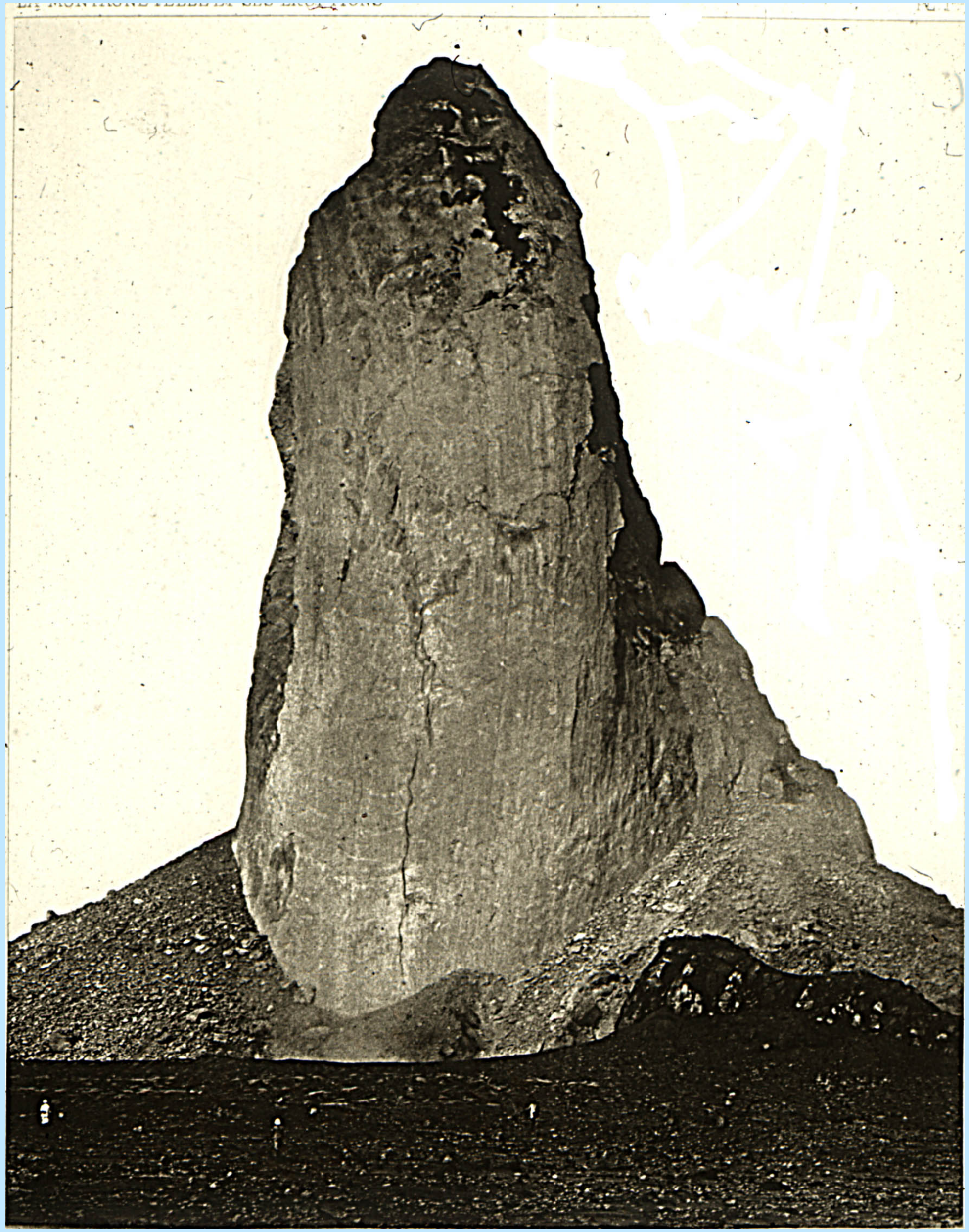
LAVA FLOWS AND DOMES

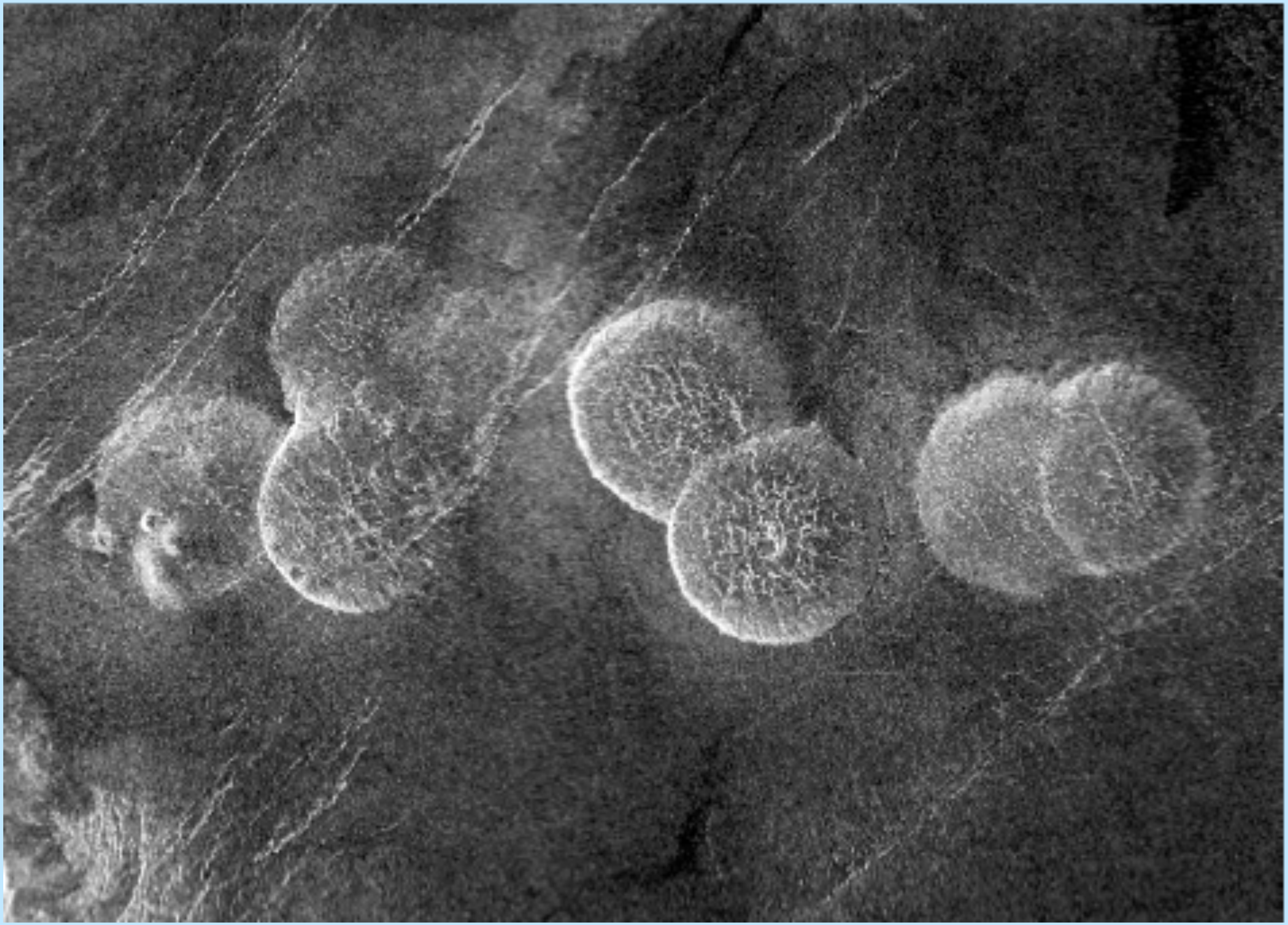
Dynamics of spreading
and
morphology

Colima

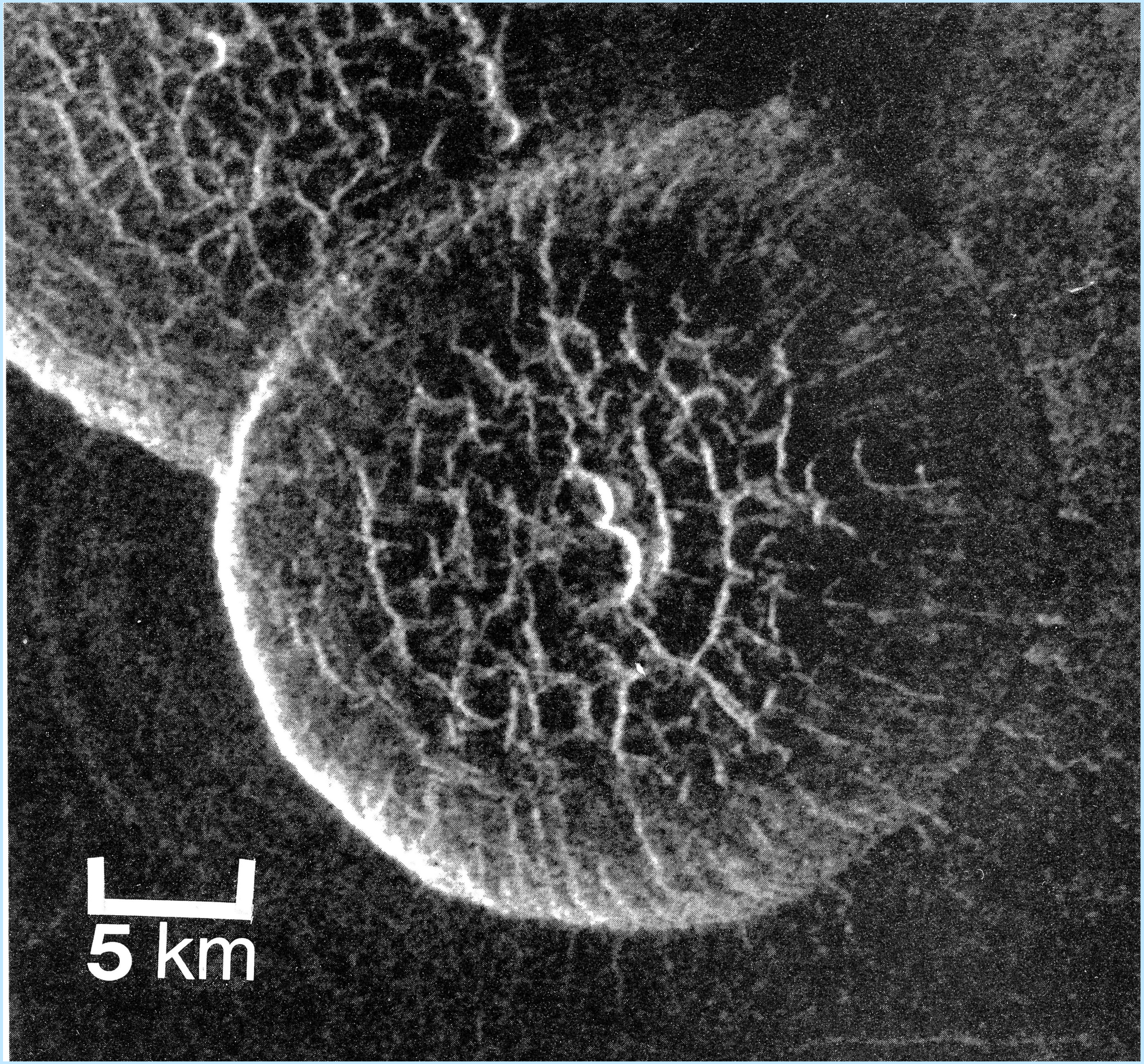
Unzen, Japan

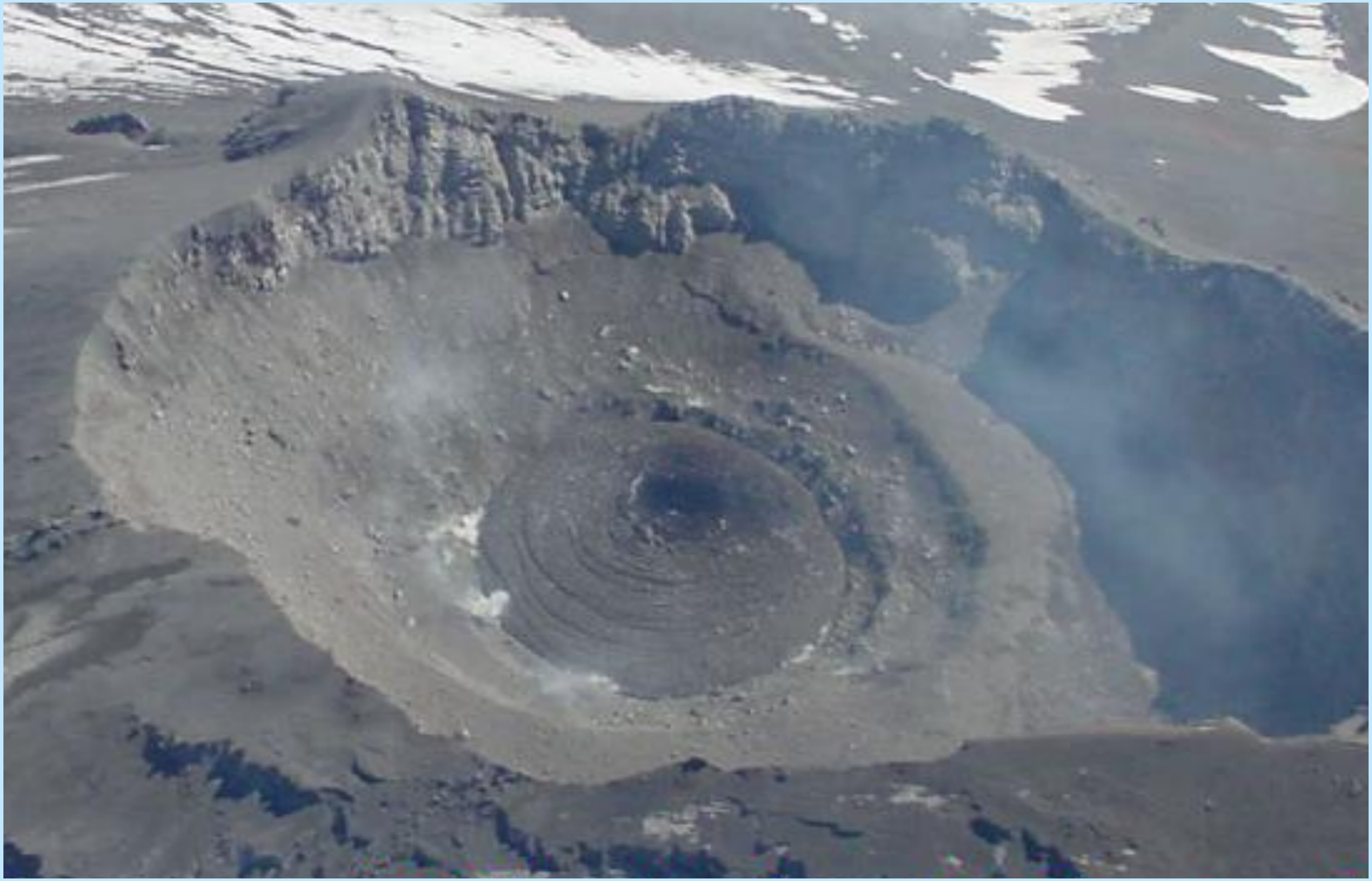


Montagne Pelée, Martinique (1902)

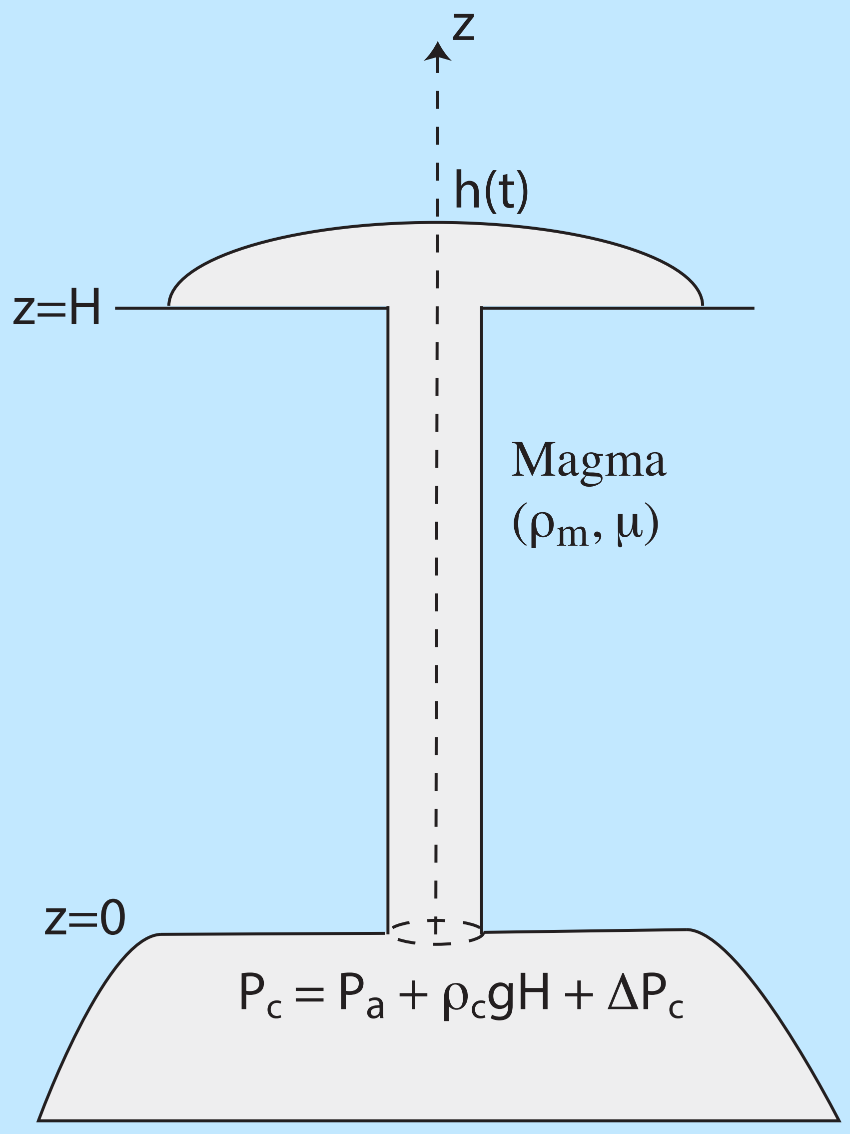


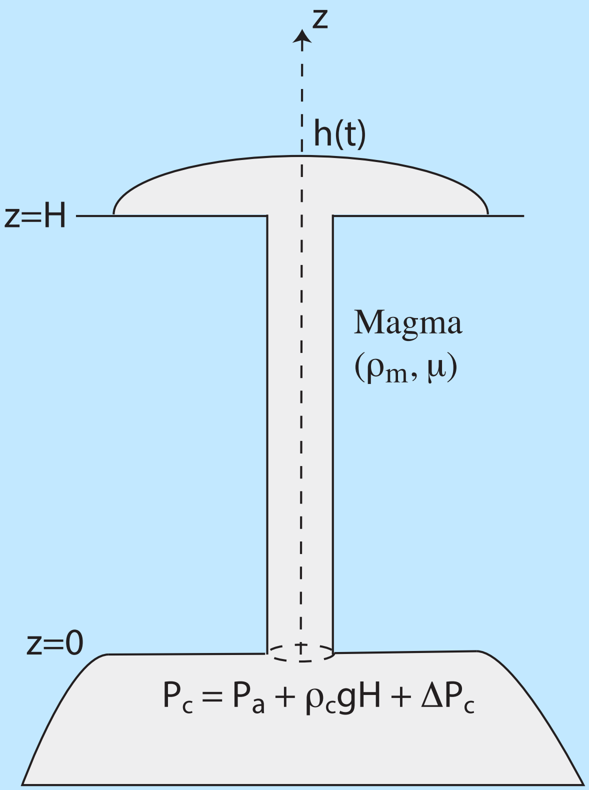
Venus


5 km



Popocatepetl, Mexico


z
h(t)
z=H
Magma
(ρ_m, μ)
z=0
$P_c = P_a + \rho_c g H + \Delta P_c$



Cylindrical conduit from $z = 0$ to $z = H$.
Lava flow or dome with thickness h.
$p(z)$ = pressure in the conduit.
Pressure at the vent:

$$p(H) = P_a + \rho_m g h$$

Pressure at $z = 0$ (top of the reservoir)
P_c = lithostatic pressure + overpressure (or underpressure) ΔP_c.

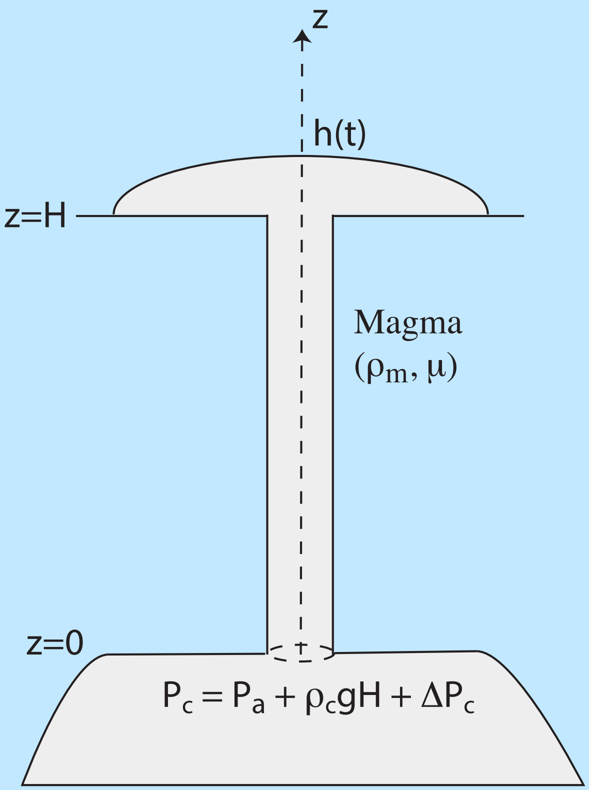
$$p(0) = P_c = P_a + \rho_c g H + \Delta P_c$$

Hydrostatic pressure component at $z = 0$.

$$P_L = P_a + \rho_m g (H + h)$$

Pressure difference that drives ascent

$$\Delta P = P_c - P_L = (\rho_c - \rho_m) H + \Delta P_c - \rho_m g h$$



Eruption stops when $\Delta P = 0$.

(1) Decreasing reservoir overpressure ΔP_c.

(2) Increasing thickness of lava at the vent.

NOTE 1: magma buoyancy ($\rho_c \geq \rho_m$) positive or negative !

Negative buoyancy leads to $\Delta P_c < 0$.

NOTE 2: we have assumed that the conduit remains open.

Calculation of the eruption rate.

Incompressible magma of density ρ_m and viscosity μ.
Flow at small Reynolds numbers (laminar regime, no inertia).
Cylondrical coordinate system (r, θ, z). Velocity components (u, v_θ, w).
Assume purely vertical flow, such that $(u, v_\theta) = (0, 0)$.
Assume that pressure and velocity do not depend on θ (no swirling motion).

Navier-Stokes equations

$$0 = \frac{\partial w}{\partial z}$$

$$0 = -\frac{\partial p}{\partial r}$$

$$0 = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \frac{\partial^2 w}{\partial z^2} \right] - \rho_m g$$

Two very useful simplifications:
w does not depend on z
p does not depend on radial distance r.
Recast the vertical momentum balance:

$$\frac{d}{dr} \left(r \frac{dw}{dr} \right) = \frac{1}{\mu} r \left(\frac{dp}{dz} + \rho_m g \right)$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{1}{\mu}r\left(\frac{dp}{dz} + \rho_m g\right)$$

Integrate once between $r = 0$ and r:

$$r\frac{dw}{dr} = \frac{r^2}{2\mu}\left(\frac{dp}{dz} + \rho_m g\right)$$

Integrate between $r = a$ and r:

$$w(r) - w(a) = \frac{1}{4\mu}\left(r^2 - a^2\right)\left(\frac{dp}{dz} + \rho_m g\right)$$

No slip at the conduit walls, such that $w(a) = 0$.

$$w = \frac{1}{4\mu}\left(r^2 - a^2\right)\left(\frac{dp}{dz} + \rho_m g\right)$$

The mass flux of magma (eruption rate):

$$Q^* = \int_0^{r=a} \rho_m w 2\pi r dr = -\rho_m \frac{\pi a^4}{8\mu}\left(\frac{dp}{dz} + \rho_m g\right)$$

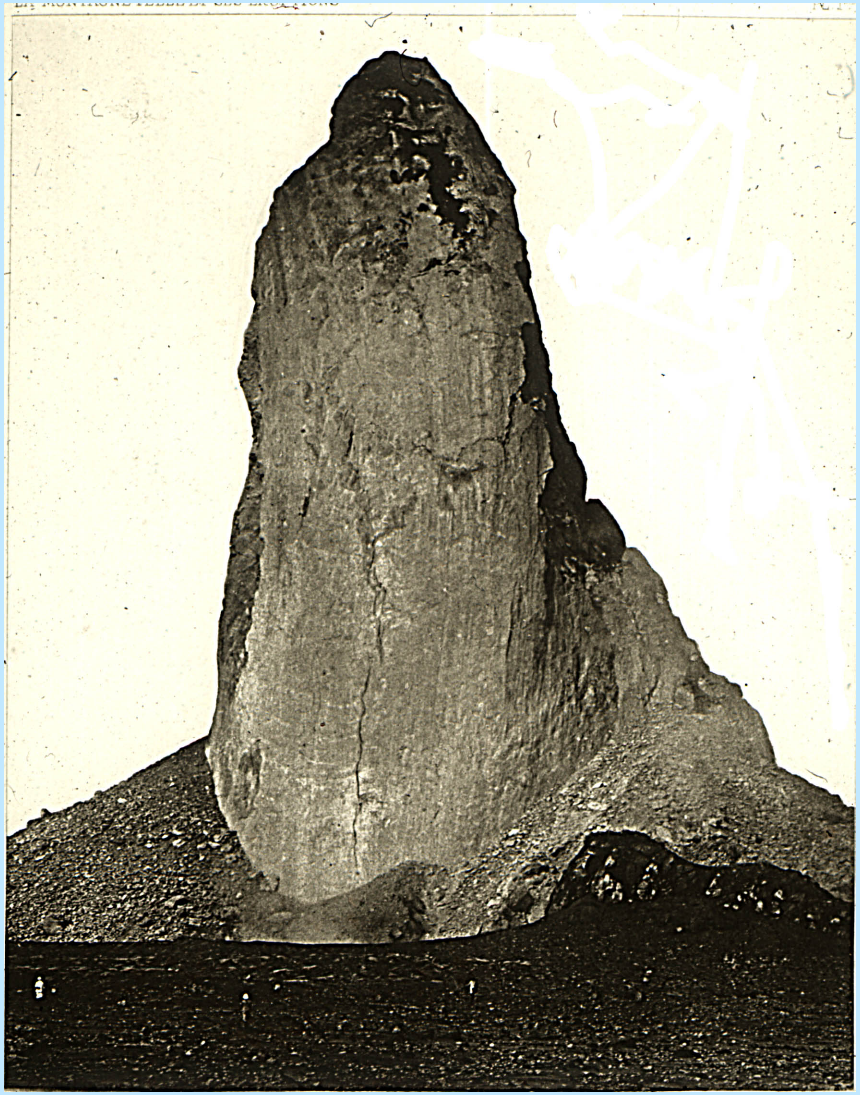
Poiseuille parabolic radial profile.
Vertical pressure gradient ?

Q^* must be constant and independent of height z (mass conservation).
Thus, dp/dz independent of z, and hence constant.
From $p(0)$ and $p(H)$

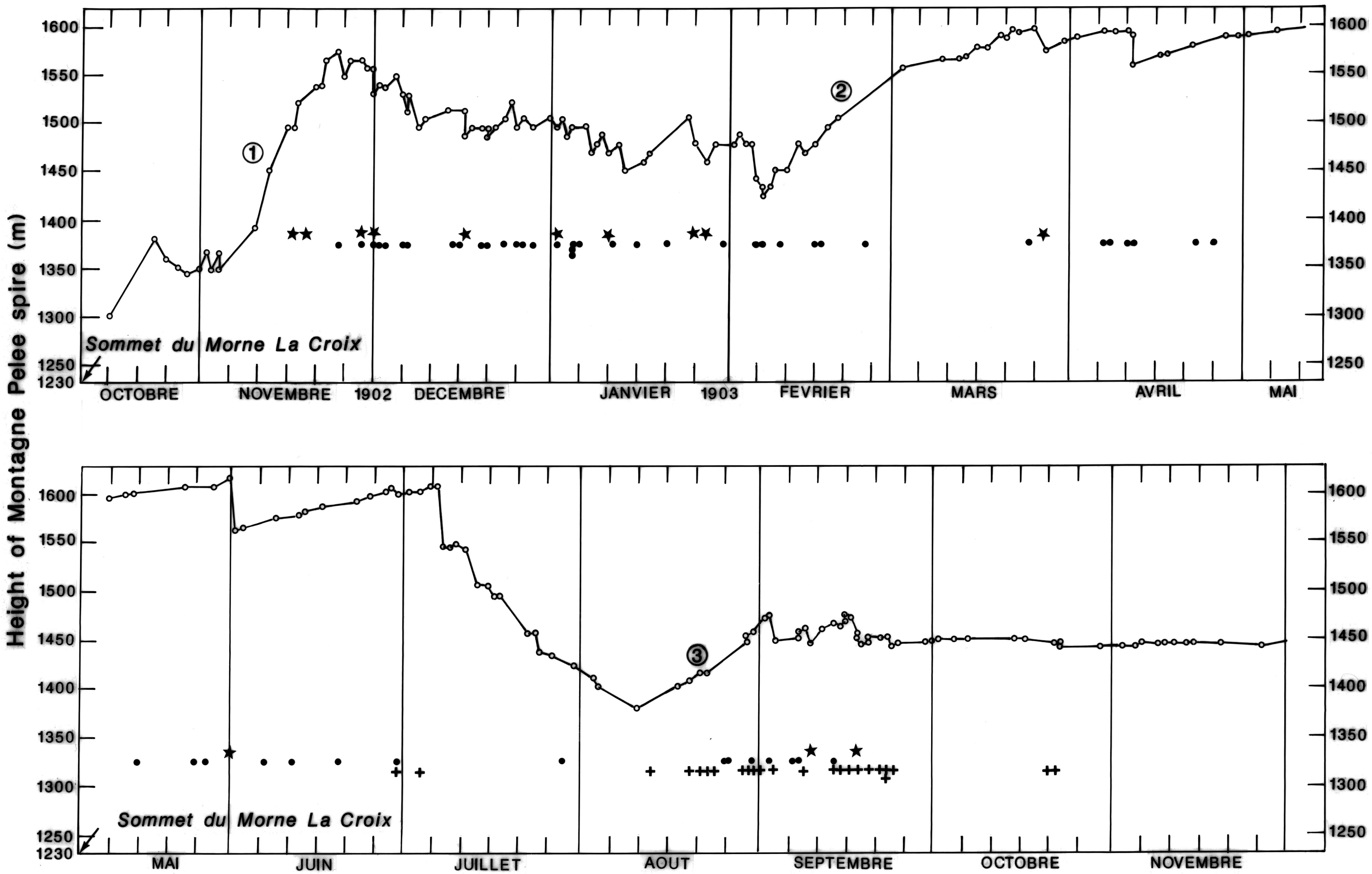
$$\frac{dp}{dz} = constant = \frac{p(H) - p(0)}{H} = -\frac{\rho_c g H + \Delta P_c - \rho_m g h}{H}$$

And hence:

$$Q^* = \rho_m \frac{\pi a^4}{8\mu} \frac{(\rho_c - \rho_m) g H + \Delta P_c - \rho_m h}{H}$$



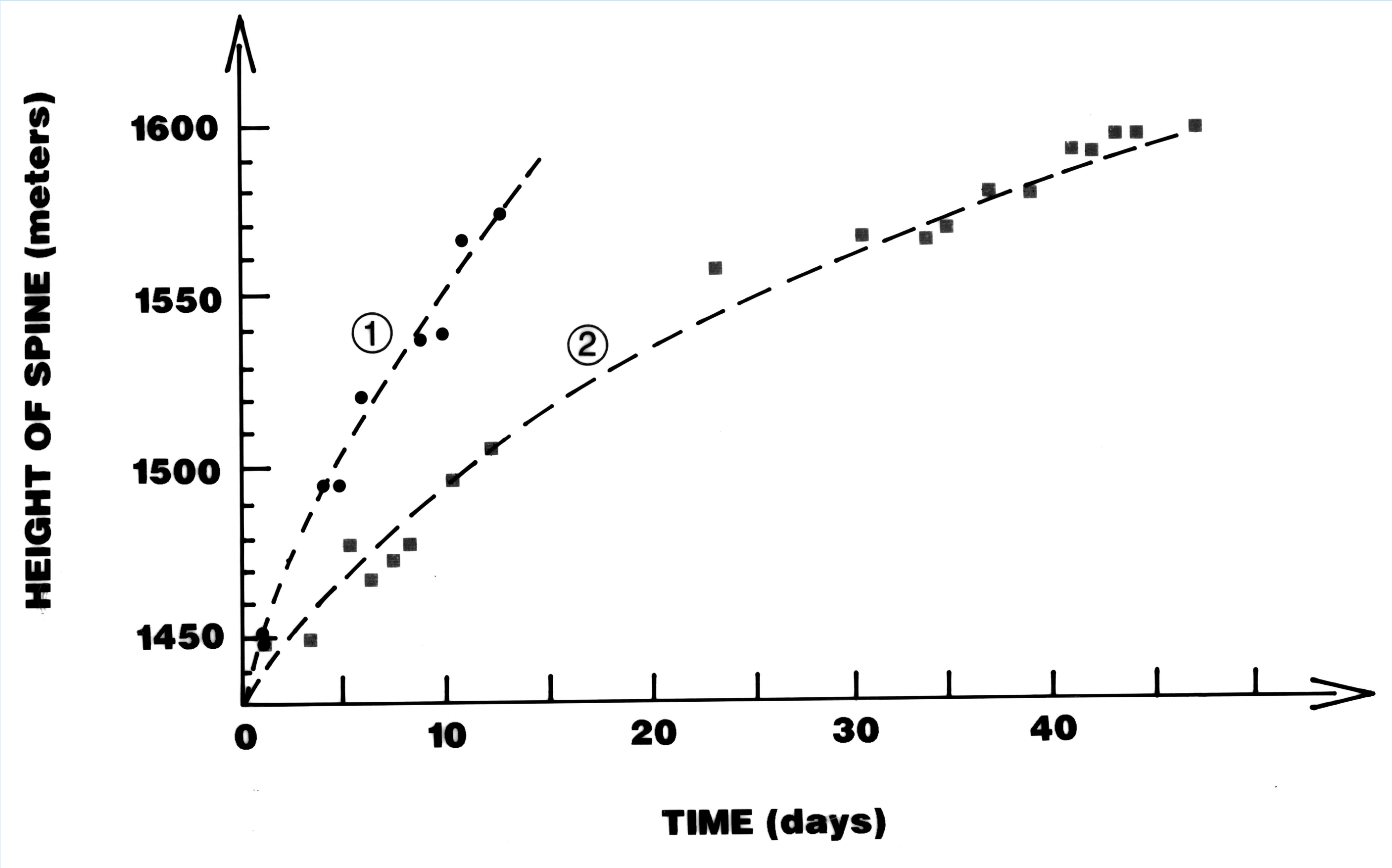
Montagne Pelée, Martinique (1902)



★ Powerful pyroclastic flows which reached the sea.

• Pyroclastic flows which descended halfway down the riviere Blanche valley

+ Pyroclastic flows descended in directions other than that of the riviere Blanche

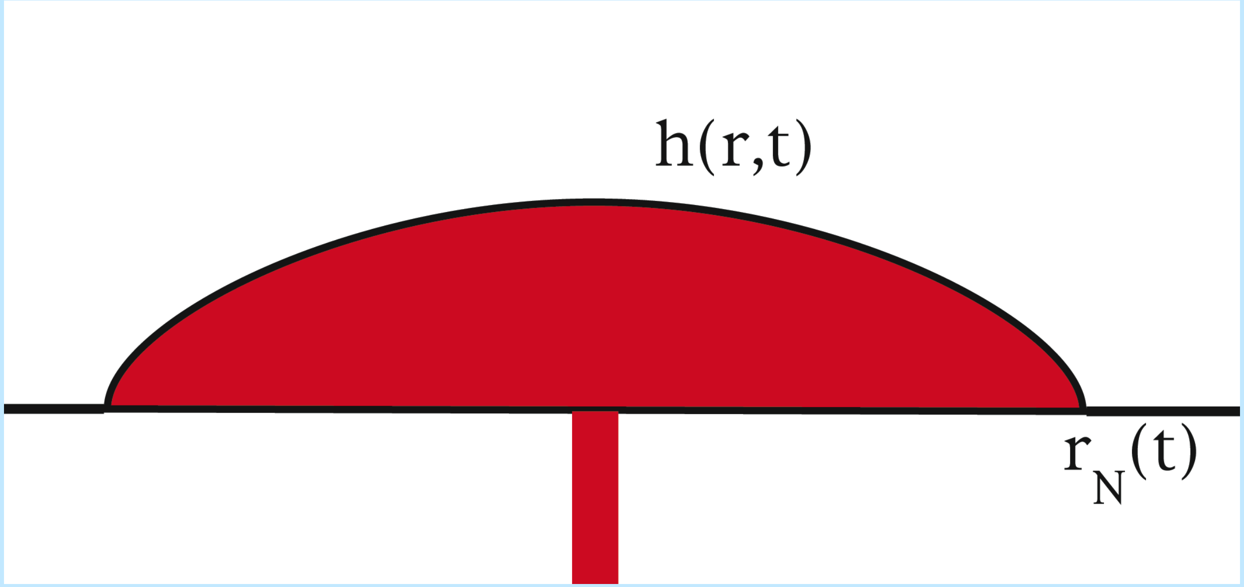


Velocity decrease from phase (1) to phase (2) implies that :

- the reservoir pressure decreased by ≈ 2 MPa,
- **there is a reservoir !**

For the total erupted volume, total $\Delta P > 30$ MPa.

Dynamics of spreading



Scales

- H for height
- R for radius
- U for horizontal velocity

Laminar regime (small Reynolds number) : no inertia.

Flow dimensions and spreading rate 1. Constant eruption rate.

Assume incompressible lava (to be discussed later).
Control variables: eruption rate Q (volumetric) + lava properties.
Global mass balance. Volume increases linearly with time.

$$V(t) = Qt \sim HR^2$$

$\sim$ symbol = proportional to.

Horizontal force balance.
Driving = pressure.
Pressure acting on a cylindrical surface with area $2\pi RH$, prop. to (HR).

$$F_D \sim (\rho_m g H) HR$$

Resisting = viscous shear at the base of the flow.
Shear stress:

$$\tau \sim \mu \frac{U}{H}$$

Acting on area πR^2.
Force balance:

$$\rho_m g H^2 R \sim \mu \frac{UR^2}{H}$$

Three unknowns, H, R, U, and only two equations.
But velocity∼spreading rate, such that $U \sim dR/dt \sim R/t$.

$$R \sim \left(\frac{\rho_m g Q^3}{\mu}\right)^{1/8} t^{1/2}$$

$$H \sim \left(\frac{\mu Q}{\rho_m g}\right)^{1/4}$$

Full solution

Cylindrical coordinate system (r, θ, z)
Assume no orthoradial velocity component: $\bar{v} = (u, 0, w)$.
Navier-Stokes equations:

$$
\begin{aligned}
0 &= \frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} \\
0 &= -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right) + \frac{\partial^2 u}{\partial z^2}\right] \\
0 &= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right] - \rho_m g
\end{aligned}
$$

$H << R$: neglect radial derivatives compared to vertical ones.
Continuity equation implies that:
$|w| \ll |u|$
Viscous stresses associated with gradients of vertical velocity are small.
Reduced equations:

$$
\begin{aligned}
0 &= \frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} \\
0 &= -\frac{\partial p}{\partial r} + \mu\frac{\partial^2 u}{\partial z^2} \\
0 &= -\frac{\partial p}{\partial z} - \rho_m g
\end{aligned}
$$

Integrate vertical momentum equation:

$$p(r,z) = P_a + \rho_m g(h - z)$$

P_a = atmospheric pressure (negligibe).

$$\frac{\partial p}{\partial r} = \rho_m h \frac{\partial h}{\partial r}$$

Flow is driven by thickness variations.

Integrate the simplified radial momentum balance.
Boundary conditions:

$$\mu \left(\frac{\partial u}{\partial z} \right)_{z=h} = 0 \quad \text{(zero shear stress at the top)}$$

$$u(r,0) = 0 \quad \text{(no slip at the base)}$$

$$u(r,z) = -\frac{\rho_m g}{2\mu} \frac{\partial h}{\partial r} z(2h - z)$$

Mass conservation constraint ?
Continuity equation allows calculation of w as a function of u.
Choose control volume : avoid mass flux through a horizontal surface.
Control volume between two vertical cylinders at radii r and $r + dr$.
Control volume $\delta V = 2\pi h r dr$.
Horizontal mass flux across a vertical cylinder = $\phi(r)$.
Mass (volume) conservation:

$$2\pi r \frac{\partial h}{\partial t} = -\frac{\partial \phi}{\partial r}$$

Using solution for u:

$$\phi(r) = -2\pi r \frac{\rho_m g}{3\mu} h^3 \frac{\partial h}{\partial r}$$

Substituting into the mass balance equation:

$$\frac{\partial h}{\partial t} - \frac{\rho_m g}{3\mu} \frac{1}{r} \frac{\partial}{\partial r} \left(h^3 \frac{\partial h}{\partial r} \right) = 0$$

Non linear !

$$\frac{\partial h}{\partial t} - \frac{\rho_m g}{3\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(h^3\frac{\partial h}{\partial r}\right) = 0$$

To be solved with global volume conservation:

$$V(t) = Qt = \int_0^{r_N(t)} h 2\pi r dr$$

Solution method: introduce similarity variable $\eta \sim r/R(t)$.
This states that the flow is $self-similar$.

$$h(r,t) = \left(\frac{3\mu Q}{\rho_m g}\right)^{1/4} H(\eta)$$
$$\eta = \left(\frac{\rho_m g Q^3}{3\mu}\right)^{-1/8} r t^{-1/2}$$

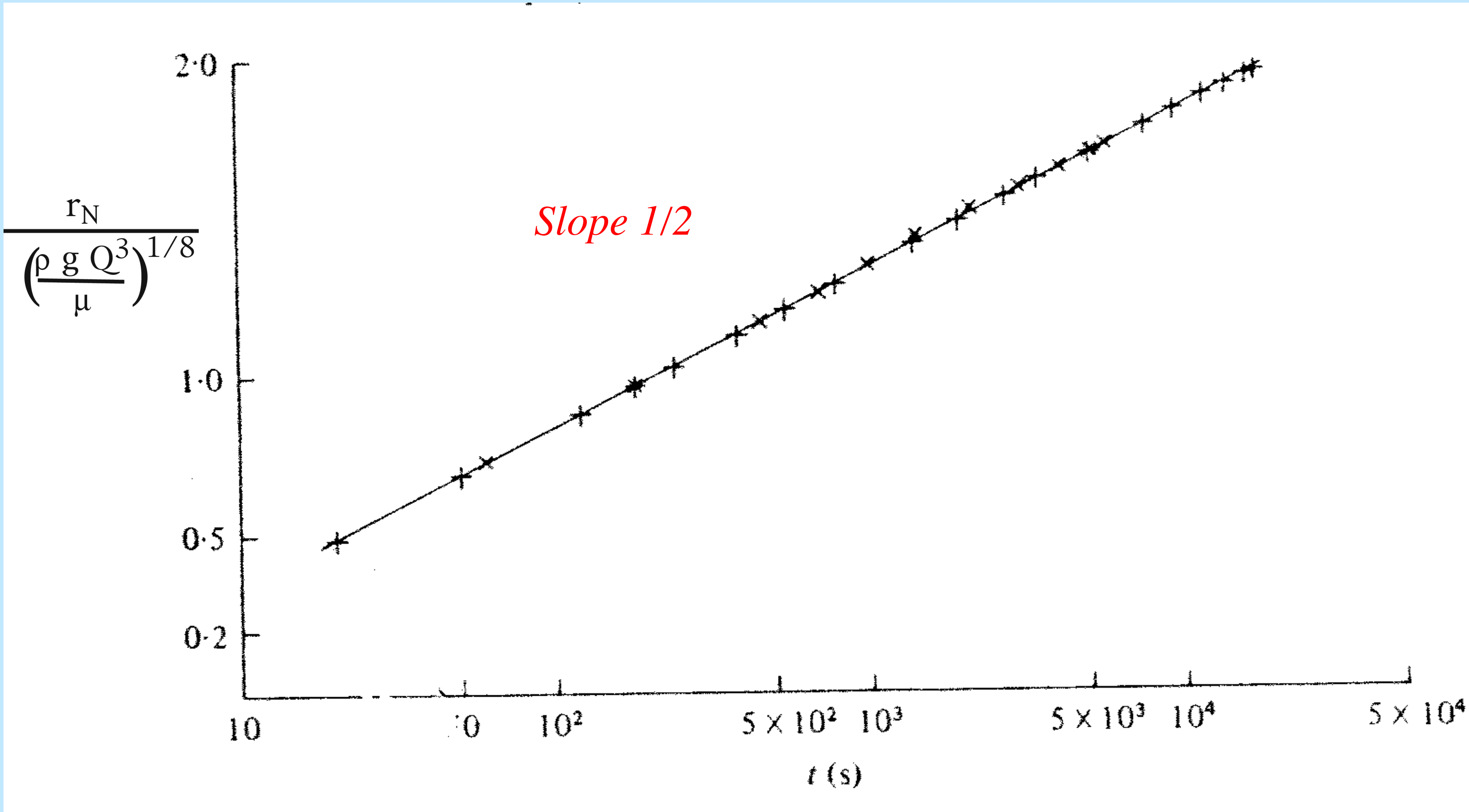
where $H(\eta)$ is a dimensionless function.
Numerical integration yields:

$$r_N(t) = (0.715...) \left(\frac{\rho_m g Q^3}{3\mu}\right)^{1/8} t^{1/2}$$

Defining $\xi = r/r_N$, an approximate solution:

$$H(\xi) = \left(\frac{3}{2}\right)^{1/3} (1-\xi)^{1/3} \left[1 + \frac{1}{12}(1-\xi) + \mathcal{O}(1-\xi)^2\right]$$

Constant eruption rate



Flow dimensions and spreading rate 2. Constant volume.

Residual spreading once the eruption has stopped.

Mass conservation :

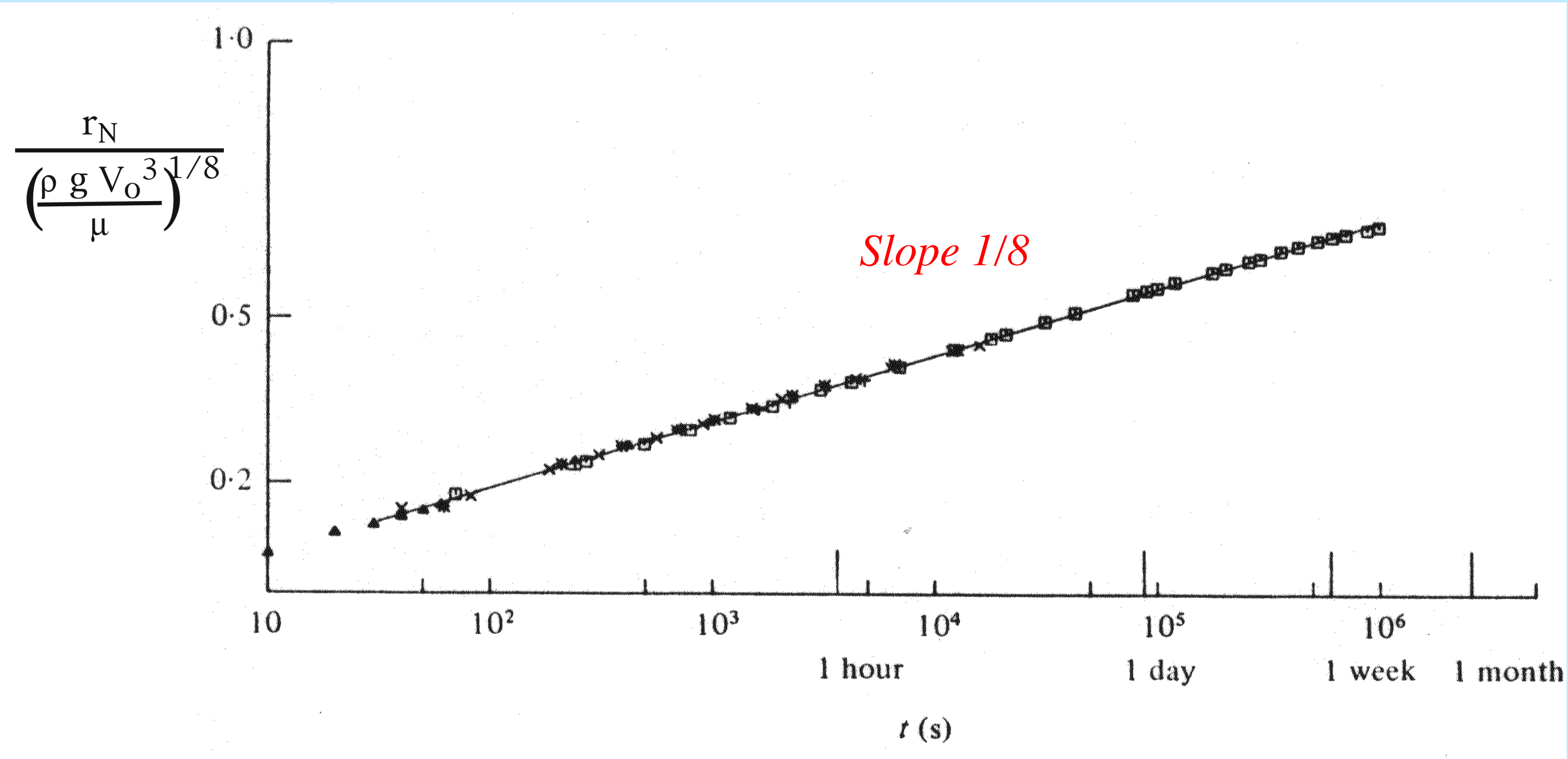
$$V_o \sim HR^2$$

Same horizontal force balance.

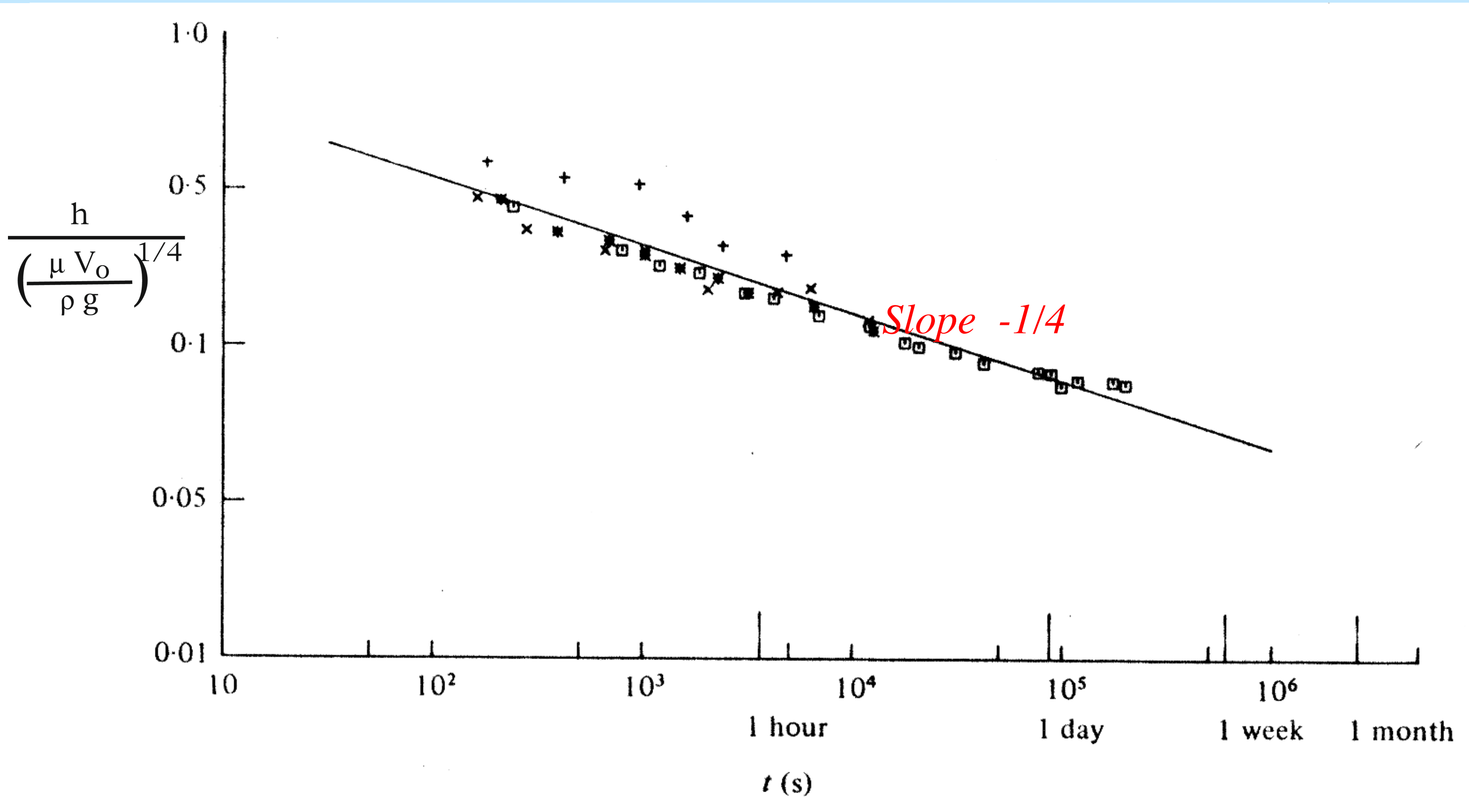
Same relationship between U and R.

$$R \sim \left(\frac{\rho_m g V_o^3}{\mu}\right)^{1/8} t^{1/8}$$

$$H \sim \left(\frac{\mu V_o}{\rho_m g}\right)^{1/4} t^{-1/4}$$



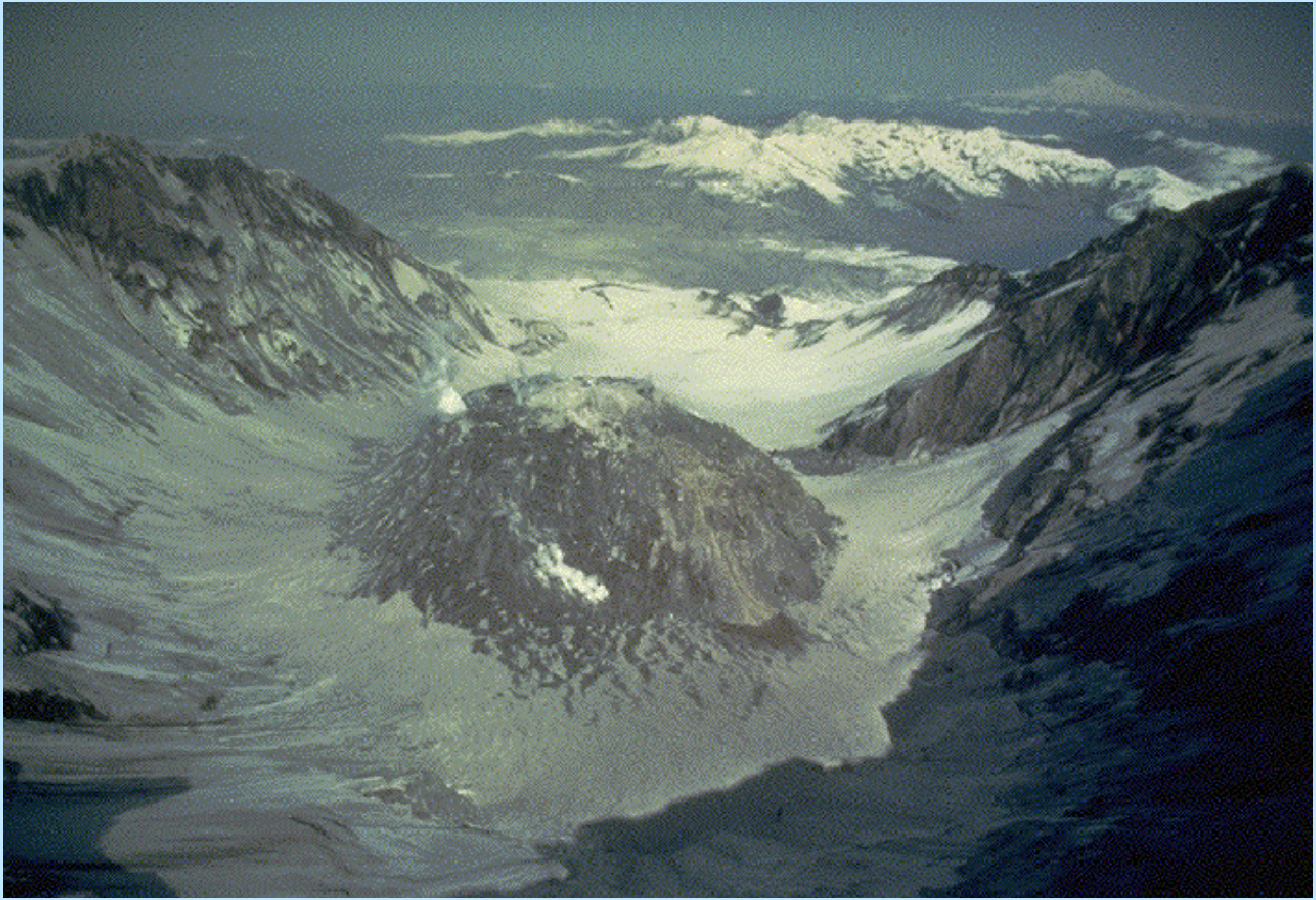
$\frac{r_N}{\left(\frac{\rho g V_o^3}{\mu}\right)^{1/8}}$
1·0
0·5
0·2
Slope 1/8
10
10²
10³
10⁴
10⁵
10⁶
1 hour
1 day
1 week
1 month
t (s)


1·0
0·5
0·1
0·05
0·01
h
μ V0
ρ g
1/4
Slope -1/4
10
10²
10³
10⁴
10⁵
10⁶
1 hour
1 day
1 week
1 month
t (s)

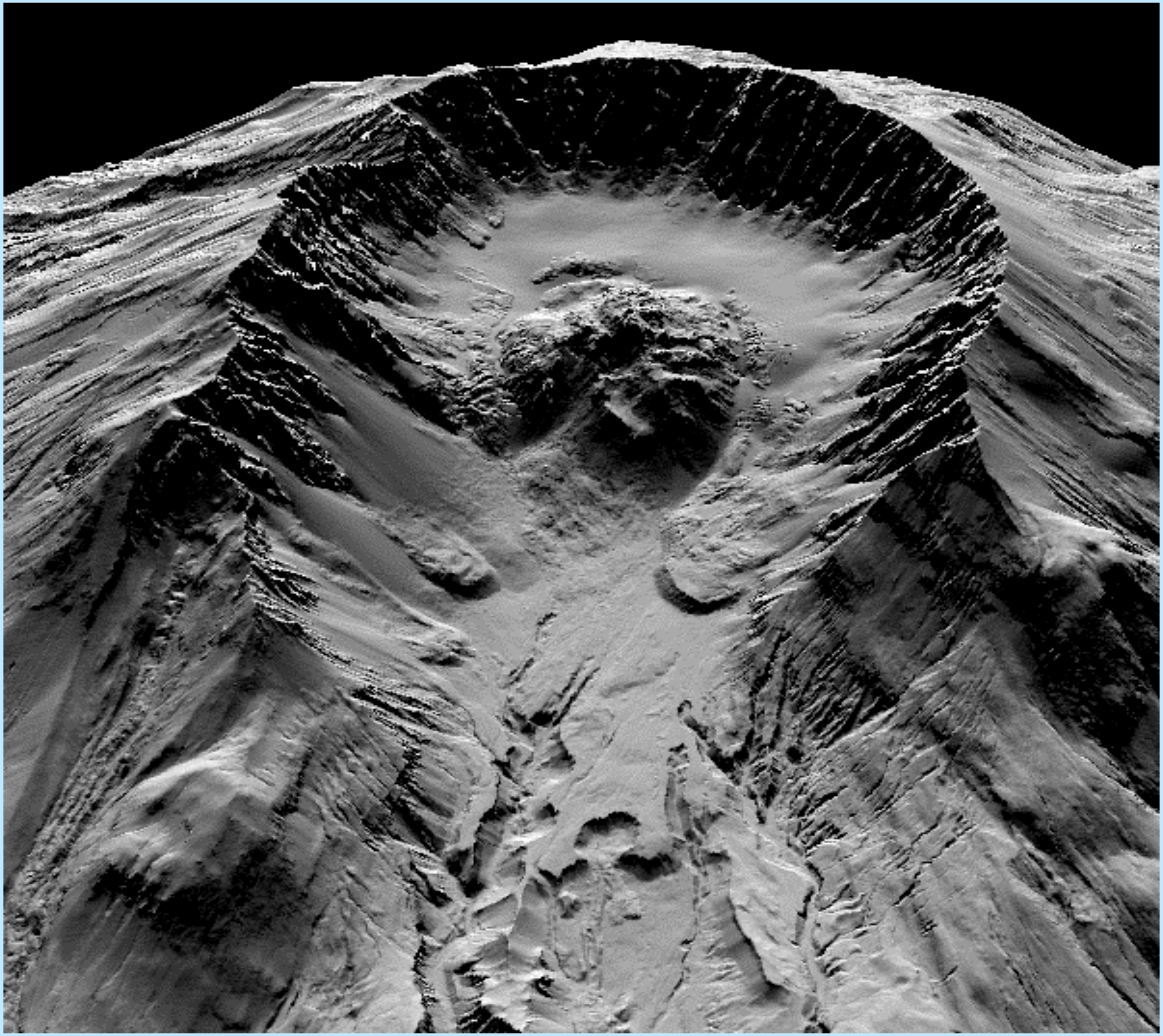
LAVA FLOW MORPHOLOGY

1. Observations
2. Physical principles
3. Laboratory experiments

Mount St Helens 1980 lava dome

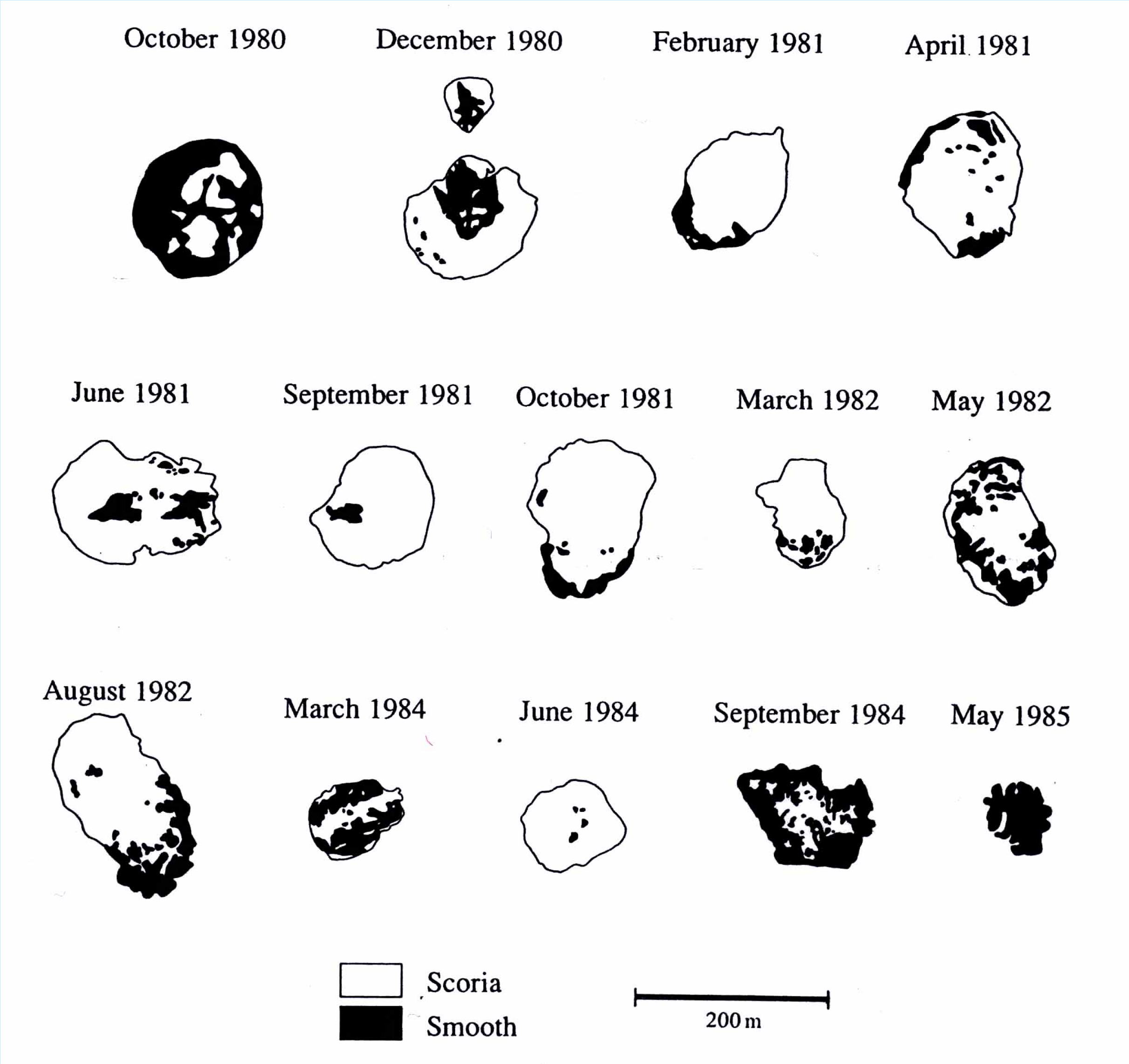


Mount St Helens dome

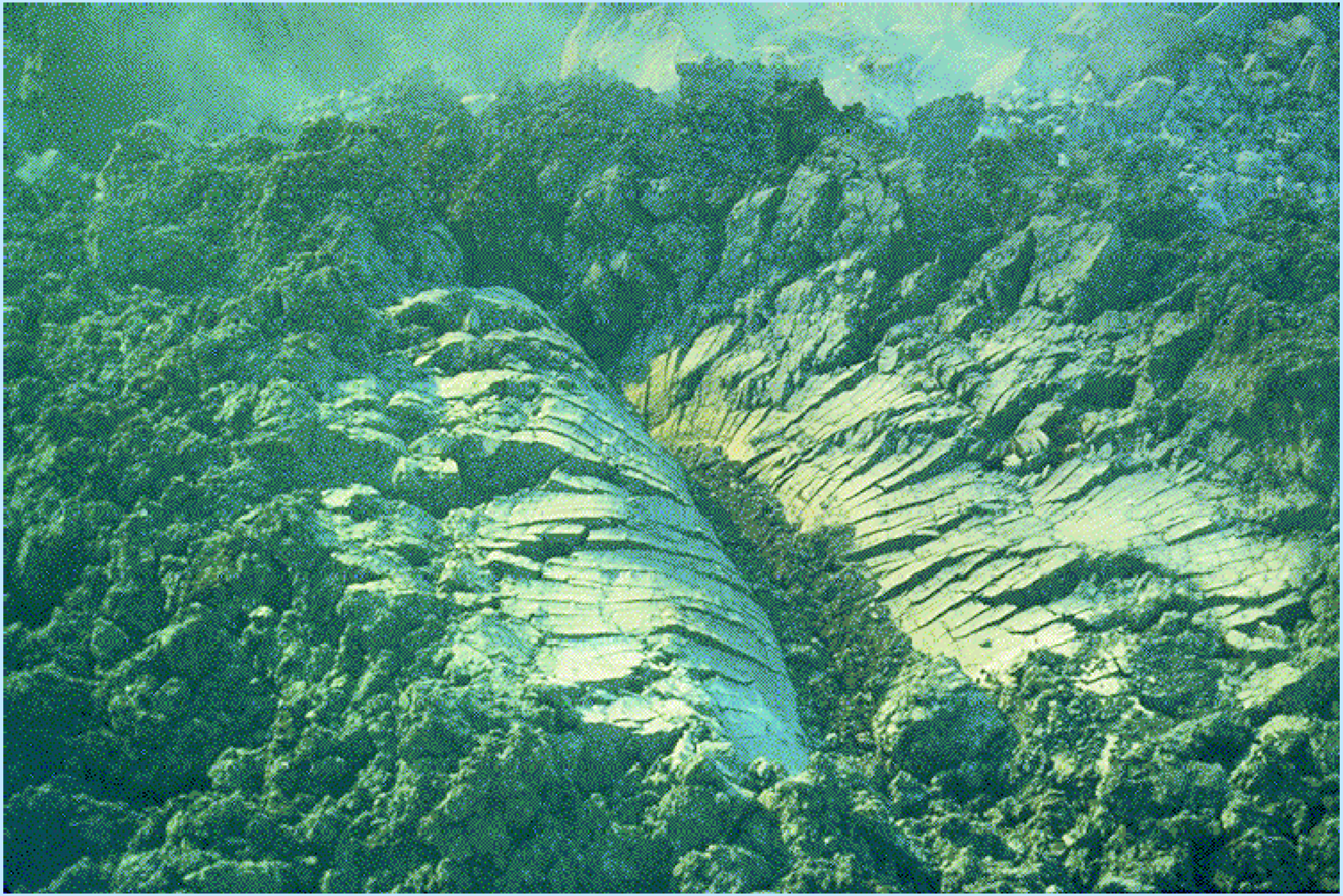


Formation of lobes

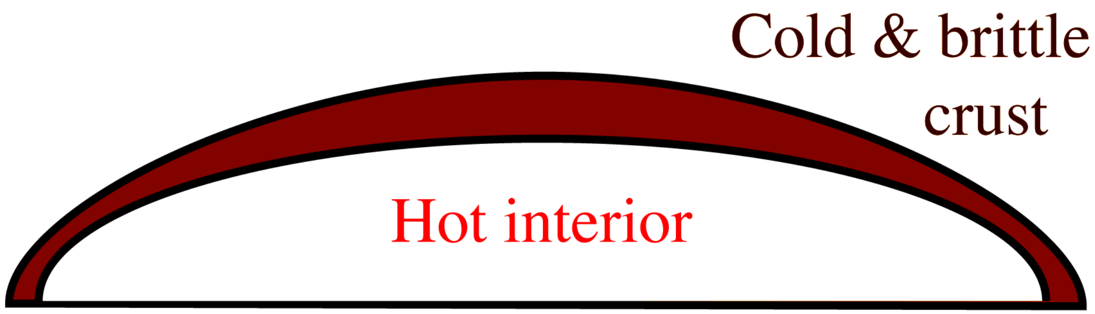
Mount St Helens



Mount St Helens



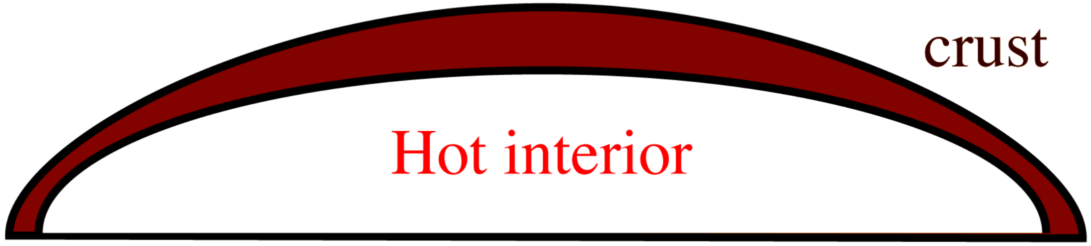
“Rifting structure”



INPUTS

Volume flow rate Q, magma viscosity μ, magma density ρ

Cooling mechanism (with relevant variables and properties)



INPUTS

Volume flow rate Q, magma viscosity μ, magma density ρ

Cooling mechanism (with relevant variables and properties)

Behaviour of flow depends on crust resistance.

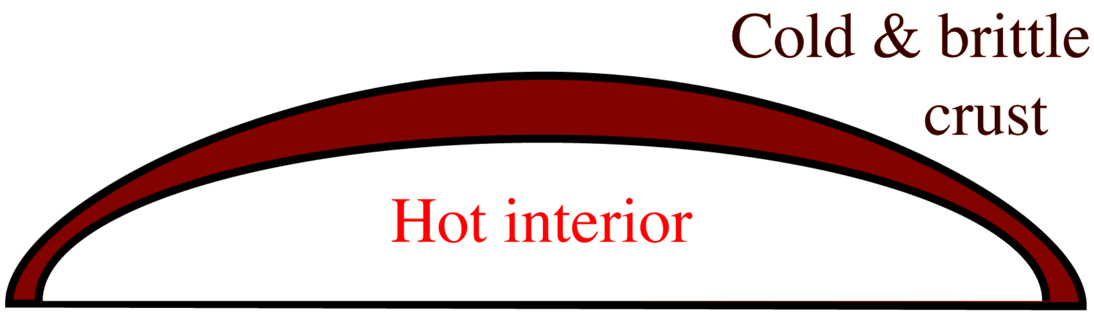
Two time-scales:

Flow time-scale τ_a

Solidification time-scale τ_s

$\tau_a >> \tau_s$: crust formation has a large influence on the flow.

$\tau_a << \tau_s$: flow is faster than crust formation.



INPUTS

Volume flow rate Q, magma viscosity μ, magma density ρ

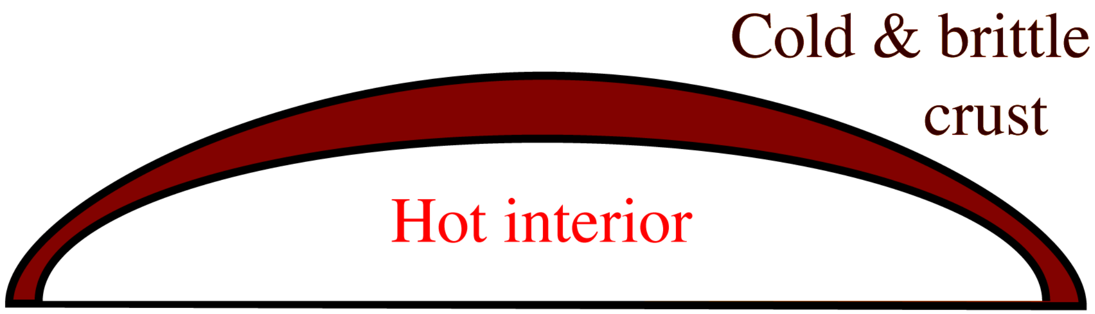
Spreading time-scale τ_a:

$$Q = \frac{dV}{dt} \sim \frac{H^3}{t}$$

Use thickness scale derived previously:

$$H \sim \left(\frac{\mu Q}{\rho_m g}\right)^{1/4}$$

$$\tau_a \sim \frac{H^3}{Q} \sim \left(\frac{\mu}{\rho_m g}\right)^{3/4} Q^{-1/4}$$

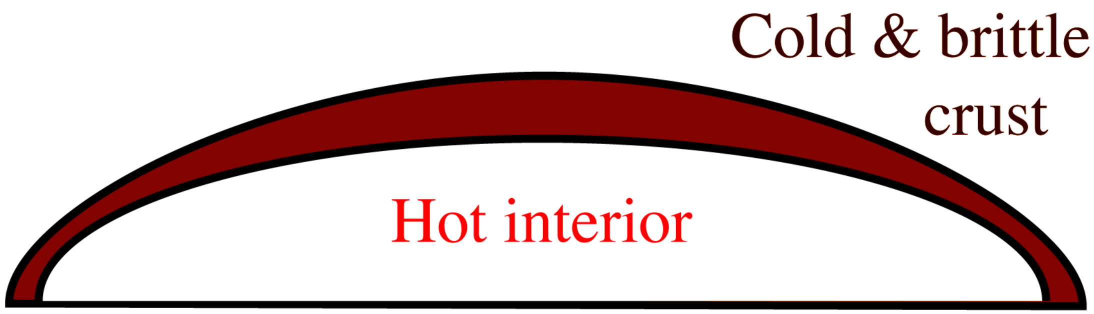


INPUTS

Volume flow rate Q, magma viscosity μ, magma density ρ

Time-scale for cooling depends on the cooling mechanism.
For diffusion:

$$\tau_S \sim \frac{H^2}{\kappa}$$



INPUTS

Volume flow rate Q, magma viscosity μ, magma density ρ

Two time-scales:

Flow time-scale τ_a

Solidification time-scale τ_s

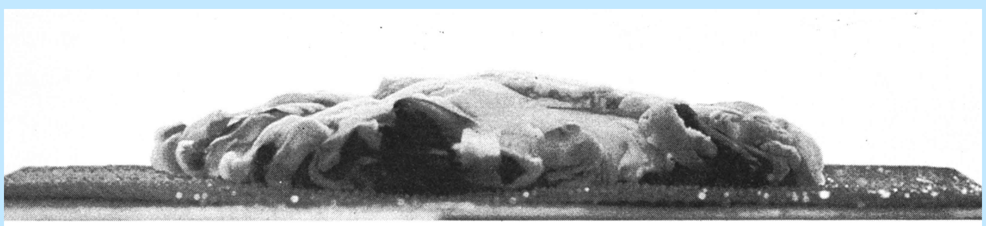
Dimensionless number

$$\Psi = \tau_s/\tau_a$$

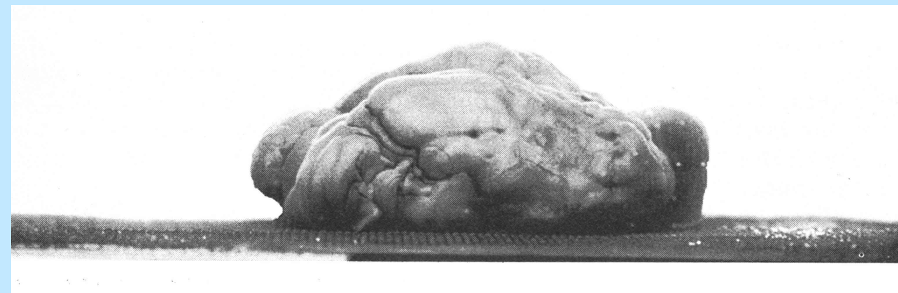
Point source (vent eruption)

$\Psi > 50$: crust has no detectable effect.

Ψ = 17: folding



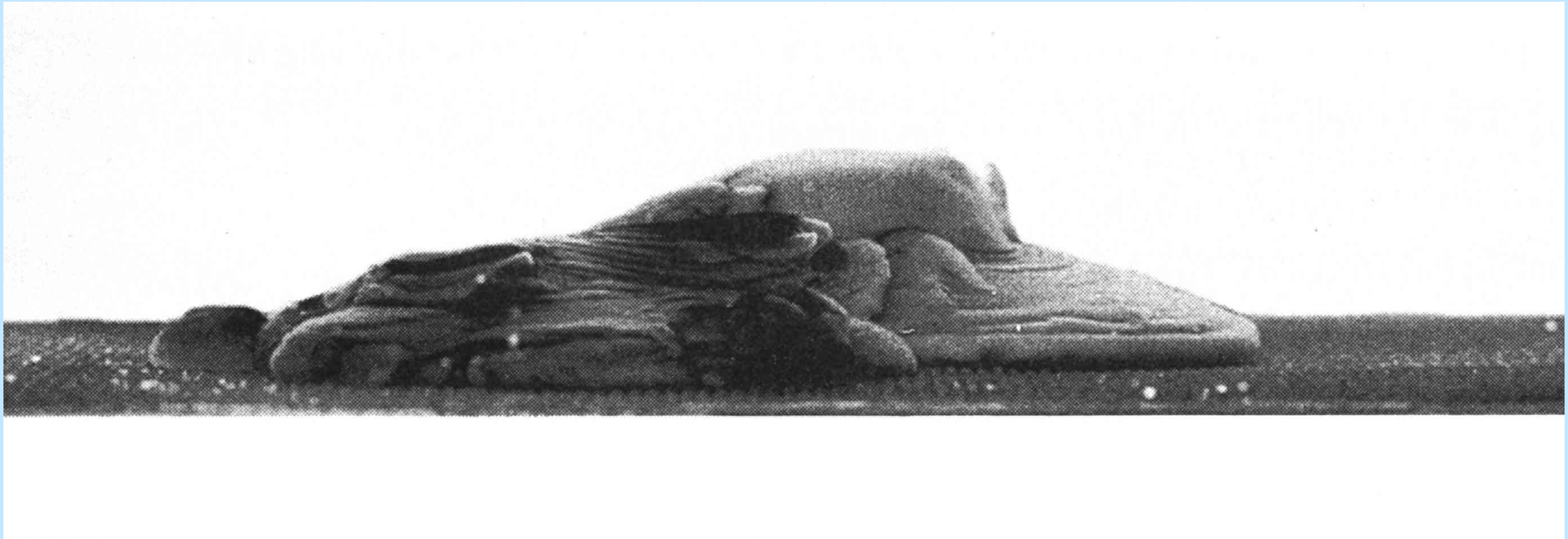
Ψ = 9: rifting



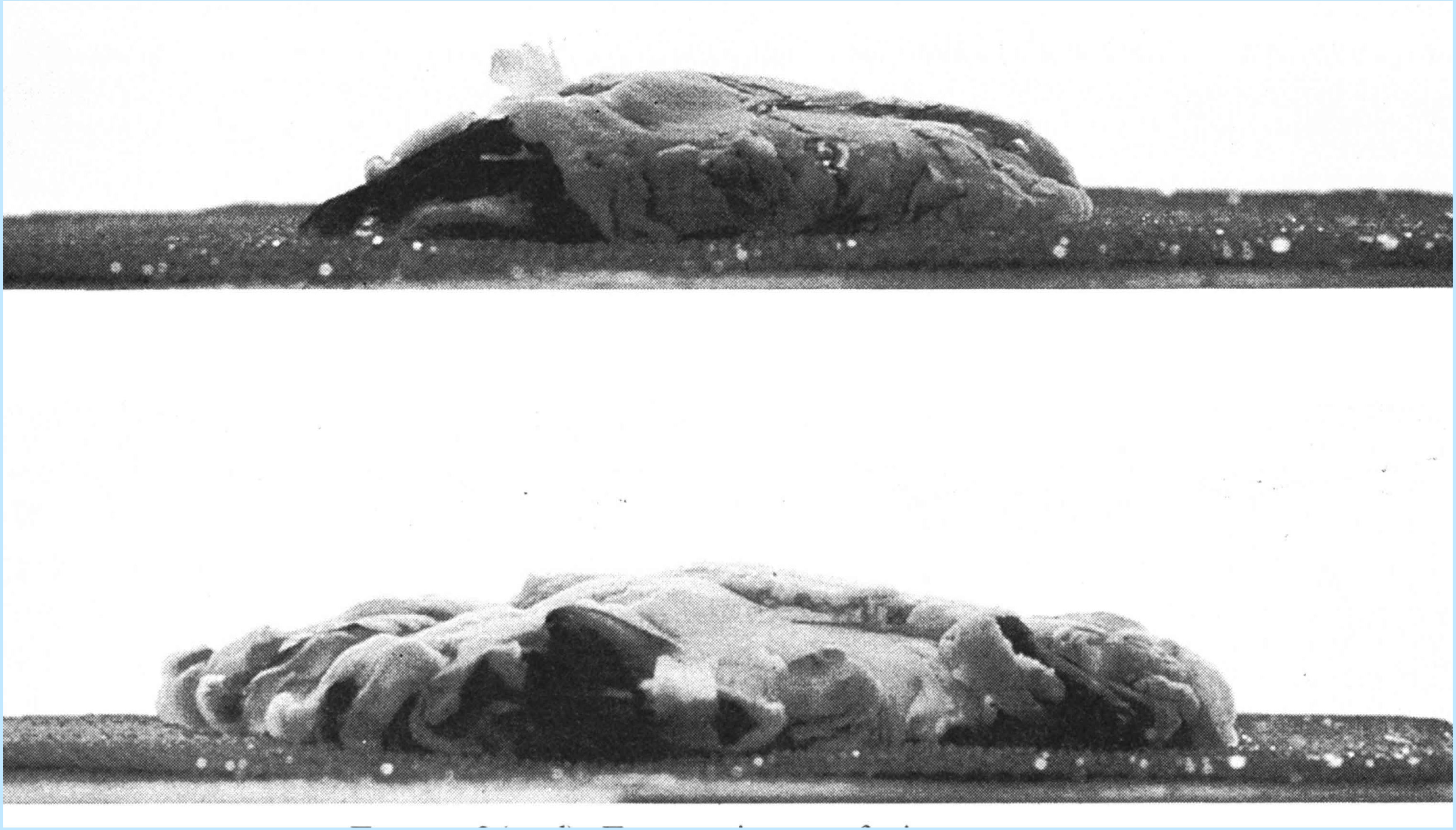
Ψ = 4: rifting and pillow (or lobe) formation

(From Griffiths & Fink, 1993)

Ψ = 17 (small crust influence : “folding”)



Ψ = 9 (moderate crust effect : “rifting”)
Time evolution



Ψ = 4 (strong crust effect)
Formation of "pillows" or "lobes"