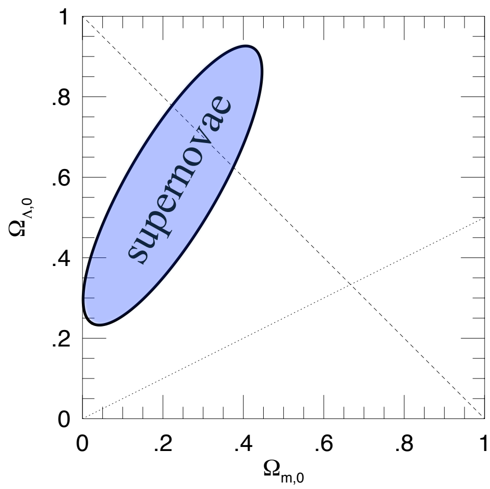
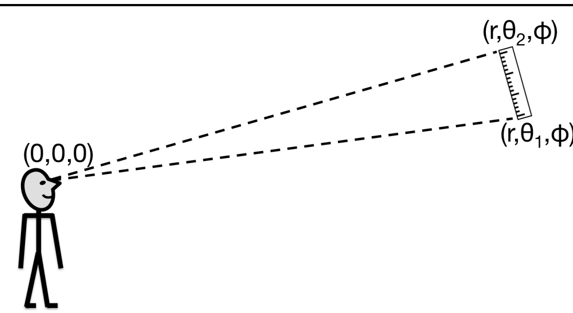


Introduction to Cosmology Part 3

Professor Barbara Ryden
Department of Astronomy
The Ohio State University

ICTP Summer School on Cosmology
2016 June 7

Learning about $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ from observing standard yardsticks (objects of known physical size ℓ).



We can measure the standard yardstick's redshift z.
We can (ideally) measure its angular diameter $\delta\theta$.

We can compute a function called the angular-diameter distance

$$d_A \equiv \frac{\ell}{\delta\theta}$$

$d_A = d_p$ if space is static and Euclidean.

The distance ds between the ends of the yardstick at time t_e:

$$ds = a(t_e) S_\kappa(r) \delta\theta$$

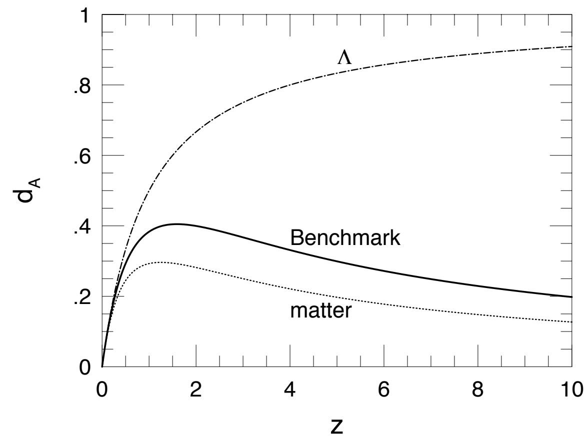
and thus

$$d_A \equiv \frac{\ell}{\delta\theta} = a(t_e) S_\kappa(r) = \frac{S_\kappa(r)}{1+z}$$

If space is nearly flat, then

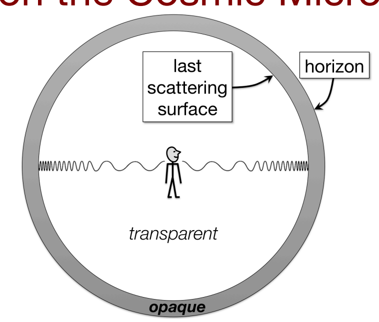
$$d_A \approx \frac{d_p(t_0)}{1+z} < d_p(t_0)$$

& angular-diameter distance is an *underestimate* of proper distance.



A preferred standard yardstick of cosmologists:

Hot and cold spots on the Cosmic Microwave Background



Typical CMB photon last scattered from a free electron when the temperature was $T_{ls} \approx 2970\text{K}$.

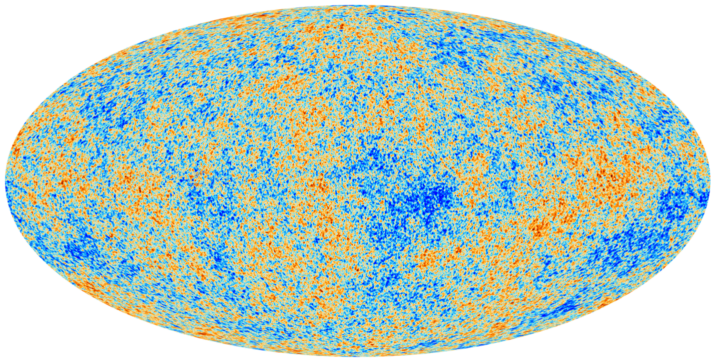
Temperature of CMB today: $T_0 = 2.7255\text{K}$

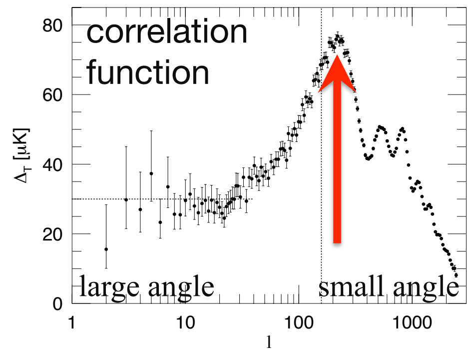
Redshift of the last scattering surface:

$$1+z_{ls} = T_{ls}/T_0 \approx 1090$$

Planck 2015: $z_{ls} = 1089.90 \pm 0.23$

Hot & cold spots on the CMB have a preferred angular scale, revealed by the temperature correlation function.

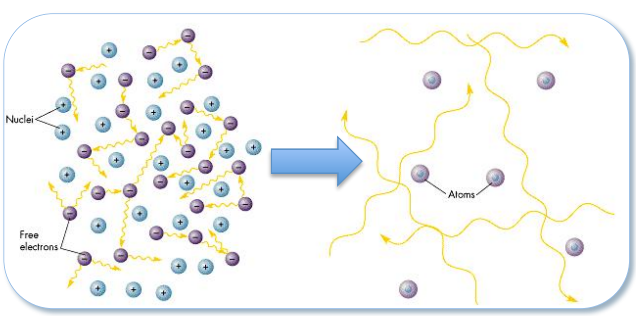




preferred angular scale = "first peak":

$\delta\theta = 0.8° = 0.014$ rad

First peak results from standing acoustic waves in the photon-baryon fluid that existed before recombination.



Physical size $\ell \approx$ sound horizon distance at the time of last scattering.

$$\ell \approx d_s(t_{ls}) = a(t_{ls})\int_0^{t_{ls}} \frac{c_s(t)dt}{a(t)}$$

$$c_s \approx c/\sqrt{3} \quad \Rightarrow \quad \ell \approx 0.145\,\mathrm{Mpc}$$

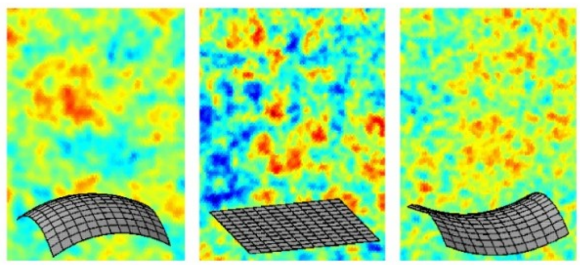
with some dependence on $\Omega_{m,0}$

Angular-diameter distance to the last scattering surface:

$$d_A \equiv \frac{\ell}{\delta\theta} \approx \frac{0.145\,\mathrm{Mpc}}{0.014} \approx 10\,\mathrm{Mpc}$$

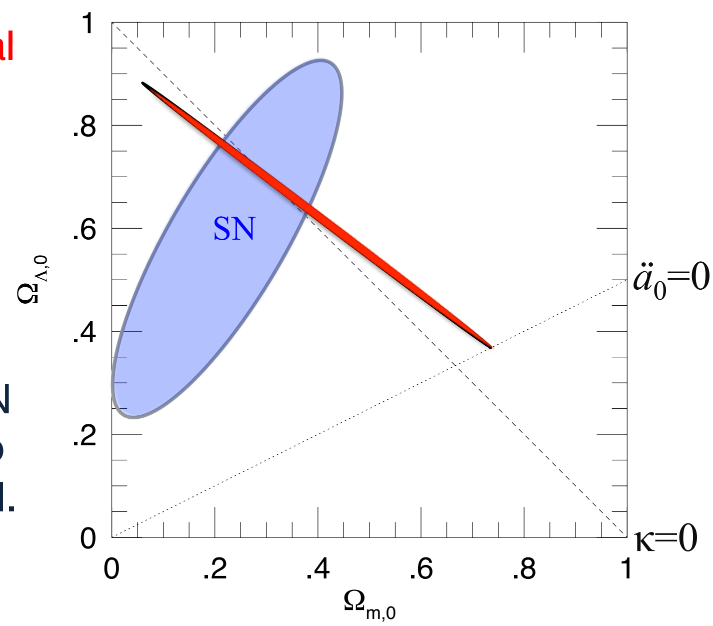
What combinations of $\Omega_{\Lambda,0}$ and $\Omega_{m,0}$ yield this angular-diameter distance for an object with $z=1090$?

$\Omega_{\Lambda,0}+\Omega_{m,0} > 1$
$\kappa = +1$
d_A too small



$\Omega_{\Lambda,0}+\Omega_{m,0} < 1$
$\kappa = -1$
d_A too large

95% confidence interval for the CMB results:



The **combination** of SN and CMB data leads to a flat Benchmark Model.

Benchmark Model: Ingredients

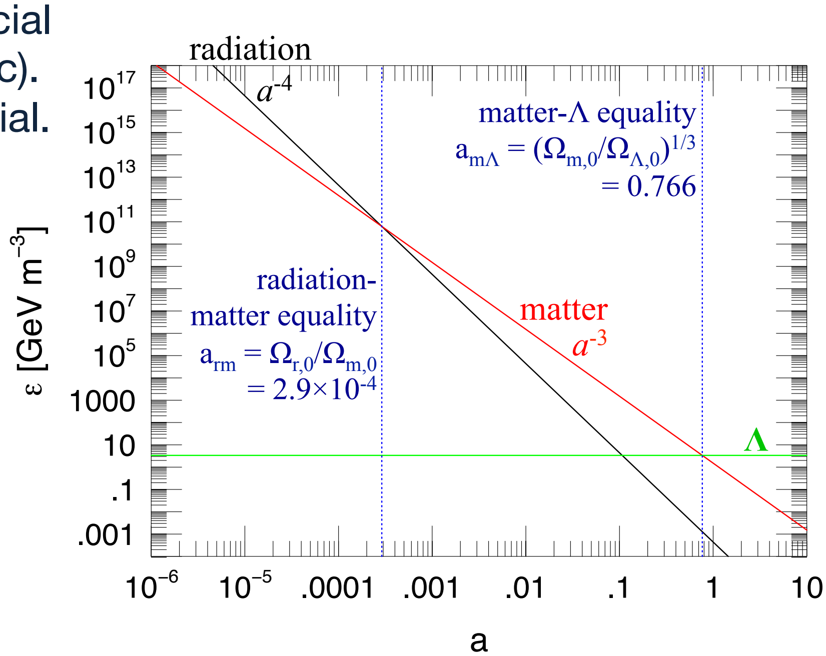
photons: $\Omega_{\gamma,0} = 5.35\times10^{-5}$
neutrinos*: $\Omega_{\nu,0} = 3.65\times10^{-5}$
total radiation: $\Omega_{r,0} = 9.0\times10^{-5}$

baryonic matter: $\Omega_{bary,0} = 0.048$
(cold) dark matter: $\Omega_{dm,0} = 0.262$
total matter: $\Omega_{m,0} = 0.31$

cosmological constant: $\Omega_{\Lambda,0} = 1-\Omega_{m,0}-\Omega_{r,0} \approx 0.69$

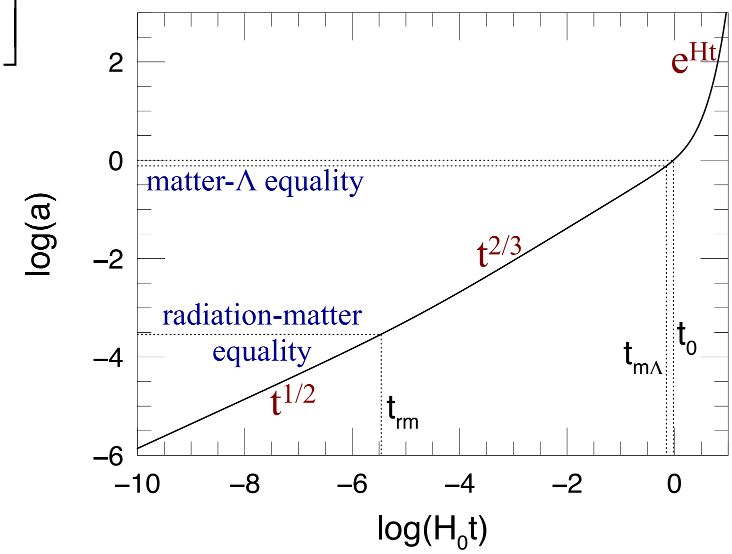
*assumes massless neutrinos

No locations are special (on scales > 100 Mpc). Some *times* are special.



Benchmark Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0}\right]$$



Benchmark age:

$$t_0 = 0.96\, H_0^{-1}$$

Benchmark Model: Special Epochs

radiation-matter equality: $z_{\rm rm} = 3440$, $t_{\rm rm} = 50{,}000$ yr

matter-lambda equality: $z_{\rm m\Lambda} = 0.31$, $t_{\rm m\Lambda} = 10.4$ Gyr

now: $z_0 = 0$, $t_0 = 13.7$ Gyr

However, these aren't *very* special (no dramatic changes).
Today, I'll discuss the extremely special epochs:

Last Scattering: t ~ 370,000 yr

Big Bang Nucleosynthesis: t ~ 3 min

1) Last Scattering

Simplifying assumption: baryonic content at last scattering was pure hydrogen.

Epoch of **recombination**: when the fractional ionization of hydrogen fell to $X = 1/2$.

$$X \equiv \frac{n_p}{n_p + n_{\mathrm{H}}} = \frac{n_p}{n_{\mathrm{bary}}} = \frac{n_e}{n_{\mathrm{bary}}}$$

What is X(t)? At what value of X does last scattering occur?

Epoch of **last scattering**: when a typical CMB photon last scattered from a free electron. (This happened at $X>0$, not necessarily at $X=1/2$.)

Fractional ionization of hydrogen is determined by the balance between *photoionization* & *radiative recombination*:

$$\mathrm{H} + \gamma \rightleftharpoons p + e^-$$

There are 1.6 billion photons per baryon. Forget collisional ionization!

The ionization energy of hydrogen is $Q = 13.6$ eV ($T = Q/\mathrm{k} = 158{,}000$ K).

However, I stated that $T_{\mathrm{ls}} = 2970\ \mathrm{K} = 0.019\ Q/\mathrm{k}$.

Why so low?

Before last scattering, H, e^-, p, and γ were in kinetic equilibrium.

The photoionization / radiative recombination equation was in a state of chemical equilibrium:

$$\mathrm{H} + \gamma \rightleftharpoons p + e^-$$

Result: the ionization state was given by the Saha equation:

$$\frac{n_{\mathrm{H}}}{n_p n_e} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kT}\right)$$

In terms of the fractional ionization X,

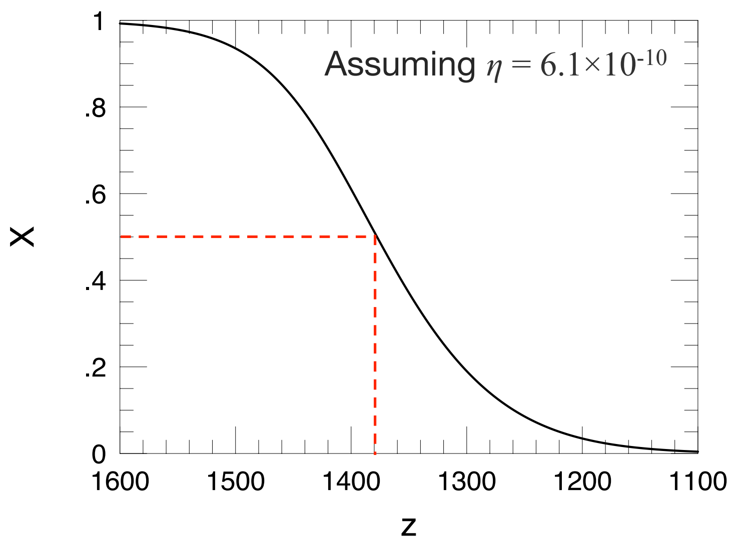
$$\frac{1-X}{X} = n_p \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kT}\right)$$

To get rid of the factor of n_p, recall that the baryon-to-photon ratio $\eta \approx 6\times10^{-10}$ is constant.

$$n_p = X n_{\mathrm{bary}} = X\eta n_\gamma = X\eta\left[2.44\left(\frac{kT}{\hbar c}\right)^3\right]$$

$$\frac{1-X}{X^2} = 3.84\eta\left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left(\frac{Q}{kT}\right)$$

Solve for X(η,T)



X=1/2 when
T_{rec} = 3760K,
corresponding to
z_{rec} = 1380,
t_{rec} = 0.25 Myr.

(This result is weakly dependent on η.)

When does the last scattering of a photon occur?

By comparing the rate of photon scattering,

$$\Gamma(z) = n_e(z)\sigma_e c = X(z)(1+z)^3 n_{\mathrm{bary},0}\sigma_e c$$

with the rate of expansion,

$$H(z) = H_0\left[\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}\right]^{1/2}$$
$$\approx H_0\Omega_{m,0}^{1/2}(1+z)^{3/2}$$

we find the last scattering occurs at
z_{ls}= 1090, when X $\approx$ 0.007.

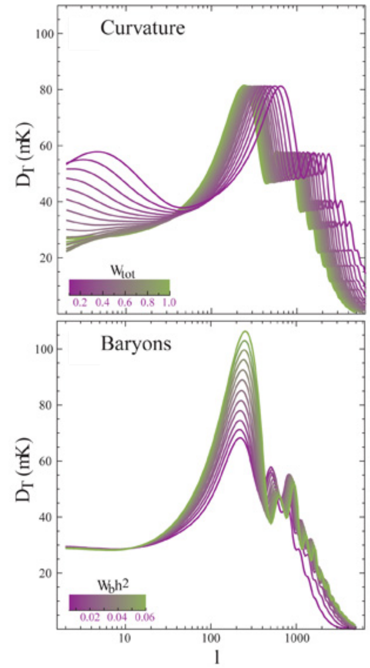
How do we know η so well?

The height of the 'first peak' in the CMB temperature fluctuations is sensitive to the baryon-to-photon ratio.

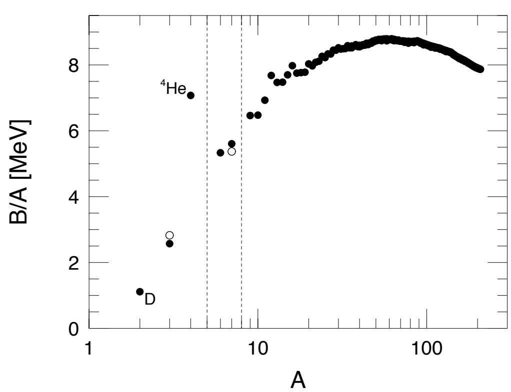
$$\eta = (6.10 \pm 0.06) \times 10^{-10}$$

$$n_{\mathrm{bary},0} = 0.251 \pm 0.003\,\mathrm{m}^{-3}$$

$$\Omega_{\mathrm{bary},0} = 0.048 \pm 0.003$$



2) Big Bang Nucleosynthesis



Early universe ($t < t_{rm} \approx 50{,}000$ yr) was radiation-dominated.

$$a(t) \propto t^{1/2}$$

$$T \sim 10^{10}\,\mathrm{K}\left(\frac{t}{1\,\mathrm{sec}}\right)^{-1/2}$$

$$kT \sim 1\,\mathrm{MeV}\left(\frac{t}{1\,\mathrm{sec}}\right)^{-1/2}$$

At $t < 1$ sec, photons were energetic enough to photodissociate atomic nuclei.

Free neutrons are unstable.

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\tau_n = 880\,\text{sec}$$

$$Q_n = m_n c^2 - m_p c^2 = 1.29\,\text{MeV}$$

At $t < 1$ sec, neutrons hadn't yet decayed.

At $t < 1$ sec, electrons & positrons were created by pair production:

$$\gamma + \gamma \rightleftharpoons e^- + e^+$$

At $t < 1$ sec, neutrinos were still coupled to baryons:

$$n + \nu_e \rightleftharpoons p + e^-$$

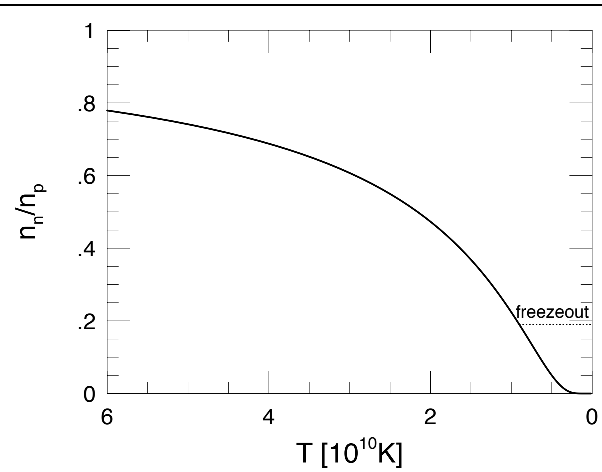
$$n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

150 MeV > kT > 0.8 MeV:

Neutrons and protons are in kinetic equilibrium.

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{(m_n - m_p)c^2}{kT}\right)$$

$$\approx \exp\left(-\frac{Q_n}{kT}\right)$$



At kT≈ 0.8 MeV, the neutron-to-proton ratio "freezes out": $\frac{n_n}{n_p} \approx \exp\left(-\frac{1.29\,\text{MeV}}{0.8\,\text{MeV}}\right) \approx 0.2$

When Big Bang Nucleosynthesis begins,
there is 1 neutron for every 5 protons.

The essential first step in BBN is the creation of a deuteron (^{2}H, or D).

$$p + n \rightleftharpoons \mathrm{D} + \gamma \qquad B_D = m_p c^2 + m_n c^2 - m_\mathrm{D} c^2 = 2.22\,\mathrm{MeV}$$

Footnote: The Sun, lacking free neutrons, makes deuterium the hard way.

$$p + p \rightarrow {}^2\mathrm{He}$$

$${}^2\mathrm{He} \rightarrow p + p \quad \tau \sim 10^{-23}\ \mathrm{s}$$

$${}^2\mathrm{He} \rightarrow \mathrm{D} + e^+ + \nu_e \quad \tau > 10^{-2}\ \mathrm{s}$$

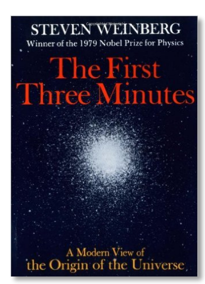
Deuterium synthesis: $p + n \rightleftharpoons \mathrm{D} + \gamma \quad [2{,}220{,}000\ \mathrm{eV}]$

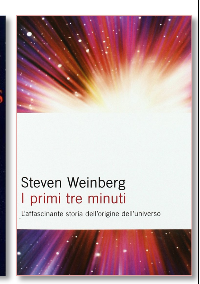
Recombination: $p + e^- \rightleftharpoons \mathrm{H} + \gamma \quad [13.6\ \mathrm{eV}]$

Rough estimate: If recombination takes place at $T_{\mathrm{rec}} = 3760\mathrm{K}$, then deuterium synthesis takes place at a temperature

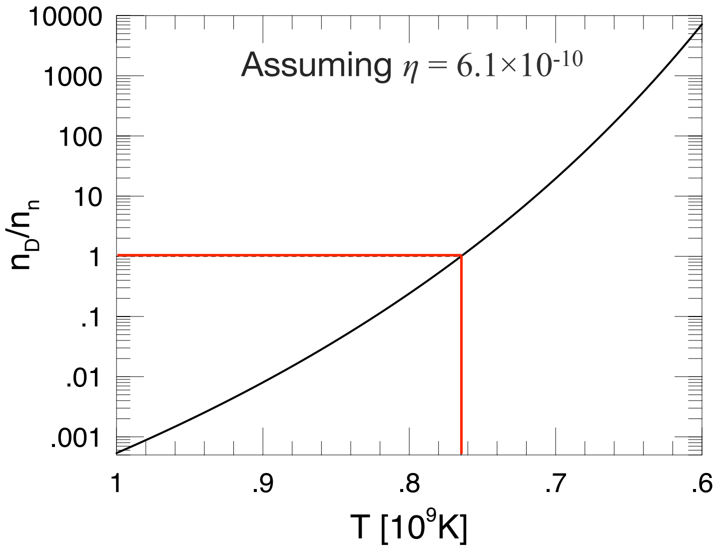
$$T_{\mathrm{nuc}} \approx \left(\frac{2{,}220{,}000}{13.6} \right) 3760\,\mathrm{K} \approx 6 \times 10^8\,\mathrm{K}$$

(This corresponds to $t \approx 270$ sec.)





Using the nucleosynthetic equivalent of the Saha equation, we find a more accurate value of $t_{nuc.}$



n_D=n_n when
$T_{nuc} = 7.6\times10^8$K,
corresponding to
$z_{nuc} \approx 3\times10^8$,
$t_{nuc} \approx 200$ sec.

(This result is weakly dependent on η.)

Deuterium is not the end of the line for BBN.
The next steps make light helium (^{3}He) and tritium (^{3}H).

For instance:

$$D + p \rightleftharpoons {}^3He + \gamma$$
$$D + n \rightleftharpoons {}^3H + \gamma$$

^{3}H decays to ^{3}He, but with τ = 18 yr >> 3 minutes.

The next steps make helium (^{4}He).

For instance:

$$^3H + p \rightleftharpoons {}^4He + \gamma$$
$$^3He + n \rightleftharpoons {}^4He + \gamma$$

^{4}He is *almost* the end of the line for BBN. There are no stable nuclei with atomic mass A=5 or A=8.

Tiny amounts of ^{6}Li are made:

$$^4\mathrm{He} + \mathrm{D} \rightleftharpoons {}^6\mathrm{Li} + \gamma$$

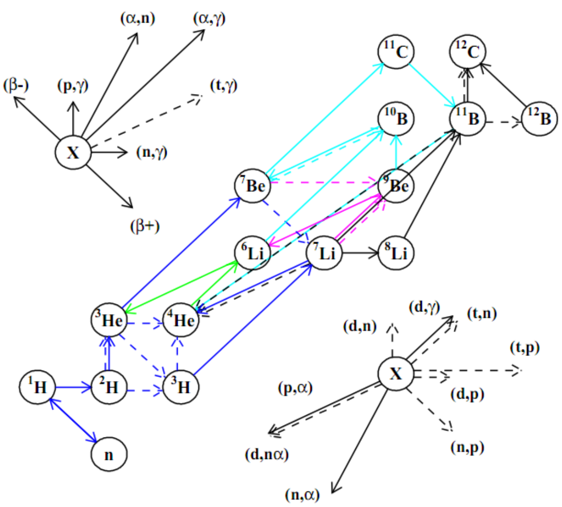
Small amounts of ^{7}Li are made:

$$^4\mathrm{He} + {}^3\mathrm{H} \rightleftharpoons {}^7\mathrm{Li} + \gamma$$

Small amounts of ^{7}Be are made:

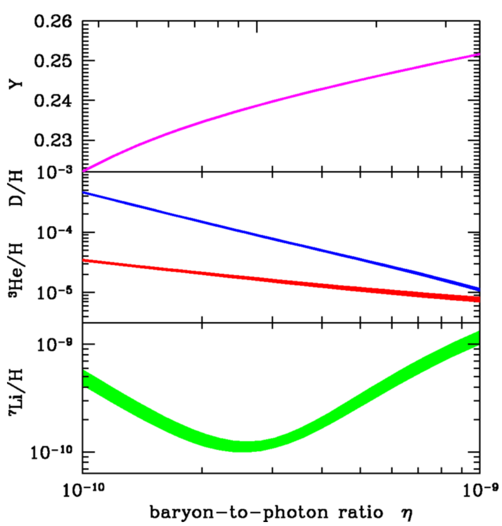
$$^4\mathrm{He} + {}^3\mathrm{He} \rightleftharpoons {}^7\mathrm{Be} + \gamma$$

(^{7}Be later decays to ^{7}Li
by electron capture.)



And that's about it...

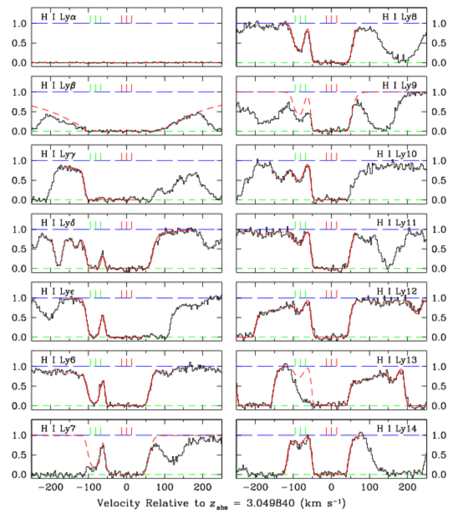
The yields of different elements depend on η,
the baryon-to-photon ratio.



The primordial deuterium to hydrogen ratio (D/H) is a particularly sensitive probe for η.

Deuterium is destroyed in stars. Where can we find primordial gas, unaltered by star formation?

Look for deuterium abundances in
"metal-poor damped Lyman alpha systems."



Pettini et al. 2012: J1419+0829

Lyα for H: 121.567 nm

Lyα for D: 121.534 nm

Best fit:
D/H = $(2.53 \pm 0.04) \times 10^{-5}$,
yielding $\eta = (6.0 \pm 0.1) \times 10^{-10}$