ICTP (Trieste) 2016: Advanced School on Physics of Volcanoes

# Fluid-filled fracture growth and related seismicity

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*Questions*

- ✓ *How do fluid-filled cracks grow ?*
- ✓ *What can we learn from shape and growth path ?*
- ✓ *What can we learn from induced seismicity?*

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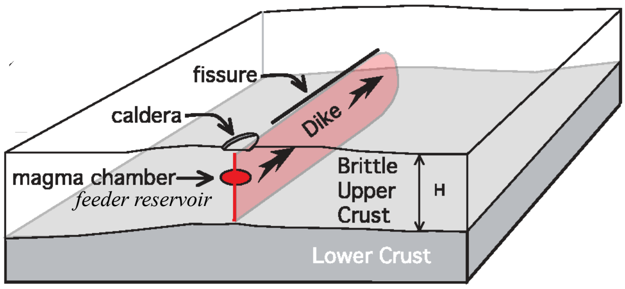
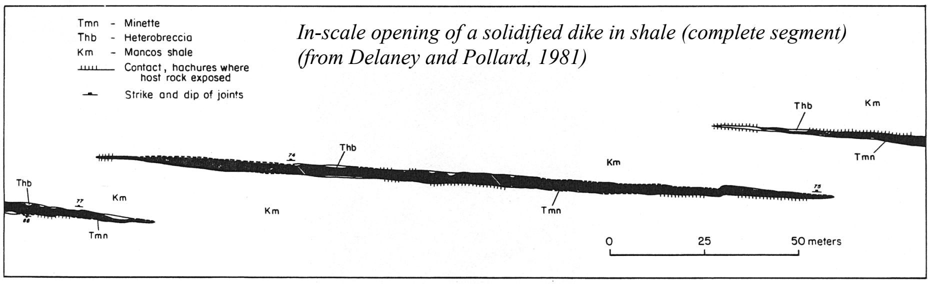
# I) How do fluid filled fractures form and grow ?

## Schematic sketch on the generation of lateral dikes

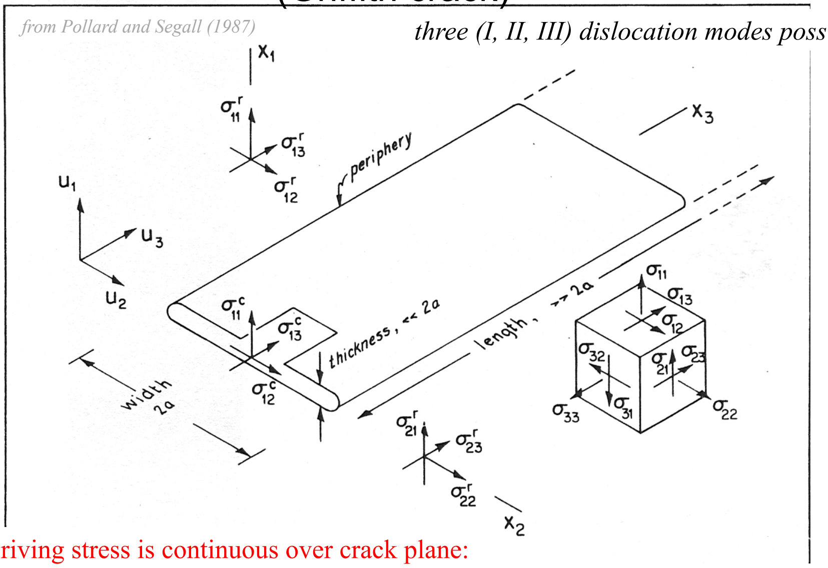


*modified from Buck et.al. (2006)*



*In-scale opening of a solidified dike in shale (complete segment) (from Delaney and Pollard, 1981)*

## Dislocation and stress of a planar 2D crack (Griffith crack)



*from Pollard and Segall (1987)*

*three (I, II, III) dislocation modes possible*

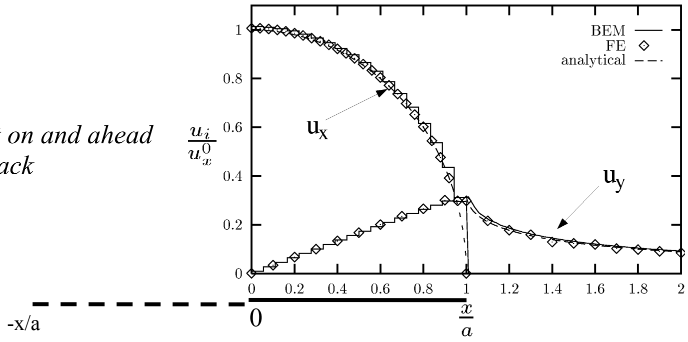
Driving stress is continuous over crack plane:

$$\left[\Delta\sigma^{(1)}, \Delta\sigma^{(2)}, \Delta\sigma^{(3)}\right] = \left[\Delta\sigma^{I}, \Delta\sigma^{II}, \Delta\sigma^{III}\right] = [(\sigma^r_{11} - \sigma^c_{11}), (\sigma^r_{12} - \sigma^c_{12}), (\sigma^r_{13} - \sigma^c_{13})]$$

# Crack opening Δu

$$\begin{Bmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{Bmatrix} = \begin{Bmatrix} \sigma^{I} \\ \sigma^{II} \\ \frac{\sigma^{III}}{1-\nu} \end{Bmatrix} \frac{2(1-\nu)}{\mathcal{N}} \sqrt{a^2 - x_2^2} \quad .$$

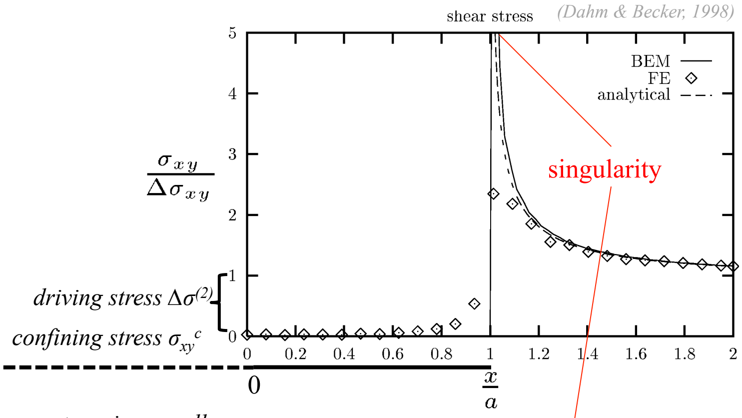
*comparison of analytical and different numerical methods (Dahm & Becker, 1998)*



*displacement on and ahead of Griffith crack*

# Stress on and ahead of Griffith crack

*(Dahm & Becker, 1998)*



*background stress* $\sigma_{xy}{}^{r}$

*driving stress* $\Delta\sigma^{(2)}$

*confining stress* $\sigma_{xy}{}^{c}$

*stress in a small distance* $r_1$ *from crack tip*

$$\sigma_{ij} = \underbrace{\Delta\sigma^{(m)}\sqrt{\pi a}}_{K} \frac{1}{\sqrt{2\pi r_1}} f_{ij}^{(m)}(\theta_1)$$

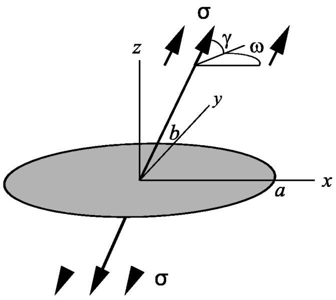
*shape function (bounded)*

*with stress intensity factor*

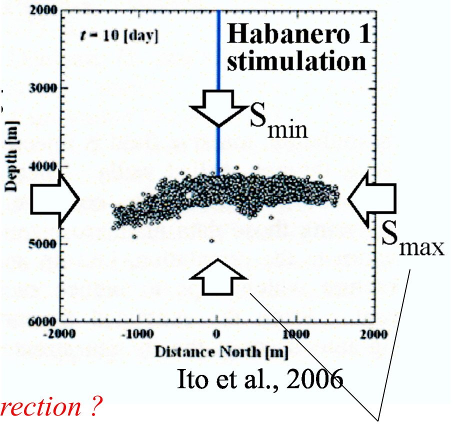
$$K = \Delta\sigma^{(m)}\sqrt{\pi a}\,.$$

# More realistic: 3D cracks (circular or elliptical)

*Penny shaped crack*



*Induced seismicity from borehole fluid injection is penny shaped*

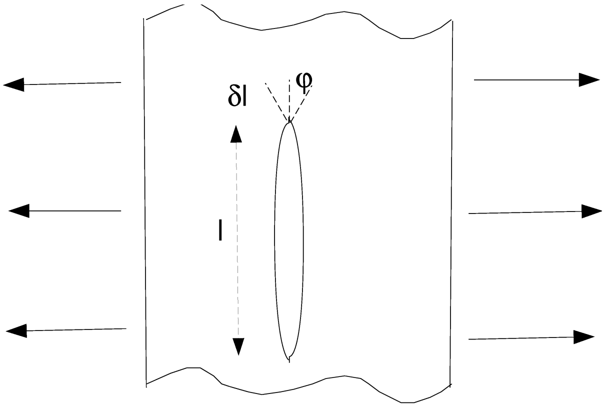


Ito et al., 2006

*Principal stresses*

- *Do fluid-filled cracks open in $S_{min}$ direction ?*
- *Do they grow in plane defined by $S_{max} - S_2$ ?*

# In which direction do cracks grow ?



*The strain energy released with incremental length growth: (δQ/δl)*

Fracture criterion: δQ/δl > threshold and δQ/δl is maximal

(alternativ: stress intensity factor K > fracture toughness $K_c$

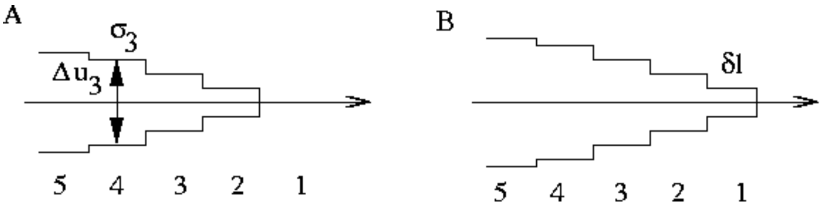
# Analytical and numerical approach of Griffith growth

Analytical: *e.g., find maximum of stress intensity K:*

*strain energy* $$Q_u = \frac{1}{2}\int_{-a}^{a} \left(\bar{\sigma}_n \Delta u_n + \bar{\sigma}_s \Delta u_s\right) dx \ ;$$

$$\frac{\delta Q}{\delta l} = \frac{(Q_U^B - Q_U^A)}{\delta l} = \mathcal{G} = \frac{(1-\nu)}{2\mathcal{N}} K_I^2 \, .$$

Numerical: *estimate $Q_u(A)$ and $Q_u(B)$ and find maximum of $[Q_u(B)-Q_u(A)]/\delta l$:*

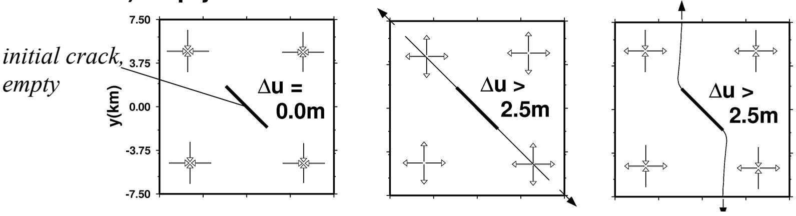


# Numerical simulation of crack growth and arrest

*Dahm (2000)*

**I) compression** **II) tension** **III) mixed load**

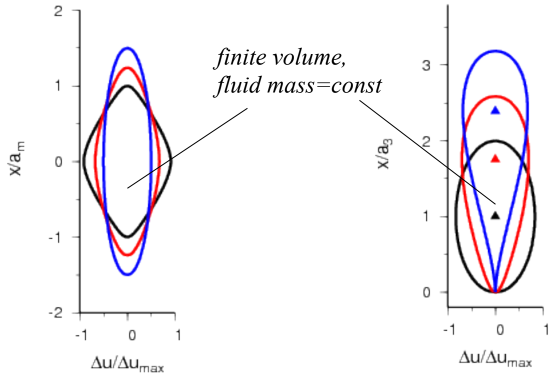
**a) empty** *no growth* *in-plane growth* *turn and growth in $S_{max}$ direction*



# Fluid-filled cracks of finite “volume”

*overpressurized, symmetric*
*crack expands (bilateral viscous flow)*

*unilateral expansion from*
*quasi-static re-adjustment*
*with apparent buoyancy*

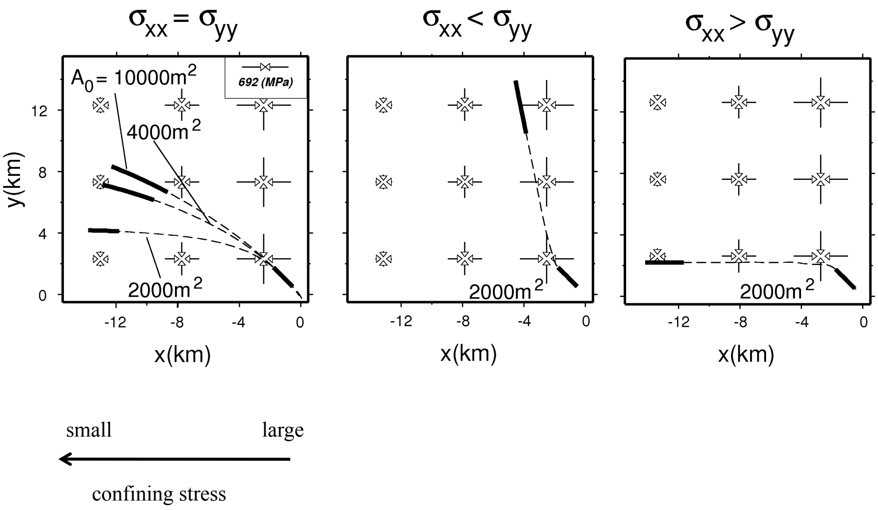


*Ambient pressure changes as*

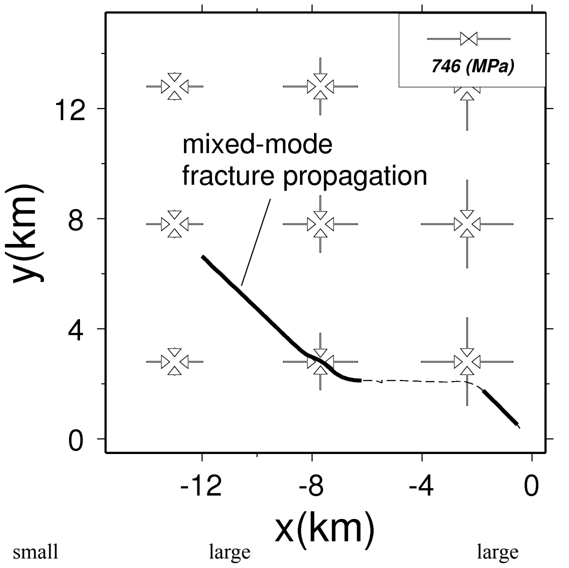
$\Delta P = -K\,\Delta V/V_0$

*(decompression leads to small volume expansion)*

# The influence of stress gradients



# Is mixed-mode propagation possible ?



small ← large

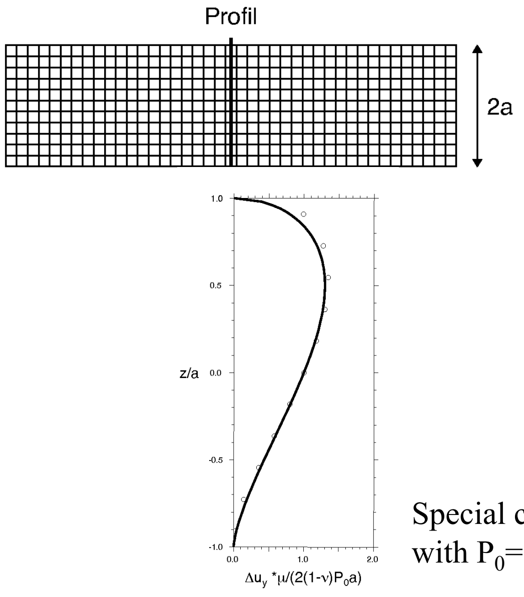
confining stress

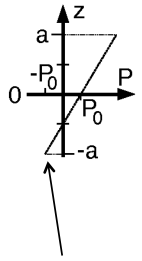
large ← small

deviatoric stress

# Fluid filled Griffith crack under linear increasing pressure

$$P=P_0+P_g\,z$$

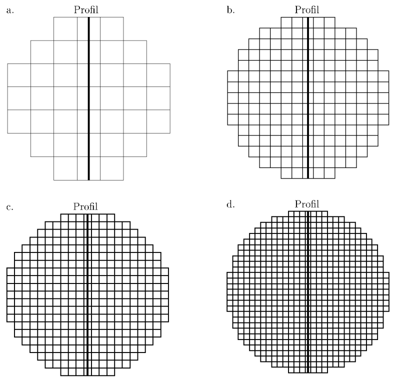


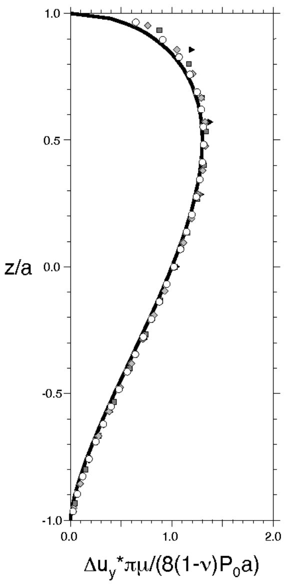


*negative driving pressure at bottom*

Special case: $P = 0.5\,a\,P_g + P_g\,z$
with $P_0 = 0.5aP_g$

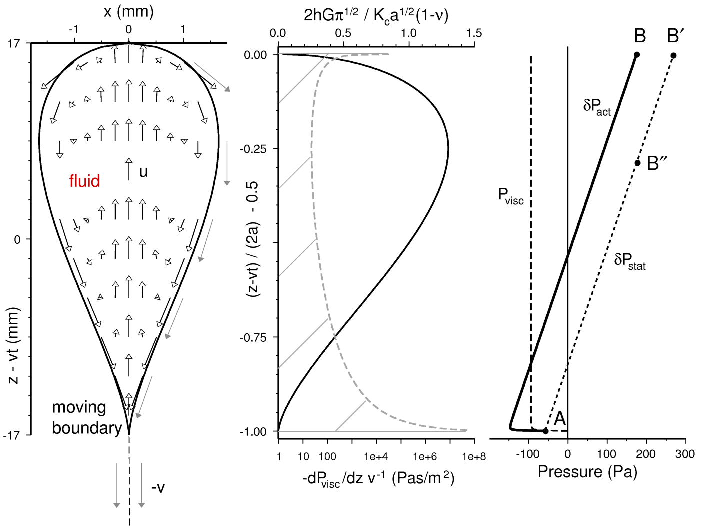
# Fluid filled penny shaped crack under linear increasing P





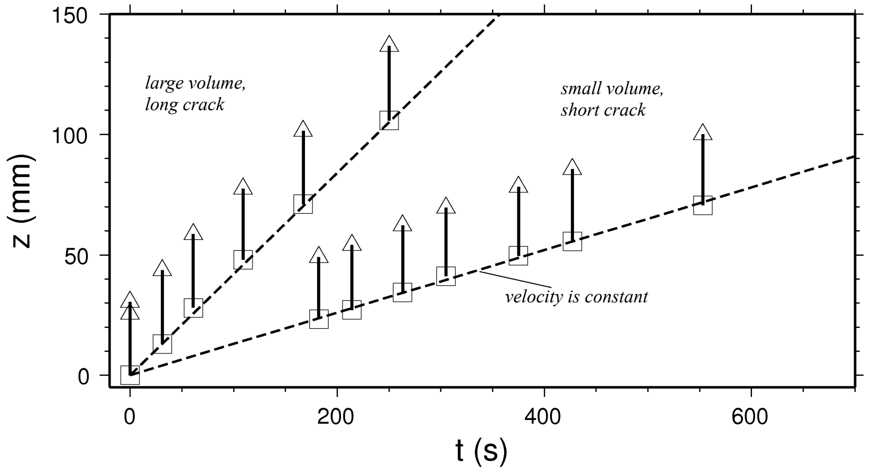
**note!** $P = 0.69\, a\, Pg + Pg\, z$

# Weertmann crack: wholesale crack “ascent”

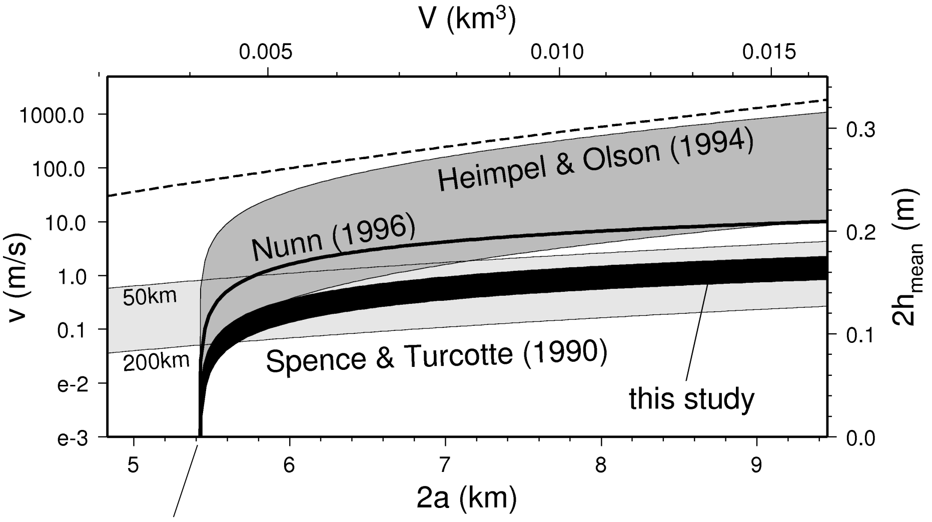


# velocity of wholesale crack ascent

*Measurements of ascent velocity of air-filled cracks in gelatine (Dahm 2000b)*



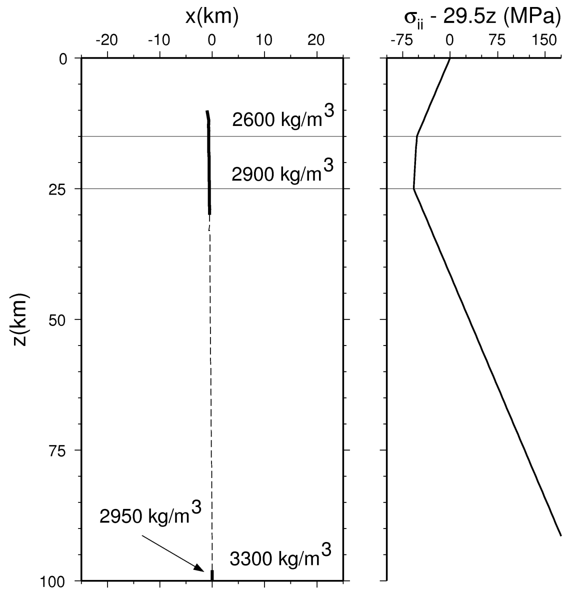
# Ascent velocity of magma-filled dikes in mantle



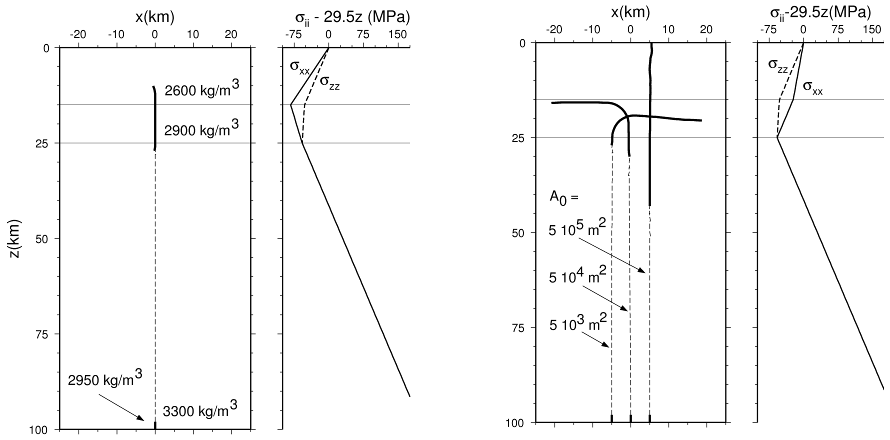
*critical length to initiate wholesale migration*

*Dahm (2000b)*

# Simulation of dike ascent in crust and mantle

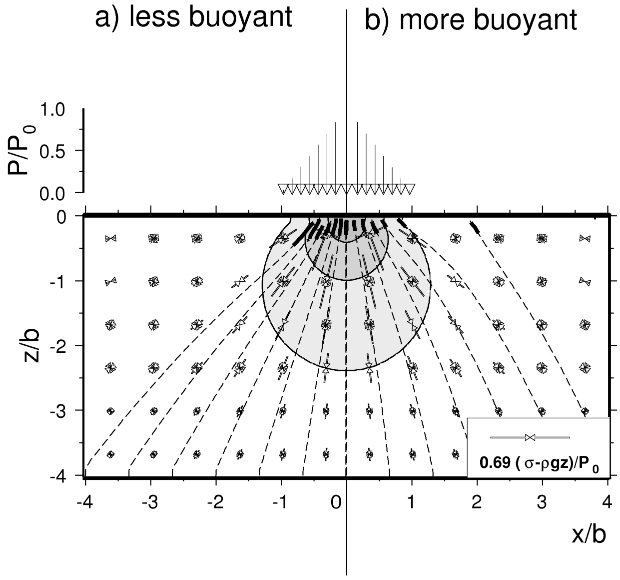


# When do sills form and magmatic underplating occurs?


a) extension

b) compression

# The apparent “attraction” depends on magma density

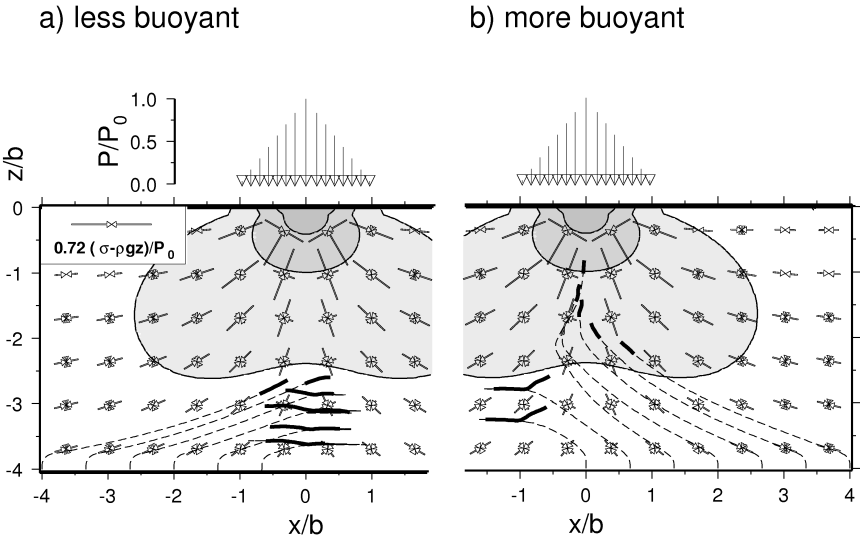

a) less buoyant

b) more buoyant

*ascent path is plotted as dashed line*

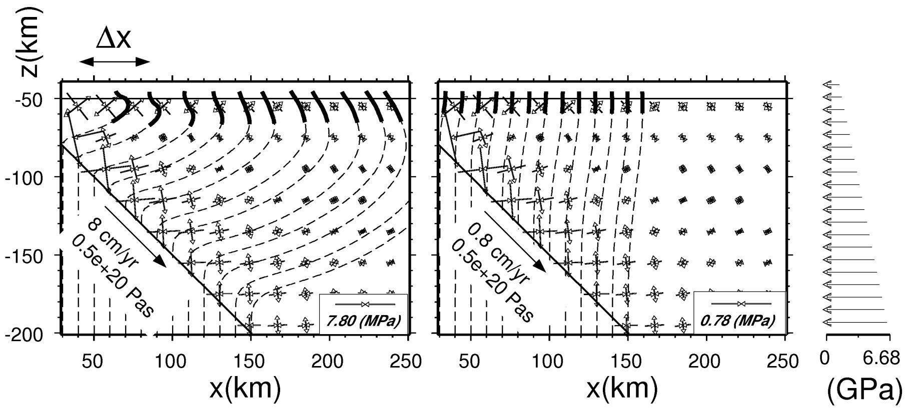
*arrested dike position by fat line*

# Dike ascent under tectonic compression


a) less buoyant

b) more buoyant

# Mantle corner flow above a subducting slab

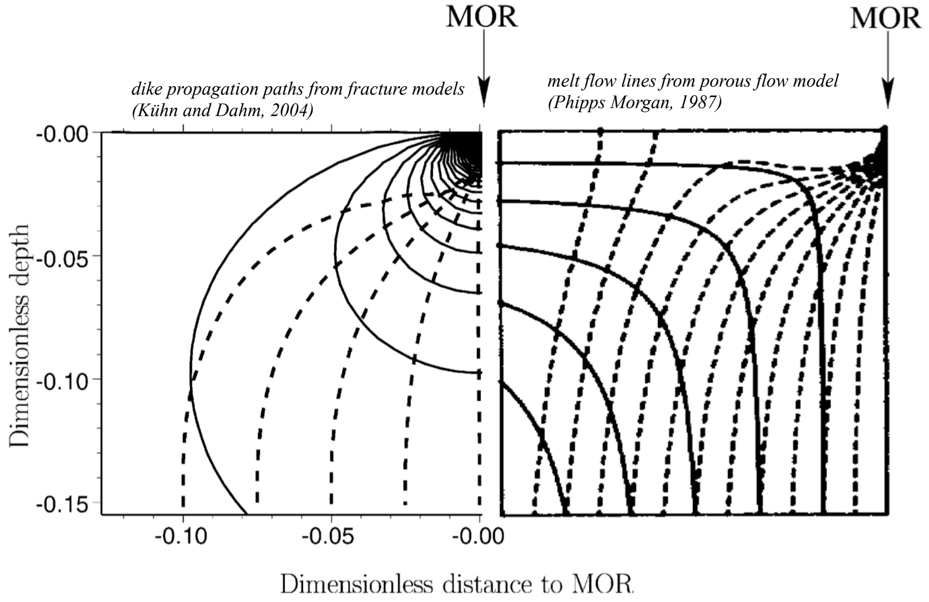


a) faster subduction

b) slower subduction

*Dahm (2000a)*

# Dike ascent in mantle beneath mod-oceanic ridges



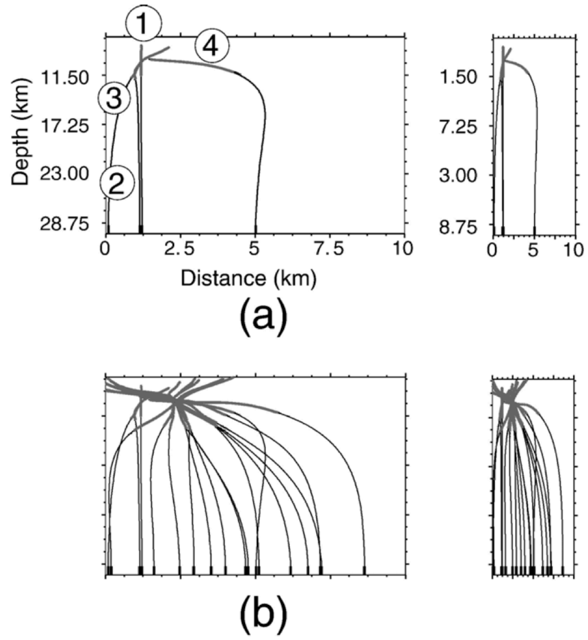
*dike propagation paths from fracture models (Kühn and Dahm, 2004)*

*melt flow lines from porous flow model (Phipps Morgan, 1987)*

# Crack-crack interaction in mud (influence of self stress)

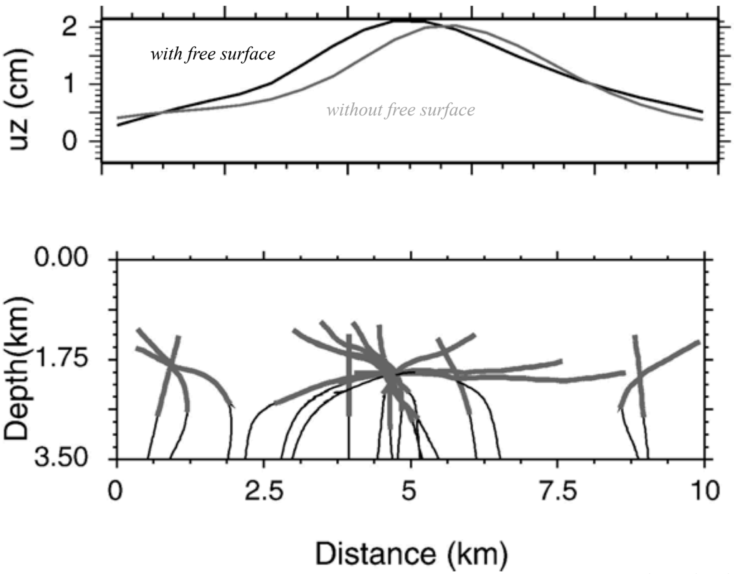
Foto by G. Müller

# Interaction of sequentially intruding dikes



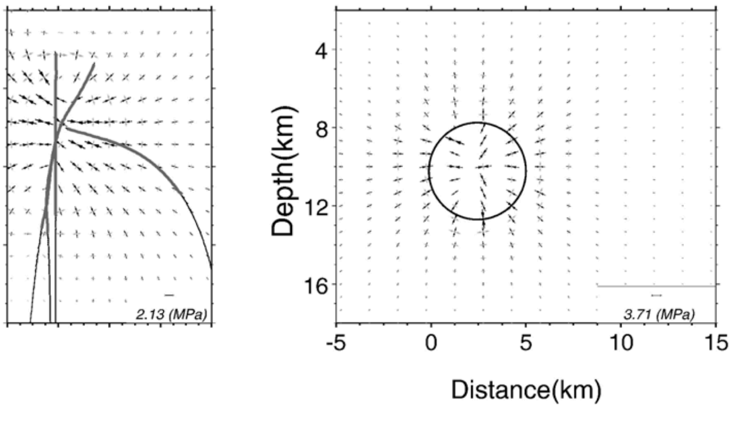
*Kühn and Dahm (2008)*

# Simulation of oceanic crust along axis magma ascent



*Kühn and Dahm (2008)*

# Interaction of dikes explains how magma chambers form



*first 3 interacting dikes*

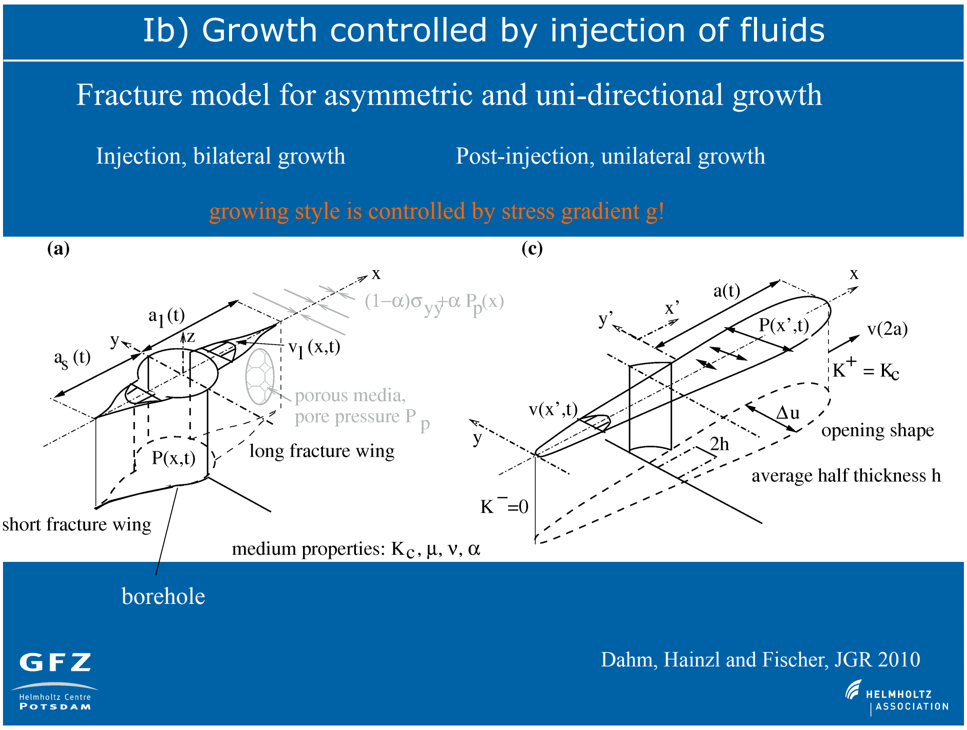
*stress field after 18 dikes*

*Kühn and Dahm (2008)*

# Ib) Growth controlled by injection of fluids

examples:

- lateral intrusions fed by central magma reservoir (rifting)
- mid-crustal earthquake swarms (from fluid intrusions)
- hydraulic fracturing (e.g. in tight gas sandstone)



## Ib) Growth controlled by injection of fluids

Fracture model for asymmetric and uni-directional growth

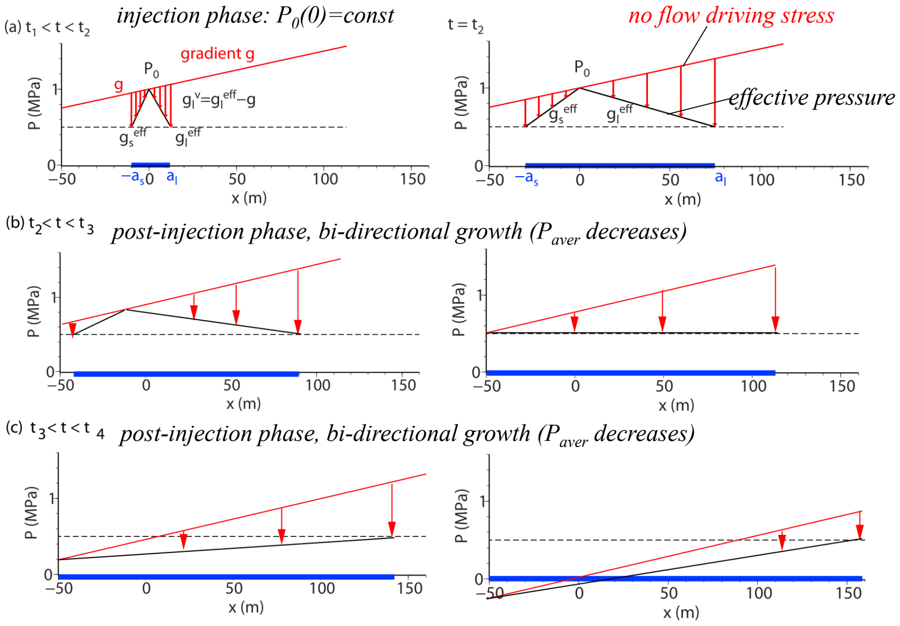
Injection, bilateral growth

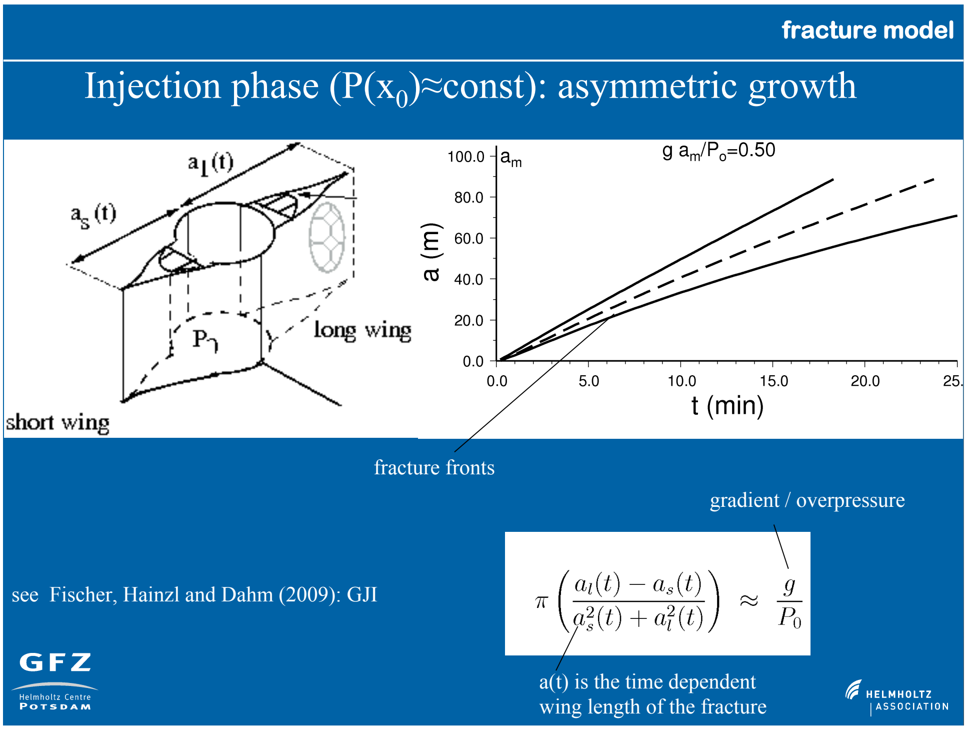
Post-injection, unilateral growth

growing style is controlled by stress gradient g!

Dahm, Hainzl and Fischer, JGR 2010

# Concept





## Injection phase ($P(x_0)\approx$const): asymmetric growth

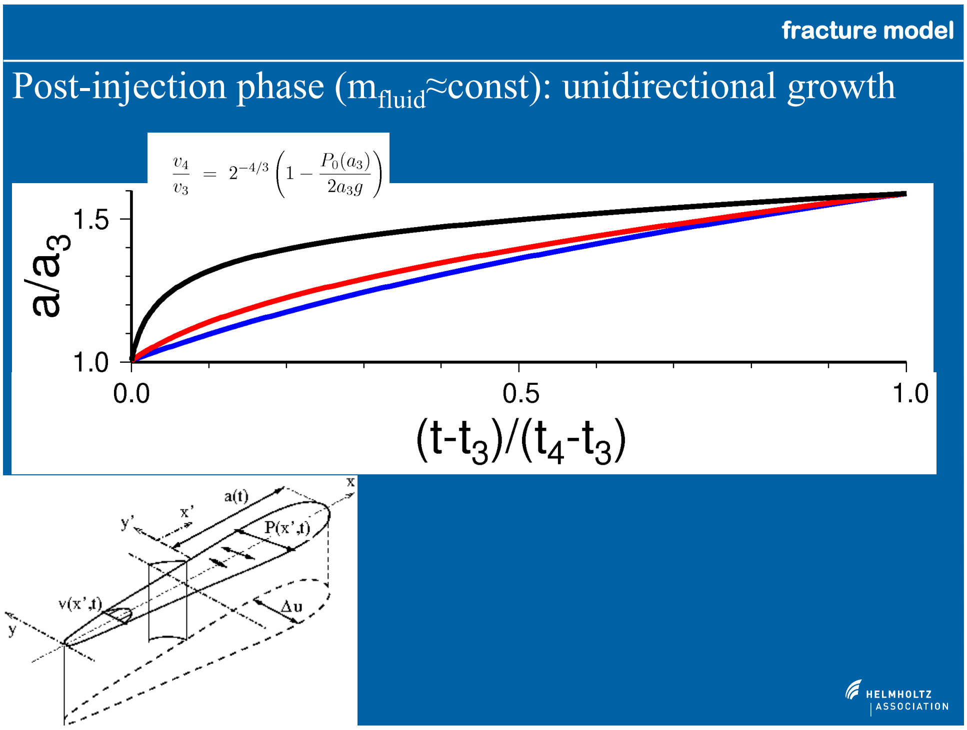
fracture fronts

gradient / overpressure

$$\pi\left(\frac{a_l(t)-a_s(t)}{a_s^2(t)+a_l^2(t)}\right)\approx\frac{g}{P_0}$$

see Fischer, Hainzl and Dahm (2009): GJI

a(t) is the time dependent wing length of the fracture



fracture model

## Post-injection phase ($m_{fluid}$≈const): unidirectional growth

$$\frac{v_4}{v_3} = 2^{-4/3}\left(1-\frac{P_0(a_3)}{2a_3 g}\right)$$

## Summary "fluid-filled crack growth"

✓ Fluid-filled crack growth is controlled by 3 factors:
- orientation of $\sigma_{least}$ (least compressive stress)
- gradients of effective driving stress (buoyancy + stress)
- self stress generated by the crack (i.e. length and shape of dike)

➔ no sharp turns, whole-sale and post-injection movement, path depends also on volume/length of dikes

✓ Growth is influenced by crack-crack interaction

➔ localized volcanic centers may result from diffuse dike ascent
sills can stop "buoyant" dikes / fluid-cracks

✓ Deviation from penny shaped cracks arise from:
- $\sigma 1 \neq \sigma 2$ (elliptical growth)
- wholesale movement if overcritical length (Weertman shape)
- confining layers and free surface

➔ vertical (dikes) and horizontal (sills) length of lateral intrusion depend (also) on effective "overpressure"

## Summary “injection-related” fractures

✓ Asymmetric bi-lateral growth during injection is possible
  - the time function depends on $K_c$ and fluid viscosity
  - ratio between long and short wing depends only on gradient

✓ After injection, self similar, bi- and unilateral expansion
  - length increase is always 1.5 of length at end of transition (injection)
  - time dependency of expansion depends on driving stress gradient

✓ Crack opening and stress buildup in rock explains
  - bilateral front of seismicity during injection
  - shape of unilateral front and backfront of seismicity in post-injection

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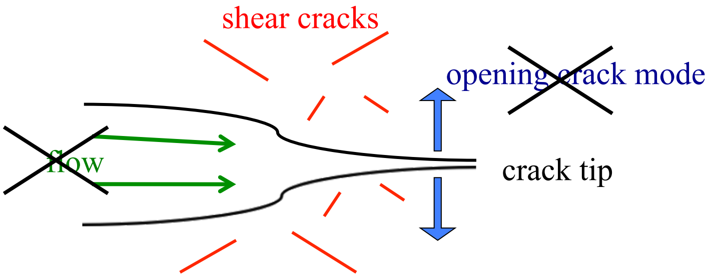
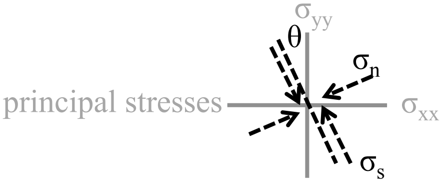
# II) How are earthquakes triggered (seismicity models)

# Key message

*Seismicity accompanying fluid-filled fracture growth is (ususally) not associated with the crack tip opening itself, but represents triggered shear cracks in the rock which experiences stress increase*



# Shear rupture is driven by “shear stress” and hindered by cohesive strength

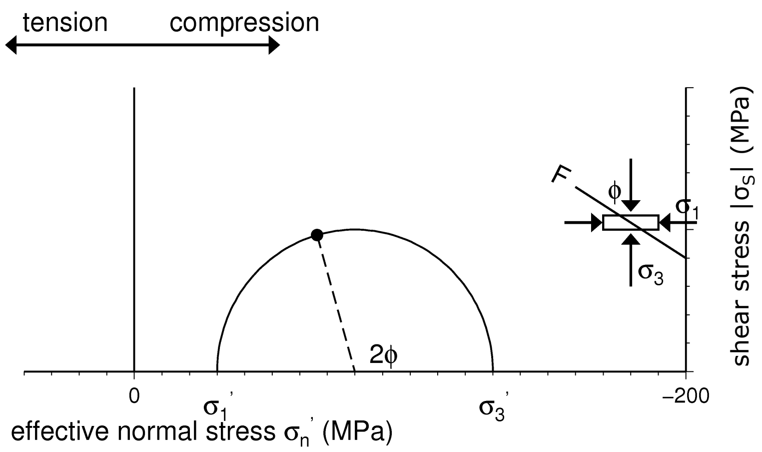


Normal and shear stress on dipping fault (Mohr circle)

$$\begin{aligned}\sigma_n &= \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\Theta \\ \sigma_s &= -\frac{(\sigma_{xx}-\sigma_{yy})}{2}\sin 2\Theta \quad .\end{aligned}$$

Here: tensional stress is positive

## (3) from failure criteria to seismicity models



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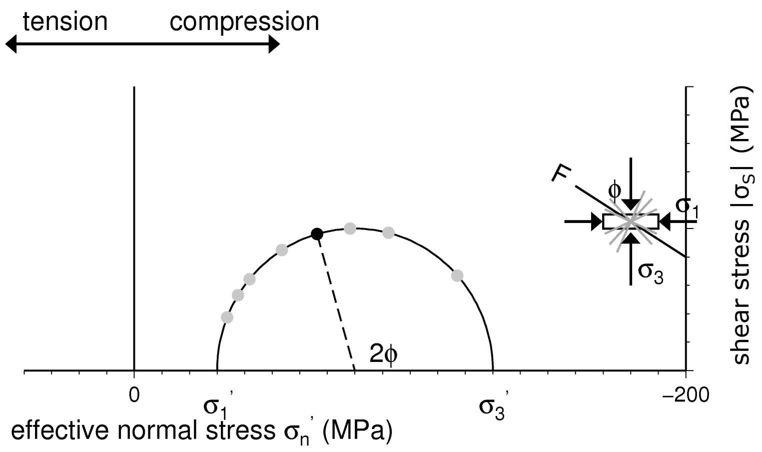
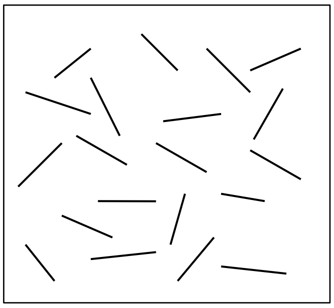
## distribution of faults in arbitrary orientations

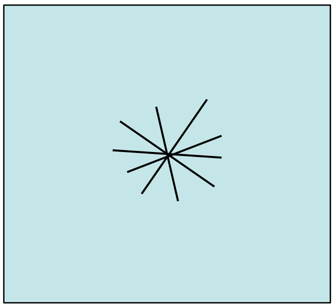


# effective media model

real rock with with distribution of cracks / flaws with random orientation

effective media model with single crack of random orientation





heterogeneous stress on micro-scale

homogeneous effective media, homogeneous stress

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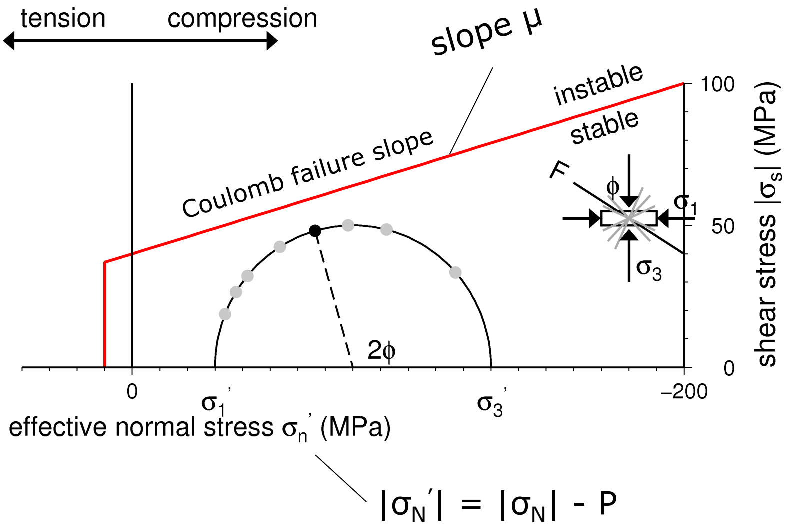
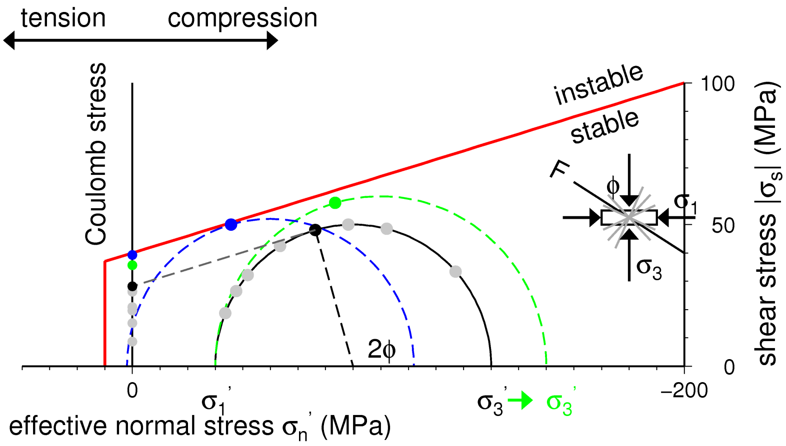
# (3) seismicity models



# (3) seismicity models



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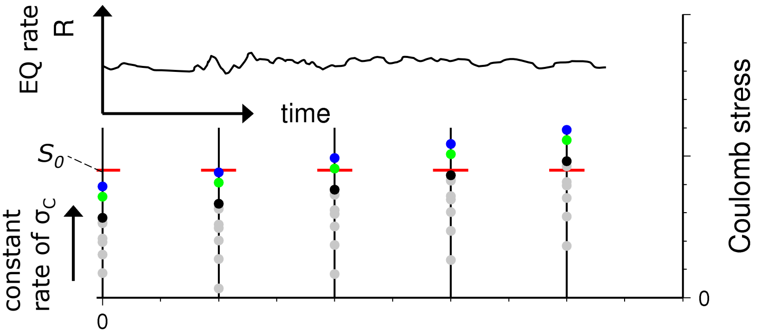
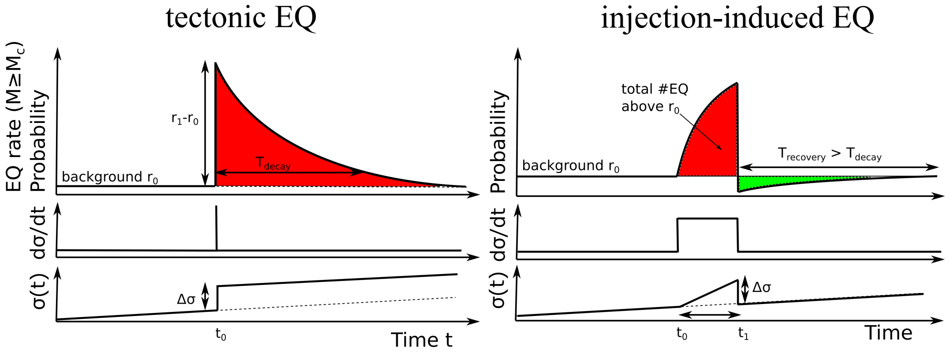
# Earthquake rate R from constant stress loading rate

Note: R is the same a the a-value in GR relation (if M=0) or seismogenic index



1. *threshold model (Coulomb failure model, CFM, or Brownian Passage Time)*
2. *frictional nucleatin phase model (e.g. rate and state, RSM)*

## EQ rate r in given rock volume experiences different stressing



- ✓ Both, stressing rate and steps (increase) lead to higher EQ rate
- ✓ Stress shadow from decreasing stress can have long memory
- ✓ Growing and propagating dikes involve positive and negative stress changes

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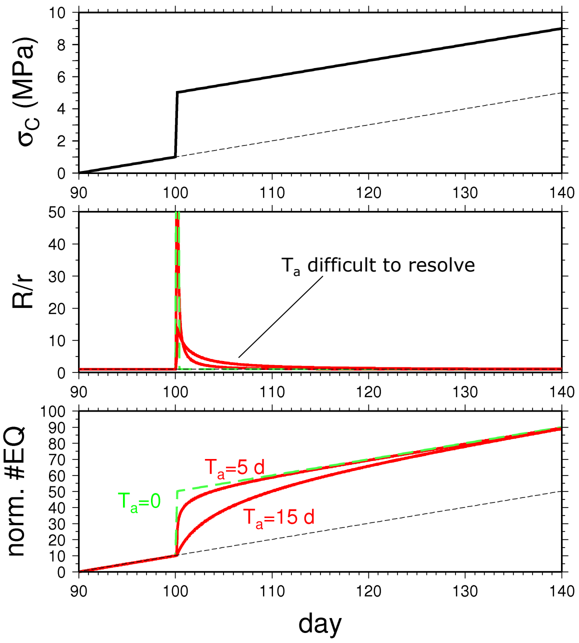
## Sudden pressure step loading (e.g. EQ aftershocks)



✓positive ΔP

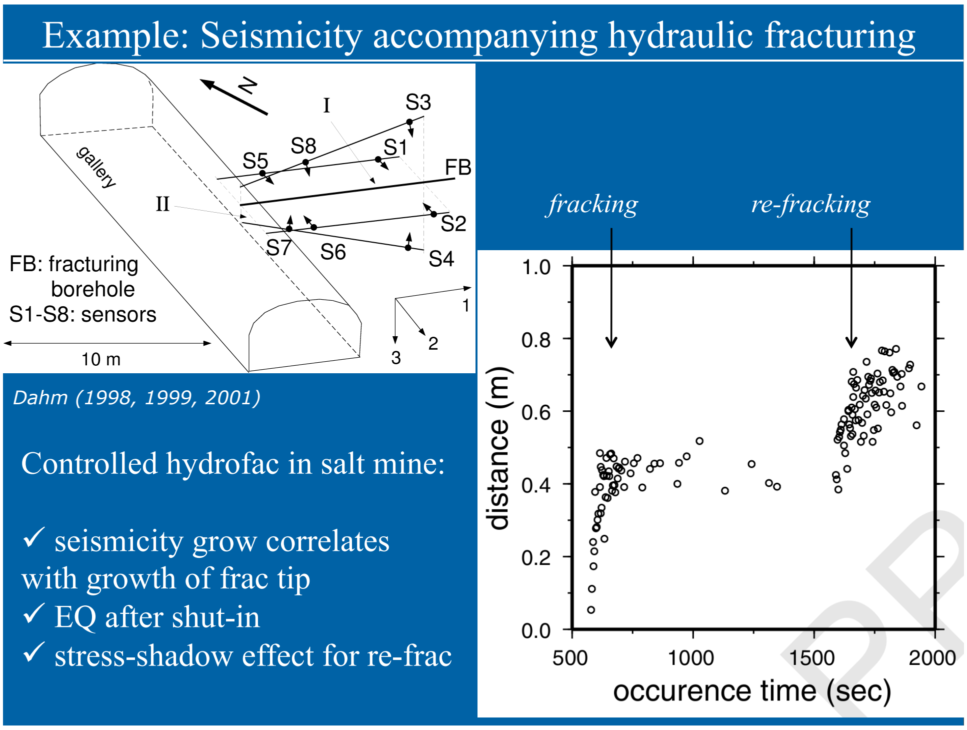
2$^{nd}$ parameter:
decay time $T_a$

transition depends $T_a$ :
Rate & state model (RSM)
limiting case for $T_a$=0:
Coulomb failure model (CFM)

Total number constant:
short $T_a$ –> high peak
long $T_a$ –> small peak

# III) Examples

## “Hydrofracture” induced seismicity



Example: Seismicity accompanying hydraulic fracturing

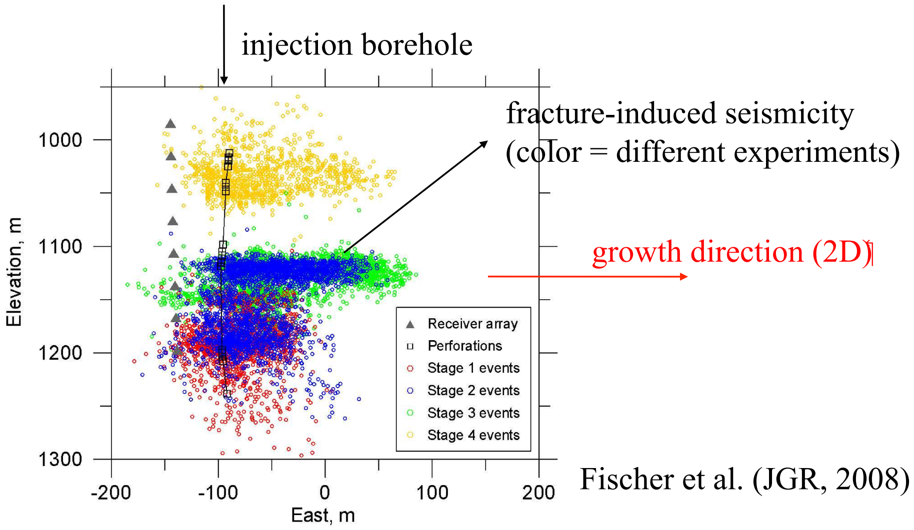
FB: fracturing borehole
S1-S8: sensors

*Dahm (1998, 1999, 2001)*

Controlled hydrofac in salt mine:

✓ seismicity grow correlates with growth of frac tip
✓ EQ after shut-in
✓ stress-shadow effect for re-frac

# Hydrofrac stimulations in Canyonsand gas field, W. Texas

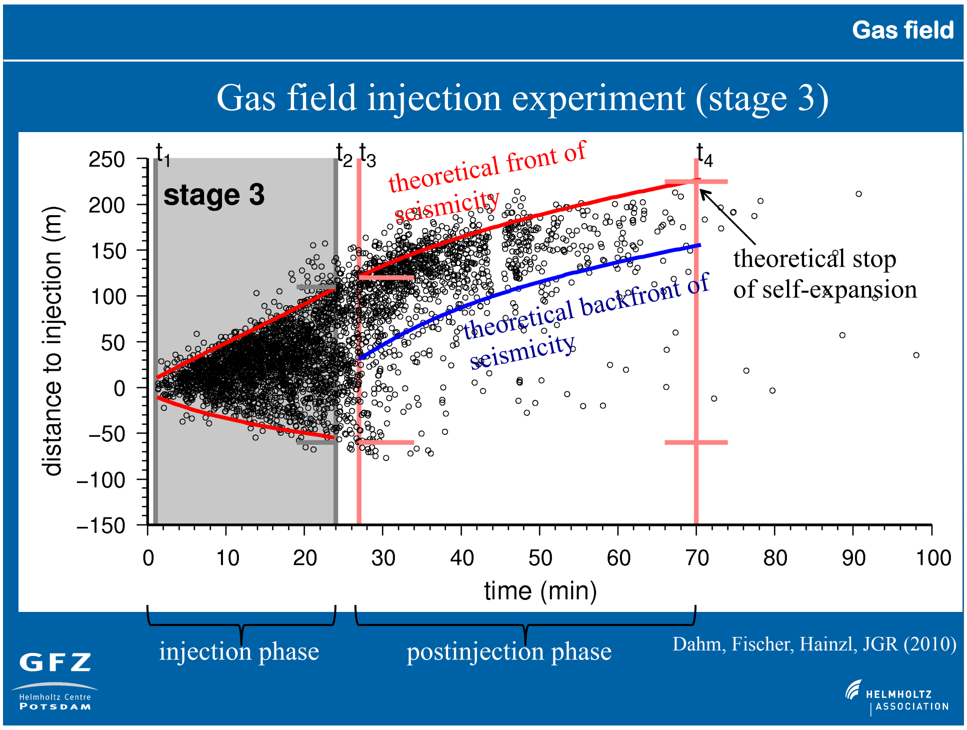


injection borehole

fracture-induced seismicity
(color = different experiments)

growth direction (2D)

Fischer et al. (JGR, 2008)



**Gas field**

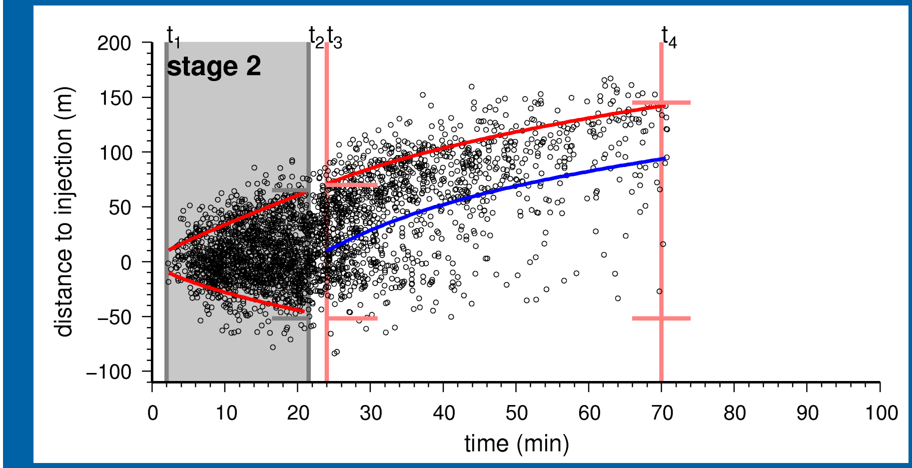
## Gas field injection experiment (stage 3)

injection phase

postinjection phase

Dahm, Fischer, Hainzl, JGR (2010)

# Gas field injection experiment (stage 2)



Dahm, Fischer, Hainzl, JGR (2010)

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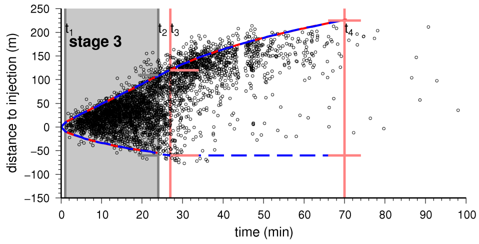
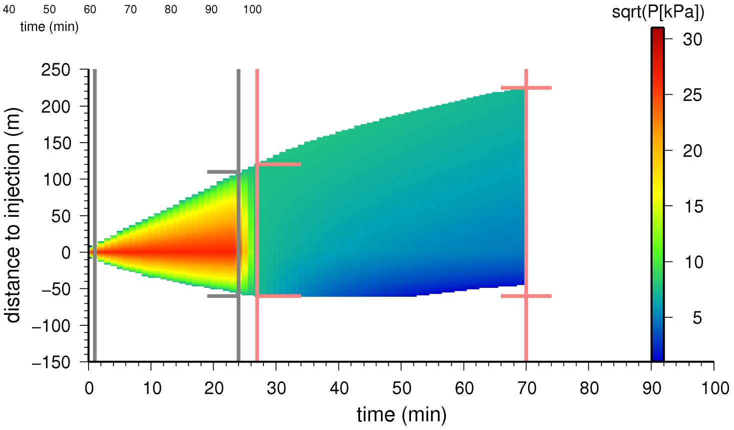
# Modeling stress changes



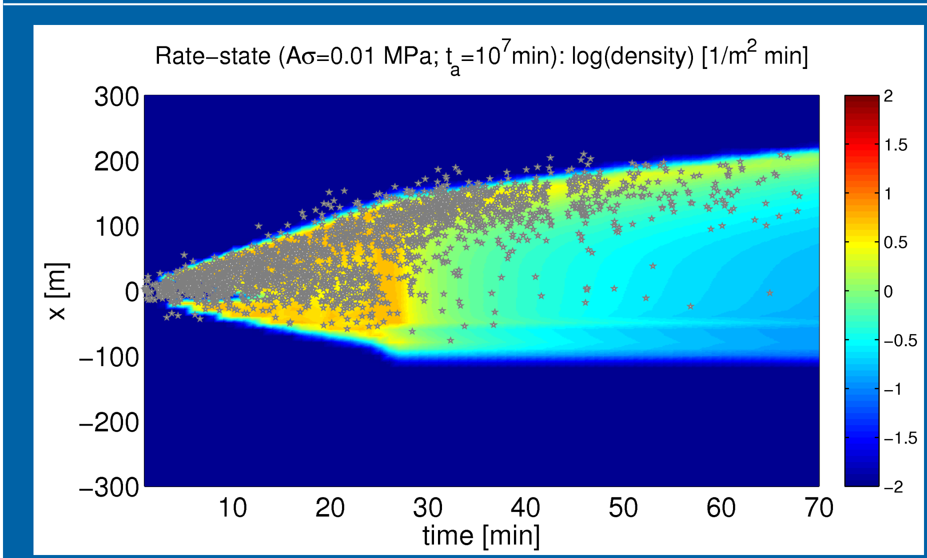
hydrofrac length

internal effective pressure



## stage 3 : rate and state, low Aσ, long decay

**stress model**



$P_0 \approx 0.7$ MPa

$dP/dx \approx 14\ P_0$/km, $K_c \approx 1$ MPa$\sqrt{\text{km}}$

$T_{inject} \approx 0.4$h, $l_{inject} \approx 0.18$ km

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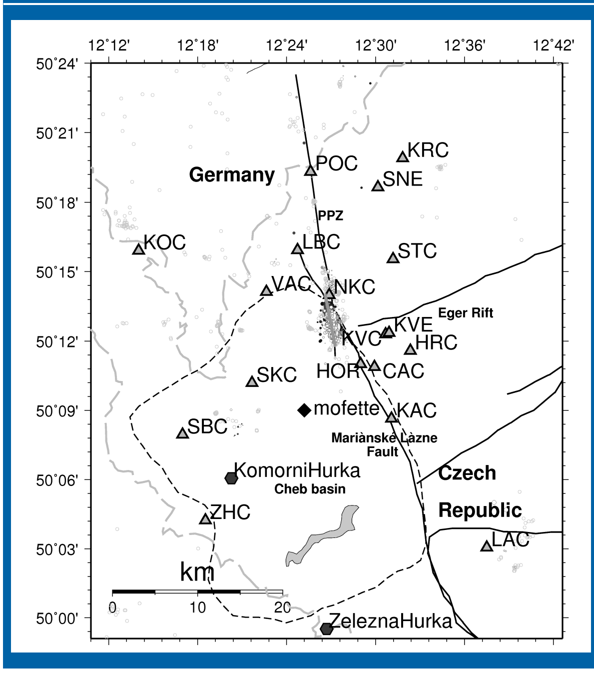
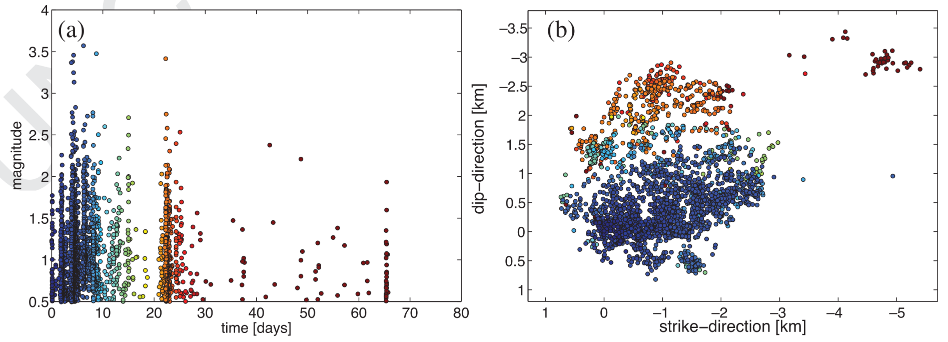
## Mid-crustal earthquake swarms (NW Bohemia)



Hainzl et al., GJI, 2012

# NW Bohemia 2008 swarm



Hainzl, Fischer, Dahm, GJI (2012)

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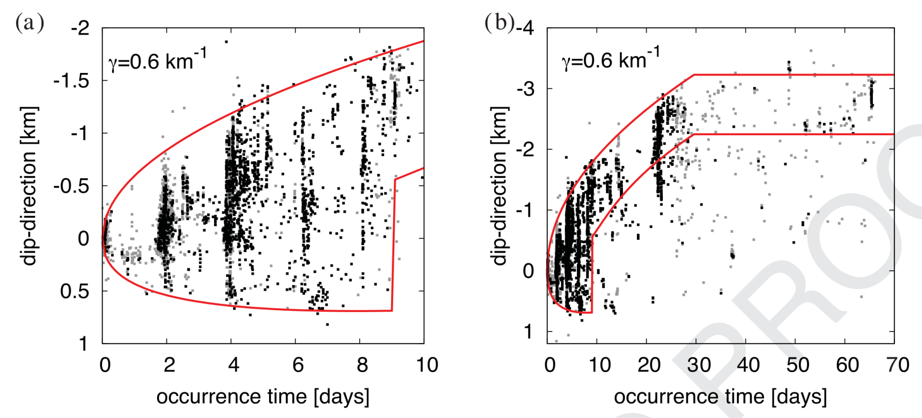
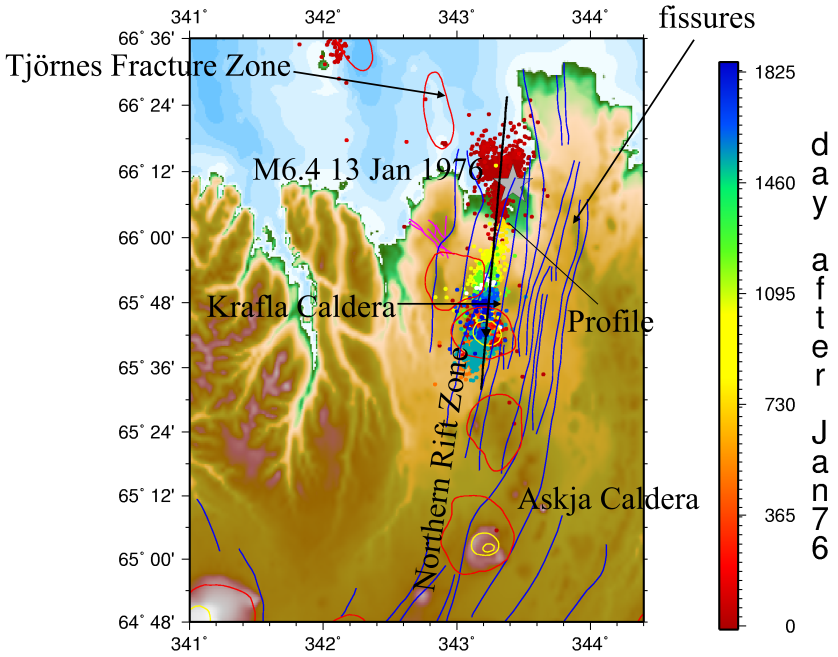
# NW Bohemia 2008 swarm



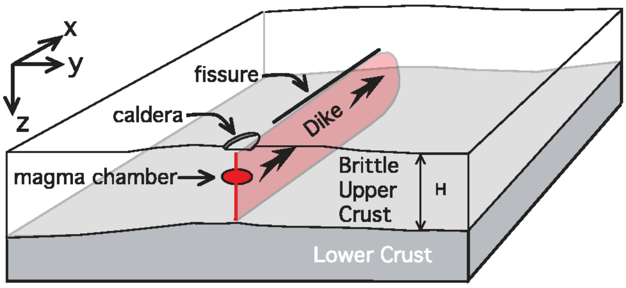
$\Delta P \approx 15$ MPa (water) or 25 MPa (gas)
$dP/dx \approx 0.6\ P_0$/km
$T_{inject} \approx 9$h, $l_{inject} \approx 2.4$ km

Hainzl, Fischer, Dahm, GJI (2012)

## Seismicity during the Krafla rifting events Dec75 - Jan81

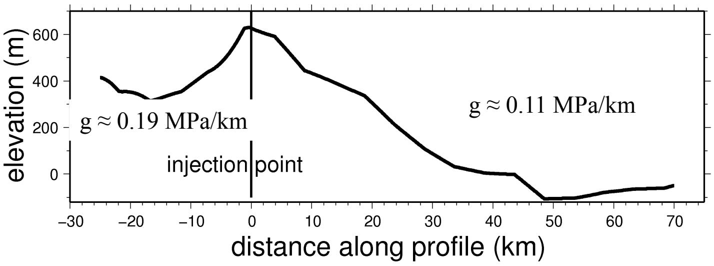


## Rifting at Krafla: topography may control stress gradients

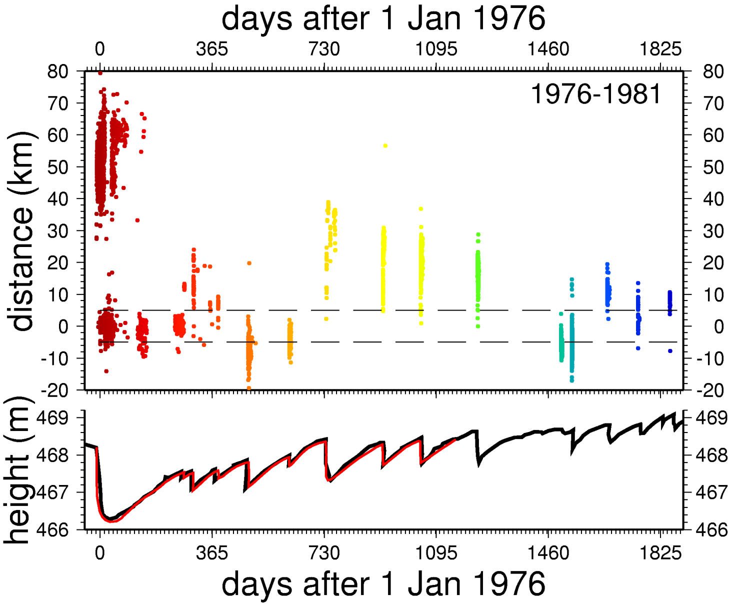


Buck et.al. (2006)

g from infinite slope model with mu=0.25 and rho=2800 kg/m^3



# Along strike seismicity and caldera deflation

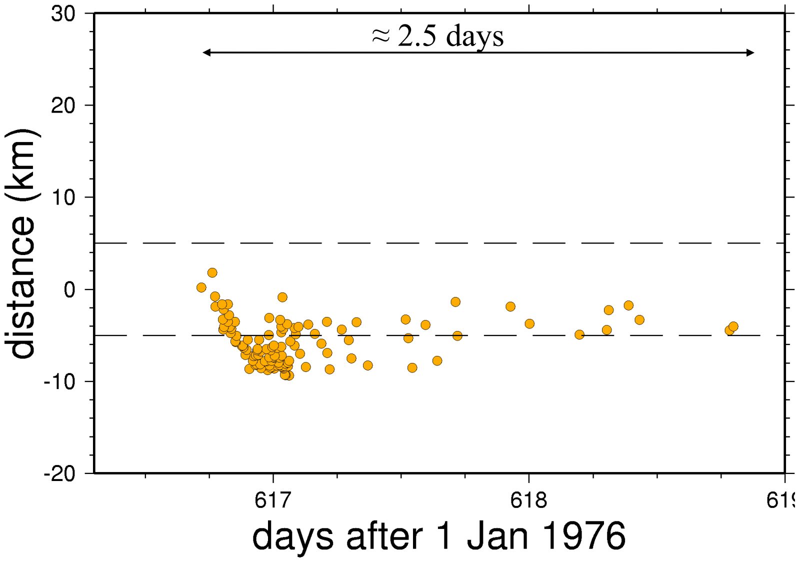


Caldera

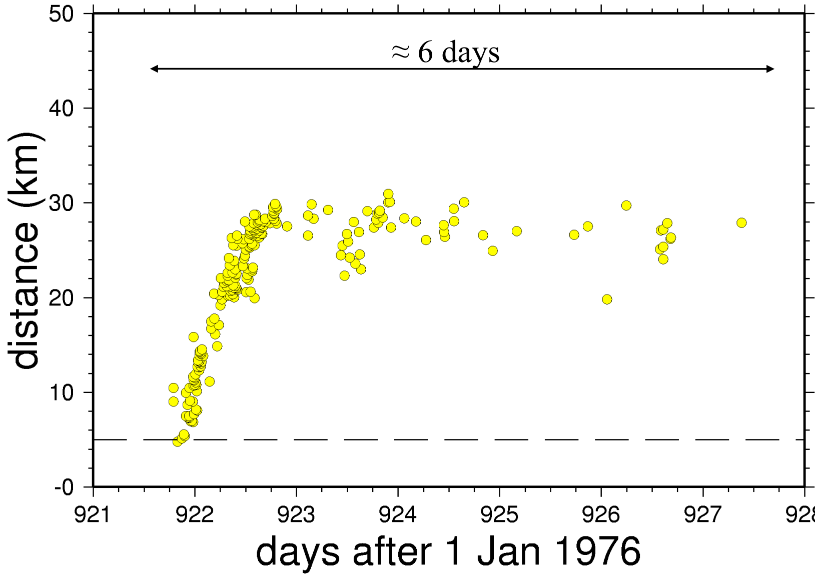
Elevation
change at
Caldera

see Einarsson, 1991, Buck et al., 2006 and references therein

# induced seismicity: Sep 77 intrusion



# induced seismicity: Jul 78 intrusion



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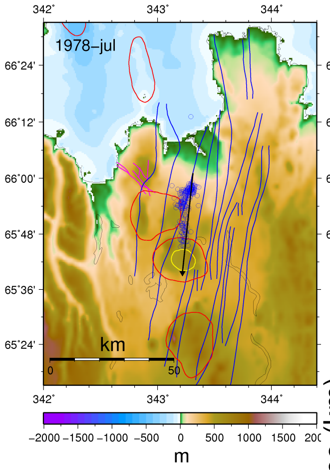
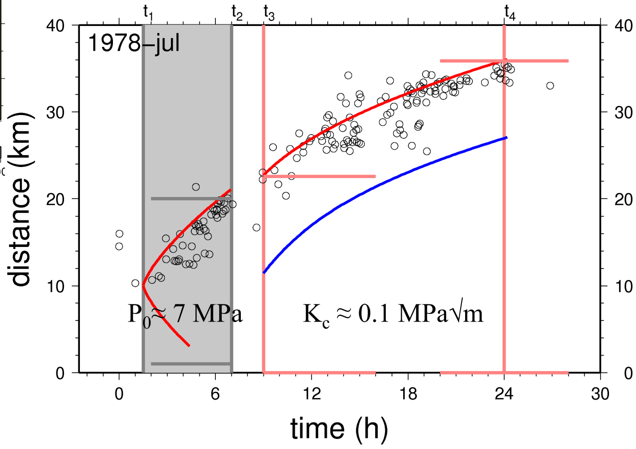


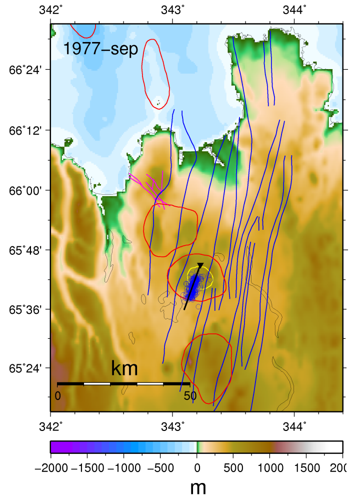
## July 1978 intrusion northward

- self-expansion starts after 9 h
- injection controlled length is 19-22 km
- final length is 34 km
- gap after7 h: re-organisation of flow

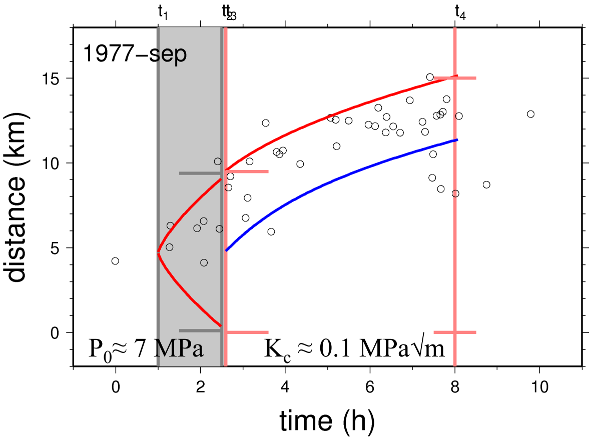


assuming g≈0.11 MPa/km
η ≈ 20 Pa s

## Sep 1977 intrusion southward



- self-expansion starts after 2.5 h
- injection controlled length ≈ 10 km
- final length is ≈ 15 km



assuming g≈0.19 MPa/km
η ≈ 20 Pa s

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# What can be learned from intrusion-induced seismicity?

- Retrieve the geometry and dynamics of the intrusion (duration of injection, overpressure, viscosity?, ...)
- Estimate the fracture properties of the medium (e.g. $K_c$)
- Estimate the stress direction and state of stress
- Estimate the permeability of the medium