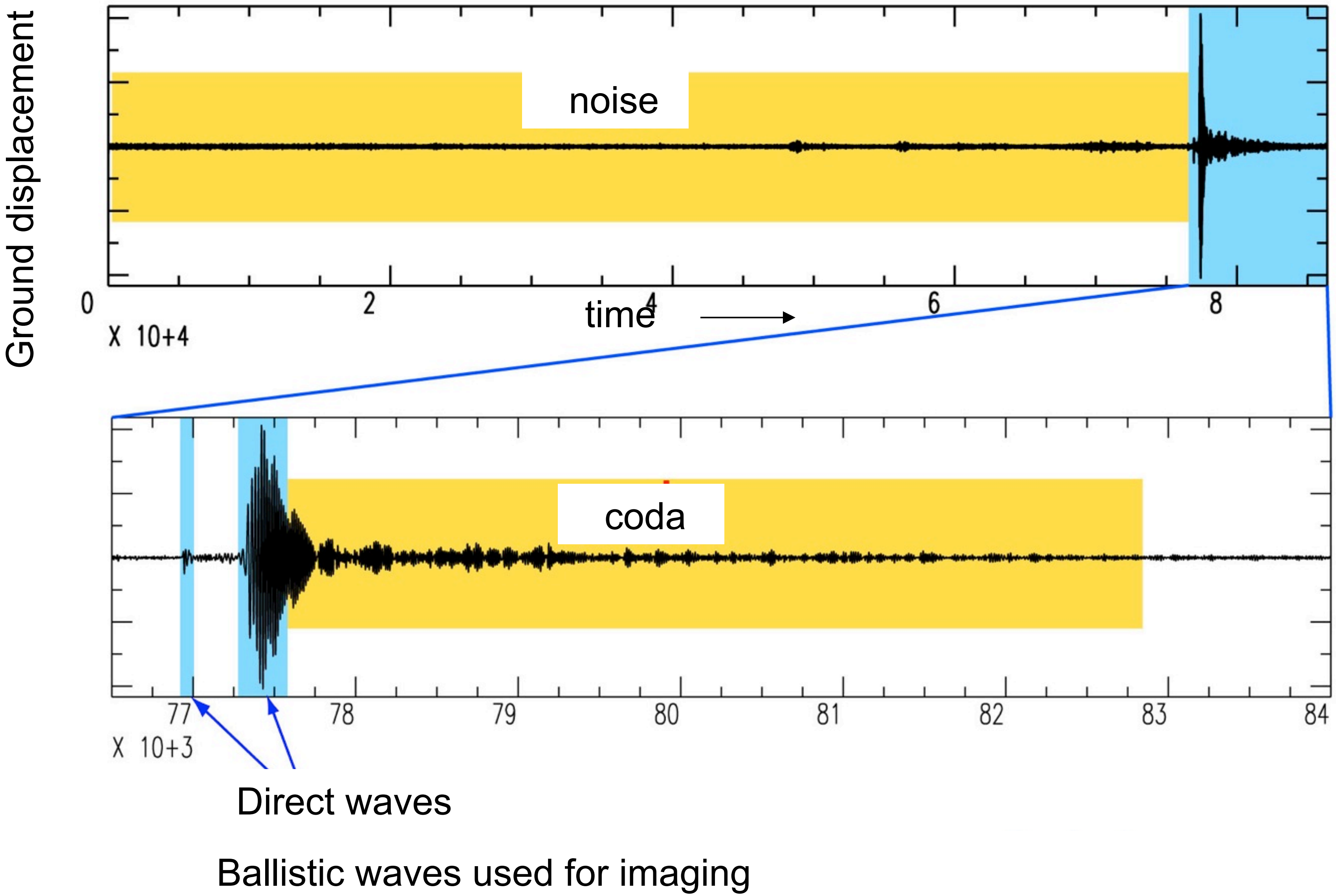
$\alpha/\beta = \sqrt{3}$



high frequency limit: half space

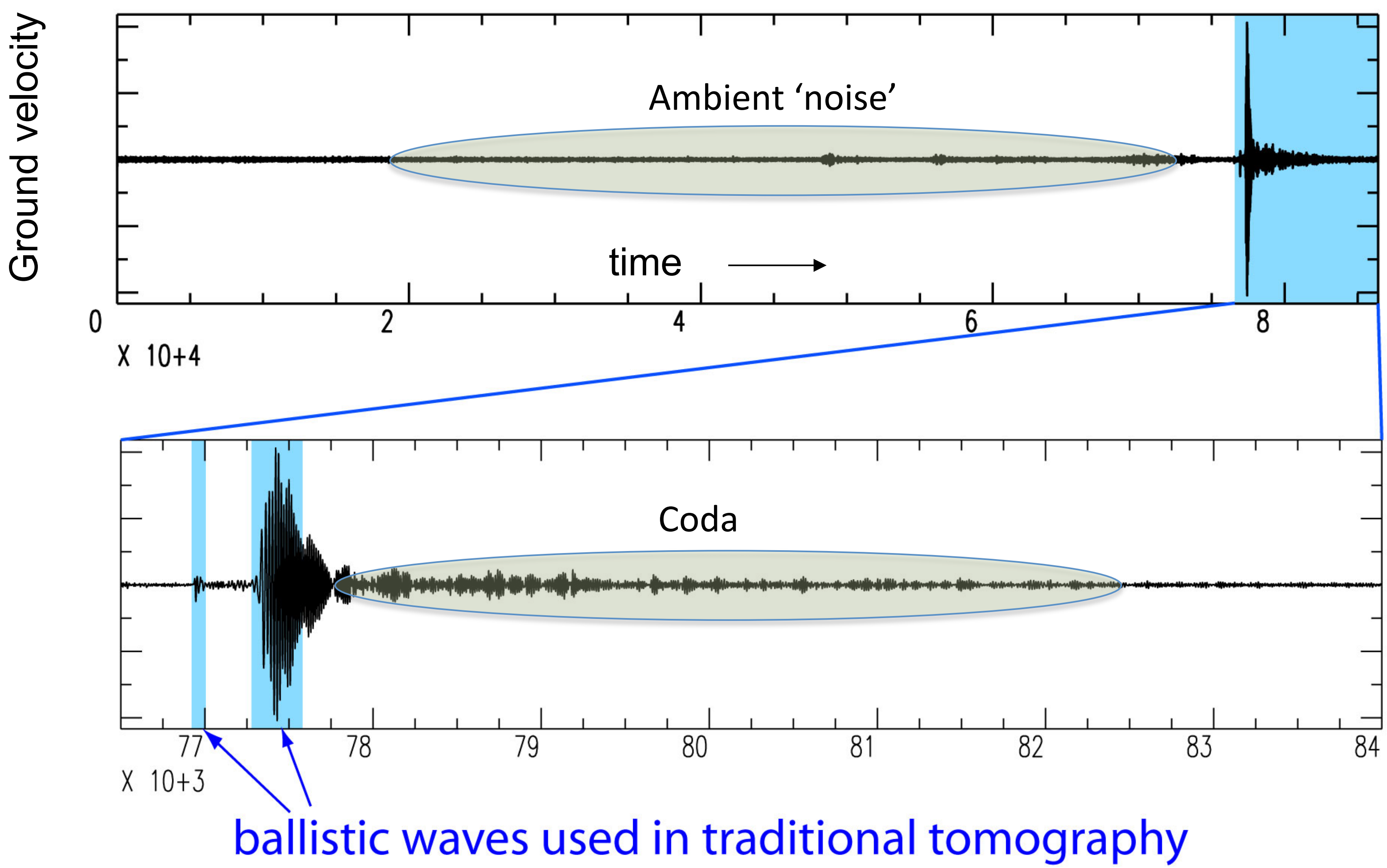
low frequency limit: half space

Margerin et al., 2009

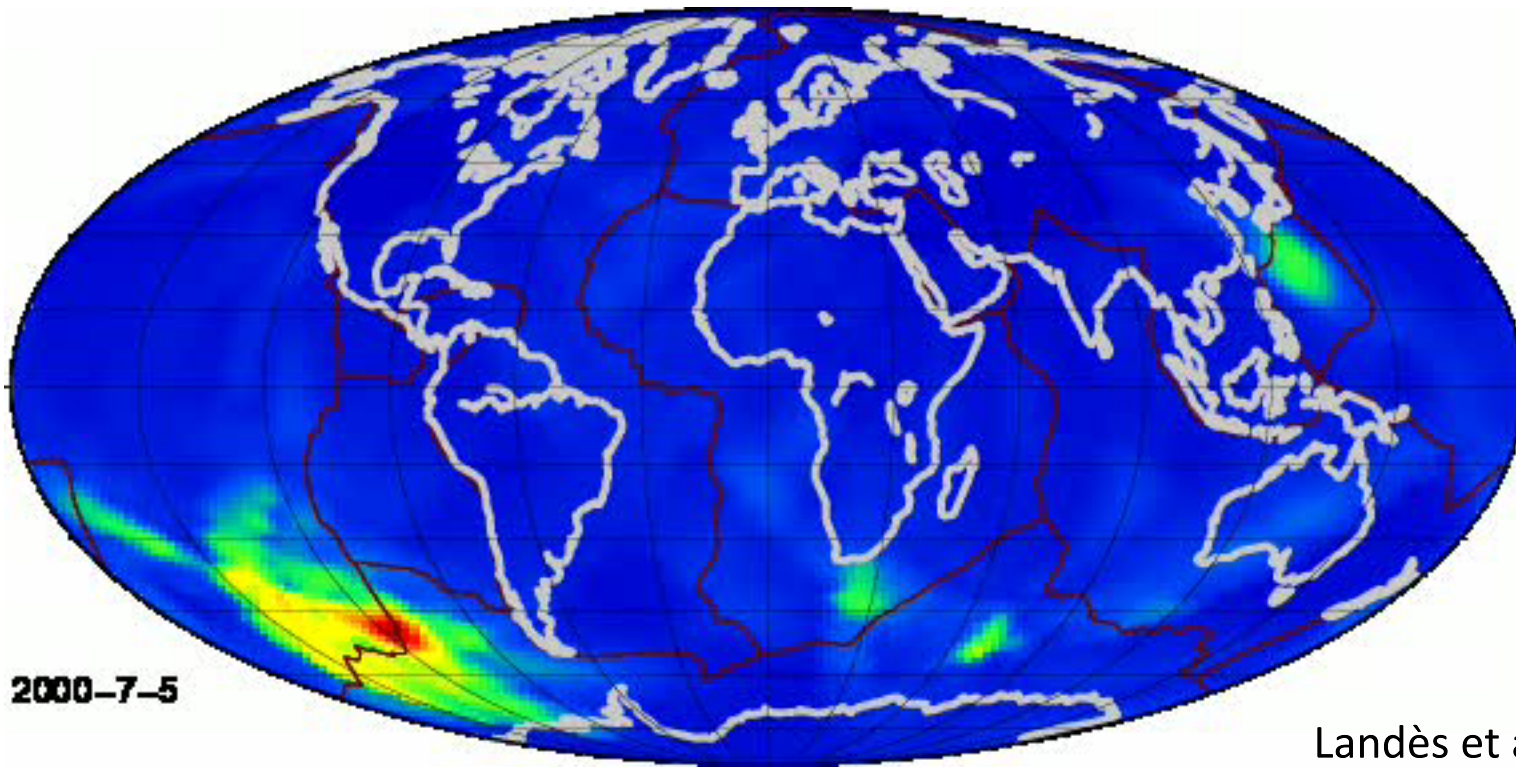

Ground displacement
noise
0
2
4
6
8
X 10+4
time
coda
77
78
79
80
81
82
83
84
X 10+3
Direct waves
Ballistic waves used for imaging

‘Noise’ = no well-defined source (analogy with coda?)

one day of seismic record



The origin of the noise in the period band 5-10s as seen by seismic arrays



Landès et al., 2010

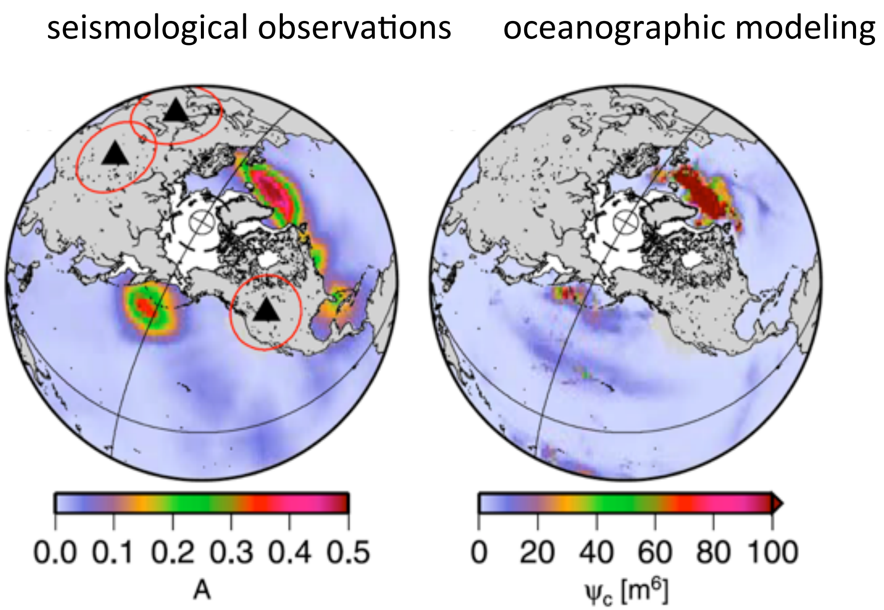
VARIABLE SOURCE LOCATIONS

At higher frequencies: human activity, wind,….

Global 'noise' sources in the microseism band
(extended ≈2-50s)

Strong contribution from oceanic waves

Example of a global comparison
(secondary microseism-
Miche/Longuet-Higgins mechanism)



Hillers et al., 2012

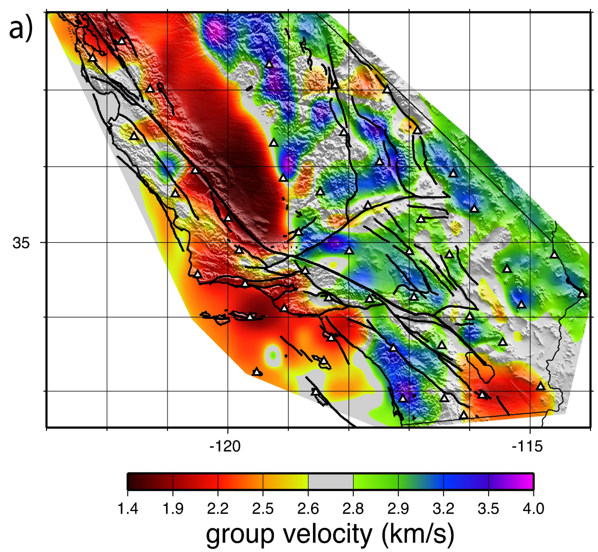
Longer periods: infragravity waves, e.g Fukao et al. 2010

+EARTHQUAKES

Cross-correlations of coda and noise records≈ Green functions = virtual seismograms

-demonstrated for the retrieval of surface waves (e.g. Paul and Campillo, 2001; Campillo and Paul, 2003; Shapiro and Campillo, 2004….) or body waves (e.g. Zhan et al., 2010 ; Poli et al., 2012).

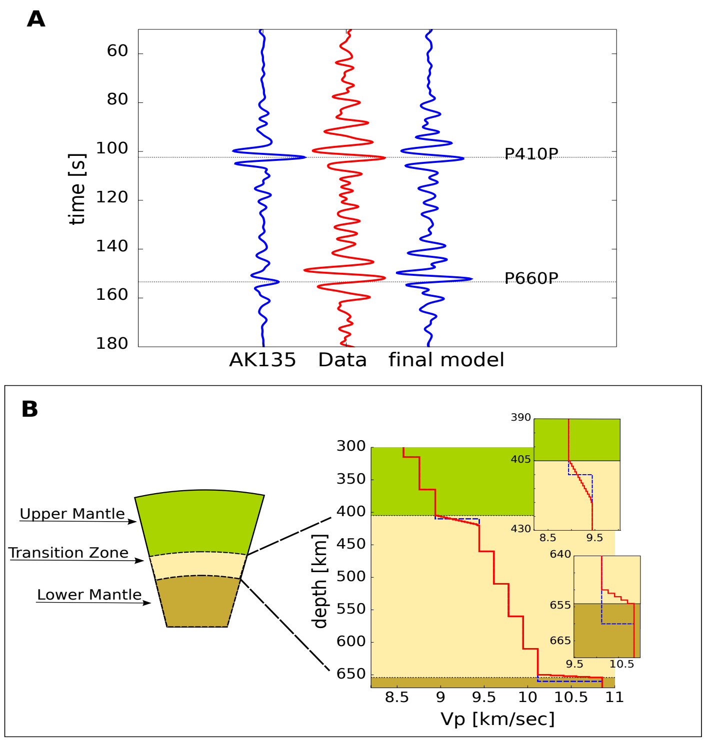
High resolution velocity map of California obtained from ambient noise (Rayleigh)
(Shapiro, Campillo, Stehly and Ritzwoller, Science 2005)



Earth's mantle discontinuities from ambient noise (phase transition ➔ (P,T))

Body waves (Poli et al., 2012)

Poli, Campillo, Pedersen. Science 2012



Large N sensor array =➔ $N^2/2$ correlations

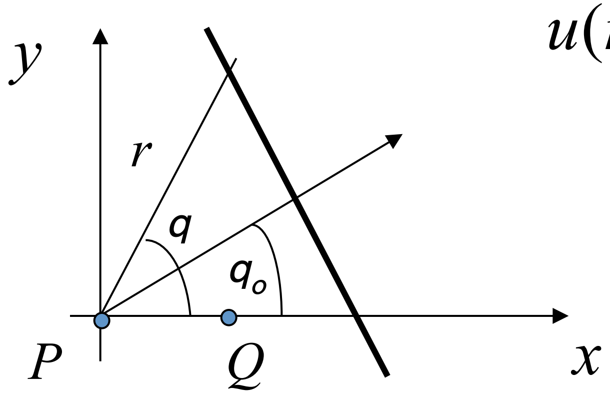
Aki (1957)

Evaluating k at different frequencies makes it possible to obtain the dispersion curve $C(\omega)$.

The method relies on the hypothesis of the stationnarity of the noise and requires specific array design to perform the azimuthal average.

Another approach consists of using only two points and to rely on long term average to produce the azimuthal average.

Let us consider a plane wave in 2D:



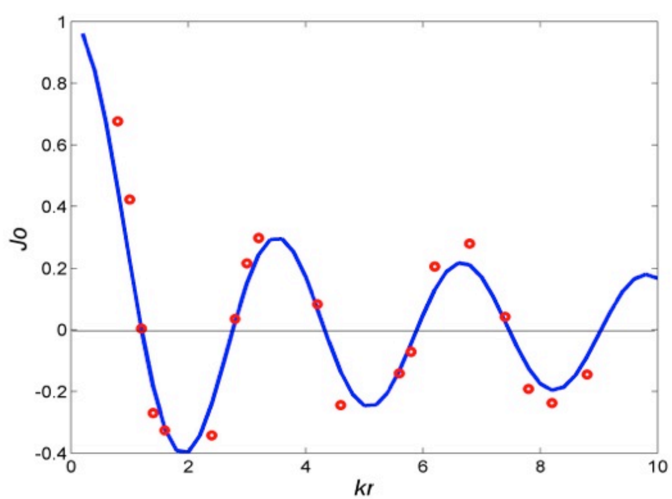
$$u(r,\theta,\omega) = F(\omega)\exp(-ikr\cos(\theta-\theta_0))$$

$$\frac{u^P u^{Q^*}}{|u^P||u^Q|} = e^{+ikr\cos\theta_0}$$

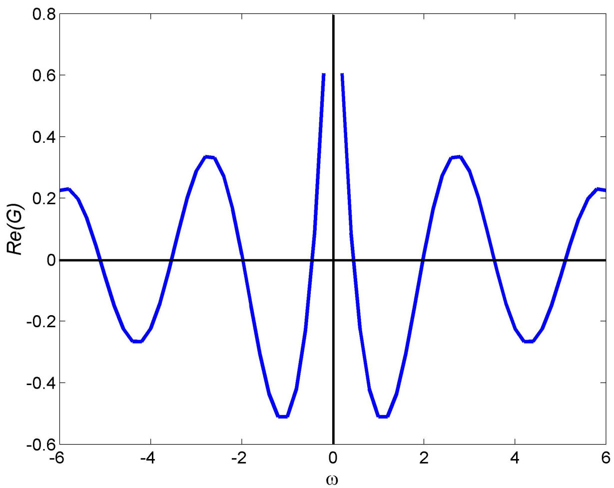
$$\boxed{\left\langle \rho(r,\omega) \right\rangle = \left\langle \frac{u^P u^{Q^*}}{\left|u^P\right|\left|u^Q\right|} \right\rangle = \left\langle e^{ikr\cos\theta_0} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} \underbrace{e^{ikr\cos\theta_0}} d\theta_0 = J_0(kr)}$$

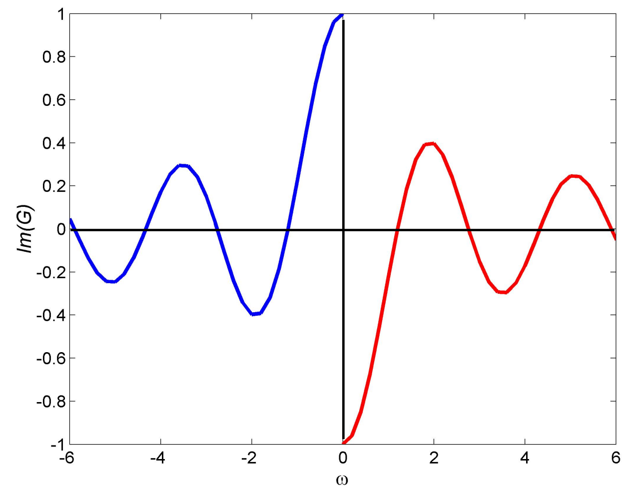
$$\frac{1}{2\pi}\int_0^{2\pi}\left(\sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m J_m(kr)\cos m\theta_0\right)d\theta_0$$

azimuthal average of the spatial cross-correlation



$$G=\frac{1}{4\mathrm{i}\mu}H_0^{(2)}\left(\frac{\omega r}{\beta}\right)=\frac{1}{4\mu}\left\{-Y_0\left(\frac{\omega r}{\beta}\right)-\mathrm{i}\,J_0\left(\frac{\omega r}{\beta}\right)\right\}$$



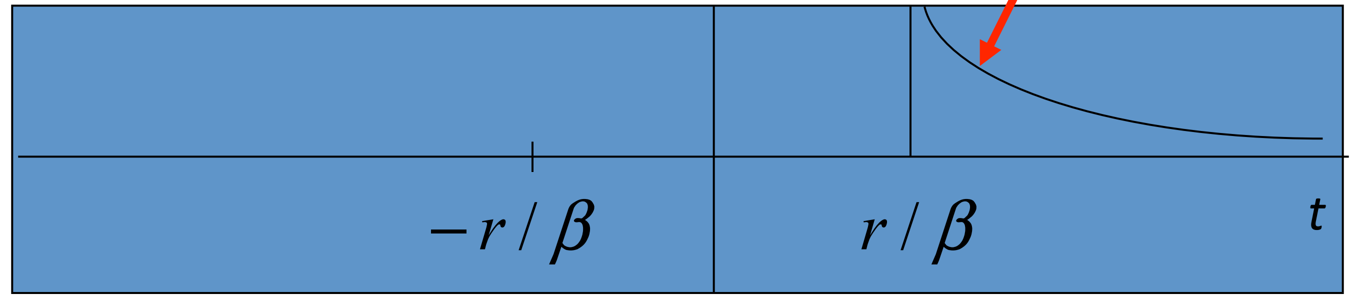


$$G(r,\omega)=\frac{-1}{4\mu}\left\{\mathcal{H}\left[\mathrm{sgn}(\omega)\boxed{J_0\left(\frac{\omega r}{\beta}\right)}\right]+i\,\mathrm{sgn}(\omega)\boxed{J_0\left(\frac{\omega r}{\beta}\right)}\right\}$$

Causality

$$G = \frac{1}{4i\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right)$$

$$G = \frac{1}{2\pi\mu} \frac{H\left(t - {}^{r}\!/_{\beta}\right)}{\sqrt{t^2 - {}^{r^2}\!/_{\beta^2}}}$$

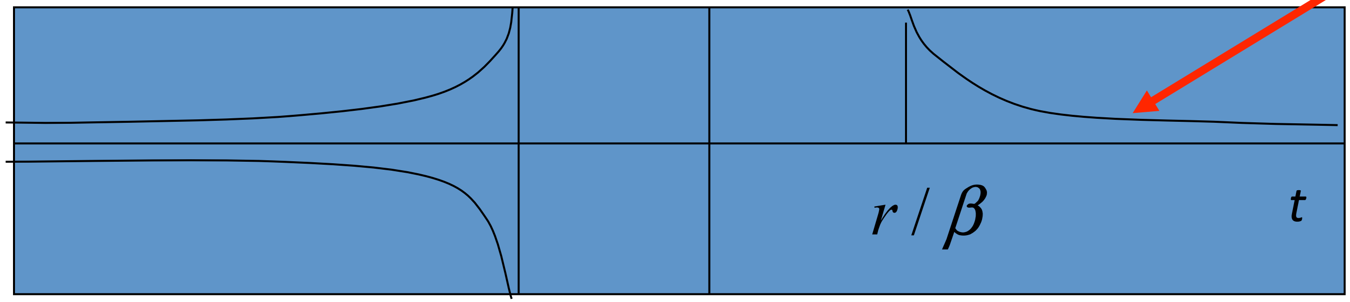


$G/2$

(Re)

(Im)

(Im, Re)



$$G_{22}(r,\omega) = \frac{1}{4\mu}\left\{-Y_0\left(\frac{\omega r}{c}\right) - iJ_0\left(\frac{\omega r}{c}\right)\right\}$$

$$J_0\left(\frac{\omega r}{c(\omega)}\right) = -4\mu\,\mathrm{Im}\left(G_{22}(r,\omega)\right) \qquad r = \left|P,Q\right|$$

$$\boxed{\boxed{\mathrm{Im}\left(G_{22}^{PQ}\right) = \frac{-1}{4\mu}\left\langle \frac{u_2(P)u_2^*(Q)}{\left|u_2(P)\right|\left|u_2(Q)\right|} \right\rangle}}$$

Green function in 2D

P-SV case

$$G_{ij} = \frac{i}{4\rho\omega^2}\left\{-\delta_{ij}k^2 H_0^{(2)}(kr) + \frac{\partial^2}{\partial x_i \partial x_l}\left[H_0^{(2)}(qr) - H_0^{(2)}(kr)\right]\delta_{lj}\right\}$$

$$G_{ij}(P,Q) = \frac{-\mathrm{i}}{8\rho}\left\{A\delta_{ij} - B\left(2\gamma_i\gamma_j - \delta_{ij}\right)\right\} \qquad \gamma_j = \frac{x_j - \xi_j}{r}$$

$$A = \frac{H_0^{(2)}(qr)}{\alpha^2} + \frac{H_0^{(2)}(kr)}{\beta^2} \qquad B = \frac{H_2^{(2)}(qr)}{\alpha^2} - \frac{H_2^{(2)}(kr)}{\beta^2}$$

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad \beta = \sqrt{\frac{\mu}{\rho}} \qquad r = |P,Q|$$

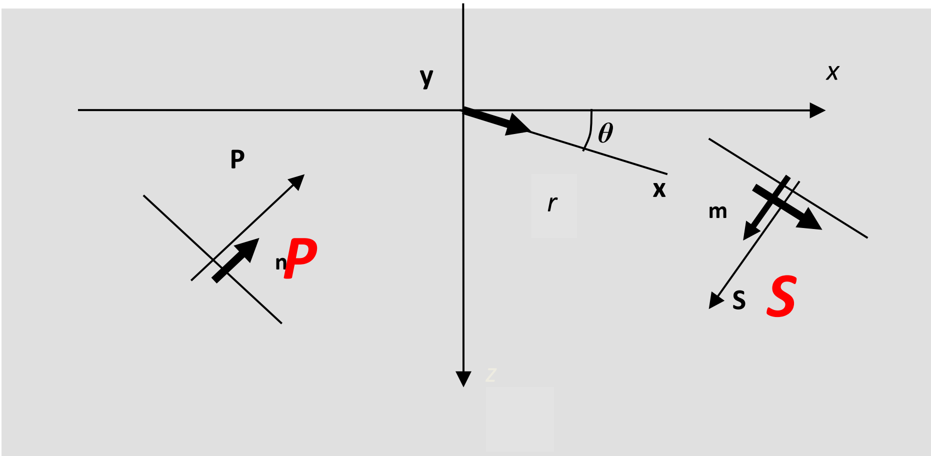
THE 2D VECTOR CASE

$$\beta^2 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (\alpha^2 - \beta^2)\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\partial^2 u_i}{\partial t^2}$$

Summation of P and S plane waves:

$$u_l(\mathbf{x}, \omega, t) = P(\omega, \phi) n_l \exp(-\mathrm{i}\frac{\omega}{\alpha} x_j n_j) + S(\omega, \psi) m_l \exp(-\mathrm{i}\frac{\omega}{\beta} x_j m_j)$$

Correlation:



$$u_l(\mathbf{y}) u_s^*(\mathbf{x}) =$$

$$(P^2 n_l n_s + SP^* m_l n_s)\exp(\mathbf{i}kr\cos[\phi - \theta]) +$$

$$(S^2 m_l m_s + PS^* n_l m_s)\exp(\mathbf{i}kr\cos[\psi - \theta])$$

Azimuthal average:

$$\langle \bullet \rangle = \frac{1}{4\pi^2} \int_0^{2\pi} d\phi \int_0^{2\pi} \bullet \, d\psi$$

$$\langle u_i(\mathbf{y}) u_j^*(\mathbf{x}) \rangle = \frac{S^2 \beta^2}{2} \left\{ A\delta_{ij} - B(2\gamma_i \gamma_j - \delta_{ij}) \right\}$$

$$A = \varepsilon \frac{J_0(qr)}{\alpha^2} + \frac{J_0(kr)}{\beta^2} \text{ and } B = \varepsilon \frac{J_2(qr)}{\alpha^2} - \frac{J_2(kr)}{\beta^2}$$

$$P^2 \alpha^2 = \varepsilon S^2 \beta^2$$

Equipartition (ε=1):

$$E_S / E_P = \left(\frac{\alpha}{\beta} \right)^2 \frac{1}{\varepsilon}$$

And finally if ε=1

$$\langle u_i(\mathbf{y}, \omega) u_j^*(\mathbf{x}, \omega) \rangle \equiv -8 E_S k^{-2} \operatorname{Im}\left[G_{ij}(\mathbf{x}, \mathbf{y}, \omega) \right]$$

Formally, same result in 3D (Sánchez-Sesma and Campillo, BSSA 2006)

Arbitrary medium: an integral representation written in the frequency domain

(see e.g. Weaver et al. 2004, or Snieder, 2007)

$$G_{12} - G_{12}^* = \boxed{\frac{4i\omega\kappa}{c}\int_{\mathcal{V}} G_{1x}G_{2x}^*\,dV} + \boxed{\oint_S \left[G_{1x}\vec{\nabla}\left(G_{2x}^*\right) - \vec{\nabla}\left(G_{1x}\right)G_{2x}^*\right]\vec{dS}}$$

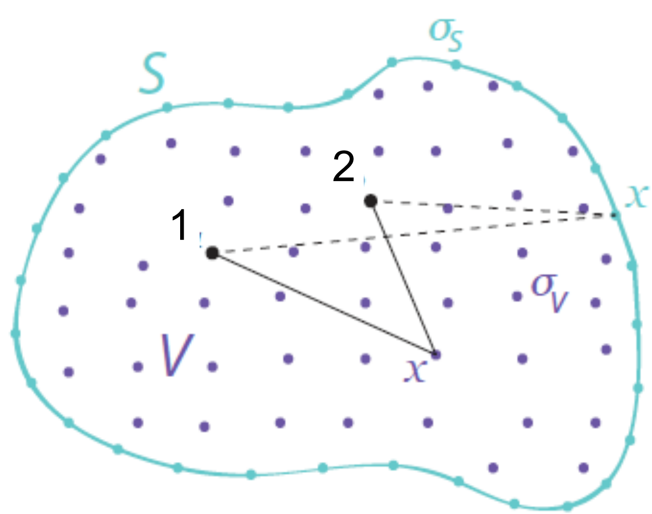
Volume term

Surface term

FT of $G(-t)$

Absorption coefficient

FT of $G(t)$



Helmholtz equation $G_{1x} = G(\vec{r}_1, \vec{x}; \omega)$

$$\Delta G_{1x} + V(\vec{x}) G_{1x} + (k + i\kappa)^2 G_{1x} = \delta(\vec{x} - \vec{r}_1)$$

where the potential $V(\vec{x})$ describes the scattering contribution does not extend to infinity.

As for the classical representation theorem, we consider a combination of the fields from source at 1 and 2 and compute the flux:

$$I = \oint_S \left[G_{1x} \vec{\nabla}\left(G_{2x}^*\right) - \vec{\nabla}\left(G_{2x}\right) G_{1x}^* \right] \overrightarrow{dS}$$

With the divergence theorem:

$$I = \int_{\mathcal{V}} \vec{\nabla} \left[G_{1x} \vec{\nabla}\left(G_{2x}^*\right) - \vec{\nabla}\left(G_{1x}\right) G_{2x}^* \right] dV$$

$$I = \int_{\mathcal{V}} \vec{\nabla}\left[G_{1x} \vec{\nabla}\left(G_{2x}^{*} \right) - \vec{\nabla}\left(G_{1x} \right) G_{2x}^{*} \right] dV$$ reduces to

$$I = \int_{\mathcal{V}} \left(G_{1x} \Delta G_{2x}^{*} - \Delta G_{1x} G_{2x}^{*} \right) dV$$

Using the definition of the GF:

$$\Delta G_{1x} = \delta\left(\vec{x} - \vec{r}_1 \right) - V\left(\vec{x} \right) G_{1x} - \left(k + i\kappa \right)^2 G_{1x}$$

we obtain:

$$I = G_{12} - G_{21}^{*} - \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^{*} \, dV$$

and finally:

$$G_{12} - G_{12}^{*} = \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^{*} \, dV + \oint_{S} \left[G_{1x} \vec{\nabla}\left(G_{2x}^{*} \right) - \vec{\nabla}\left(G_{1x} \right) G_{2x}^{*} \right] \overrightarrow{dS}$$

Surface term: $$G_{12} - G_{12}^{*} = \oint_S \left[G_{1x} \vec{\nabla}\left(G_{2x}^{*}\right) - \vec{\nabla}\left(G_{1x}\right) G_{2x}^{*} \right] \overrightarrow{dS}$$

κ =0 (no attenuation)

No source in the bulk

Surface term:

$$G_{12} - G_{12}^{*} = \oint_{S} \left[G_{1x} \vec{\nabla} \left(G_{2x}^{*} \right) - \vec{\nabla} \left(G_{1x} \right) G_{2x}^{*} \right] \overrightarrow{dS}$$

If the surface is taken in the far field of the medium heterogeneities

$$G_{1x} \sim \frac{1}{4\pi \left| \vec{x} - \vec{r}_1 \right|} \exp\left(-ik \left| \vec{x} - \vec{r}_1 \right| \right) \text{ and } \vec{\nabla} \left(G_{1x} \right) \sim i\vec{k}\, G_{1x}$$

and we obtain another widely used integral relation:

$$G_{12} - G_{12}^{*} = -2i \frac{\omega}{c} \oint_{S} G_{1x} G_{2x}^{*} \, dS$$

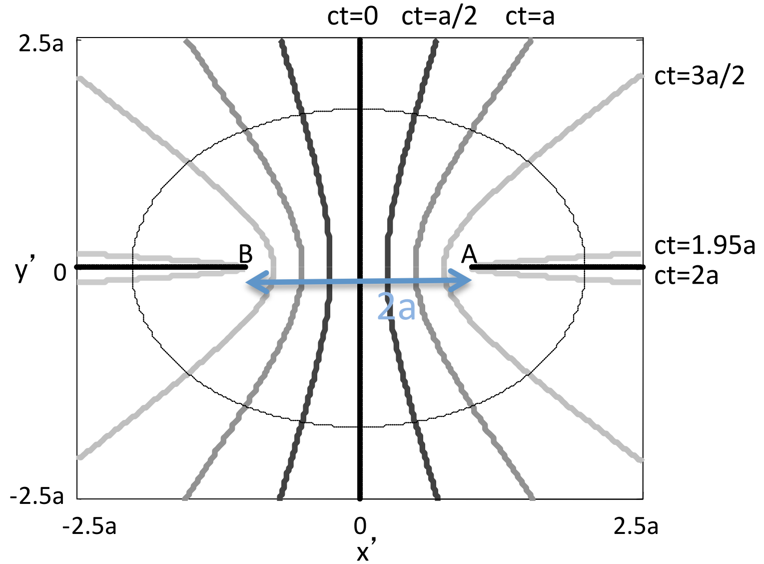
Volume term: $$G_{12} - G_{12}^{*} = \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^{*} \, dV$$

κ is finite (attenuation)

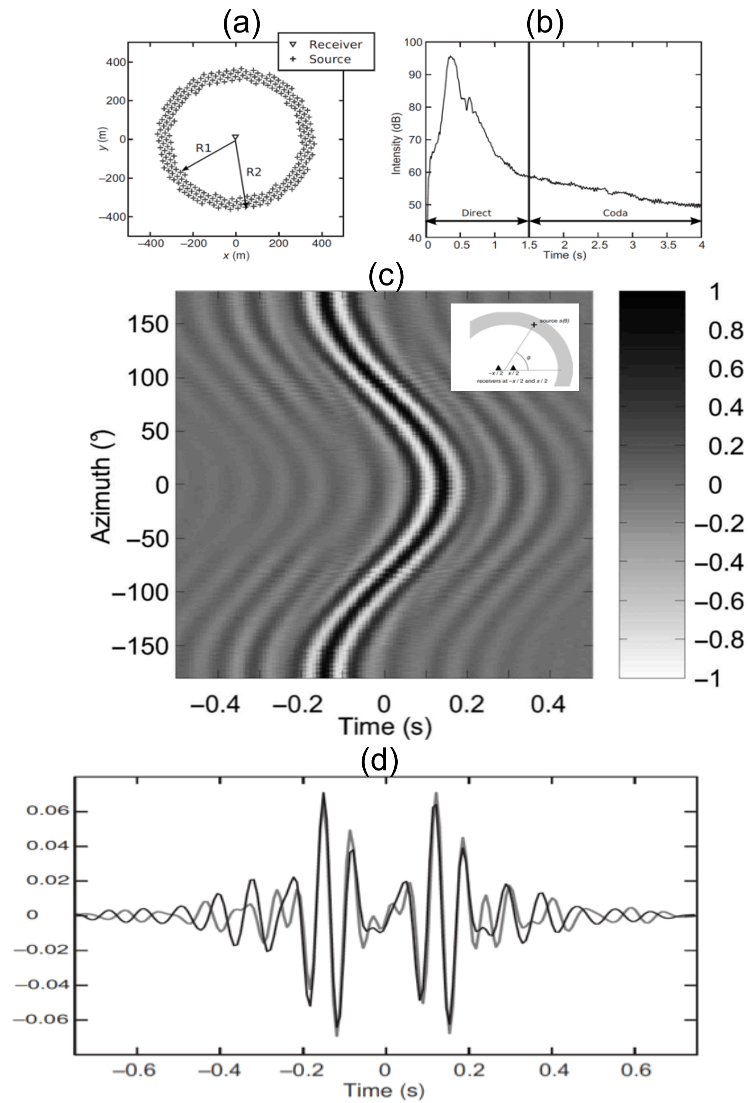
S is assumed to be sufficiently far away, for its contribution to be neglected (spreading and attenuation)

Location of the sources that contribute to the correlation: the end fire lobes

Difference of travel time between A and B
wrt the position of the source

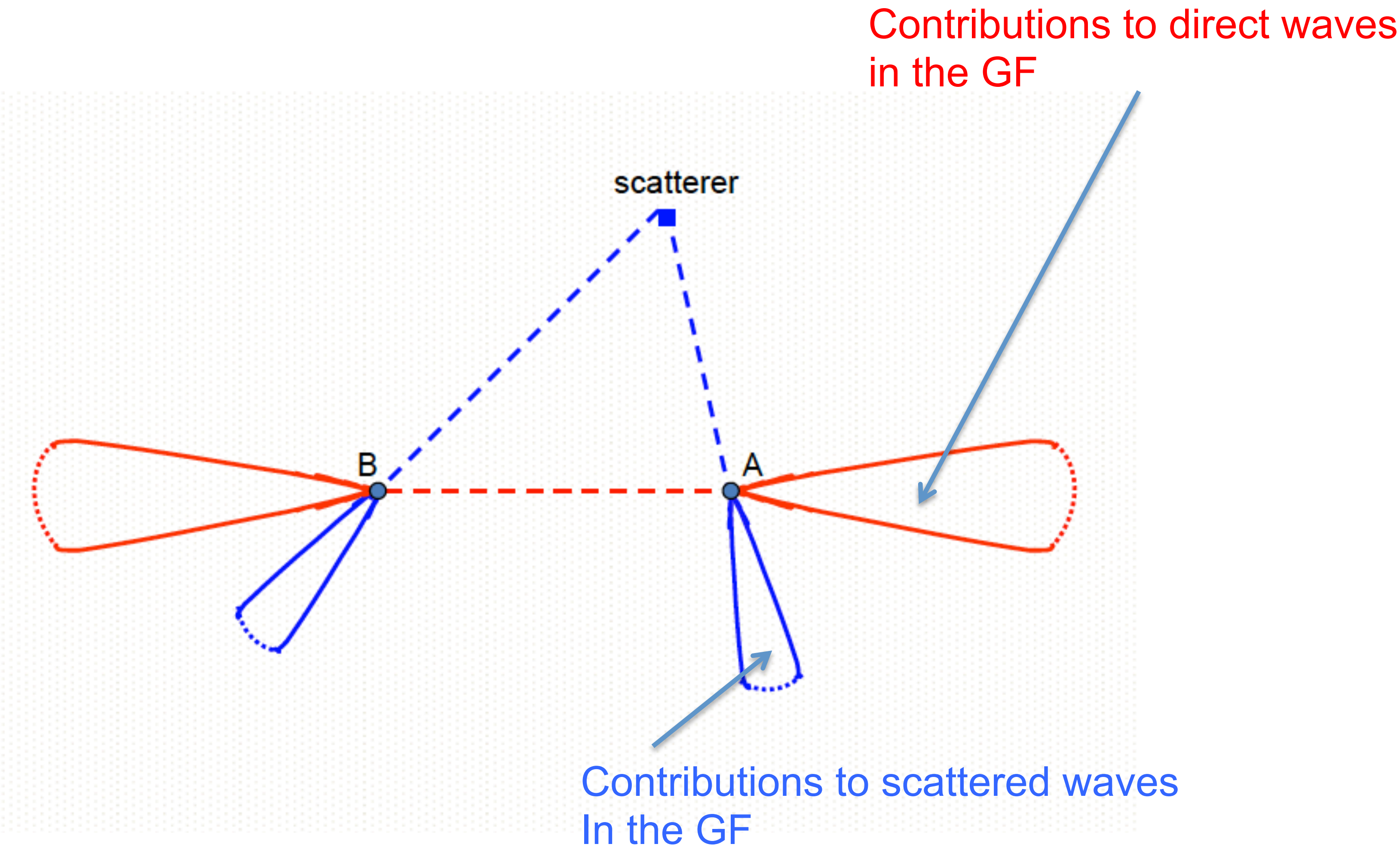


Stationary phase and end fire lobes



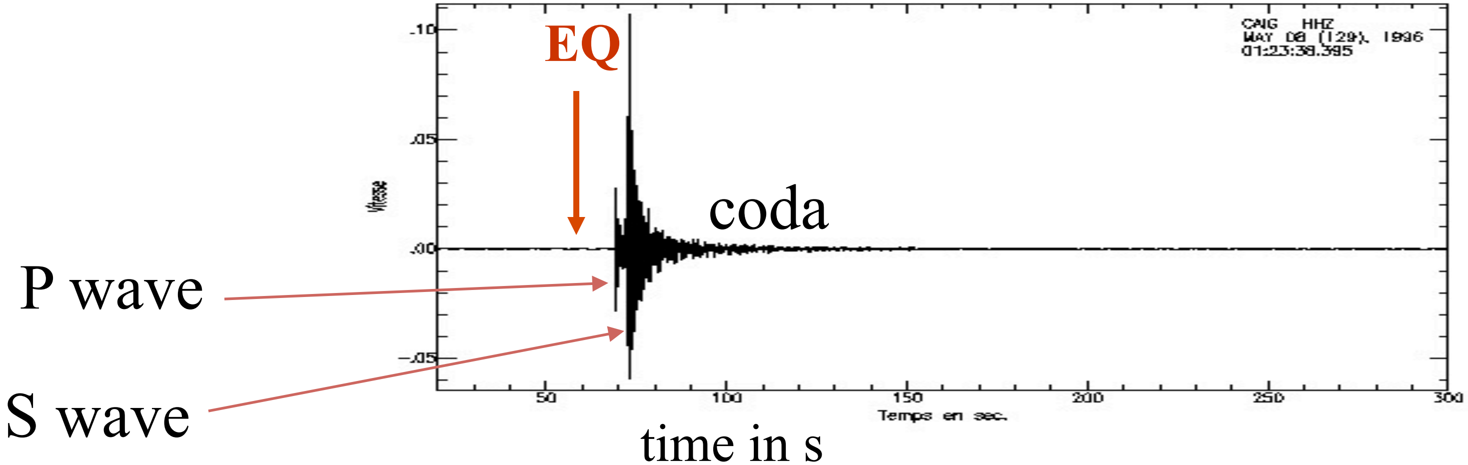
From Gouédard et al., 200X

End fire lobes

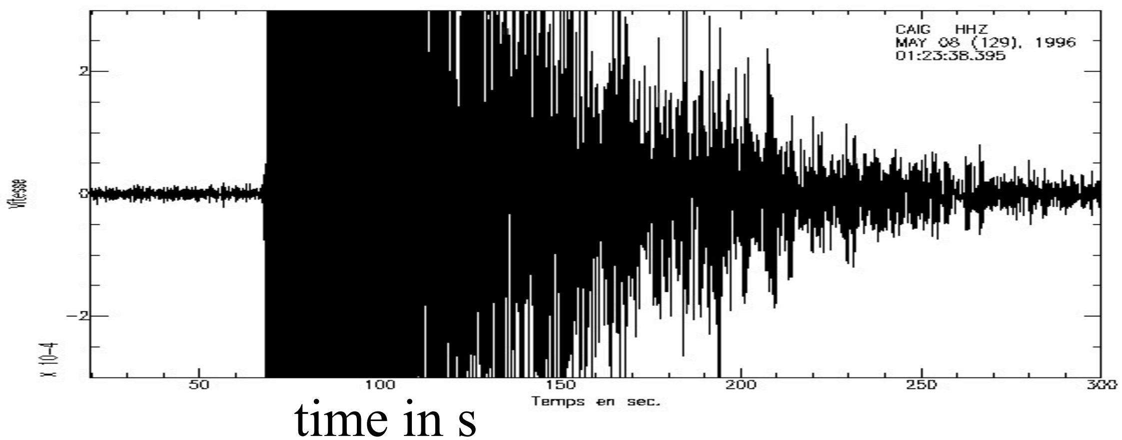


Extension to scattered waves

Correlations of coda records



scale x 1000:



An argument independant of the representation theorems
Multiple scattering and equipartition: the simplest case (finite body)

$$\phi(\vec{r};t) = \sum_n a_n U_n(\vec{r}) \cos(\omega_n t)$$

Equipartion
All modes excited at the same level

$$< a_n a_m^* >= F(\omega_n) \delta_{nm}$$

correlation

$$C_{1,2}(t) = \frac{1}{T} \int_0^T \phi(\vec{r}_1, \tau) \phi(\vec{r}_2, t+\tau) d\tau$$

Assuming a long recording interval T, this reduces to:

$$C_{1,2}(t) = \frac{1}{2} \sum_n F(\omega_n) U_n(\vec{r}_1) U_n(\vec{r}_2) \cos(\omega_n t)$$

Compare with:

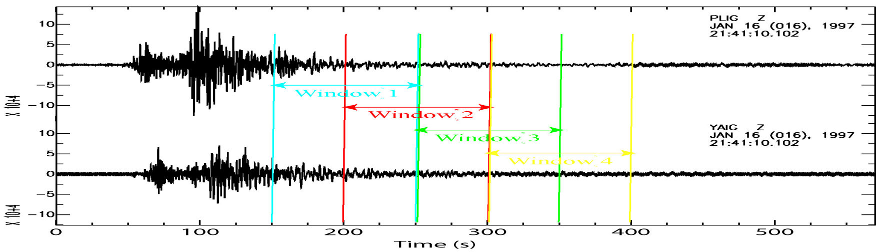
$$G(\vec{r}_1, \vec{r}_2; t) = \sum_n U_n(\vec{r}_1) U_n(\vec{r}_2) \frac{\sin(\omega_n t)}{\omega_n} \Theta(t)$$

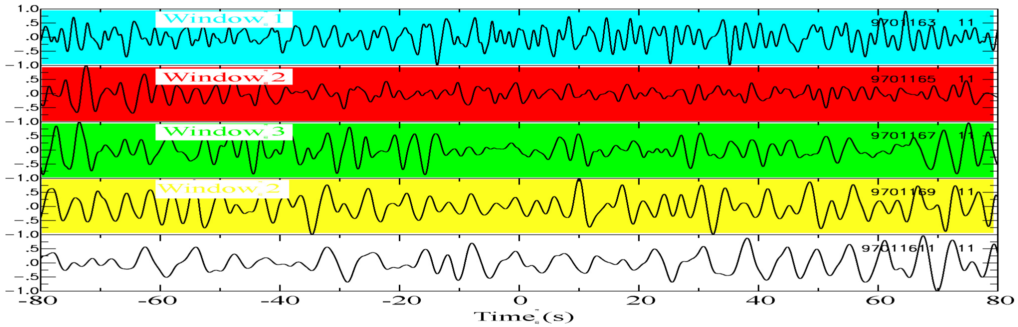
1 derivative 2 causality

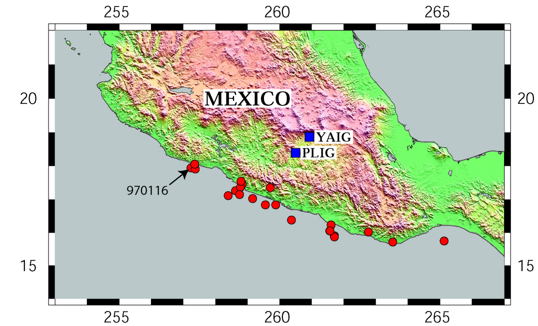
➔ Long range correlation in seismic coda= Green function *(Paul and Campillo, AGU 2001, Campillo and Paul, Science 2003)*

Seismological application: coda waves

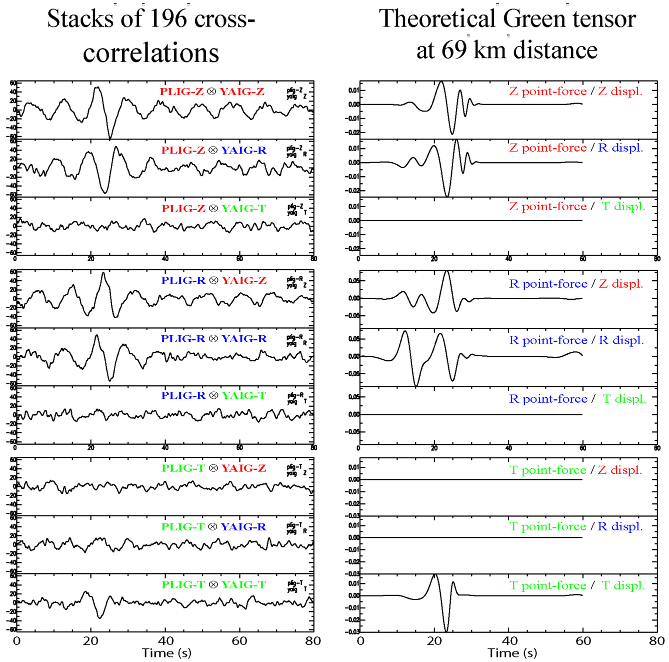
Individual cross-correlations:fluctuations dominate.







Emergence of the Green function



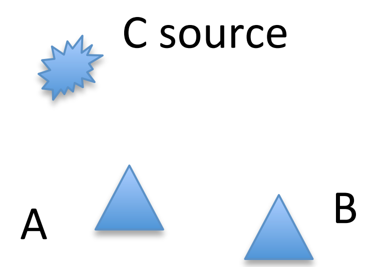
After averaging over 100 EQs➔

(Paul and Campillo, AGU 2001; Campillo and Paul, Science, 2003)

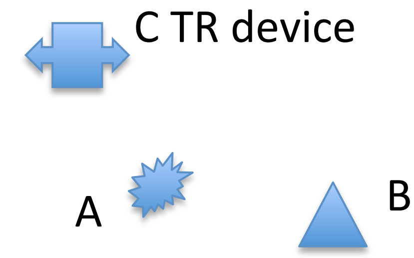
Physical interpretations

Time reversal

MOVIE : revers_water



Correlation and Time reversal
Focusing/virtual source in A



Equivalence in a reciprocal medium

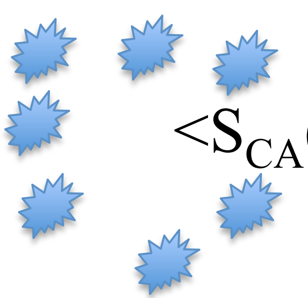
- C source
- A et B receivers

- A source
- C receiver ($S_{AC} = S_{CA}$)
- C emits the time reversed signal
- B receiver

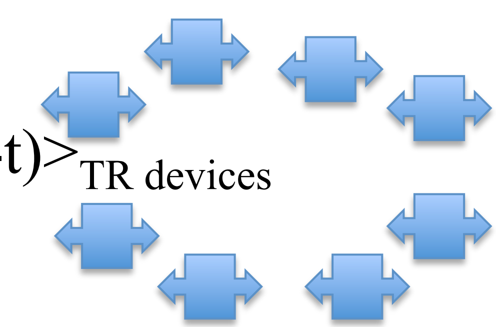
- Correlation :

$S_{CA}(t) \times S_{CB}(t) = S_{CA}(t) \otimes S_{CB}(-t)$

- Convolution :



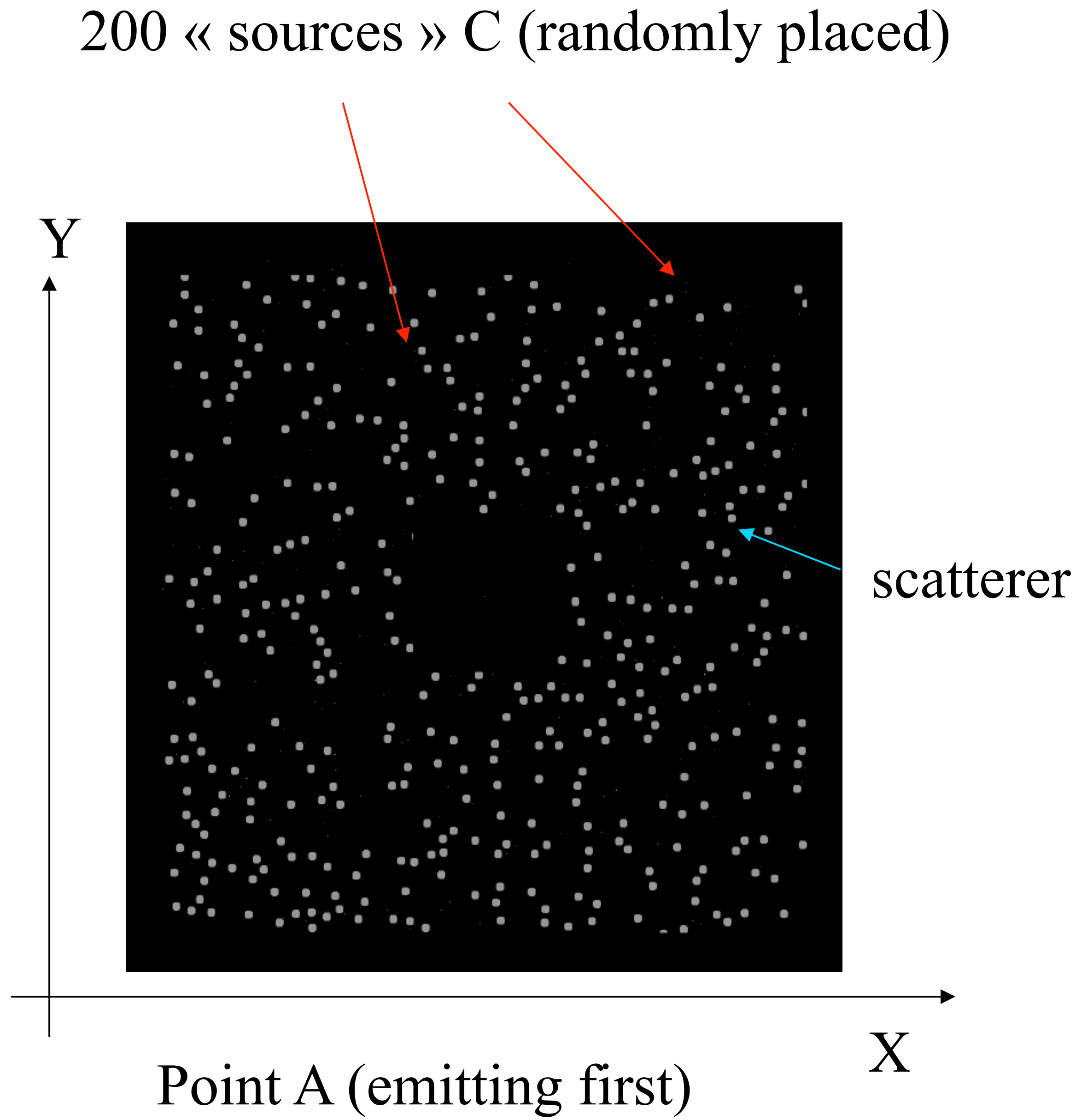
$$<S_{CA}(t) \times S_{CB}(t)>_{sources} \quad = \quad <S_{CA}(t) \otimes S_{CB}(-t)>_{TR\ devices}$$



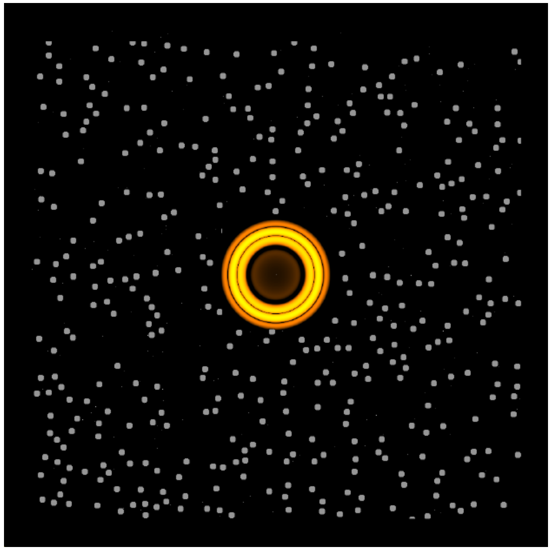
Ambient noise based method → using a (broken) time reversal mirror

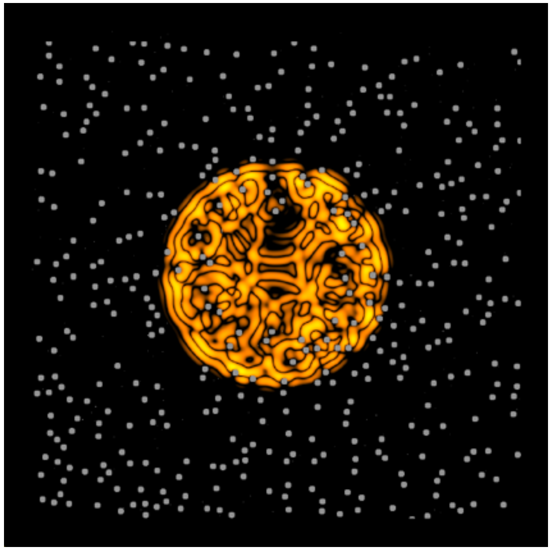
Derode et al., 2003++++

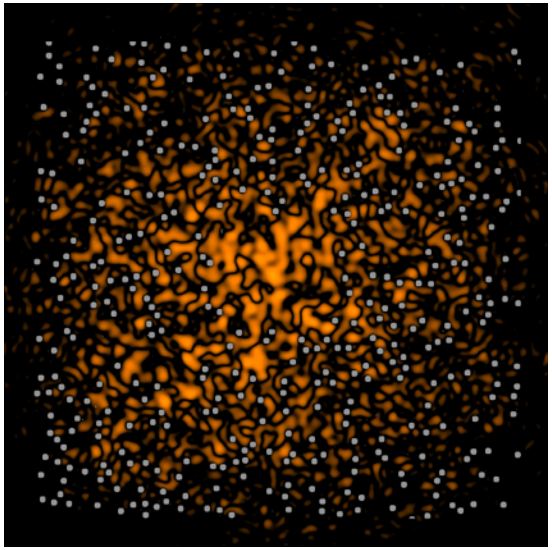
Numerical 2D FD simulation



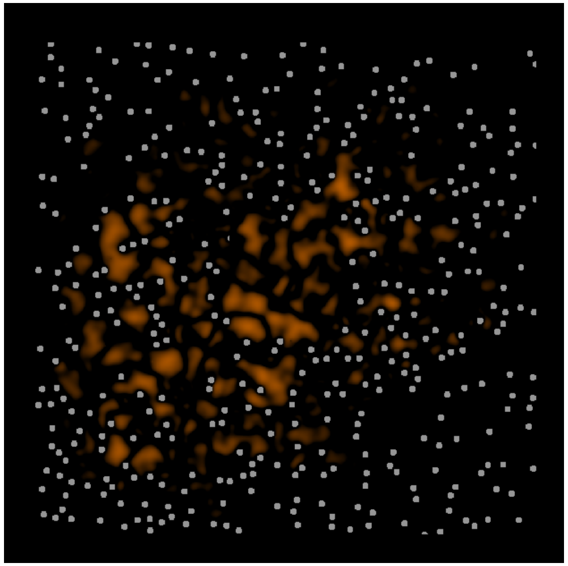
A pulse is emited in A
and recorded at point randomly
distributed



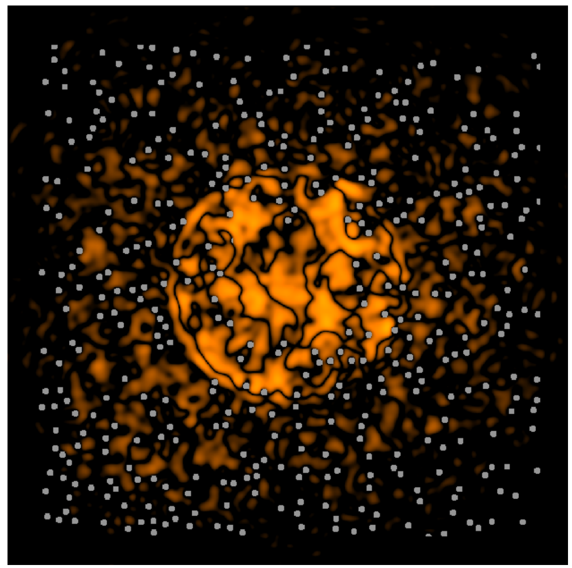




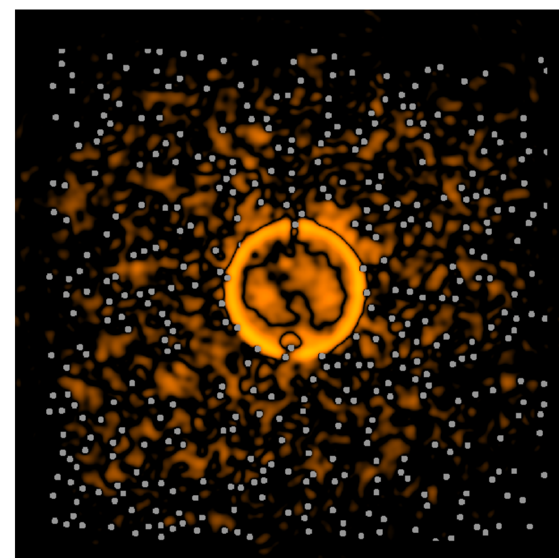
time



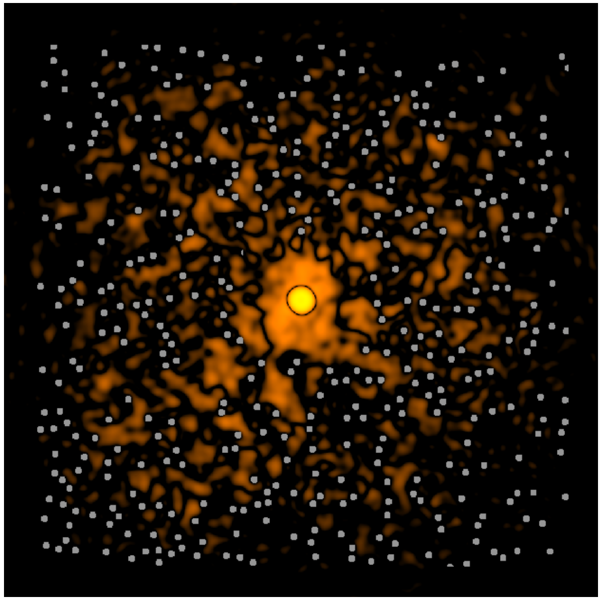
Re-emission from the points ‘C’ of the time-reversed signals
(map of cross-correlations)



Constructive interferences of time-reversed field

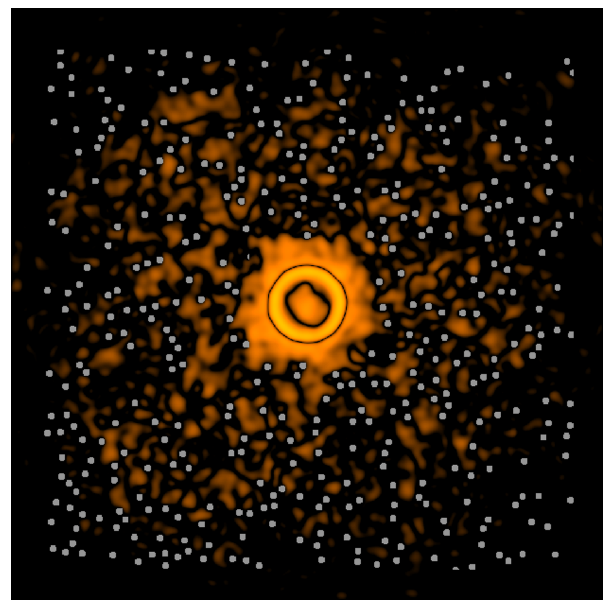


Converging field
: G(-t)

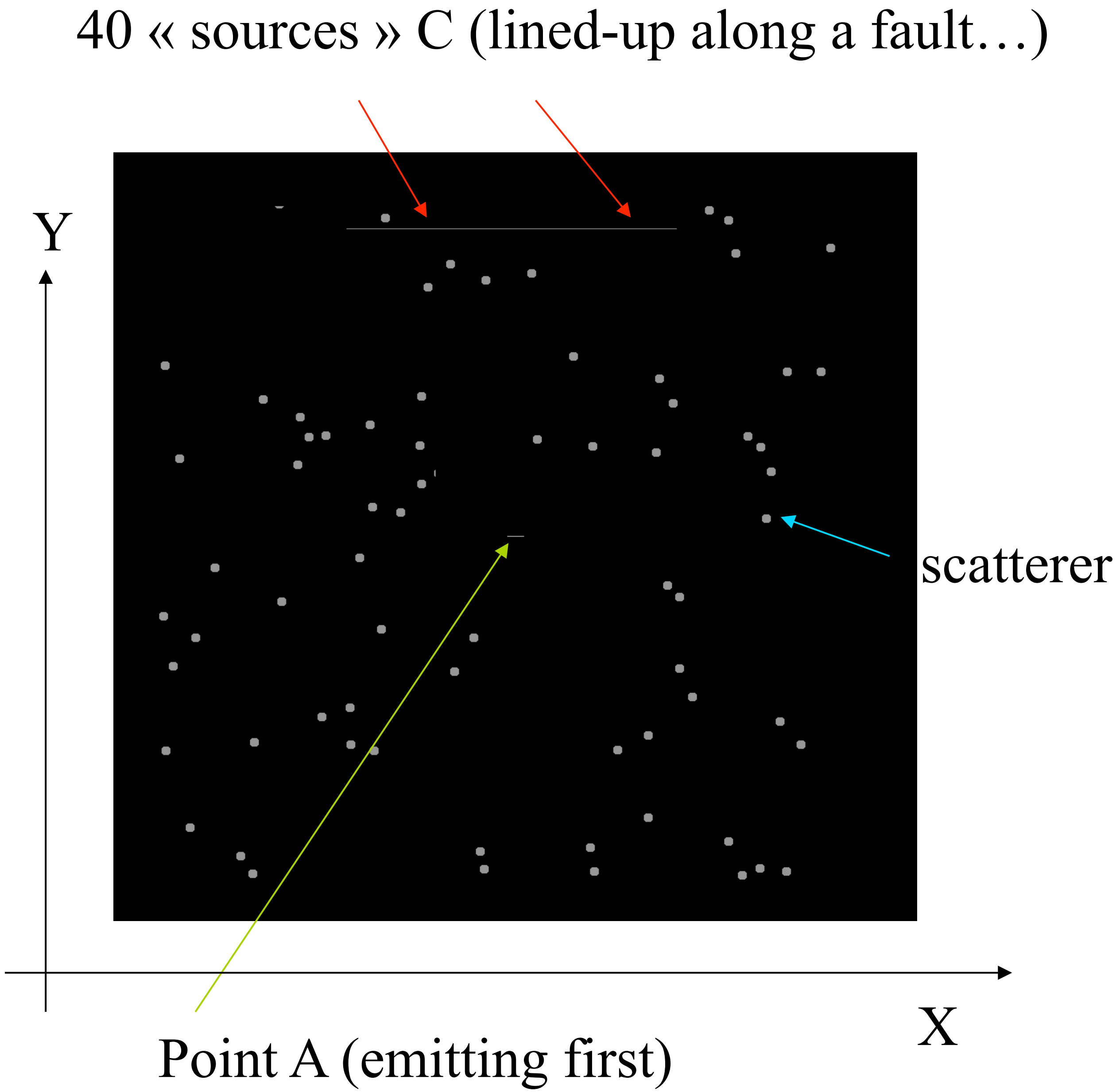


Nearly perfect refocalisation

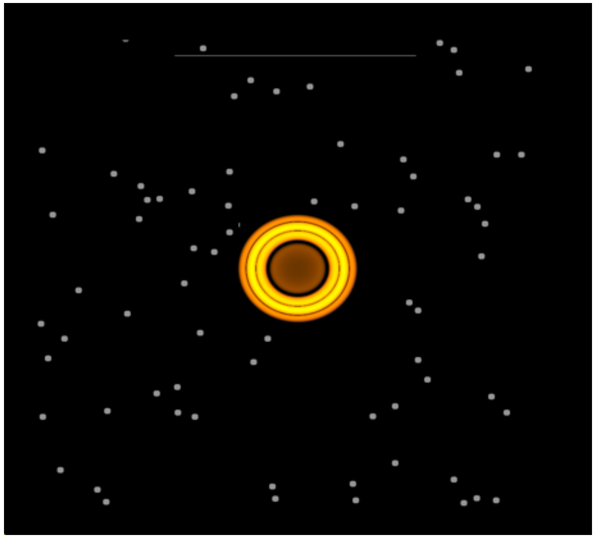
Re-emission from A :
G (t)



A more realistic configuration of sources

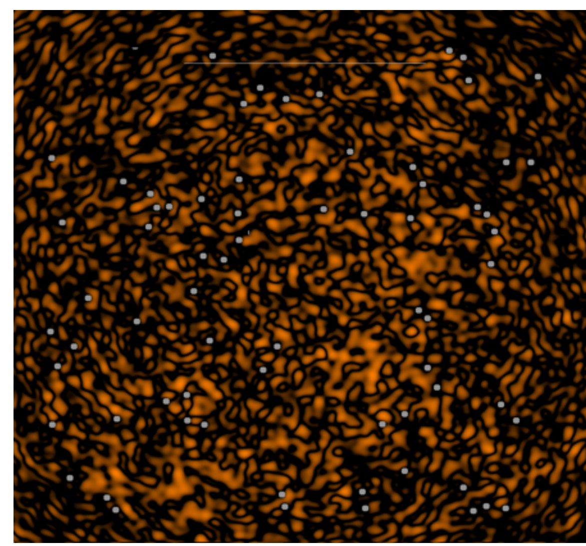


Time reversal experiment

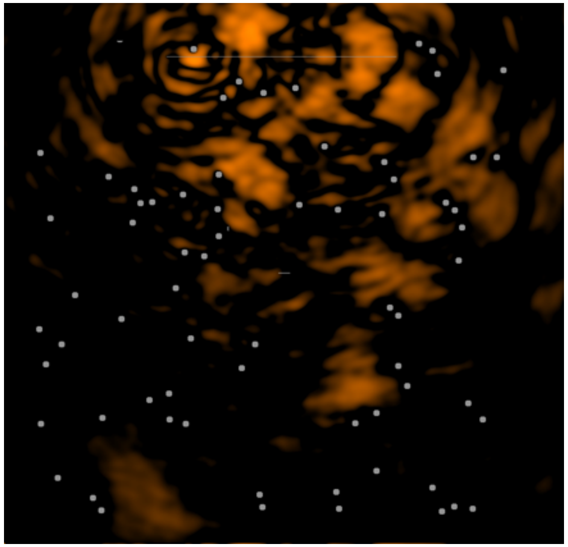


A send a pulse

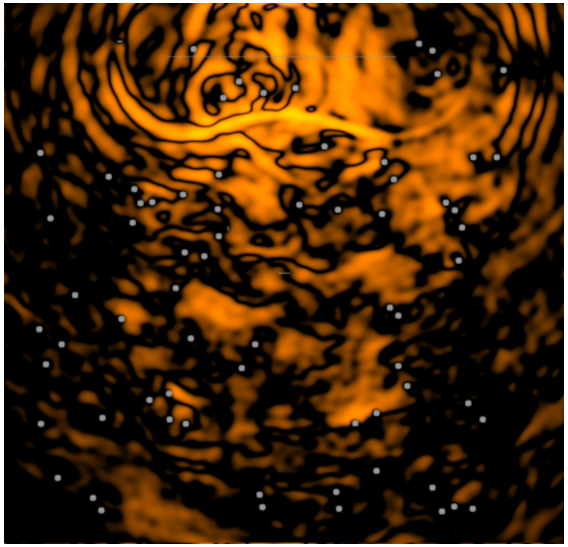
Scattering effects



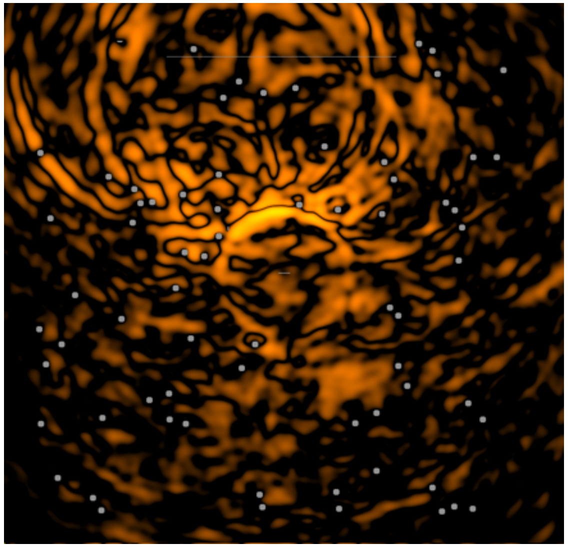
Diffuse field is
also recorded



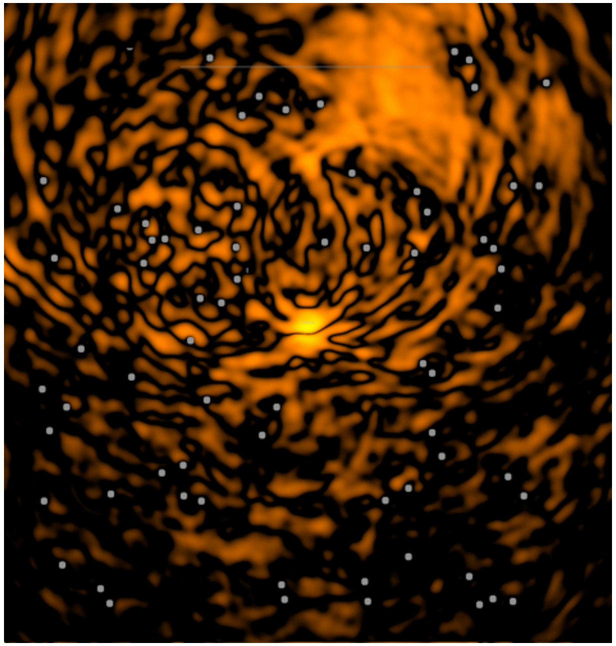
Re-emission



Converging field : G(-t)

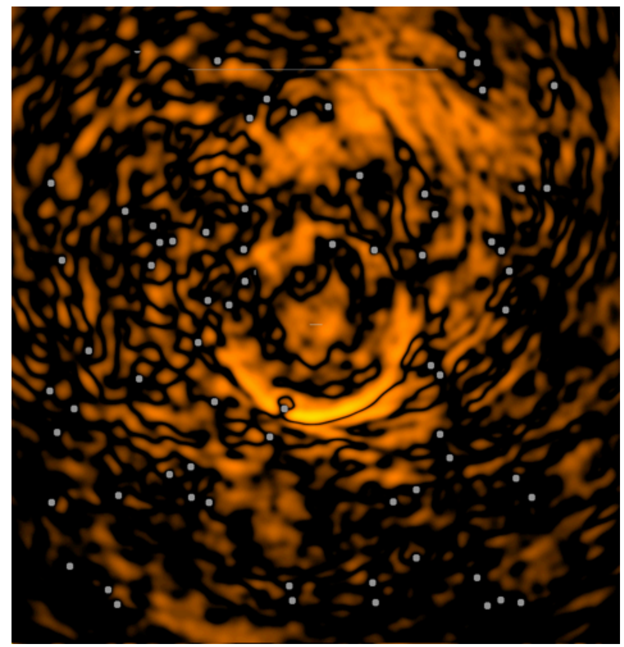


time



Partial focalisation

Diverging field : $h_{AB}(t)$



The symmetry of the Green function is lost!