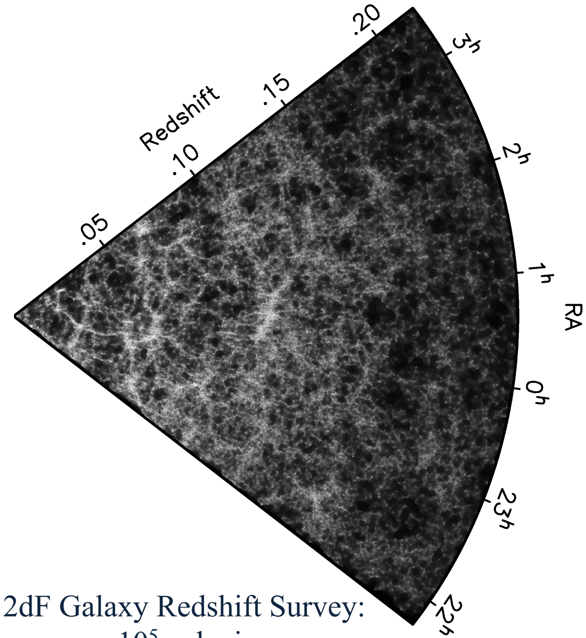

2dF Galaxy Redshift Survey:
~10^5 galaxies

Introduction to Cosmology Part 4

Professor Barbara Ryden
Department of Astronomy
The Ohio State University

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There are a few unsatisfactory aspects of the standard Hot Big Bang model; these led to the concept of cosmic inflation.

Flatness problem: Space is nearly flat today, and was even flatter in the past.

Horizon problem: The universe is nearly homogeneous on scales that are not causally connected in the standard Big Bang Model.

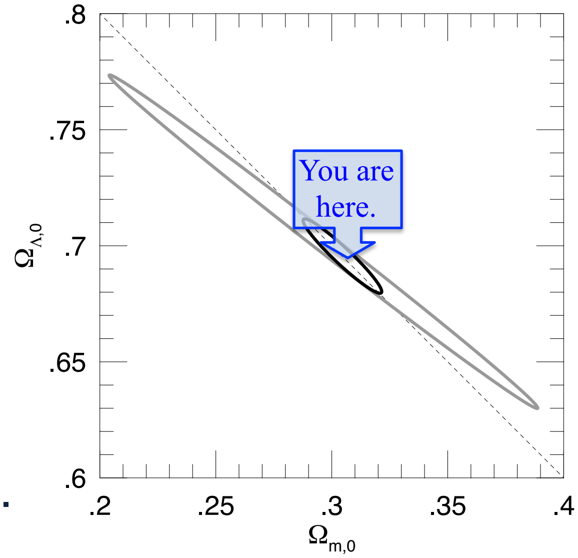
Combining SNIa, CMB, and baryon acoustic oscillations,

$$|1-\Omega_0| \leq 0.005$$

Friedmann equation tells us

$$|1-\Omega(t)| = \left(\frac{c/H(t)}{a(t)R_0}\right)^2$$

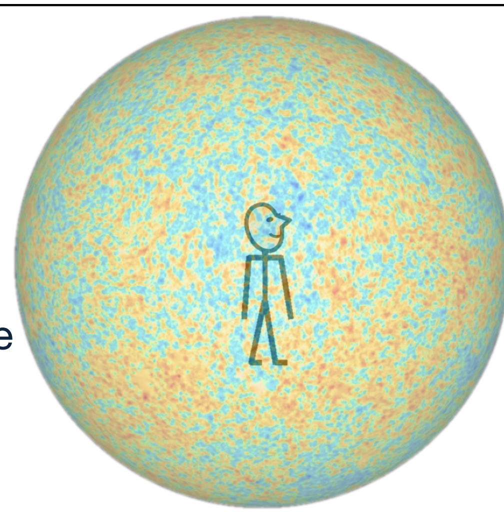
When radiation & matter are dominant, $|1\text{-}\Omega|$ increases with time.



deuterium synthesis: $|1-\Omega_{\rm nuc}| \leq 10^{-15}$

Planck time: $|1-\Omega_P| \leq 2\times 10^{-62}$

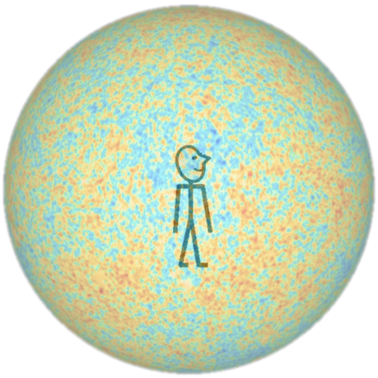
Horizon problem: consider looking out at the last scattering surface.



In the standard Hot Big Bang model (no inflation), particle horizon distance at the time of last scattering was:

$$d_{\rm hor}(t_{ls}) = a(t_{ls})c\int_0^{t_{ls}} \frac{dt}{a(t)}$$

$$= 2.24ct_{ls} = 0.25\,\mathrm{Mpc}$$



The angular-diameter distance to the last scattering surface is

$$d_A \approx 10\,\mathrm{Mpc}$$

The observed angular size of a patch $d_{\mathrm{hor}} = 0.25$ Mpc across is

$$\theta_{\mathrm{hor}} = \frac{d_{\mathrm{hor}}}{d_A} \approx \frac{0.25\,\mathrm{Mpc}}{10\,\mathrm{Mpc}} \approx 0.025\,\mathrm{rad} \approx 1.4^\circ$$

Without inflation, points more than ~1.4° apart were outside each others' horizon.
But... we see they had the same temperature to within ~10^{-5}.

Inflation: during the very early universe, there was a temporary era when $\ddot{a} > 0$.

$t_i \sim t_{GUT} \sim 10^{-36}$ sec ?

Toy model: Exponential expansion began at time t_i, with Hubble constant $H_i \sim (\Lambda_i/3)^{1/2} \sim t_i^{-1}$, and ended at $t_f > t_i$. The energy density of Λ_i is then transfered to relativistic particles in a "reheating" process.

(Physical mechanisms for inflation will be discussed next week...)

How does inflation solve the flatness problem?

$$|1-\Omega(t)| = \left(\frac{c/H(t)}{a(t)R_0}\right)^2$$

During exponential inflation,

$$|1-\Omega(t)| = \left(\frac{c/H_i}{a_i e^{H_i(t-t_i)} R_0}\right)^2 \propto e^{-2H_i t}$$

If the universe had $|1-\Omega| \sim 1$ before inflation,
and if inflation started at $t_i \sim 10^{-36}$ sec, then
$N > 60$ e-foldings are needed to match today's flatness.

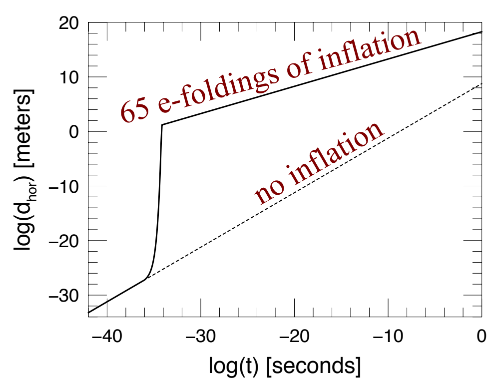
How does inflation solve the horizon problem?

During exponential inflation,
the particle horizon size
grows exponentially.

$$d_{\text{hor}}(t_i) \sim 2ct_i \sim 6\times 10^{-28}\,\text{m}$$

$$d_{\text{hor}}(t_f) \sim e^N 2ct_i \;[>7\,\text{cm if } N>60]$$

$$d_{\text{hor}}(t_{ls}) = \frac{a(t_{ls})}{a(t_f)} d_{\text{hor}}(t_f) \;[>1\text{Mpc if } N>60]$$



$N > 63$ e-foldings ensure the last scattering surface is isotropic.

Inflation, by increasing the particle horizon size, prevents the CMB from having large temperature fluctuations ($\delta T/T \sim 1$).

Inflation, by inflating quantum perturbations to macroscopic scales, also causes the observed small temperature fluctuations ($\delta T/T \sim 10^{-5}$).

Quantum perturbations in the "inflaton" field
➔ small variations δN in the e-foldings of inflation
➔ slight differences in the time of reheating
➔ small fluctuations in the post-inflation density ε

When dark matter decouples from other components of the universe ($t \sim 1$ sec for WIMPs), it has low-amplitude density fluctuations:

$$\rho(\vec{r},t) = \bar{\rho}(t)\left[1+\delta(\vec{r},t)\right]$$

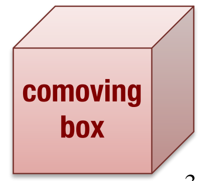
spatially averaged density ($\bar{\rho}(t)$)

fluctuation ($\delta(\vec{r},t)$): initially $|\delta| << 1$

We expect that the density field δ resulting from inflation will be a Gaussian random field.

$$p(\delta) \propto \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

Take the Fourier transform of δ:



$$\delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{r}) e^{i\vec{k}\cdot\vec{r}} d^3 r$$

$$V(t) \propto a(t)^3$$

Each Fourier component can be written as

$$\delta_{\vec{k}} = \left|\delta_{\vec{k}}\right| e^{i\varphi_{\vec{k}}}$$

If $\delta(\mathrm{r})$ is a Gaussian random field, then the phases φ_k are uncorrelated, and all useful information is in the power spectrum

$$P(k) = \left\langle \left|\delta_{\vec{k}}\right|^2 \right\rangle$$

Prediction: inflationary density perturbations should have a power spectrum

$$P(k) \propto k^n$$

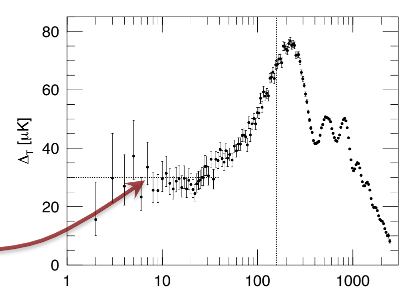
with $\mathrm{n} \approx 1$. (If $\mathrm{n} = 1$ exactly, this is called a Harrison-Zel'dovich spectrum.)

Observable consequences: spheres of mean mass M have

$$\delta M / M \propto M^{-(3+n)/6}$$

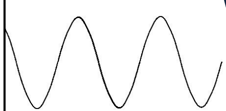
$$\delta\phi \propto \delta M / r \propto M^{(1-n)/6}$$

Planck: $n = 0.97 \pm 0.01$

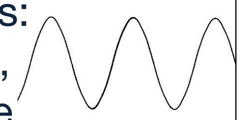


The initial $P \sim k^{0.97}$ spectrum is modified on small scales during the era of radiation domination.

When the physical size of a perturbation, $\lambda \sim a(t) 2\pi/k$, is larger than the Hubble distance, $c/H(t) \sim 2ct$, its amplitude grows. **Why?**

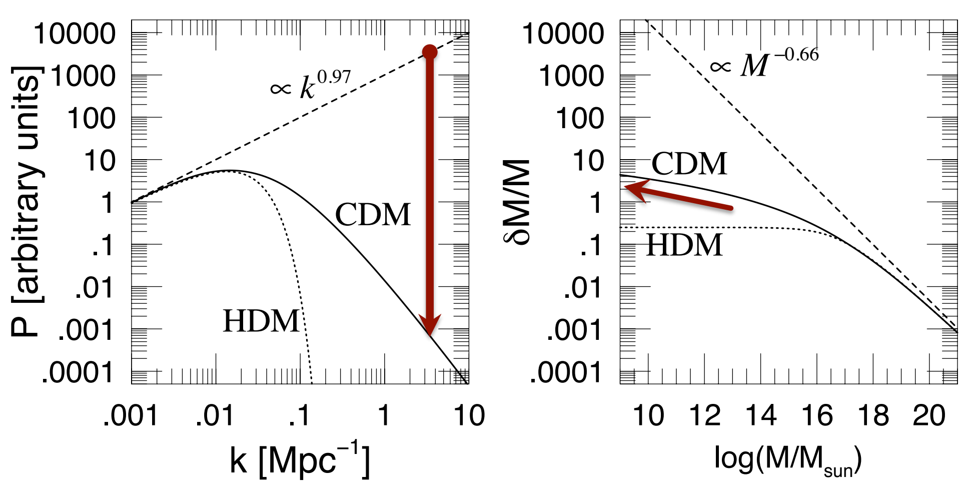


Wave crests are out of contact with troughs: crests act like a patch of an $\Omega>1$ universe, troughs act like a patch of an $\Omega<1$ universe.



$$\lambda \propto a(t) \propto t^{1/2} \qquad c / H(t) \propto t$$

Eventually, λ is overtaken by c/H, and the amplitude of the perturbation freezes out.



Suppression of power is greatest on small length scales (large comoving wavenumber).

$\delta M/M$ is largest on small length scales ("bottom up" structure formation).

During the matter-dominated era, density fluctuations in dark matter evolve by gravitational instability: "The rich get richer, the poor get poorer."

[Evolution of baryonic fluctuations is more complicated, because of interactions with photons.]

Simplest case of gravitational instability: low-amplitude density fluctuations in completely pressureless, completely dark matter.

Applying linear perturbation theory ($|\delta| << 1$) to the acceleration equation, we find [for $\lambda < c/H(t)$]:

$$\ddot{\delta} + 2H(t)\dot{\delta} = 4\pi G\bar{\rho}(t)\delta(t)$$

Applies even when universe is not matter-dominated.

Static universe ($H = 0$, constant mean density):

$$\ddot{\delta} = 4\pi G\bar{\rho}\delta(t)$$

Well-known solution: exponential growth (or decay)

$$\delta(t) = A_1 e^{t/t_{\rm dyn}} + A_2 e^{-t/t_{\rm dyn}},$$

$$\text{where } t_{\rm dyn} = (4\pi G\bar{\rho})^{-1/2}$$

Growth of density perturbations: $\ddot{\delta} + 2H(t)\dot{\delta} = 4\pi G\bar{\rho}(t)\delta(t)$

Hubble expansion

Self-gravity

Hubble expansion decreases density on a time scale $1/H(t) \sim (G\varepsilon/c^2)^{-1/2}$

Self-gravity increases density on a time scale $t_{dyn} \sim (G\rho)^{-1/2}$

If matter is not dominant, $\rho \ll \varepsilon/c^2$, $t_{dyn} \gg 1/H$, and perturbations grow *extremely slowly*.

If matter is dominant, $\rho \approx \varepsilon/c^2$, $t_{dyn} \sim 1/H$, and perturbations grow as a power law (not exponentially).

A flat, matter-dominated universe: $\Omega_m = 1$, $H(t) = (2/3)t^{-1}$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} = \frac{2}{3t^2}\delta$$

Power-law solutions (verify by substitution!)

$$\delta = C_1 t^{2/3} + C_2 t^{-1}$$

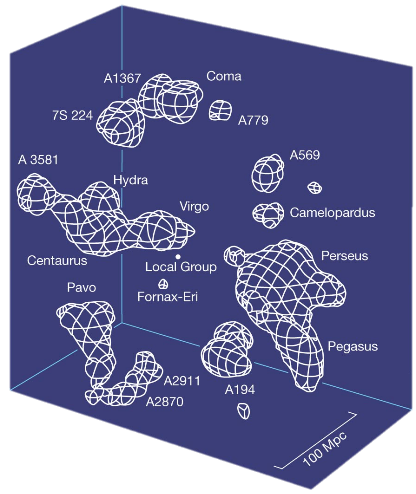
When only the growing mode remains,

$$\delta \propto t^{2/3} \propto a(t) \propto (1+z)^{-1}$$

Maximum growth: $\dfrac{\delta_{m\Lambda}}{\delta_{rm}} \approx \dfrac{1+z_{rm}}{1+z_{m\Lambda}} \approx \dfrac{3440}{1.31} \approx 2600$

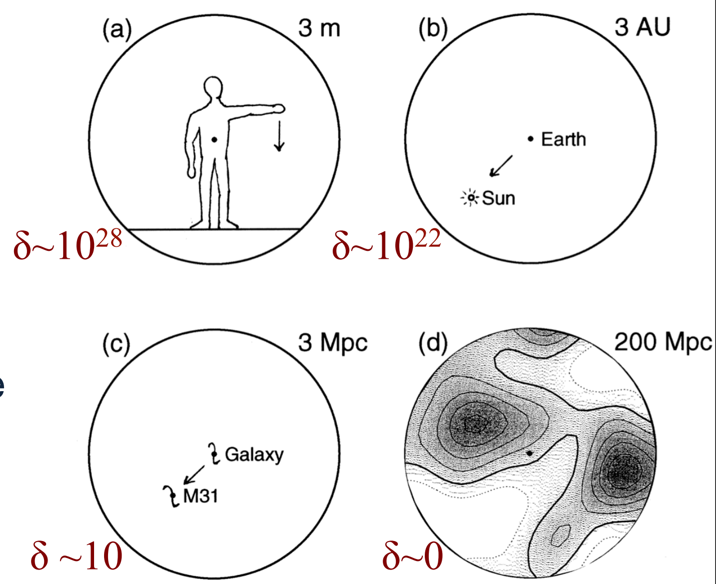
A region of space with $\delta_{rm} < 1/2600 \sim 4\times10^{-4}$ at radiation-matter equality never reaches $\delta \sim 1$, the overdensity at which collapse begins.

We are entering the Λ-dominated era: the biggest collapsing objects that we see today (superclusters) are the biggest there will ever be.

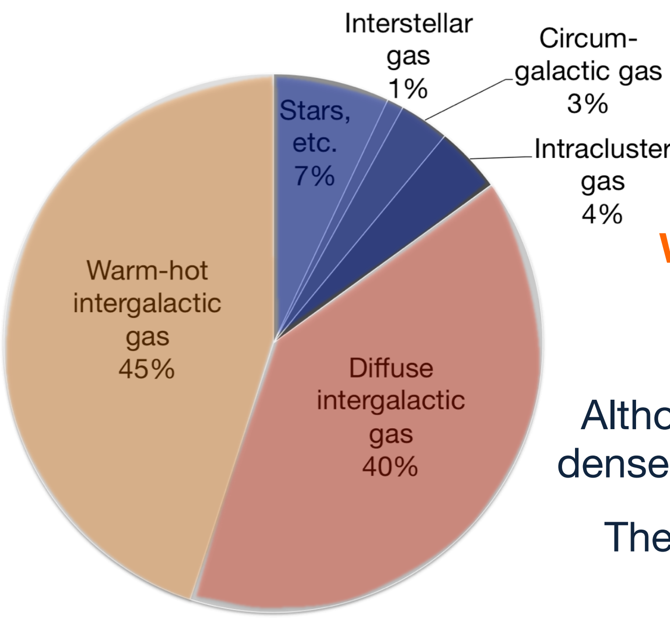


On small scales, today's universe is very inhomogeneous.

Baryonic matter can reach very high densities because it can radiate away excess thermal energy.



Baryonic matter today:



Bound baryons

Diffuse intergalactic gas:
$\delta_{bary} \leq 0$, $T < 10^5$K, ionized

Warm-hot intergalactic gas:
$\delta_{bary} \sim 30$, $T \sim 10^6$K, ionized

Although baryons **can** make very dense objects, most baryons **don't.**

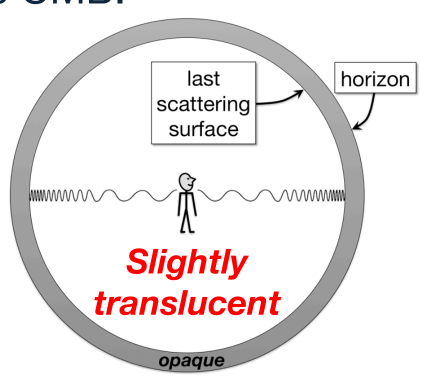
The baryons are ionized again!
How did that happen?

The baryons are ionized again!
When did that happen?

Recombination occurred at $t_{rec} \sim 0.25$ Myr.

The time of reionization can be deduced from looking at the CMB.

Ionized intergalactic gas provides a foreground screen of free electrons that can scatter CMB photons.



Because of the translucent foreground material, our view of the last scattering surface is slightly blurred.

The most recent analysis of the Planck results (Aghanim et al., arXiv:1605.02985) yields

$\tau = 0.055 \pm 0.009$

(That is, about 1 in 18 CMB photons has scattered.)

The rate at which a CMB photon scatters from free electrons in reionized gas:

$$\Gamma(t) = n_e(t)\sigma_e c$$

If the baryonic gas is reionized starting at a time t_R, then the optical depth of the reionized gas is:

$$\tau = \int_{t_R}^{t_0} \Gamma(t)\,dt = \sigma_e c \int_{t_R}^{t_0} n_e(t)\,dt$$

Simplifying assumptions: pure hydrogen undergoes instant total reionization at $t = t_R$.

With these assumptions, at $t > t_R$ we find

$$n_e = n_p = \frac{n_{\text{bary},0}}{a(t)^3}$$

The optical depth is then

$$\tau = \Gamma_0 \int_{t_R}^{t_0} \frac{dt}{a(t)^3}$$

where $\Gamma_0 = \sigma_e c n_{\text{bary},0} = 1.6 \times 10^{-4}\,\text{Gyr}^{-1} \approx 0.002 H_0$

There's an analytic solution!

$$\tau = \frac{2}{3\Omega_{m,0}} \frac{\Gamma_0}{H_0} \left(\left[\Omega_{m,0}(1+z_R)^3 + \Omega_{\Lambda,0} \right]^{1/2} - 1 \right)$$

Using $H_0 = 68$ km s^{-1} Mpc^{-1}, $\Omega_{\Lambda,0}=0.69$, $\Omega_{m,0}=0.31$, and $\tau=0.055\pm0.009$,

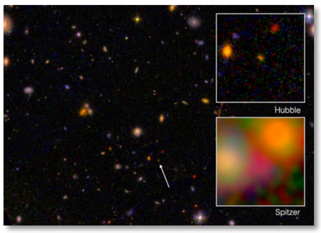
we find that reionization took place at a redshift

$$z_R = 6.9 \pm 0.8 \qquad (t_R \approx 0.65 \text{ Gyr})$$

The "era of neutrality" was a brief interlude in the history of the universe: $t_R - t_{rec} \sim 0.05 t_0$

So... what happened around $z \sim 7$ that could have reionized the universe?

Hint: the highest redshift galaxies known are at $z > 7$.

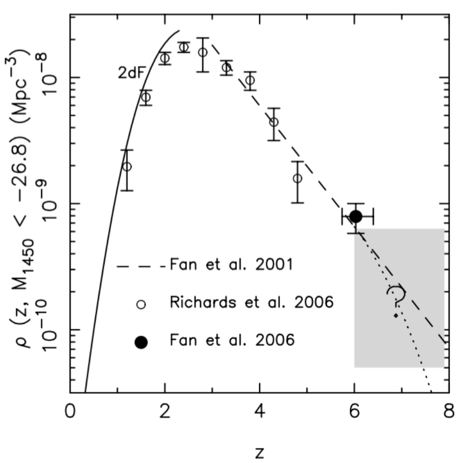


$$z = 8.68; \quad a(t_e) = 1/(1+z) = 0.103$$

Galaxies contain two sources of ionizing photons: hot stars and active galactic nuclei (AGN).

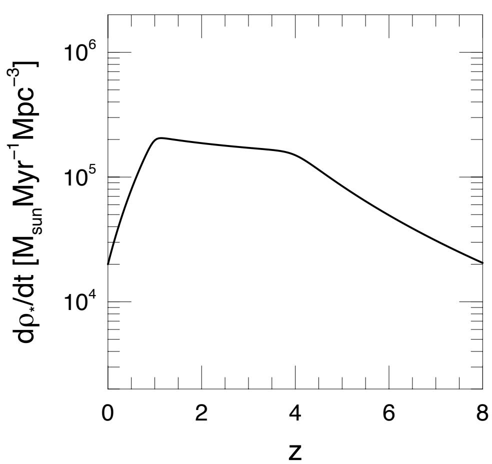
Intergalactic gas was *photoionized*.

(comoving) number density of quasars (= luminous AGN)



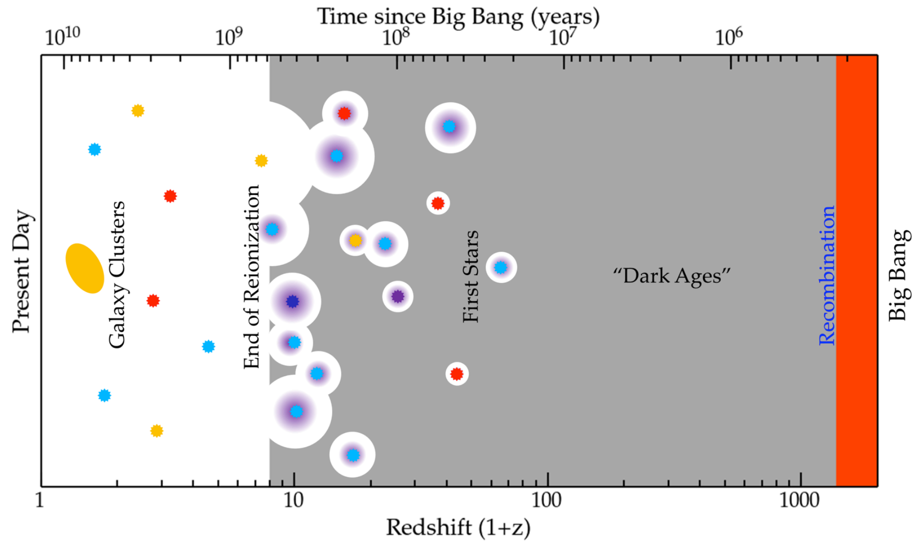
Probably too few AGN at z>7 to do the job.

(comoving) star formation rate

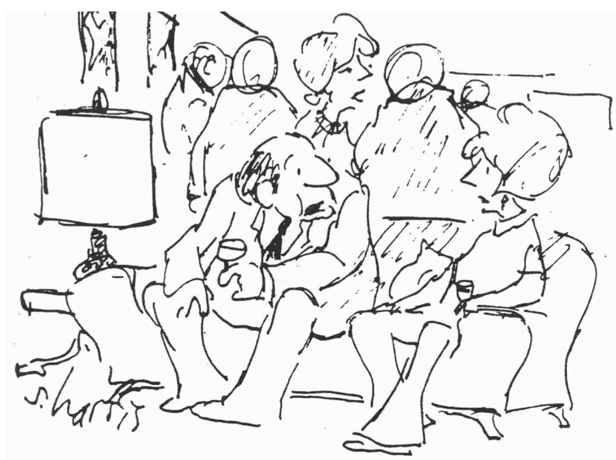


Probably (maybe?) enough hot stars.

Reionization: much work remains to be done.



Cosmology in general: much work remains to be done.



"My big mistake was going into cosmology just for the money."