Dark Matter
Lecture 2: Theoretical Models

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ICTP Summer School on Cosmology
Trieste
6 June 2016

Goals (Lecture 2)

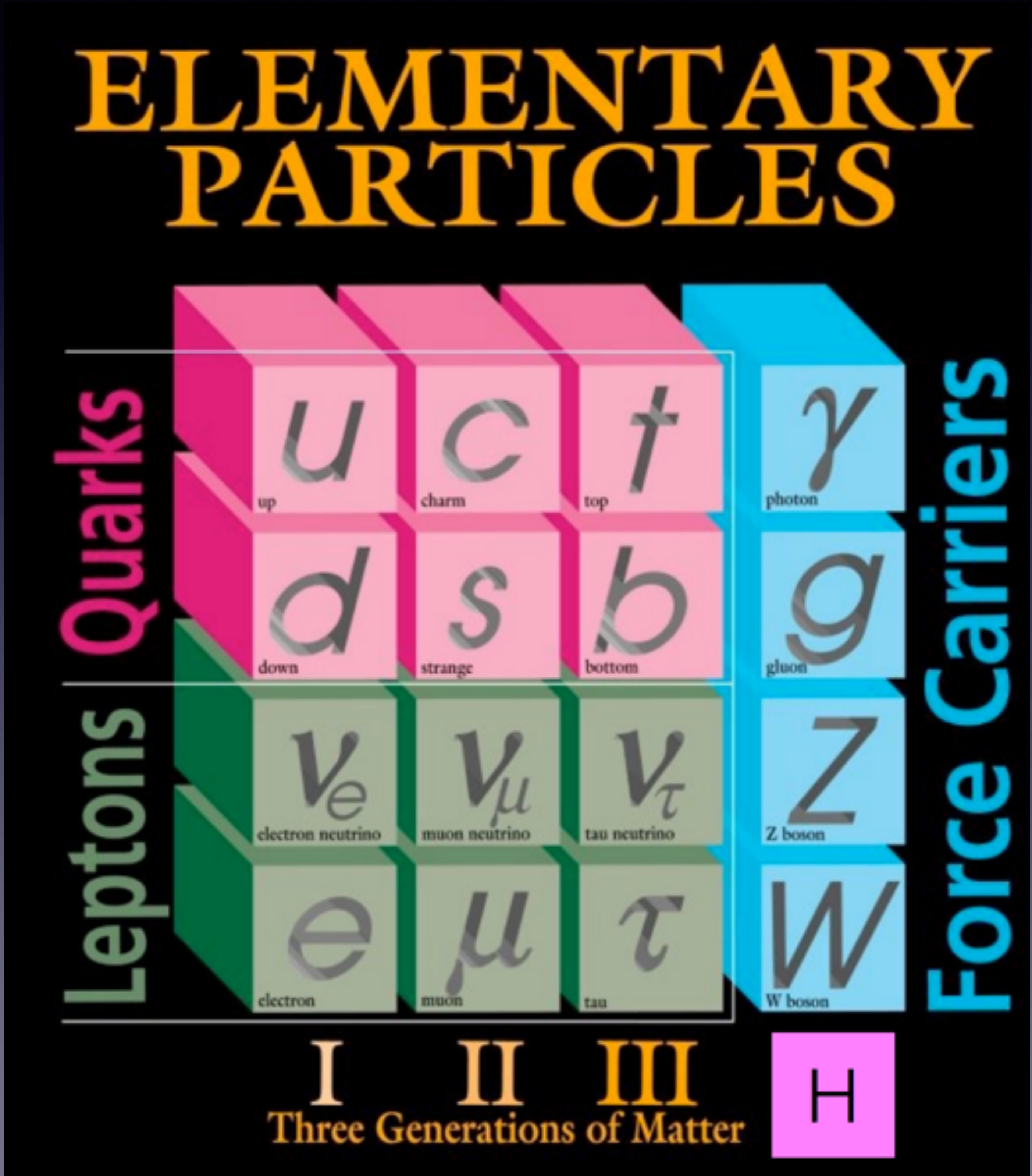
- Describe characteristics needed for a dark matter particle, and their implications:
  - Stabilization
  - Relic density
- Outline and explain the cosmology and key properties of the following scenarios:
  - Weakly Interacting Massive Particles (WIMPs)
  - Axions

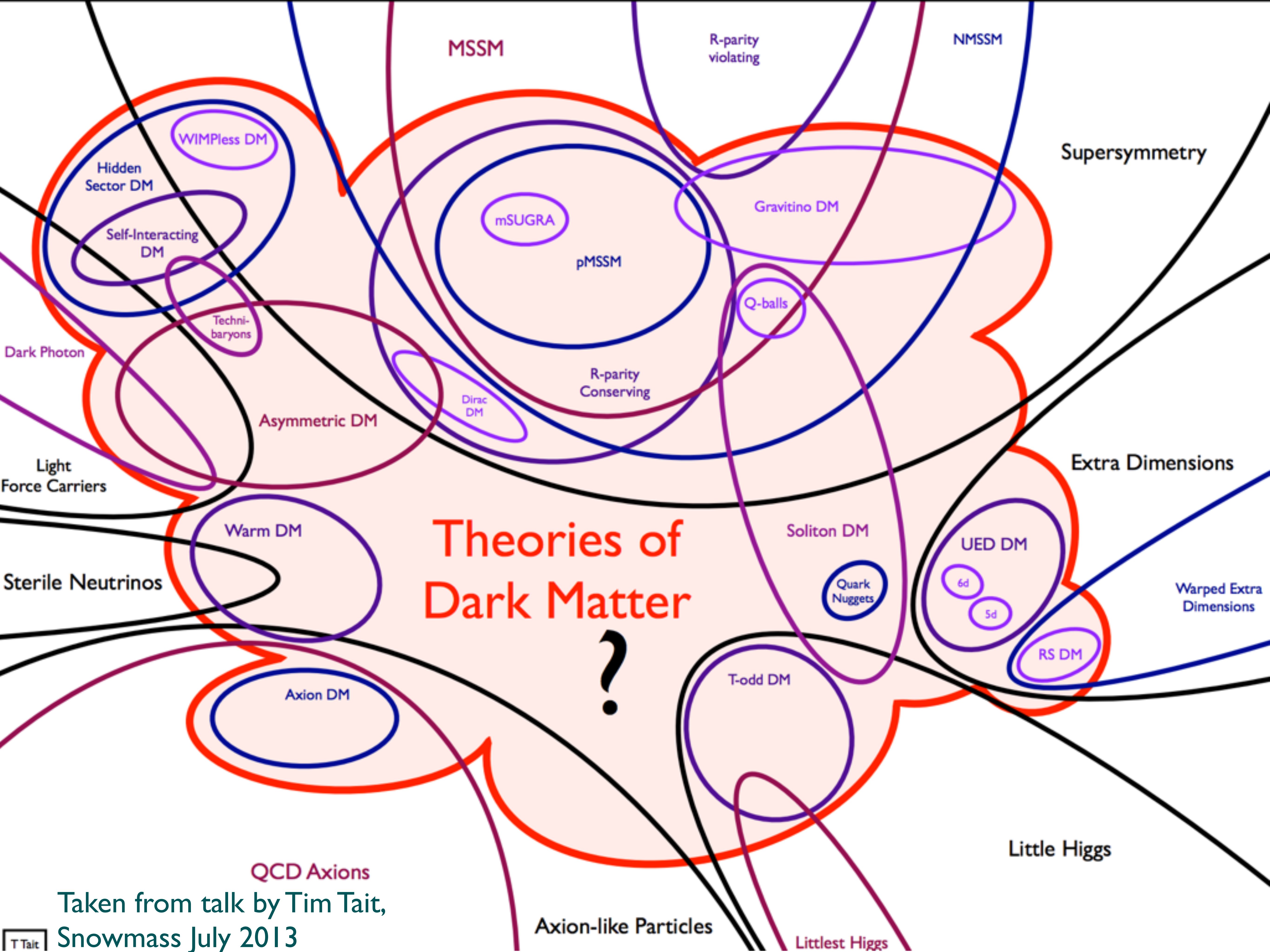
Recap from Lecture 1

- The distribution and gravitational effects of dark matter can be a powerful probe of dark-matter properties and interactions, independent of any interaction with the known particles.
- We know that dark matter is:
  - Still around today, i.e. stable on timescales ~age of the universe (rotation curves)
  - "Collisionless" - electrically neutral and interactions are fairly weak (Bullet Cluster)
  - "Cold" / slightly warm - small free-streaming length in epoch of structure formation (matter power spectrum, Lyman-alpha forest)
- No particle in the Standard Model of particle physics (the "SM") matches these properties.

Beyond the SM

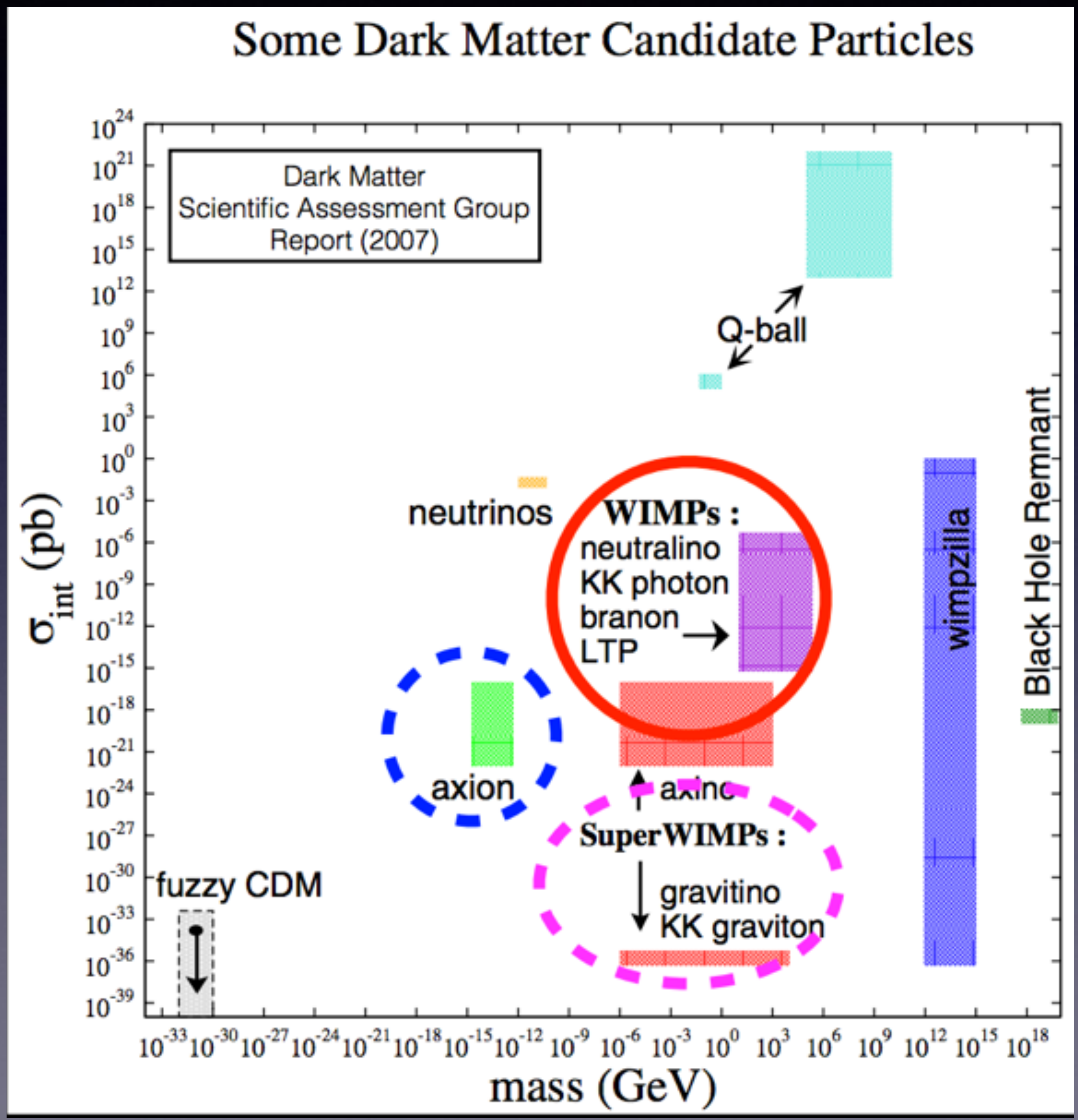
- Photons, leptons, hadrons and W bosons shine too brightly / are charged.
- Z and Higgs bosons are neutral but short-lived.
- Neutrinos are neutral and stable, but too light. They would be hot dark matter - cannot comprise all DM.





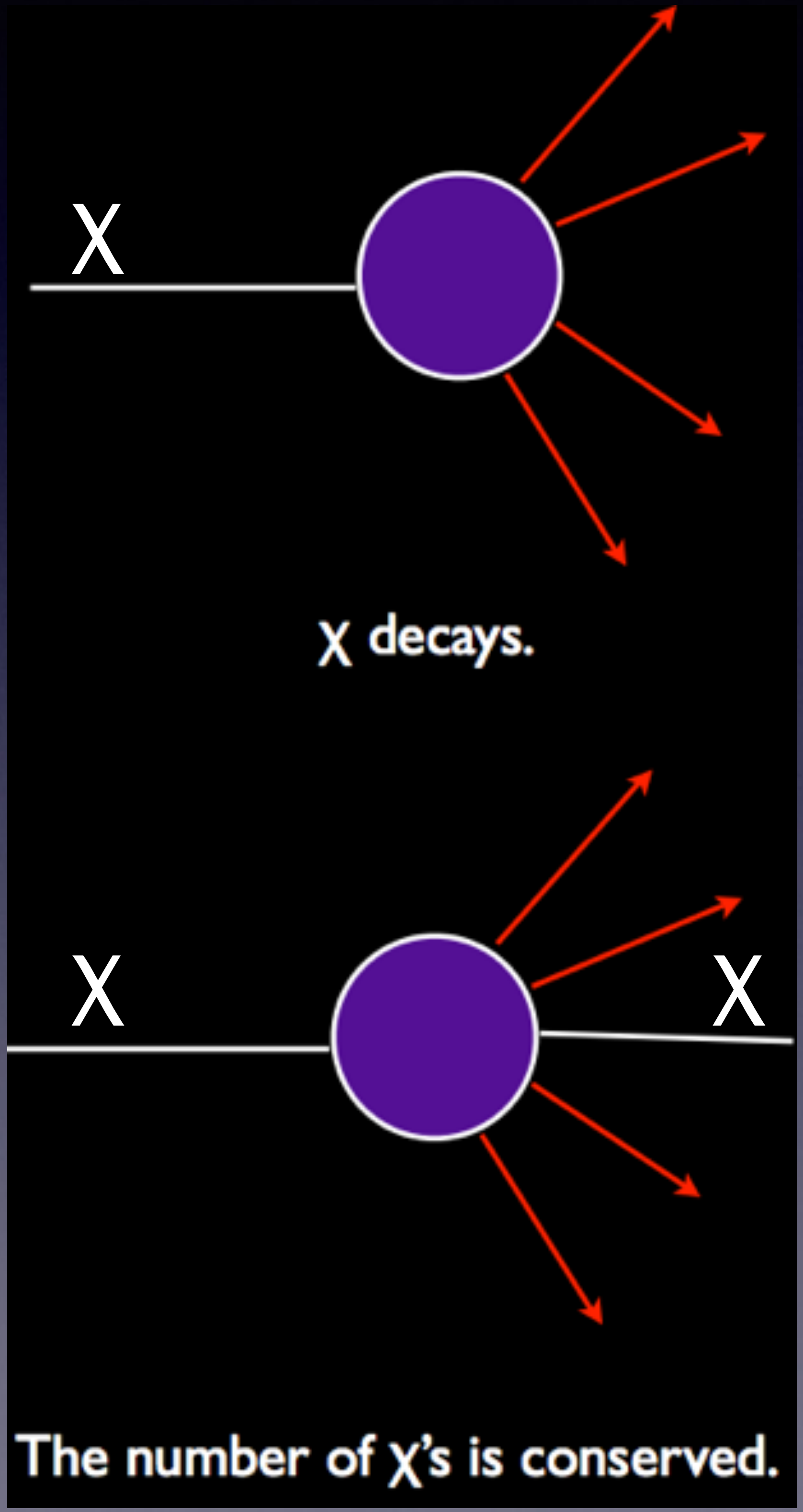
Taken from talk by Tim Tait, Snowmass July 2013

Huge range of possibilities



Stability

- One mystery is why dark matter is stable - especially if it is heavy enough to be “cold” in the early universe
- Sets stringent limits on DM-SM interactions:
  - Easiest route: impose some kind of symmetry to prevent DM from decaying
  - Simplest example is a new kind of “parity” - Z_2 discrete symmetry, forces coupling to SM fields to involve pairs of DM particles.
  - Many more examples!



The dark matter abundance

- Any DM model must explain the abundance of dark matter at the epoch of last scattering, precisely measured (from the CMB) to be:

$$\Omega_c h^2 = 0.1186 \pm 0.0020 \qquad h = H_0/(100\text{km/s/Mpc}) = 0.6781 \pm 0.0092$$

Q: THERMAL OR NON-THERMAL?

Was the dark matter in thermal equilibrium with the Standard Model during the radiation-dominated epoch?

THERMAL

Explain how the early abundance of dark matter was depleted

-Asymmetric: small asymmetry between dark matter and antiparticle sets final abundance

-Symmetric: interactions set final abundance

NON-THERMAL

Explain how the required amount of dark matter was produced

-Initial condition from reheating?

-Misalignment mechanism

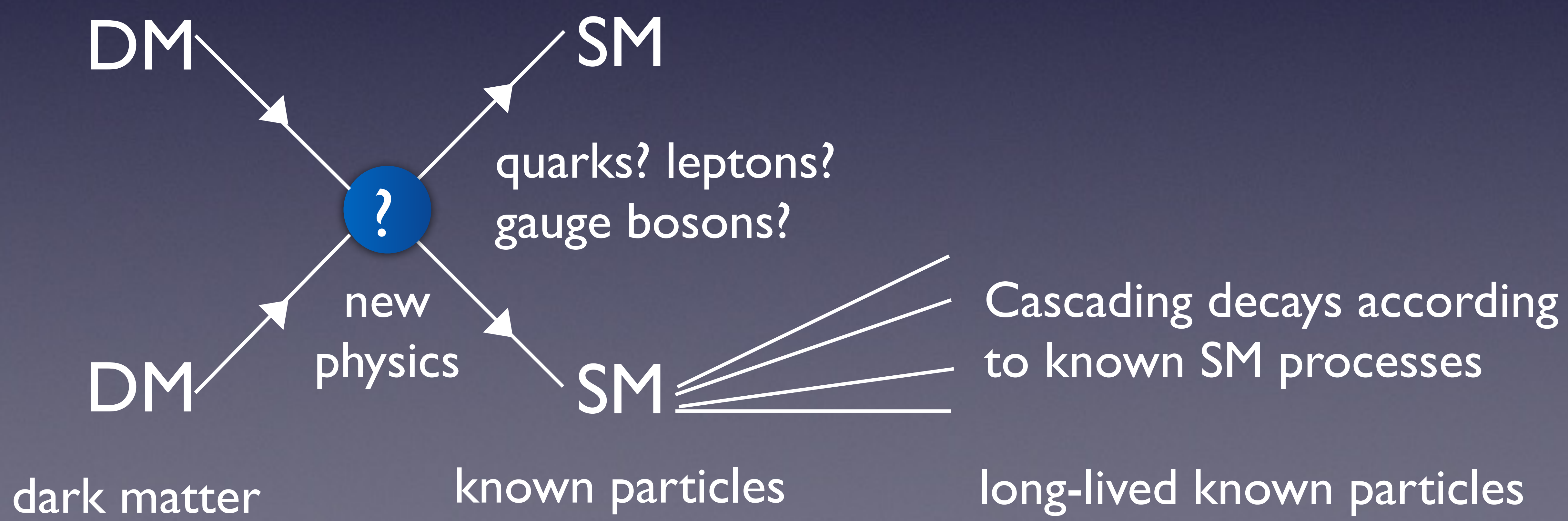
-Phase transition

-Thermal parent - interactions with state in thermal equilibrium determine its abundance

Weakly Interacting Massive Particles (WIMPs)

Thermal abundance

- Suppose dark matter:
  - can annihilate to Standard Model particles
  - was at some point kept in thermal equilibrium with the Standard Model by annihilation



Thermal freezeout

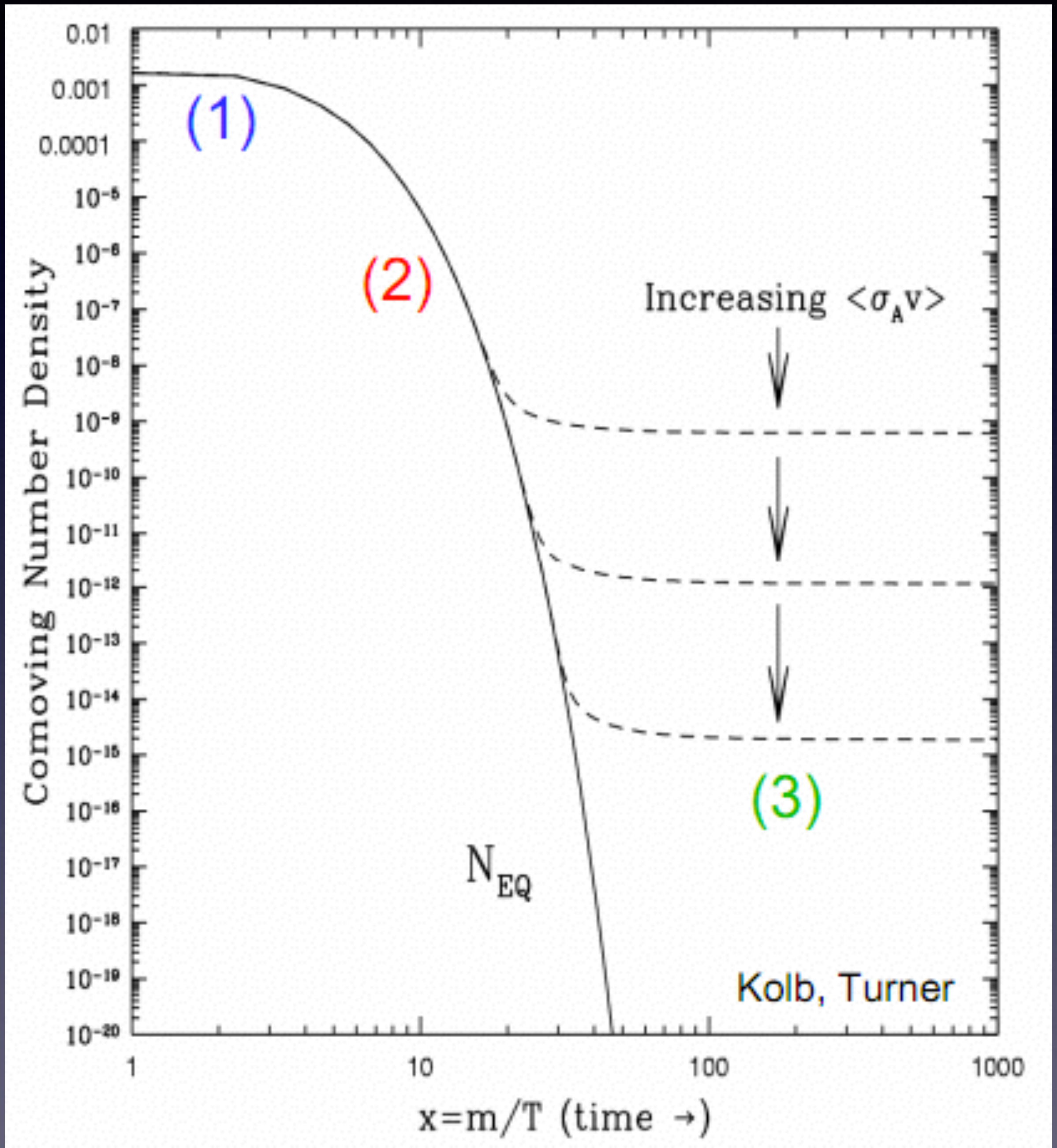
- In the early universe, let the DM particle be thermally coupled to the SM. Can annihilate to SM particles, or SM particles can collide and produce it.

$$\chi\chi \leftrightarrow \mathrm{SM}\,\mathrm{SM} \quad (1)$$

- Temperature(universe) < particle mass => can still annihilate, but can't be produced.

$$\chi\chi \longrightarrow \mathrm{SM}\,\mathrm{SM}$$
$$\chi\chi \not\leftarrow \mathrm{SM}\,\mathrm{SM} \quad (2)$$

- Abundance falls exponentially, cut off when timescale for annihilation ~ Hubble time. The *comoving* dark matter density then freezes out.



So (known) late-time density is set by annihilation rate.

$$\langle \sigma v \rangle \sim 3 \times 10^{-26} \mathrm{cm}^3/\mathrm{s} \sim \pi\alpha^2/(100\,\mathrm{GeV})^2 \quad (3)$$

Outline of calculation

- Ingredients: annihilation rate for identical particles given by

$$\text{annihilations / dt / dV} = n^2 \langle \sigma v \rangle / 2$$

- Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left[n^2 - n_{\rm eq}^2 \right]$$

- Equilibrium density (Boltzmann distribution):

$$n_{\rm eq} = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

- Temperature of universe (assume radiation domination):

$$H^2 \propto \rho \propto T^4 \Rightarrow T \propto \sqrt{H} \propto t^{-1/2}$$

Estimating freezeout

- For precision solution, can solve this differential equation numerically
- But we can get a simple estimate of important quantities analytically.
  - Freezeout occurs when timescale for expansion ~ timescale for collision: $H \sim n\langle\sigma v\rangle$
  - Up to freezeout, n~n_{eq}, so we require $H \sim g(mT/2\pi)^{3/2}e^{-m/T}\langle\sigma v\rangle$
- Defining x=m/T, we have $H(m)x^{-2} = g(m^2/2\pi)^{3/2}x^{-3/2}e^{-x}\langle\sigma v\rangle$
- Transcendental equation $e^{-x} = x^{-1/2}/C$ has approximate solution

$$x \sim \ln C \sim \ln\left(g(m^2/2\pi)^{3/2}\langle\sigma v\rangle/H(m)\right)$$

note: only depends on m and cross section logarithmically

Estimating freezeout II

- Abundance at freeze-out:

$$n \sim g(m^2/2\pi)^{3/2} x^{-3/2} e^{-x} \sim H(m) x_f^{-2} / \langle \sigma v \rangle$$

- For comparison, photon abundance at freezeout:

$$n_\gamma \sim T^3 \sim m^3/x_f^3 \Rightarrow n/n_\gamma \sim \left(H(m)/m^2\right)(x_f/m)/\langle \sigma v \rangle$$

- To match measurements of DM mass density from CMB (comparable to critical density and baryon density), DM number density ~9 orders of magnitude below photon number density if $m_{DM} = m_{proton}$. At higher DM mass, number density must be lower (keeping mass density = mass x number density fixed).

$$H(m) \sim m^2/m_{\mathrm{Pl}} \Rightarrow 10^{-9}\mathrm{GeV}\, m_{\mathrm{Pl}} \sim x_f/\langle \sigma v \rangle \Rightarrow \langle \sigma v \rangle \sim x_f 10^{-10} \mathrm{GeV}^{-2}$$

- Taking $x_f \sim 1$ as a first approximation (since x_f is a log quantity, can't be too large) gives us a first estimate for cross section.

Cross section & mass scale

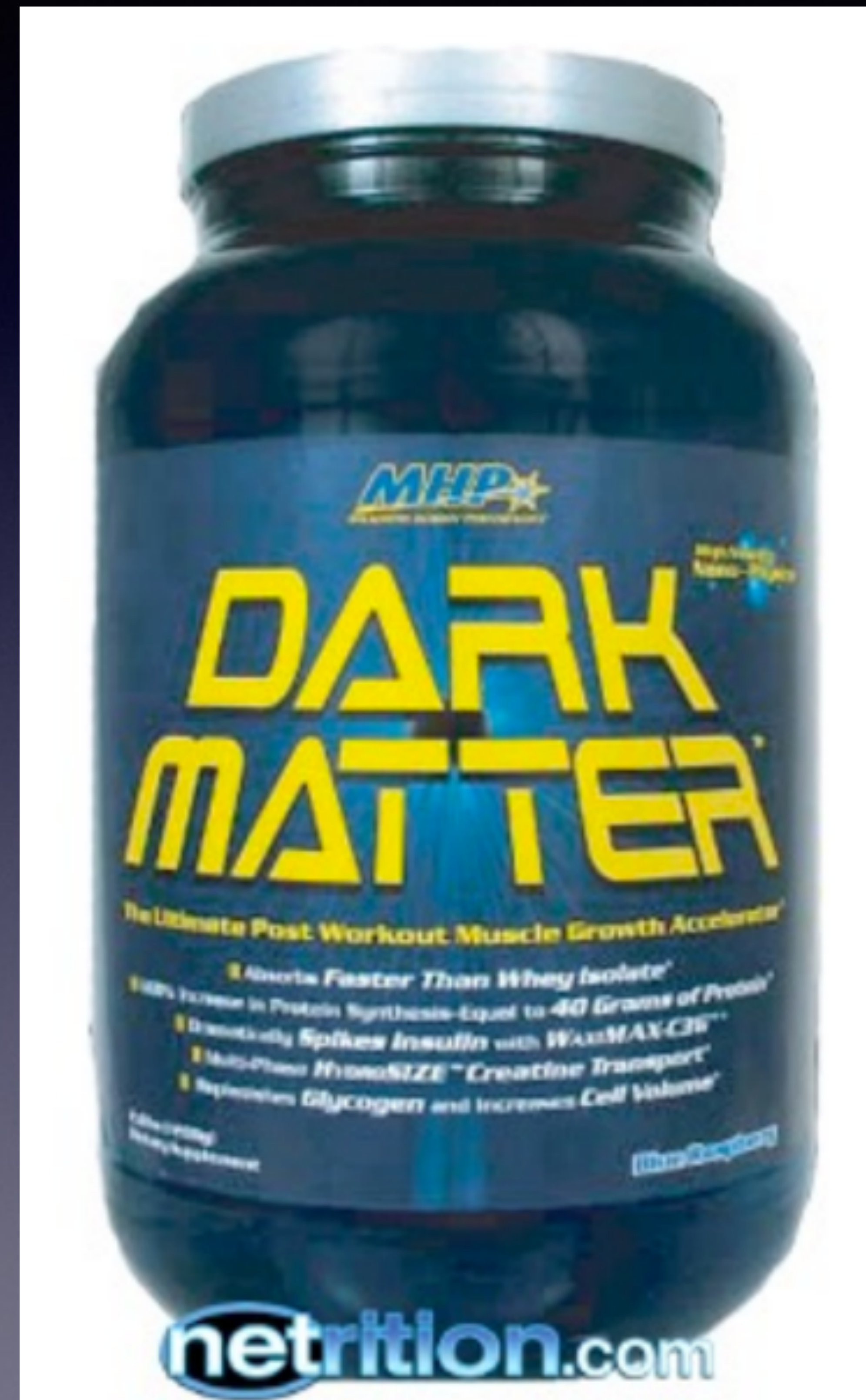
- Let us estimate $\langle\sigma v\rangle \sim \alpha^2/m^2$, $\alpha \sim 10^{-2}$
- Then from first estimate for cross section, natural mass scale is m~1000 GeV.
- Plug this back into formula for x_f; we find x_f~25.

  $$x_f \sim \ln\left(g/(2\pi)^{3/2} m m_{\rm Pl} \langle\sigma v\rangle\right)$$
- This gives us a better cross section estimate:

  $$\langle\sigma v\rangle \sim 2\times10^{-9}{\rm GeV}^{-2} \approx 2\times10^{-26}{\rm cm}^3/{\rm s}$$
- Corresponds to mass scale of a few hundred GeV, details depending on coupling and prefactors.

The WIMP miracle

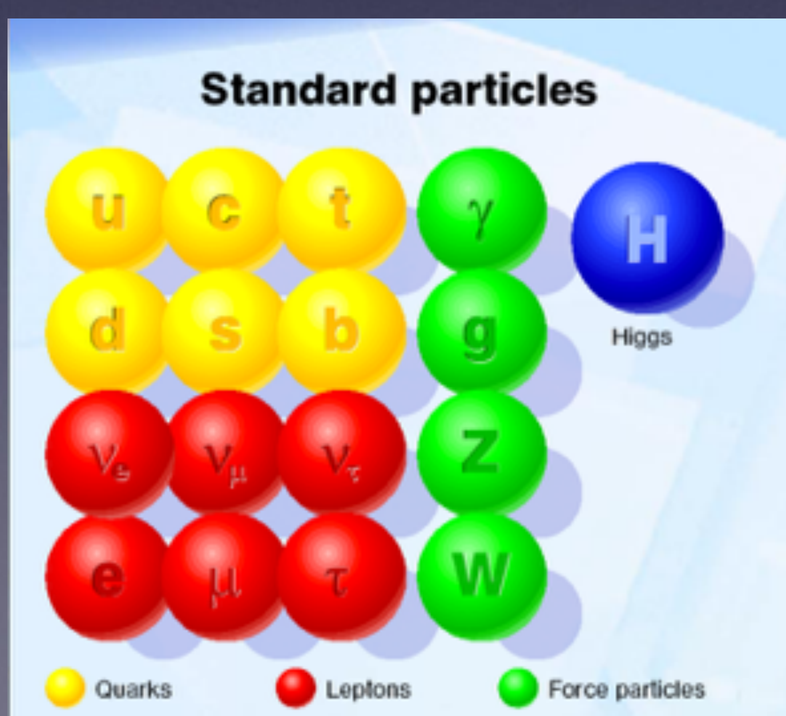
- In a thermal scenario, weak-scale annihilation cross section naturally yields the observed abundance of dark matter.
- Suggestive of new physics not too far above the weak scale.
- Stable WIMPs automatically occur in many scenarios for physics beyond the Standard Model, in particular in supersymmetry.
- However, simplest scenarios are challenged by lack of detection on other fronts; often need some extra ingredient to get the correct abundance.

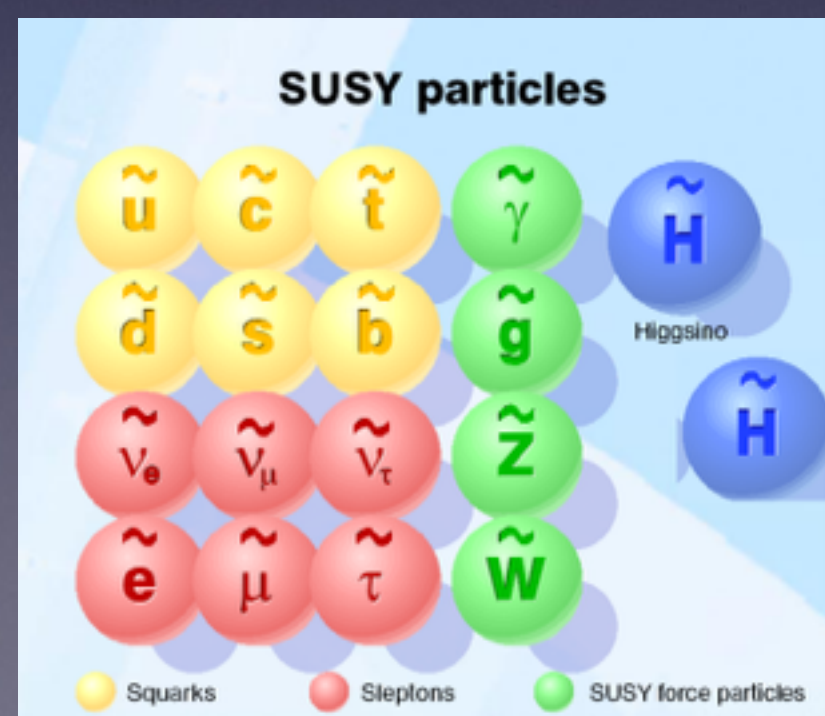


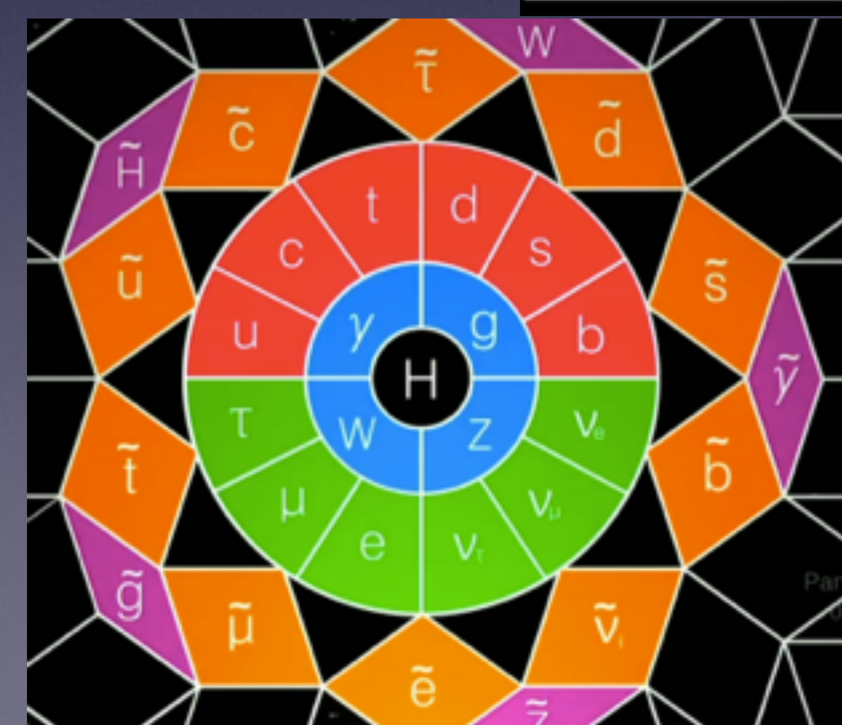
Supersymmetry (SUSY)

- Most famous dark matter candidate is the Lightest Supersymmetric Particle (LSP).
- In supersymmetric theories, every particle has a superpartner.
- Fermions have boson superpartners and vice versa.
- These additional particles cancel what would otherwise be very large predicted contributions to the Higgs mass - motivated independently of DM.
- For an in-depth introduction to SUSY, see e.g. Martin hep-ph/9709356.

Particle Fever
D.E. Kaplan

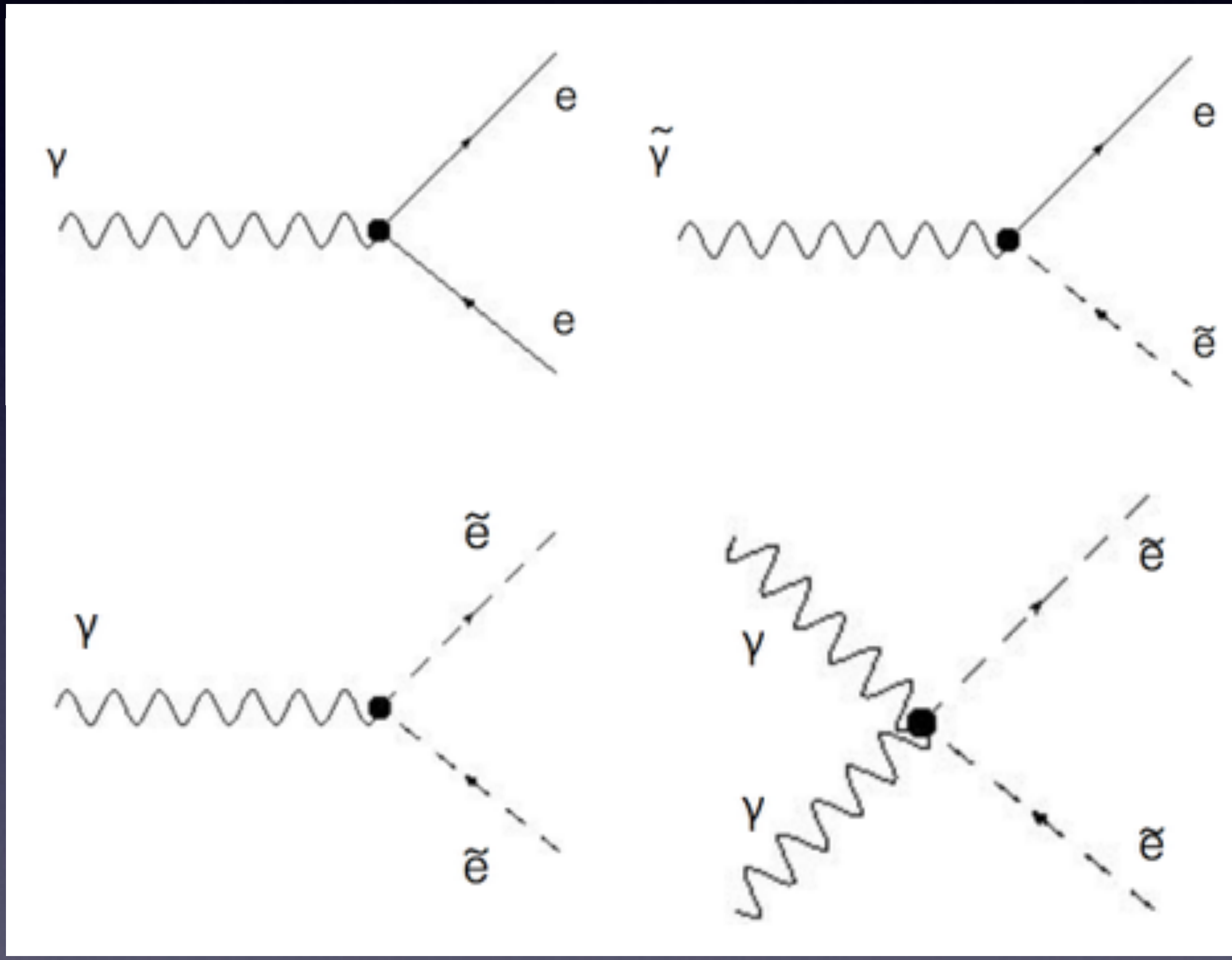






Supersymmetry (II)

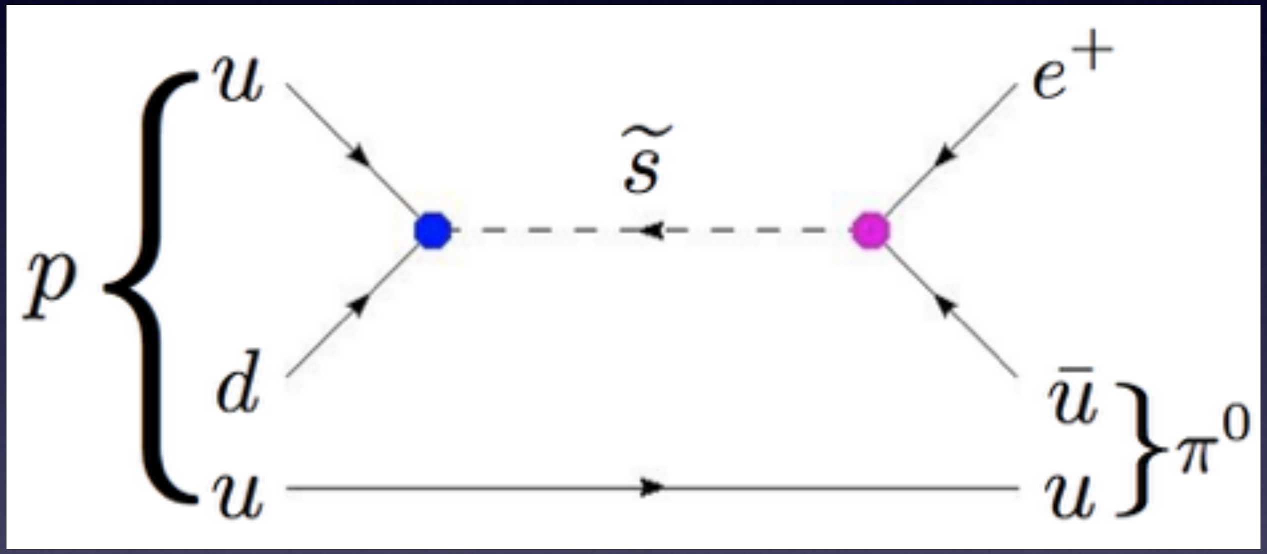
- In unbroken supersymmetry, particles and superpartners have same mass and closely related interactions.
- This symmetry must be broken as clearly superpartners do not have mass equal to their (known) counterparts!
- But if we break it "softly", while masses are separated, interactions remain fixed by supersymmetry.
- SUSY theories also inherit huge structure from the Standard Model.
- Consequently many quantities in SUSY theories can be calculated from just the masses of the superpartners.



Example of interactions related by supersymmetry - taken from talk by Tim Tait, August '15

R-parity

- These SUSY interactions naively imply some peculiar behavior!
- For example, they could make protons decay quickly:



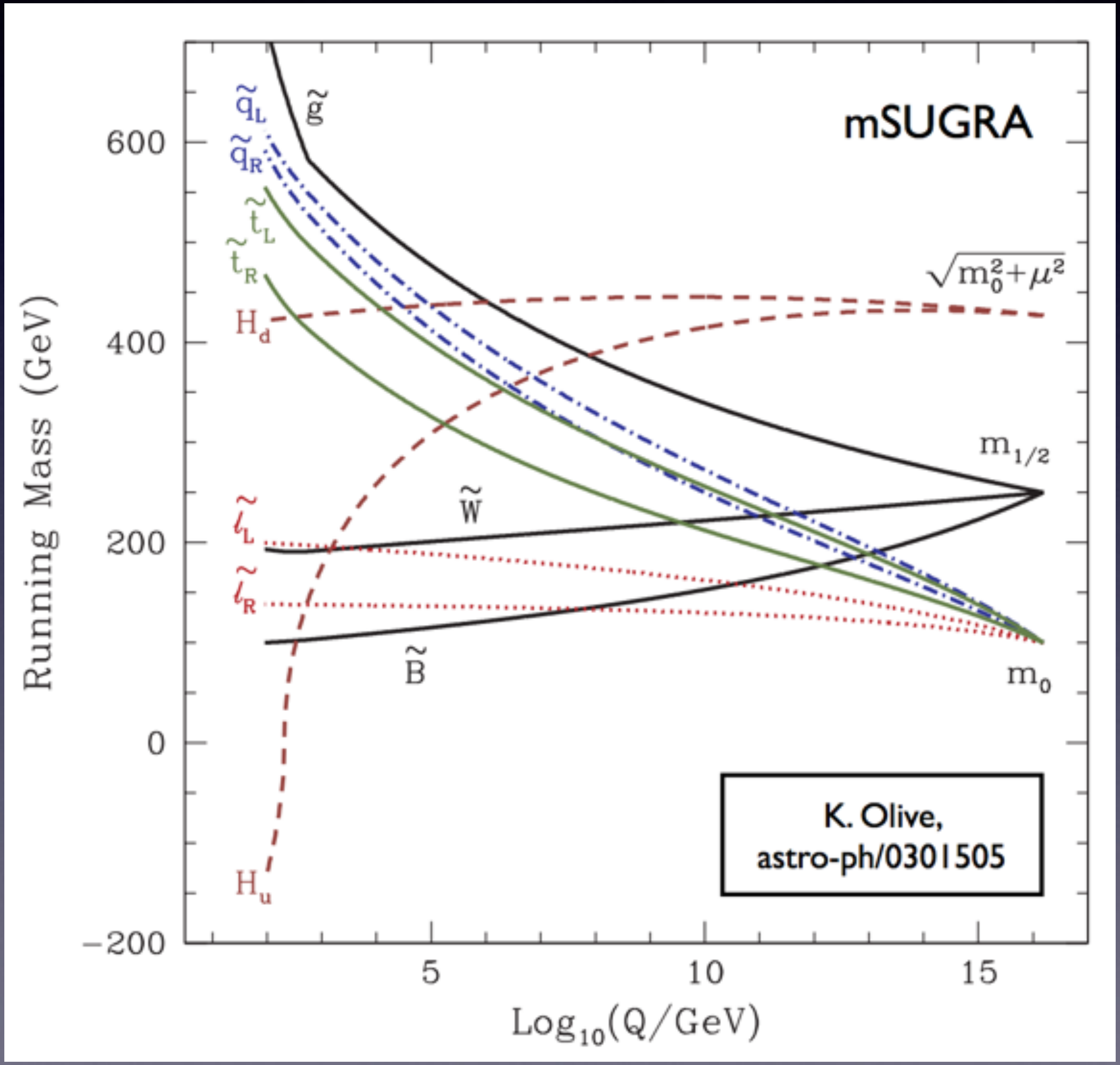
- This is pretty clearly not observed (experimental limit: lifetime $>10^{33}$ years).
- Usual approach is to impose a symmetry called R-parity, so the superpartners can only couple in pairs to the ordinary particles.
  - Superpartners have R-parity = -1
  - Ordinary particles have R-parity = +1
  - Product of R-parities before and after an interaction must be conserved

The LSP

- But then lightest particle with R-parity odd (i.e. = -1) cannot decay
    - can't produce any other particles with R-parity -1 (kinematically forbidden)
    - can't decay just to SM particles (violates R-parity)
- Avoiding proton decay gives us a stable DM candidate!
- Furthermore, any R-parity-odd particles in the early universe must eventually produce stable R-parity-odd particles by decays.
- But does the LSP satisfy other requirements for DM?

The LSP as dark matter

- First question: is it neutral?
- SUSY models in general have many parameters and the model of supersymmetry breaking matters.
- For a given model, we can compute the spectrum of superpartners, identify the lightest particle, and check its properties.



Neutralino dark matter

- Neutral fermionic superpartners = superpartners of the neutral gauge bosons
  - Higgsino = superpartner of the Higgs(es)
  - Wino = corresponds to electrically neutral gauge boson of electroweak SU(2) gauge group (there is also a "chargino" which corresponds to the charged components)
  - Bino = corresponds to gauge boson of the electroweak U(1) gauge group
- In general the physical states (of definite mass) correspond to mixtures of these - details depend on the model
- The lowest-mass such admixture determines the interactions of the DM candidate

The problem with binos

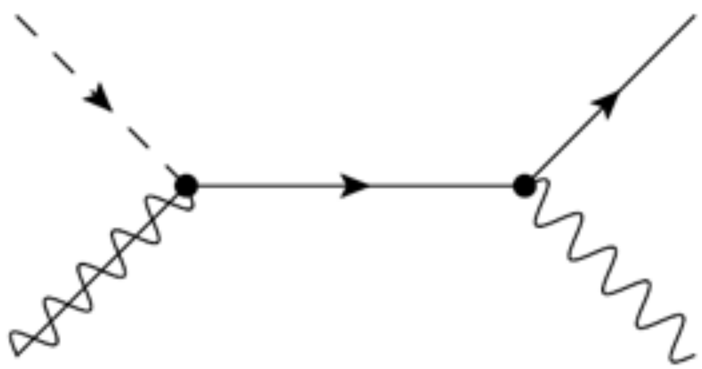
- A common (not universal) situation is for the LSP to behave mostly like a bino (with small wino/higgsino components)
  - Interactions with SM then mostly involve the scalar partners of the fermions ("sfermions")
  - Main annihilation channels for binos produce SM fermions, via interactions with sfermions.
  - But these annihilations are <u>suppressed</u> by m_f^2/m_{DM}^2 - smaller than "typical" weak-scale cross section.
- Low annihilation cross sections mean there is typically <u>too much dark matter</u> in the late universe - WIMP miracle doesn't hold up, despite weak-scale masses, due to parametric suppression.

Relic density from SUSY

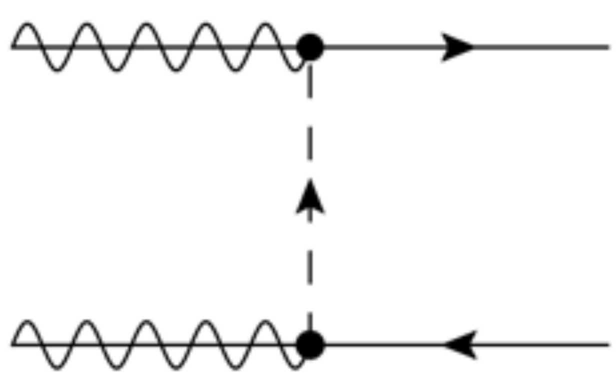
- Four standard ways to fix this problem in mSUGRA, simplified example SUSY model:
  - “bulk region” - make sfermions light (larger annihilation cross sections)
  - “focus point” - reduce bino fraction so other, unsuppressed annihilation modes dominate
  - “funnel region” - annihilation through Higgs, near twice mass of DM, gives alternative, unsuppressed decay mode
  - “coannihilation region” - DM is not the only particle involved in freeze-out, need to include interactions with other near-degenerate particles

Credit to Tim Tait for these slides

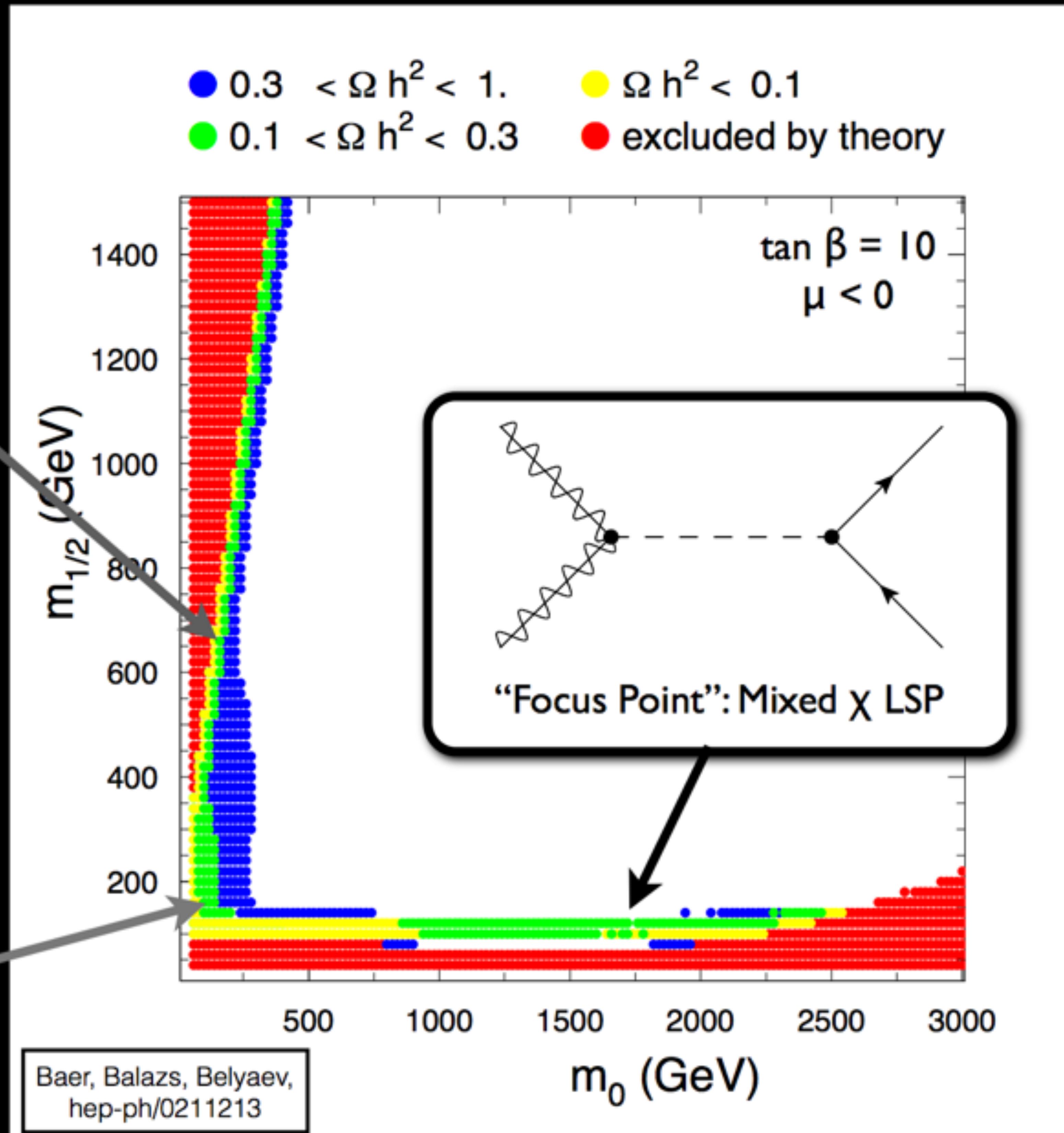
mSUGRA



"Coannihilation Region": Degenerate stau active during freeze-out



"Bulk Region": Light sfermions (~excluded by LHC)



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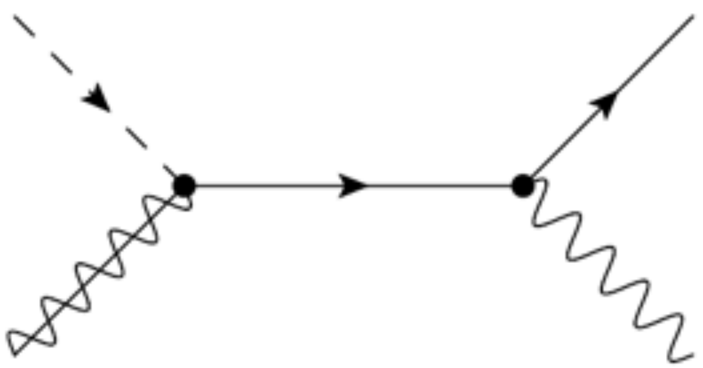
mSUGRA



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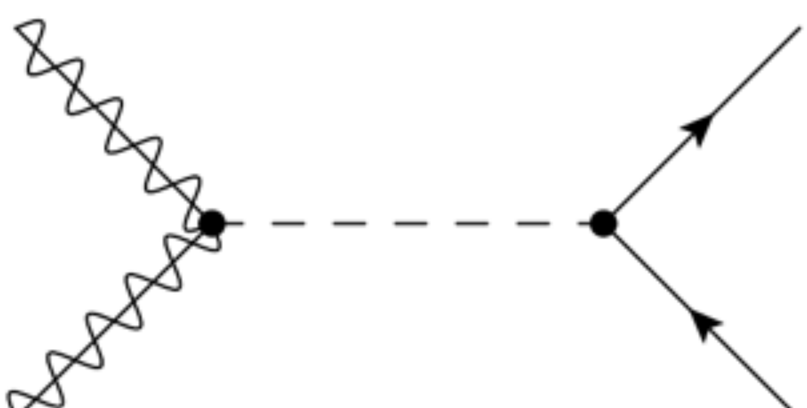
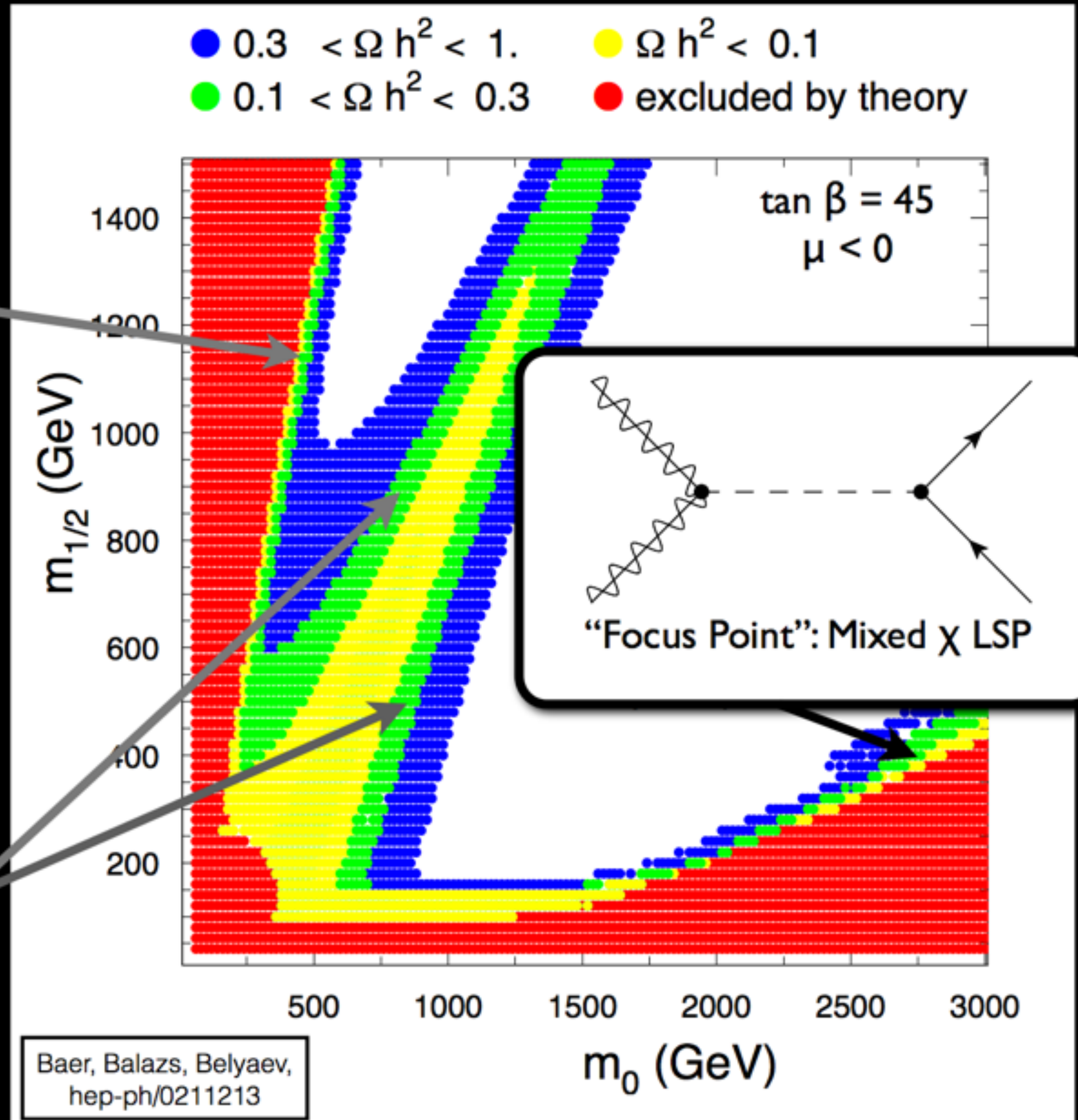
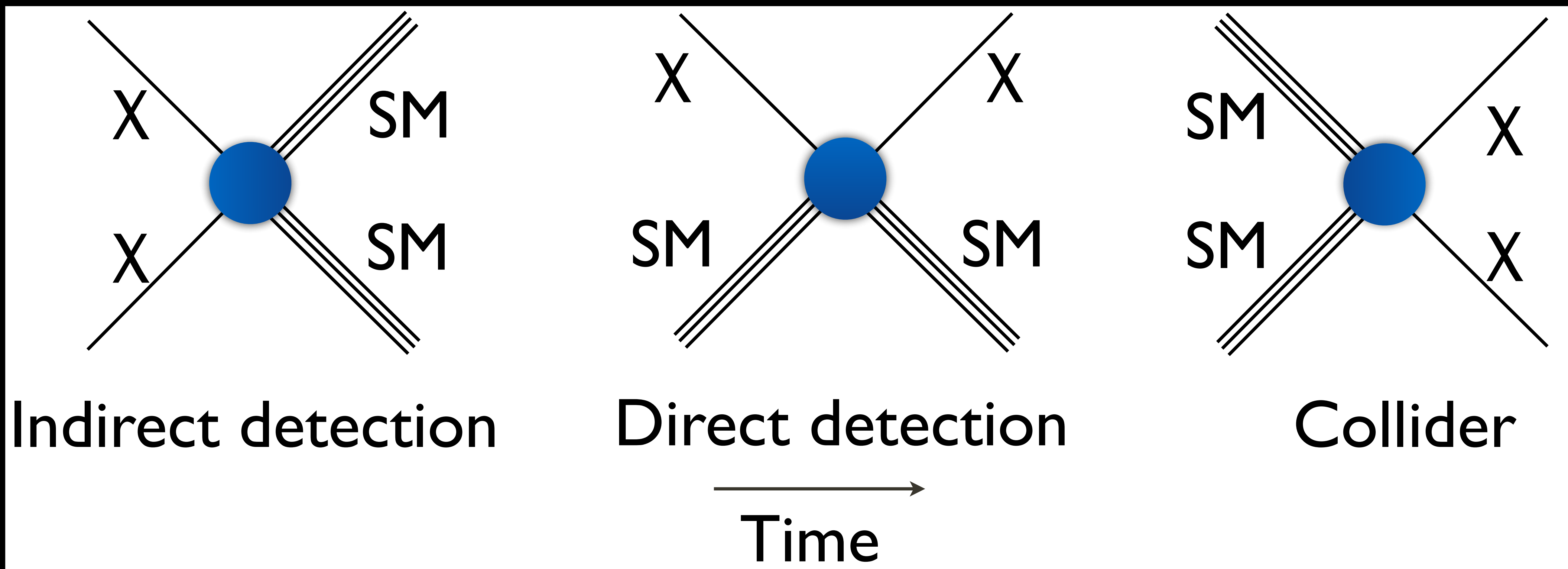


"Funnel Region": Higgs close to on-shell in decay



Hunting the WIMP



- Beyond motivations from SUSY and thermal freezeout, WIMPs are popular candidates because they have many observable signatures.
- Indirect detection: look for SM particles - electrons/positrons, photons, neutrinos, protons/antiprotons - produced in WIMP collisions or decay, with sensitive telescopes.
- Direct detection: look for nuclear recoils from WIMPs hitting SM particles with sensitive underground detectors.
- Colliders: produce DM particles in high-energy collisions and look for missing energy (e.g. at the LHC), or search for new light dark-sector particles.

Axion dark matter

The strong CP problem

- The Standard Model Lagrangian, describing all known particle interactions, in principle should have a term of the form:

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$ gluon field strength

- A term like this can be generated by CP violation elsewhere in the Standard Model, in the terms describing the quarks - no reason for it to vanish.

- But this term induces a neutron electric dipole moment:

$$d_n = 5.2 \times 10^{-16} \mathrm{e\,cm}$$

- Experimentally, we know that:

$$d_n < 3 \times 10^{-26} \mathrm{e\,cm} \qquad \Rightarrow \theta \lesssim 10^{-10}$$

- Why is this value so small?

The axion proposal

- Replace the parameter θ by a dynamical field, call it (by convention) a/f_a where a is the field and $1/f_a$ a coupling.
- Now we just need to explain why a would evolve toward a very small value.
- But the energy stored in this field depends on the value of a - potential energy changes as a evolves.
- We can work out this effective potential (I won't give the calculation here - see e.g. Dine's TASI lectures hep-ph/0011376 for much more detail on the strong CP problem) and find:

$$V(a) = -m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \cos(a/f_a)$$

$$f_\pi \approx 93\mathrm{MeV}$$

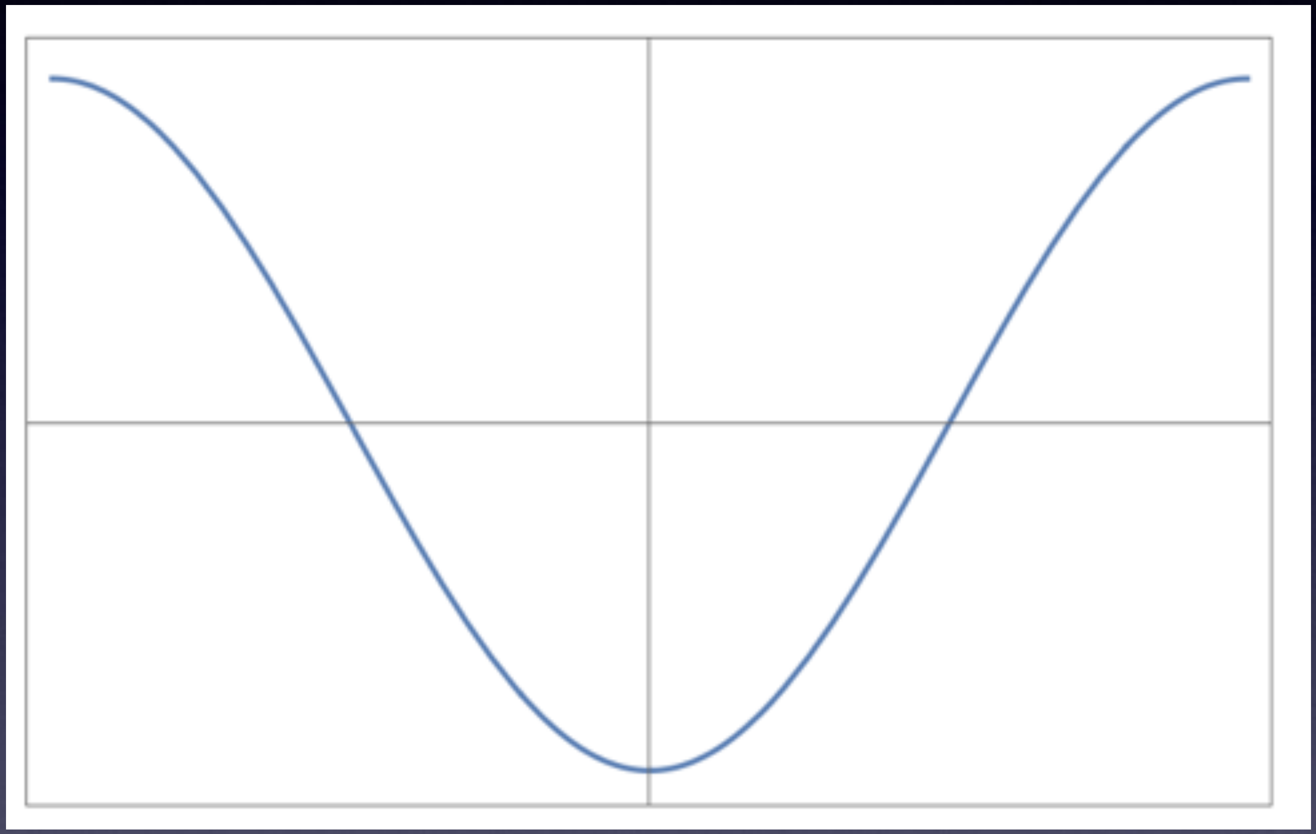
pion decay constant

$$m_\pi \approx 135\mathrm{MeV}$$

pion mass

The axion potential

- Field should evolve toward small values of this potential.
- Minima occur at $a/f_a = 2n\pi$; let's look at n=0.
- The potential is parabolic - coefficient of a^2 term gives axion mass.



$$V(a) = m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} + \frac{1}{2} a^2 \left(\frac{f_\pi}{f_a}\right)^2 m_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} + \mathcal{O}(a^4)$$

$$m_a = \frac{f_\pi m_\pi}{f_a} \left(\frac{m_u m_d}{(m_u + m_d)^2}\right)^{1/4} \approx 0.6\mathrm{meV} \left(\frac{10^{10}\mathrm{GeV}}{f_a}\right)$$

Axion properties

- Axion coupling to Standard Model fields is controlled by the coupling f_a, although exact couplings depend on details of model.
  - "DFSZ axion" - axion couples to photons, gluons, leptons, quarks
  - "KSVZ axion / hadronic axion" - axion couples to photons and gluons, but at lowest order no coupling to leptons or light quarks
- Axion mass is inversely proportional to their coupling to Standard Model fields - weakly coupled axions can be <u>very</u> light.
- One might think this makes them poor DM candidates - too hot?

Thermal axions

- Coupling for axions can be very weak
  - In contrast to WIMPs, question is not "when did they fall out of equilibrium" but "were they ever in equilibrium"?
- Axions produced in early universe by interactions of photons, pions
- Axions can also decay - and are produced singly, not in pairs (no symmetry keeping them stable)
  - Need to check lifetime is >> age of universe
  - Solve Boltzmann equation including decay + all production processes

Thermal axions as hot dark matter

- Timescale for decay to photons is approximately given by:

$$\tau \sim 10^{24} s \left(\frac{m_A}{\mathrm{eV}}\right)^{-5}$$

- Age of universe ~ 10^{10} yr ~ π x 10^{17} s => for axions to be around today, must be lighter than ~20 eV (unless decay suppressed in specific model)
  - Side note: at axion masses between about 20 eV and 300 keV, the photons from this process would disrupt nucleosynthesis!
- Solving Boltzmann equation, axions could attain thermal equilibrium if $m_a > 10^{-3}$-10^{-2} eV
- In this case, very roughly, fraction of critical density in axions:

$$\Omega_{\mathrm{axions}} \sim \mathcal{O}\left(\frac{m_a}{100\mathrm{eV}}\right)$$

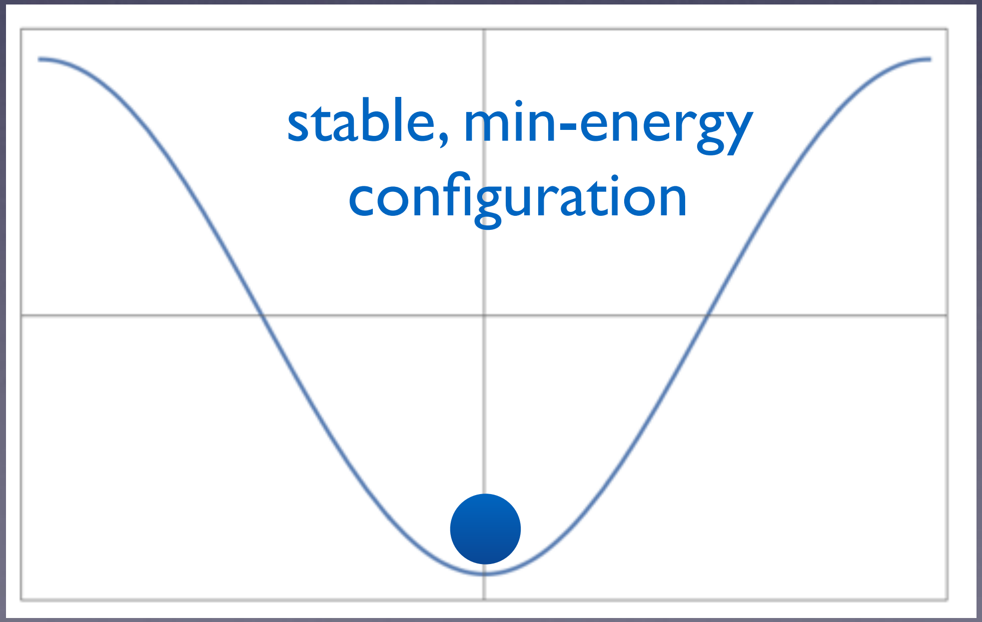
hot dark matter - needs to be small fraction of total DM
$m_a < 1$ eV is OK

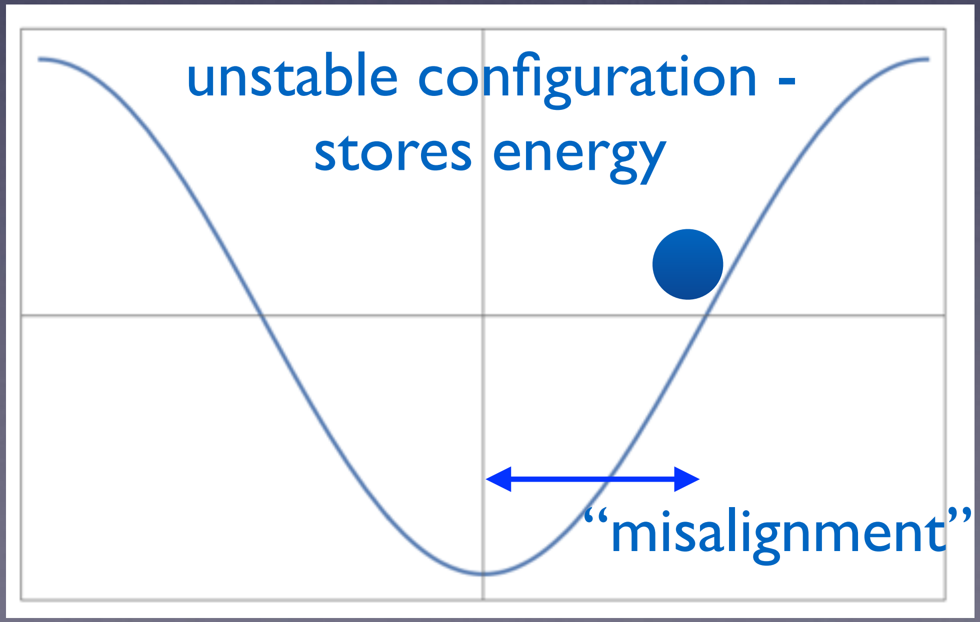
Non-thermal axions as cold dark matter

- But what if axions never equilibrate with SM?
- Sufficiently cold, light axions behave like a classical scalar field, evolving in axion potential - not individual particles

Q: How does the field evolve?

A: If initially displaced from minimum of potential (by some "misalignment angle"), must "roll" toward that minimum





An evolving scalar field

$$\frac{d^2a}{dt^2} + 3H\frac{da}{dt} + m_a^2 a = 0$$

equations of motion for scalar field in FRW
note here *a* = axion field, not scale factor

↑ (m_a^2) describes shape of potential near minimum

- For $m_a << H$, approximate solution with d*a*/dt = 0 - field does not evolve
- For $m_a > H$, field begins to oscillate in potential - like simple harmonic oscillator with H-dependent friction term ("Hubble friction"). For large t solution has approximate form:

$$a(t) = \Theta_0 f(t) \cos(m_a t)$$

f(t) slowly varying compared to oscillations

↑ (Θ_0) misalignment angle

- Solving for f(t) we find that in both radiation and matter-dominated epochs, f(t) scales like $1/(\text{scale factor})^{3/2}$.
- Energy density stored in axion field falls off like $f(t)^2 \sim 1/(\text{scale factor})^3$. Same behavior as matter - can act as cold dark matter.

Axion relic density

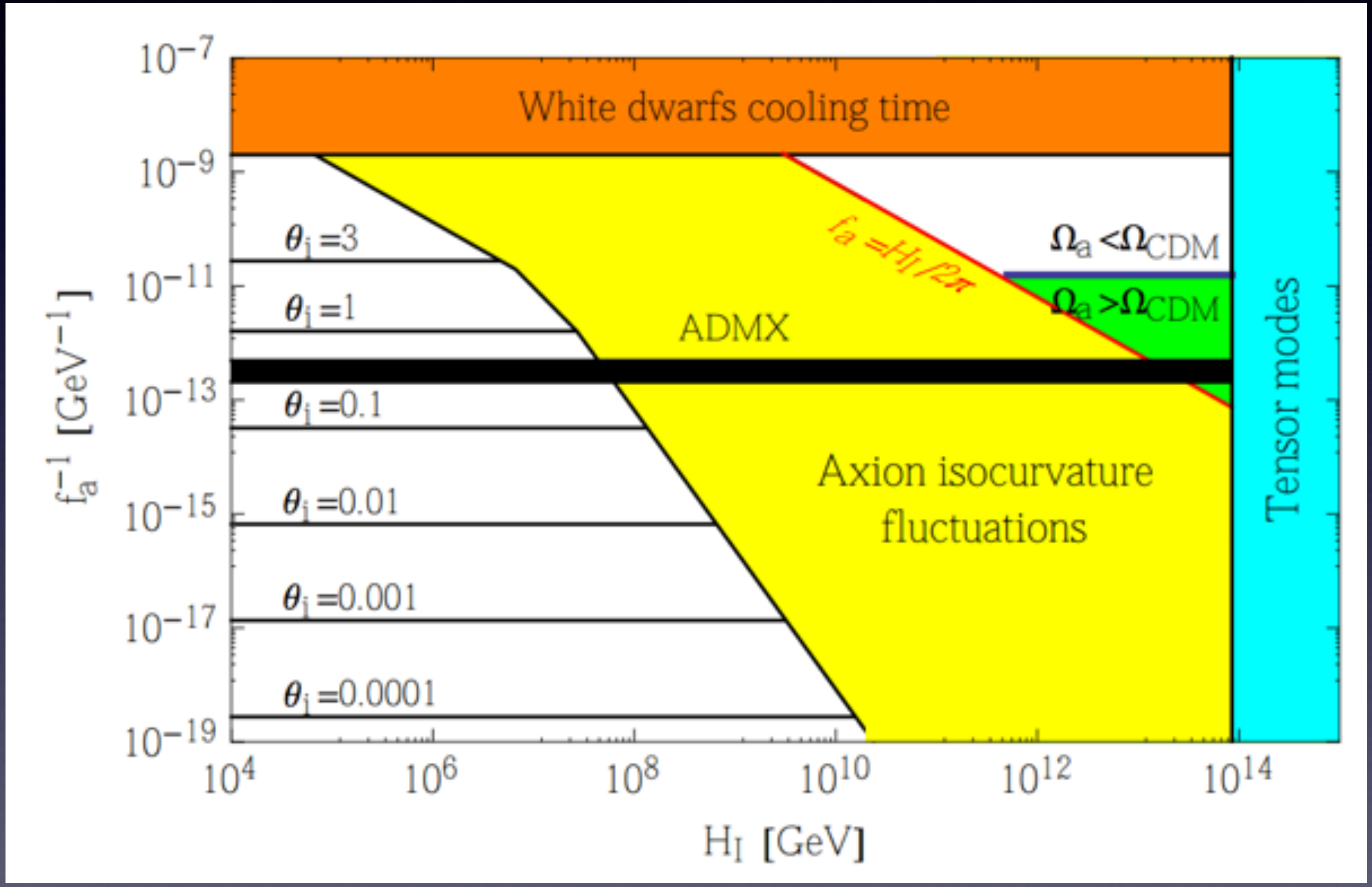
- Careful relic density calculation requires solving equation of motion including temperature dependence of axion mass, QCD phase transition, etc.
- Fraction of critical density:

$$\Omega_{\rm axions} \approx \Omega_{\rm DM} \Theta_0^2 \left(\frac{f_a}{5 \times 10^{11} {\rm GeV}} \right)^{1.184}$$

- Lighter axions = higher f_a = more weakly coupled = larger relic density
- Relic density can always be suppressed by small initial misalignment angle
- But misalignment angle cannot be much larger than 1 - axions must have f_a of order 10^{11} GeV or higher (m_a~0.1 meV or smaller) to be all the DM.

Axions and inflation

Baer '15 (1510.07501)



H_I = Hubble scale of inflation

- What value should we expect the misalignment angle to take?
- If axions are produced / misalignment angle is set only <u>after</u> inflation, i.e. $H_I >> f_a$, different patches of cosmos likely have different misalignment angles - take average of random sample
- If misalignment angle is set (in patches) <u>before</u> inflation, each such patch gets blown up at inflation - everywhere in our Hubble volume should have same angle
  - "anthropic axion"?

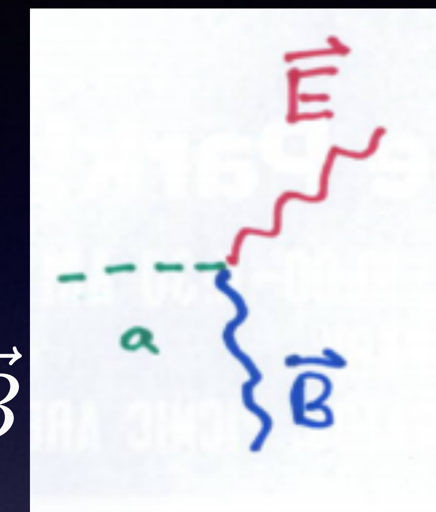
There are stringent constraints on scenarios where axion is all the DM and the energy scale of inflation is high - see Hertzberg, Tegmark & Wilczek '08
Learning about inflation may tell us about axions! (or vice versa)

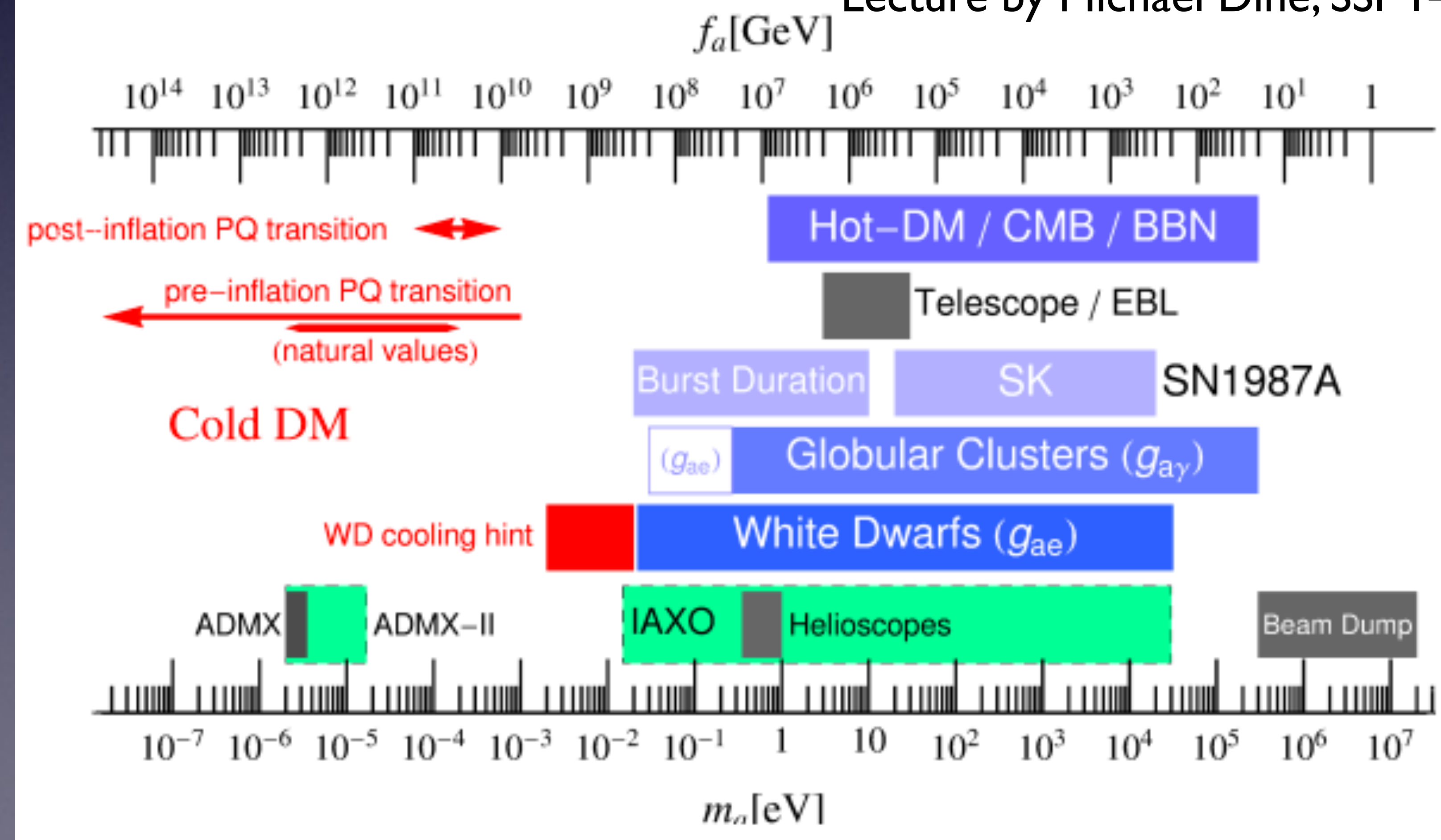
Searching for axions

- Main observable property of axions (except possibly for gravitational effects) is their coupling to the photon
- Axions can convert <u>into</u> photons in the presence of a magnetic field
- Axions interact much more weakly than photons - can carry energy through regions where photons would be absorbed
  - Can induce strong B-field, look for signs of axion production
  - Or study astrophysical systems where photon absorption is high

axion-photon interaction in presence of magnetic field

$$G_{a\gamma\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu} = G_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$





Lecture by Michael Dine, SSI ‘14

Summary

- We have discussed the basic properties and cosmology of two major categories of DM models
  - Weakly Interacting Massive Particles (WIMPs)
  - axions
- These furnish examples of non-thermal vs thermal production of the observed relic density
- Very different mass scales (<meV vs GeV-TeV)
- Very different couplings to known particles + detection methods (to be discussed in more depth in later lectures)
- These are not all-encompassing examples! There are (many) models which don’t fit into either category - but these two broad scenarios are most popular, and give a sense of the scope of possibilities.